Answer Sheet: Factoring Polynomials Homework

July 12, 2024

Answers

1. Factor the following quadratic equations using the factoring method discussed in class.

   (a) $8x^2 + 22x + 15 = 0$

   $$A \times C = 8 \times 15 = 120$$
   Factors of 120 that add up to 22: 10 and 12
   $$8x^2 + 10x + 12x + 15 = 0$$
   $$2x(4x + 5) + 3(4x + 5) = 0$$
   $$(4x + 5)(2x + 3) = 0$$

   (b) $3x^2 + 11x + 6 = 0$

   $$A \times C = 3 \times 6 = 18$$
   Factors of 18 that add up to 11: 9 and 2
   $$3x^2 + 9x + 2x + 6 = 0$$
   $$3x(x + 3) + 2(x + 3) = 0$$
   $$(x + 3)(3x + 2) = 0$$

   (c) $7x^2 + 19x + 6 = 0$

   $$A \times C = 7 \times 6 = 42$$
   Factors of 42 that add up to 19: 14 and 3
   $$7x^2 + 14x + 3x + 6 = 0$$
   $$7x(x + 2) + 3(x + 2) = 0$$
   $$(x + 2)(7x + 3) = 0$$

   (d) $6x^2 + 17x + 12 = 0$

   $$A \times C = 6 \times 12 = 72$$
   Factors of 72 that add up to 17: 9 and 8
   $$6x^2 + 9x + 8x + 12 = 0$$
   $$3x(2x + 3) + 4(2x + 3) = 0$$
   $$(2x + 3)(3x + 4) = 0$$

(e) $5x^2 + 16x + 12 = 0$

$$A \times C = 5 \times 12 = 60$$

Factors of 60 that add up to 16: 10 and 6

$$5x^2 + 10x + 6x + 12 = 0$$
$$5x(x + 2) + 6(x + 2) = 0$$
$$(x + 2)(5x + 6) = 0$$

2. Solve the following quadratic equations by factoring.

(a) $2x^2 + 9x + 7 = 0$

$$A \times C = 2 \times 7 = 14$$

Factors of 14 that add up to 9: 7 and 2

$$2x^2 + 7x + 2x + 7 = 0$$
$$x(2x + 7) + 1(2x + 7) = 0$$
$$(2x + 7)(x + 1) = 0$$

Solutions: $x = -\frac{7}{2}, -1$

(b) $4x^2 + 15x + 9 = 0$

$$A \times C = 4 \times 9 = 36$$

Factors of 36 that add up to 15: 12 and 3

$$4x^2 + 12x + 3x + 9 = 0$$
$$4x(x + 3) + 3(x + 3) = 0$$
$$(x + 3)(4x + 3) = 0$$

Solutions: $x = -3, -\frac{3}{4}$

(c) $9x^2 + 30x + 21 = 0$

$$A \times C = 9 \times 21 = 189$$

Factors of 189 that add up to 30: 21 and 9

$$9x^2 + 21x + 9x + 21 = 0$$
$$3x(3x + 7) + 3(3x + 7) = 0$$
$$(3x + 7)(3x + 3) = 0$$

Solutions: $x = -\frac{7}{3}, -1$

(d) $12x^2 + 23x + 10 = 0$

$$A \times C = 12 \times 10 = 120$$

Factors of 120 that add up to 23: 15 and 8

$$12x^2 + 15x + 8x + 10 = 0$$
$$3x(4x + 5) + 2(4x + 5) = 0$$
$$(4x + 5)(3x + 2) = 0$$

Solutions: $x = -\frac{5}{4}, -\frac{2}{3}$

(e) $10x^2 + 27x + 18 = 0$

$$A \times C = 10 \times 18 = 180$$

Factors of 180 that add up to 27: 15 and 12

$$10x^2 + 15x + 12x + 18 = 0$$
$$5x(2x + 3) + 6(2x + 3) = 0$$
$$(2x + 3)(5x + 6) = 0$$

Solutions: $x = -\frac{3}{2}, -\frac{6}{5}$

3. For each of the following quadratic equations, verify your factored form by expanding back to the original polynomial.

(a) $6x^2 + 13x + 6 = 0$

$$\begin{aligned}(2x + 3)(3x + 2) &= 6x^2 + 4x + 9x + 6 \\ &= 6x^2 + 13x + 6\end{aligned}$$

(b) $8x^2 + 14x + 3 = 0$

$$\begin{aligned}(4x + 1)(2x + 3) &= 8x^2 + 12x + 2x + 3 \\ &= 8x^2 + 14x + 3\end{aligned}$$

(c) $5x^2 + 24x + 16 = 0$

$$\begin{aligned}(x + 8)(5x + 2) &= 5x^2 + 2x + 40x + 16 \\ &= 5x^2 + 24x + 16\end{aligned}$$

(d) $3x^2 + 19x + 20 = 0$

$$\begin{aligned}(3x + 4)(x + 5) &= 3x^2 + 15x + 4x + 20 \\ &= 3x^2 + 19x + 20\end{aligned}$$

(e) $7x^2 + 31x + 20 = 0$

$$\begin{aligned}(x + 4)(7x + 5) &= 7x^2 + 5x + 28x + 20 \\ &= 7x^2 + 31x + 20\end{aligned}$$

4. Challenge problems: Factor the following quadratic equations where the leading coefficient is not 1.

   (a) $9x^2 + 24x + 16 = 0$

$$A \times C = 9 \times 16 = 144$$

Factors of 144 that add up to 24: 12 and 12

$$9x^2 + 12x + 12x + 16 = 0$$
$$3x(3x + 4) + 4(3x + 4) = 0$$
$$(3x + 4)(3x + 4) = 0$$

   (b) $12x^2 + 25x + 12 = 0$

$$A \times C = 12 \times 12 = 144$$

Factors of 144 that add up to 25: 9 and 16

$$12x^2 + 9x + 16x + 12 = 0$$
$$3x(4x + 3) + 4(4x + 3) = 0$$
$$(4x + 3)(3x + 4) = 0$$

   (c) $15x^2 + 26x + 8 = 0$

$$A \times C = 15 \times 8 = 120$$

Factors of 120 that add up to 26: 10 and 12

$$15x^2 + 10x + 12x + 8 = 0$$
$$5x(3x + 2) + 4(3x + 2) = 0$$
$$(3x + 2)(5x + 4) = 0$$