Università degli Studi di Padova
Dipartimento di Fisica e Astronomia "Galileo Galilei"
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Course: Stochastic Methods for Finance

A study of the Value at Risk of a balanced portfolio

GitHub Repo: https://github.com/aidinattar/Financial-Mathematics

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Abstract

Value at risk is a measure of the risk of loss for investments. It estimates how much a set of investments might lose (with a given probability), given normal market conditions, in a set period such as a day. VaR is typically used by firms and regulators in the financial industry to gauge the number of assets needed to cover possible losses. In this report, we will implement several methods to compute the value at risk, in particular the parametric model, the MonteCarlo model, the historical model and the historical simulation model. We will implement those methods for a balanced portfolio of two assets: Twitter and Vodafone, considering the last six months' prices. The results we obtain are in accord with what we expected.

Keywords
Value-at-Risk, Parametric, Monte-Carlo, Historical-Simulation

1 Introduction

Value at Risk (VaR) is an attempt to provide a single number summarizing the total risk in a portfolio of financial assets. It has become widely used by corporate treasurers and fund managers as well as by institutions.

The VaR of the portfolio is a function of two parameters: the time horizon (N days) and the confidence level (X%). It is the loss level over N days that has a probability of only (100-X)% of being exceeded. When N days is the time horizon and X% is the confidence level, VaR is the loss corresponding to the (100-X)th percentile of the distribution of the gain in the value of the portfolio over the next N days.

VaR depends on two parameters: the time horizon N, measured in days, and the confidence level X. In practice, analysts set $N = 1$ in the first instance when VaR is estimated for market risk. This is because there is not usually enough data available to estimate directly the behaviour of market variables over periods longer than 1 day. The usual assumption is

$$\text{N-day VaR} = \text{1-day VaR} \times \sqrt{N} \tag{1}$$

This formula is exactly true when the changes in the value of the portfolio on successive days have independent identical normal distributions with mean zero. In other cases, it is an approximation [1].

There are many ways one can use to compute the VaR, in particular in this report we use:

- Parametric normal VaR, which is computed assuming an asset returns follow a normal distribution and calculates the probability distribution of the change in the value of the portfolio analytically. This method is followed both using a flat estimation of the volatility and following a RiskMetrics EWMA (Exponential weighted moving average);
- Montecarlo simulation, which consists in running a huge number of random simulations according to a specific standard deviation, covering a wide range of possible scenarios. These variables are drawn from pre-specified probability distributions that are assumed to be known, including the analytical function and its parameters. Thus, Monte Carlo simulations inherently try to recreate the distribution of the return of a position, from which VaR can be computed.
- Historical Simulation differs from the parametric method because we don't assume any theoretical distribution, but we use the distribution of real values we have and we take the corresponding percentile.
- Historical VaR: We assume that the expected change in the value of the portfolio is zero (This is OK for short periods), then we assume that the change in the value of the portfolio is normally distributed. Considering the daily volatility we can retrieve the VaR.

2 Materials & Methods

For this task, we used a balanced portfolio of two assets. A balanced investment strategy combines asset classes in a portfolio in an attempt to balance risk and return. To build this type of portfolio we take two assets and every day we balance the investment based on the daily return, e.g. if the first asset gains 1% and the second asset lose 1%, we disinvest the gained value from the former and invest it in the latter, such that every day we have the same investment in each asset. This means that for regards to the returns, which we are interested in, they will be computed as the weighted average of the daily returns of the assets. Although typically, balanced portfolios are divided between stocks and bonds, either equally or with a slight tilt, such as 60% in stocks and 40% in bonds, we used two stocks for simplicity.

The stocks used are Twitter and Vodafone. Here we briefly describe the two assets.

Twitter, Inc. operates as a platform for public self-expression and conversation in real-time. The company's primary product is Twitter, a platform that allows users to consume, cre-

Figure 1: Stock price trends for the last 6 months. The boxes represent the spread between the open and close values and the lines represent the spread between the low and high values. Increasing candles are drawn in green whereas decreasing are drawn in red.

ate, distribute, and discover content. It also provides promoted products that enable advertisers to promote brands, products, and services, as well as enable advertisers to target an audience based on various factors, including who an account follows and actions taken on its platform, such as Tweets created and engagement with Tweets. Its promoted products consist of promoted ads and Twitter Amplify, Follower Ads, and Twitter takeover. In addition, the company offers monetization products for creators, including Tips to directly send small one-time payments on Twitter using various payment methods, including bitcoin; Super Follows, a paid monthly subscription, which includes bonus content, exclusive previews, and perks as a way to support and connect with creators on Twitter; and Ticketed Spaces to support creators on Twitter for their time and effort in hosting, speaking, and moderating the public conversation on Twitter Spaces. Further, it offers products for developers and data partners comprising Twitter Developer Platform, a platform that enables developers to build tools for people and businesses using its public application programming interface; and paid access to Twitter data for partners with commercial use cases. Twitter, Inc. was founded in 2006 and is based in San Francisco, California.

