Precalculus: Functions and Their Graphs - Homework Solutions

Instructor

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Solutions

1. The slope is $m = \frac{8-4}{3-1} = \frac{4}{2} = 2$. Using point-slope form with (1, 4): $y - 4 = 2(x - 1)$ $y = 2x - 2 + 4$ $y = 2x + 2$

2. Given $f(x) = x^2 - 2x + 3$ and $g(x) = x + 1$:

   (a) $(f \circ g)(x) = f(g(x)) = (x + 1)^2 - 2(x + 1) + 3 = x^2 + 2x + 1 - 2x - 2 + 3 = x^2 + 2$

   (b) $(g \circ f)(x) = g(f(x)) = (x^2 - 2x + 3) + 1 = x^2 - 2x + 4$

   (c) $(f \circ g)(2) = 2^2 + 2 = 6$

3. Starting with $f(x) = |x|$, to get $g(x) = |x - 1| + 2$: 1. Shift 1 unit right: $|x - 1|$ 2. Shift 2 units up: $|x - 1| + 2$

4. Let $f(x) = x^2 + 1$ and $g(x) = 2x - 3$.

   (a) $(f + g)(x) = f(x) + g(x) = (x^2 + 1) + (2x - 3) = x^2 + 2x - 2$

   (b) $(f \cdot g)(x) = f(x) \cdot g(x) = (x^2 + 1)(2x - 3) = 2x^3 - 3x^2 + 2x - 3$

   (c) $(f + g)(3) = 3^2 + 2(3) - 2 = 9 + 6 - 2 = 13$ $(f \cdot g)(3) = (3^2 + 1)(2(3) - 3) = 10 \cdot 3 = 30$

5. For $f(x) = 3x + 4$: $y = 3x + 4$ $x = \frac{y-4}{3}$ $f^{-1}(x) = \frac{x-4}{3}$
   $f^{-1}(10) = \frac{10-4}{3} = \frac{6}{3} = 2$

6. For $h(x) = x^2 - 4x - 5$:

   (a) y-intercept: $h(0) = 0^2 - 4(0) - 5 = -5$, so (0, -5)

   (b) x-intercepts: $x^2 - 4x - 5 = 0$ $(x - 5)(x + 1) = 0$ $x = 5$ or $x = -1$

   (c) Vertex: $x = -b/(2a) = -(-4)/(2(1)) = 2$ $y = 2^2 - 4(2) - 5 = 4 - 8 - 5 = -9$ Vertex is (2, -9)

   (d) The parabola opens upward because the coefficient of x^2 is positive.

7. For $f(x) = \frac{x+2}{x-1}$:

   (a) Domain: All real numbers except 1 (where denominator equals zero)

(b) Vertical asymptote: $x = 1$ Horizontal asymptote: As $x \to \infty$, $y \to 1$

(c) [Graph would be sketched here]

8. $f(x) = x^3 - x$ is one-to-one. Proof: If $f(a) = f(b)$, then $a^3 - a = b^3 - b$. $a^3 - b^3 = a - b$ $(a-b)(a^2+ab+b^2) = a-b$ $(a-b)(a^2+ab+b^2-1) = 0$ Either $a-b = 0$ or $a^2+ab+b^2-1 = 0$. The second equation has no real solutions for $a \neq b$. Therefore, $a = b$, proving the function is one-to-one.

9. For $f(x) = \sqrt{x+2}$: Domain: $x + 2 \geq 0$, so $x \geq -2$ Range: $y \geq 0$, so $[0, \infty)$

10. For $f(x) = 2^x$, to get $g(x) = 2^{x+1} - 1$: 1. Shift 1 unit left: 2^{x+1} 2. Stretch vertically by a factor of 2: $2 \cdot 2^x$ 3. Shift 1 unit down: $2^{x+1} - 1$

11. $\log_2(x+3) = 4$ $2^4 = x + 3$ $16 = x + 3$ $x = 13$

12. $x^2 - 2x - 3 = 2x + 1$ $x^2 - 4x - 4 = 0$ $(x-4)(x+0) = 0$ $x = 4$ or $x = 0$

13. Given $f(x) = 3x-1$ and $g(x) = \frac{x}{2}+1$: $(f \circ g)(x) = f(g(x)) = 3(\frac{x}{2}+1)-1 = \frac{3x}{2}+3-1 = \frac{3x}{2}+2$ $(g \circ f)(x) = g(f(x)) = \frac{3x-1}{2} + 1 = \frac{3x-1+2}{2} = \frac{3x+1}{2}$

14. $y = |x^2 - 4|$ has: - y-intercept at (0, 4) - x-intercepts at (-2, 0) and (2, 0) - Vertex at (0, 4) - Minimum points at (-2, 0) and (2, 0)

15. The graph of $f(x) = \frac{1}{x-2}$ is the graph of $y = \frac{1}{x}$ shifted 2 units to the right.