Factoring Polynomials Worksheet

Ryan Lee

July 12, 2024

Introduction to Factoring Polynomials

All quadratics can be expressed in the form $Ax^2 + Bx + C = 0$. In this worksheet, we will learn a method for factoring these quadratics.

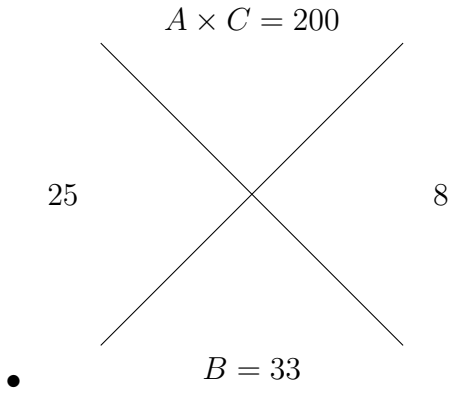
Step-by-Step Method

Let's start with an example where $A = 10$, $B = 33$, and $C = 20$.

1. Multiply A and C: $10 \times 20 = 200$.
2. Find two numbers that multiply to $A \times C$ and add up to B: $25 \times 8 = 200$ and $25 + 8 = 33$.
3. Rewrite the original equation using these numbers in place of B:
$$10x^2 + 33x + 20 \rightarrow 10x^2 + 25x + 8x + 20 = 0.$$
4. Factor by grouping:
$$10x^2 + 25x + 8x + 20 \rightarrow 5x(2x + 5) + 4(2x + 5).$$
5. Factor out the common binomial:
$$5x(2x + 5) + 4(2x + 5) \rightarrow (2x + 5)(5x + 4) = 0.$$

Visualization using the X-method:

- Draw an X, with $A \times C$ (200) at the top and B (33) at the bottom.
- Find the two numbers that fit these criteria and place them on the sides of the X.



Now, let's see another example where $A = 6$, $B = 11$, and $C = 3$.

1. Multiply A and C: $6 \times 3 = 18$.
2. Find two numbers that multiply to $A \times C$ and add up to B: $9 \times 2 = 18$ and $9 + 2 = 11$.
3. Rewrite the original equation using these numbers in place of B:

$$6x^2 + 11x + 3 \rightarrow 6x^2 + 9x + 2x + 3 = 0.$$

4. Factor by grouping:

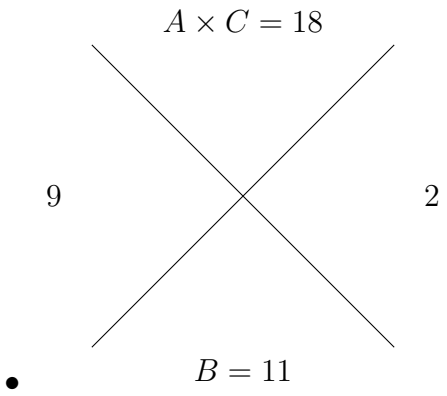
$$6x^2 + 9x + 2x + 3 \rightarrow 3x(2x + 3) + 1(2x + 3).$$

5. Factor out the common binomial:

$$3x(2x + 3) + 1(2x + 3) \rightarrow (2x + 3)(3x + 1) = 0.$$

Visualization using the X-method::

- Draw an X, with $A \times C$ (28) at the top and B (11) at the bottom.
- Find the two numbers that fit these criteria and place them on the sides of the X.



Practice Problems

Factor the following polynomials using the method described above. Show all your work.

1. $6x^2 + 13x + 6 = 0$
2. $2x^2 + 15x + 7 = 0$
3. $4x^2 + 19x + 12 = 0$
4. $3x^2 + 14x + 8 = 0$
5. $5x^2 + 29x + 20 = 0$

Solutions

1. $6x^2 + 13x + 6 = 0$

   $A \times C = 6 \times 6 = 36$

   Find two numbers that multiply to 36 and add to 13: 9 and 4.

   $6x^2 + 9x + 4x + 6 = 0$

   $3x(2x + 3) + 2(2x + 3) = 0$

   $(2x + 3)(3x + 2) = 0$

2. $2x^2 + 15x + 7 = 0$

   $A \times C = 2 \times 7 = 14$

   Find two numbers that multiply to 14 and add to 15: 14 and 1.

   $2x^2 + 14x + x + 7 = 0$

   $2x(x + 7) + 1(x + 7) = 0$

   $(x + 7)(2x + 1) = 0$

3. $4x^2 + 19x + 12 = 0$

   $A \times C = 4 \times 12 = 48$

   Find two numbers that multiply to 48 and add to 19: 16 and 3.

   $4x^2 + 16x + 3x + 12 = 0$

   $4x(x + 4) + 3(x + 4) = 0$

   $(x + 4)(4x + 3) = 0$

4. $3x^2 + 14x + 8 = 0$

   $A \times C = 3 \times 8 = 24$

   Find two numbers that multiply to 24 and add to 14: 12 and 2.

   $3x^2 + 12x + 2x + 8 = 0$

   $3x(x + 4) + 2(x + 4) = 0$

   $(x + 4)(3x + 2) = 0$

5. $5x^2 + 29x + 20 = 0$

   $A \times C = 5 \times 20 = 100$

   Find two numbers that multiply to 100 and add to 29: 25 and 4.

   $5x^2 + 25x + 4x + 20 = 0$

   $5x(x + 5) + 4(x + 5) = 0$

   $(x + 5)(5x + 4) = 0$