Vodafone Group Public Limited Company engages in telecommunication services in Europe and internationally. The company offers mobile services that enable customers to call, text, and access data; fixed line services, including broadband, television (TV) offerings, and voice; and convergence services under the GigaKombi and Vodafone One names to customers. It also provides value added services, such as Internet of Things (IoT) comprising logistics and fleet management, smart metering, insurance, cloud, and security services; and automotive and health solutions. In addition, the company offers M-Pesa, an African payment platform, which provides money transfer, financial, and business and merchant payment services; and various services to operators through its partner market agreements. Vodafone Group Public Limited Company has a strategic partnership with Open Fiber. As of March 31, 2021, it had approximately 315 million mobile customers, 28 million fixed broadband customers, and 22 million TV customers. The company was incorporated in 1984 and is based in Newbury, the United Kingdom.

In Figure 1 we report the stock prices for the last 6 months of the two assets considered.

Here we briefly describe the process followed for each method.

2.1 Parametric normal VaR with flat σ

This method is based on the assumption that the returns follow a Normal distribution. The method consists of the computation of the average and the variance of the daily returns and using them to build the corresponding normal distribution $\mathcal{N}(\mu, \sigma^2)$. From this, we can infer the VaR as the quantile corresponding to the confidence level selected multiplied by the investment.

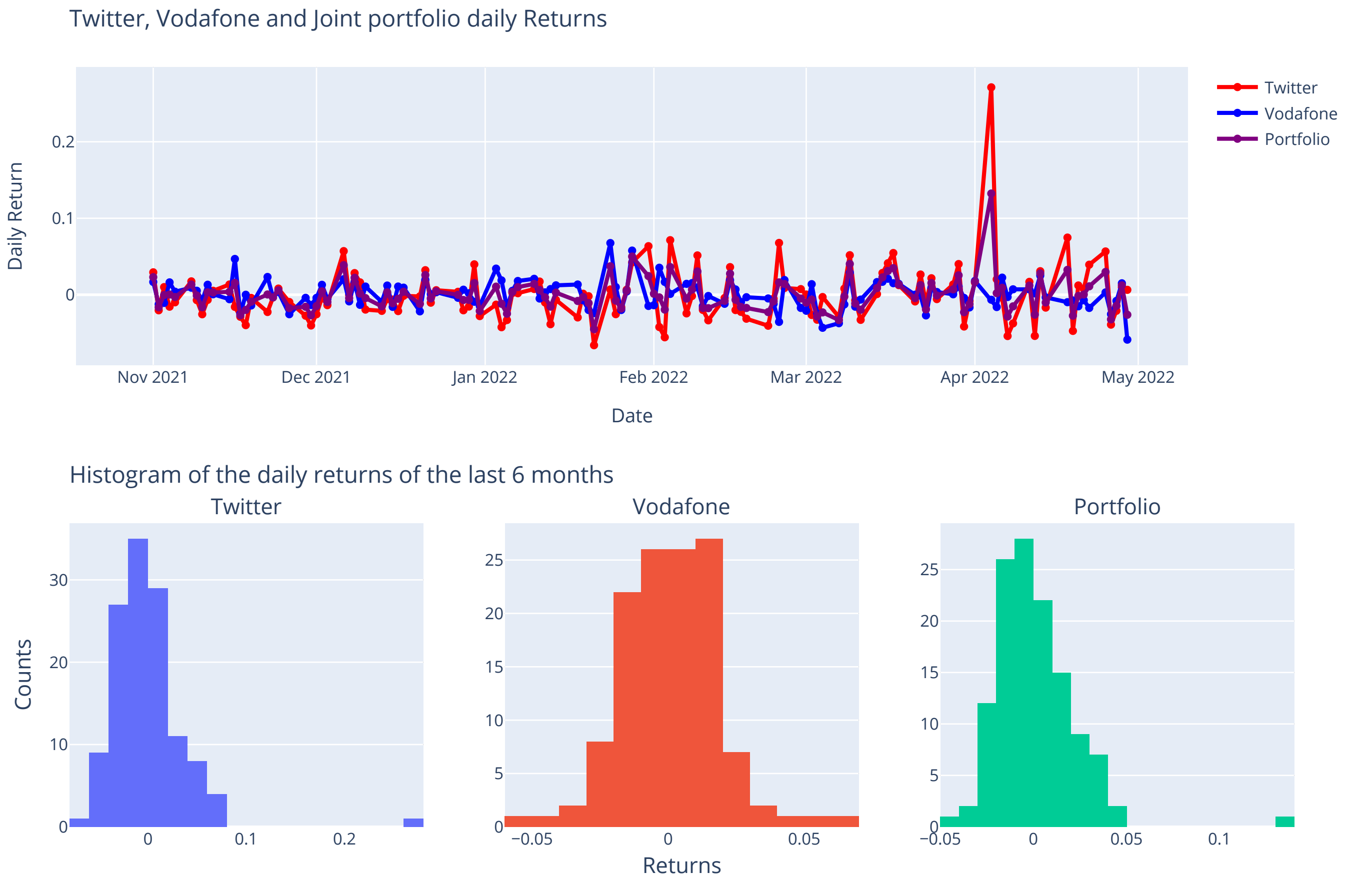

Figure 2: Top (a): Daily Returns for each asset. Bottom (b): histograms of the daily returns for each asset.

To retrieve the N-days VaR, according to equation 1, we just need to multiply by the number of days.

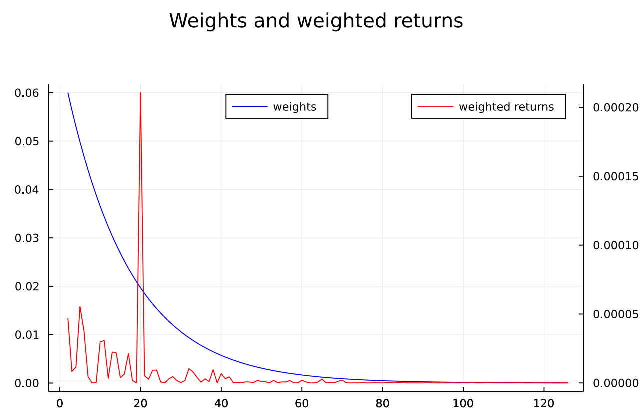

Figure 3: Weights of the EWMA approach and the weighted values for the daily returns.

2.2 Parametric normal VaR with EWMA σ

What we said earlier uses a flat estimation for the σ. Here, instead, we want to avoid giving the same weight to each point. This assumption makes sense since it is reasonable to think that closer values should count more than values far in the past. To do what we said heuristically we use a RiskMetrics EWMA.

The exponentially weighted moving average (EWMA) introduces lambda ($= .94$), which is called the smoothing parameter. Lambda must be less than one. Under that condition, instead of equal weights, each squared return is weighted by a multiplier as follows:

$$Return^2_{weighted} = (1 - \lambda)\lambda^i \times Return^2 \tag{2}$$

where i is the index running over all the returns, meaning that for the closest return the weight will be .06 and will exponentially decrease. In Figure 3 we report the weights curve and the weighted values of the daily returns of the portfolio.

2.3 Monte-Carlo VaR

To calculate VaR using MonteCarlo simulation we value the portfolio today, we sample once from the multivariate distributions of the Δx_i (defined as $\Delta S_i/S_i$). We then use the Δx_i to determine market variables at end of one day and evaluate the portfolio at the end of the day. We can now compute the ΔP (value of the portfolio).

Repeating this process many times we can build up a prob-

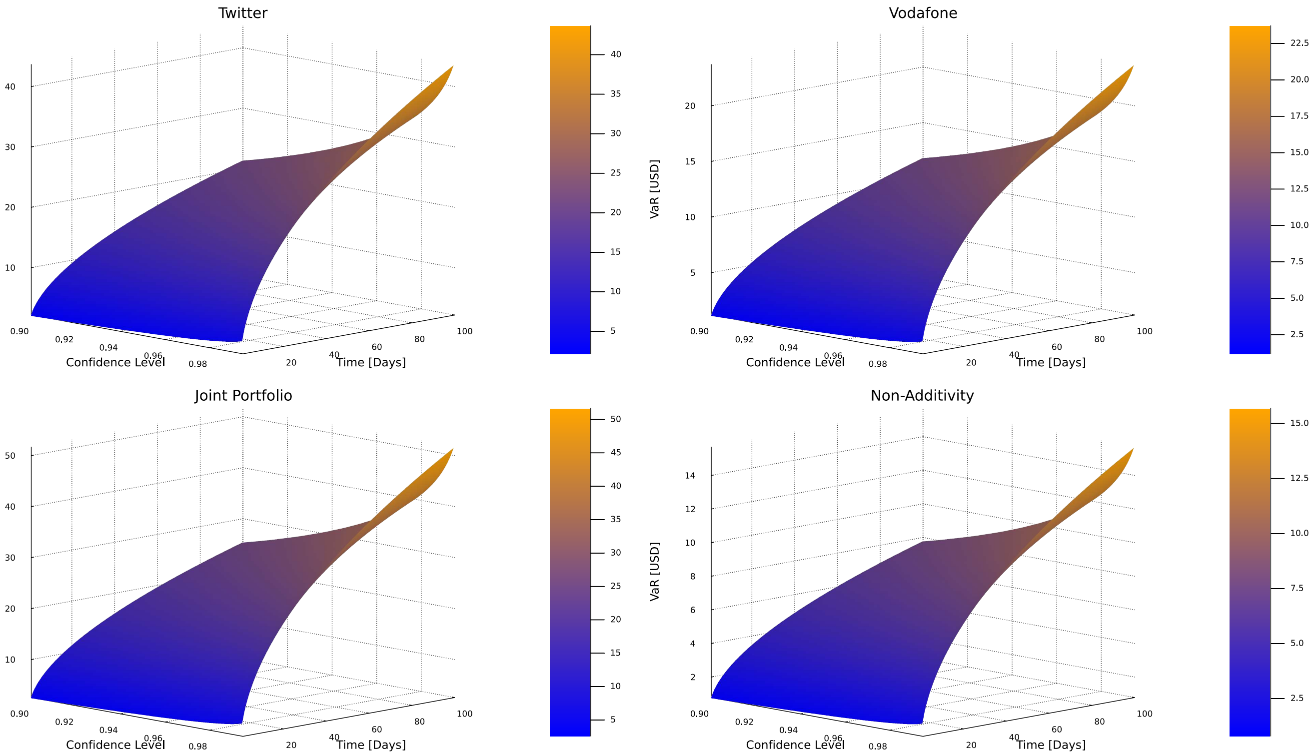

Figure 4: Value at risk for the parametric normal case: top left and top right there is the single asset VaR, bottom left joint parametric normal VaR and bottom right the difference between the sum of the two single VaRs and the Joint.

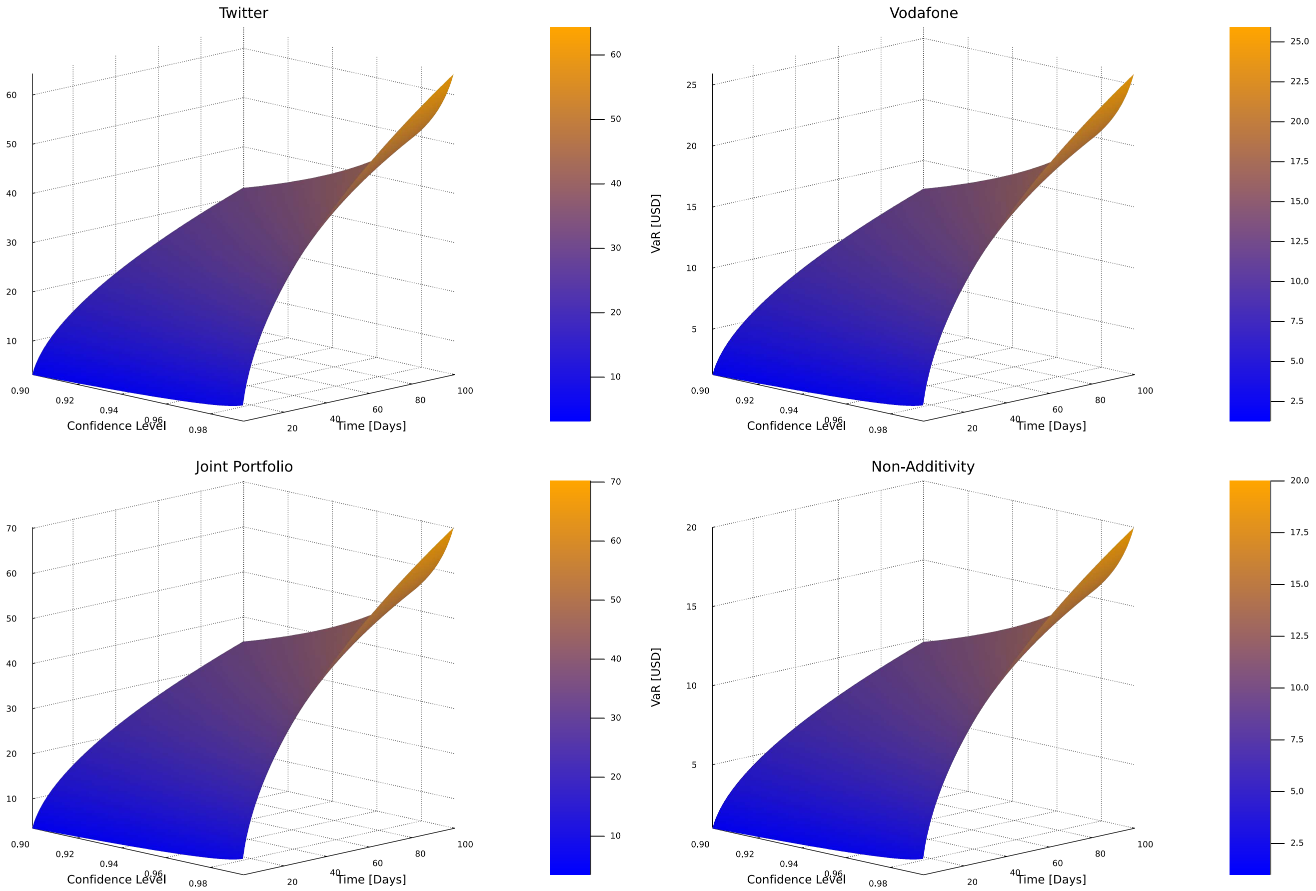

Figure 5: Value at risk for the parametric normal case using the RiskMetrics EWMA: top left and top right there is the single asset VaR, bottom left joint parametric normal VaR and bottom right the difference between the sum of the two single VaRs and the Joint.

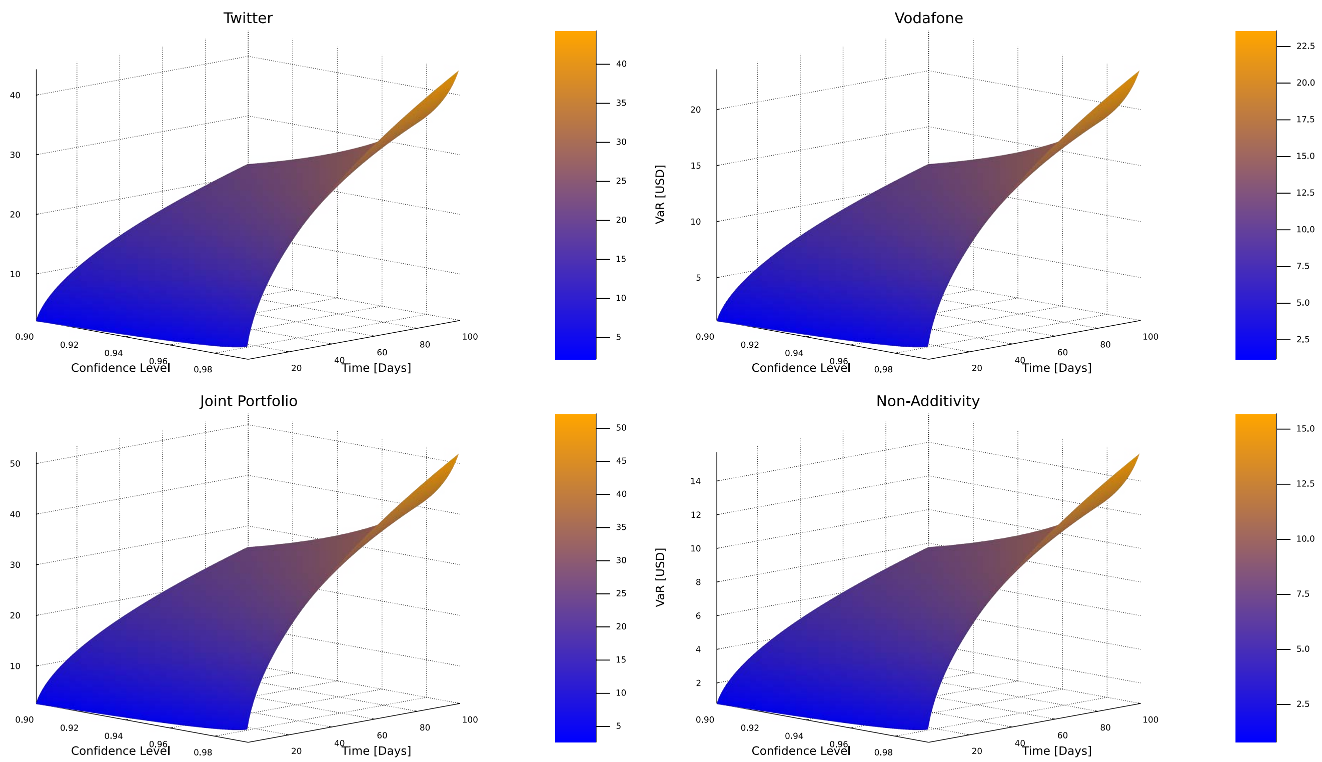

Figure 6: Value at risk for the MonteCarlo method: top left and top right there is the single asset VaR, bottom left joint VaR and bottom right the difference between the sum of the two single VaRs and the Joint.

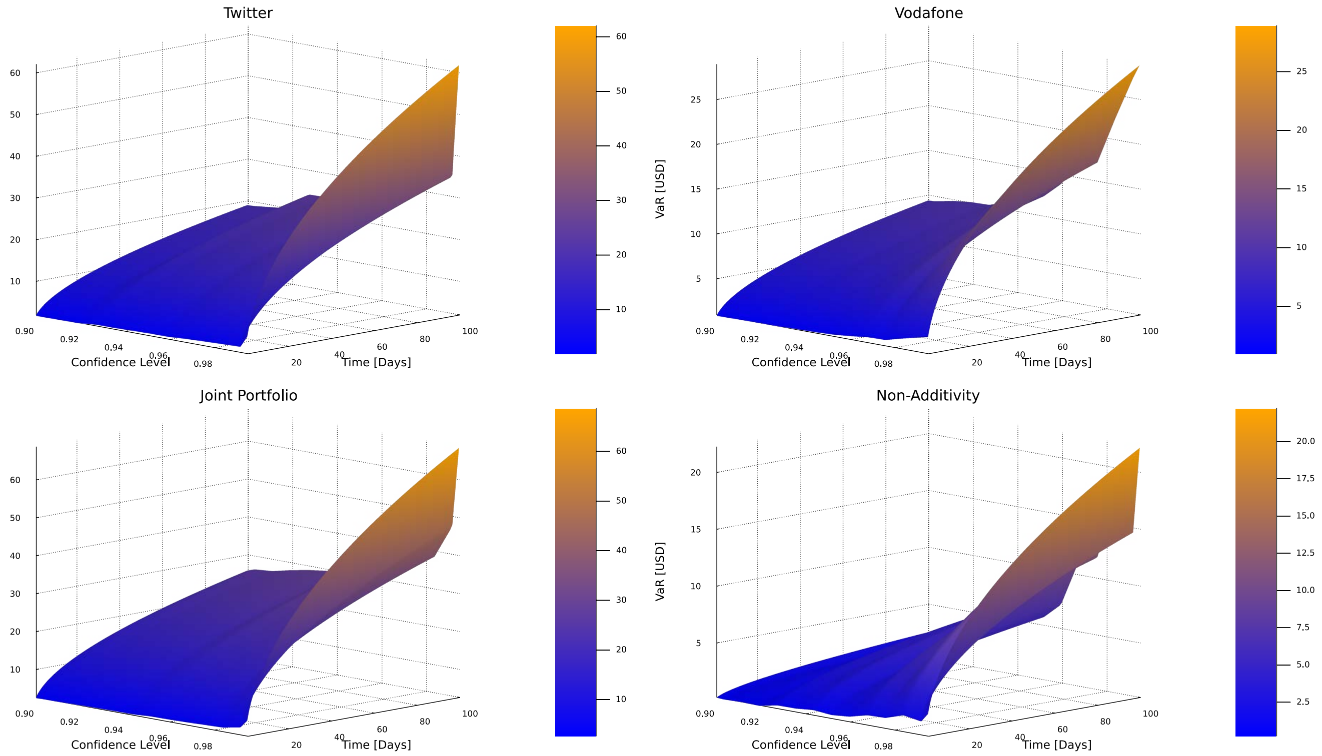

Figure 7: Value at risk for the historical simulation case: top left and top right there is the single asset VaR, bottom left joint VaR and bottom right the difference between the sum of the two single VaRs and the Joint.

ability distribution for ΔP and find VaR as the appropriate percentile of the distribution (eventually multiplied by $\sqrt{N}$ according to 1).

The interest rate was taken from the `global-rates.com` site.

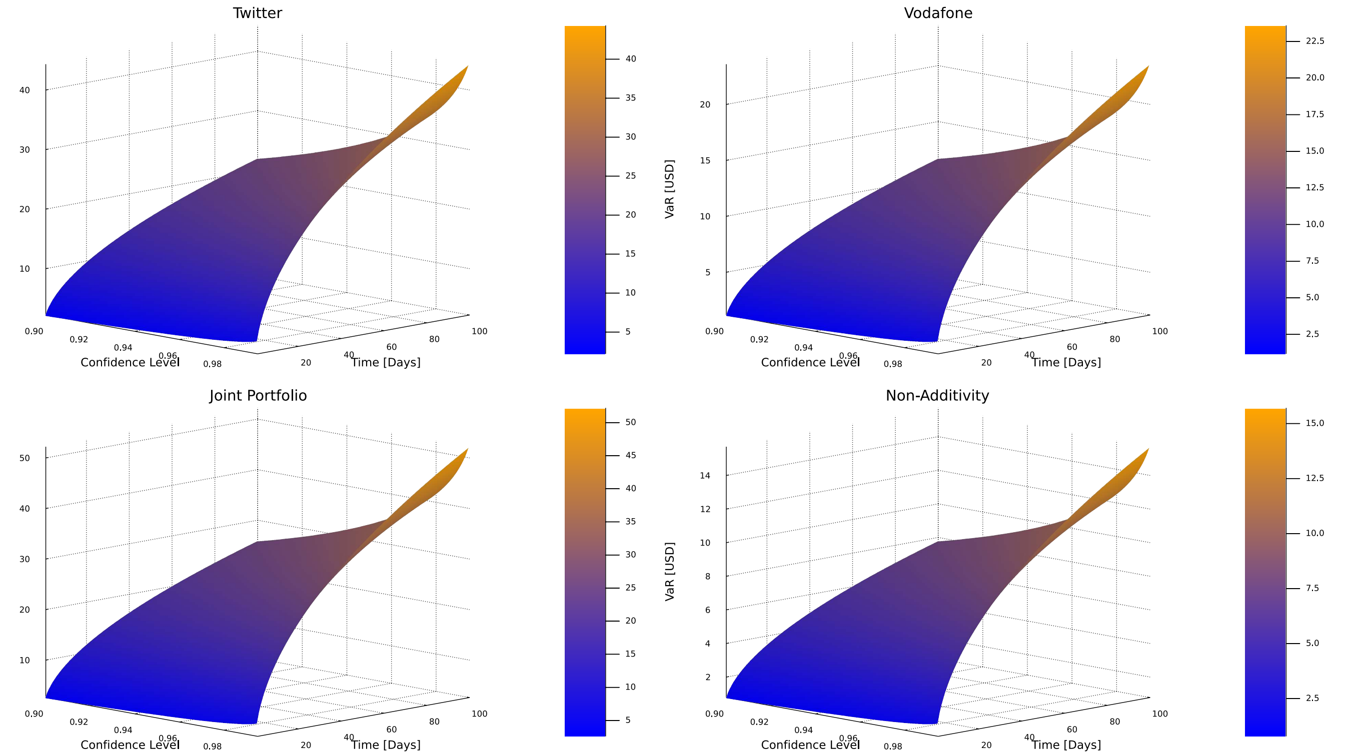

Figure 8: Value at risk for the historical case: top left and top right there is the single asset VaR, bottom left joint VaR and bottom right the difference between the sum of the two single VaRs and the Joint.

2.4 Historical Simulation VaR

Historical simulation is one popular way of estimating VaR. It involves using past data as a guide to what will happen in the future. In particular, we take a time interval in the past and consider the daily returns for this interval. We want then to find the VaR corresponding to a X confidence level: we take the $(1 - X) \times N_{days}$-th smaller value. This method corresponds to the parametric computation in the case of an ideal normal distribution of the returns.

2.5 Historical VaR

We consider the daily volatility as the daily risk. Assuming that the expected change in the value of the portfolio is zero and that the change in the value of the portfolio is normally distributed, we can compute the VaR as follows:

$$VaR = Investment \times \sigma \times \sqrt{T} \times q(1 - X) \tag{3}$$

with q the inverse cumulative of the normal distribution $\mathcal{N}(0, 1)$.

3 Results & Discussion

Here we report the main results of the study.

The initial position considered is a total of 100 USD, divided equally between the two stocks.

We report in Figures 4, 5, 6, 7, and 8 for each of the methods described earlier the surfaces of VaR as a function of the confidence level and the time considered. For each method, we report both the values for the single and joint VaR. We add also the plot of the absolute difference between the sum of the single VaRs and the joint value. We see that, since the latter is always different from zero we can appreciate the non-additivity. In particular, the sum of the single VaRs is larger than the joint portfolio's VaR, proving that diversifying the investments through the building of a balanced portfolio can significantly limit the risk.

In Table 1 we report the values of VaR computed for each method in some significant time values and for the confidence levels $.95, .99, .995$, which are widely used by investment banks and funds.

Both the charts and the tables show the pattern we expect: VaR increases both with time and confidence level. The increase in time is since the uncertainty increases as the time considered increases. In mathematical terms, according to 1, the dependence should be $\propto \sqrt{N}$. On the other hand, the increase in the confidence level is merely a probabilistic fact: we are considering a more and more extreme part of the tail of the distribution considered.

We can also appreciate the different values for each method: this difference can be significant, in particular, the largest difference is between the historical simulation and the Monte-Carlo simulation.

In Figure 9 we report the chart of the outcomes of the simulations. In particular, the histograms represent the simulation of the multiplicator of the initial investment, e.g. if we have 2 as multiplicator the final value will be two times the initial investment. The histograms are divided by the time considered, going from 1 day to one year. As we expect the curve for less time is more and more picked, while for longer times it becomes more

Table 1: VaR for each method used in USD

		Twitter Confidence level			Vodafone Confidence level			Portfolio Confidence level		
Method	Time[days]	.95	.99	.995	.95	.99	.995	.95	.99	.995
Parametric method with flat σ	1	3.10	4.38	4.85	1.52	2.15	2.37	3.53	5.01	5.55
	10	9.80	13.86	15.34	4.81	6.78	7.51	11.17	15.83	17.54
	50	21.90	30.98	34.31	10.76	15.17	16.79	24.97	35.41	39.23
Parametric method with RiskMetrics EWMA	1	4.09	5.81	6.43	1.65	2.34	2.59	4.459	6.34	7.03
	10	12.92	18.36	20.35	5.19	7.39	8.20	14.10	20.04	22.22
	50	28.90	41.05	45.50	11.60	16.53	18.33	31.53	44.81	49.68
Historical Method	1	3.09	4.37	4.84	1.51	2.14	2.37	3.56	5.03	5.58
	10	9.79	13.84	15.34	4.78	6.77	7.49	11.27	15.93	17.64
	50	21.91	30.99	34.32	10.70	15.13	16.75	25.19	35.64	39.45
Historical Simulation	1	2.10	2.76	2.98	1.26	2.09	2.45	2.68	3.29	3.76
	10	6.66	8.73	9.42	3.99	6.60	7.76	8.48	10.40	11.90
	50	14.88	19.52	21.05	8.92	14.76	17.35	18.96	23.27	26.62
MonteCarlo simulation	1	6.16	6.67	6.69	2.96	3.10	3.12	6.65	7.39	7.74
	30	22.92	24.35	24.40	12.29	12.79	12.85	27.27	29.90	31.08
	90	33.01	34.53	34.58	19.41	20.10	20.18	42.63	46.17	47.73

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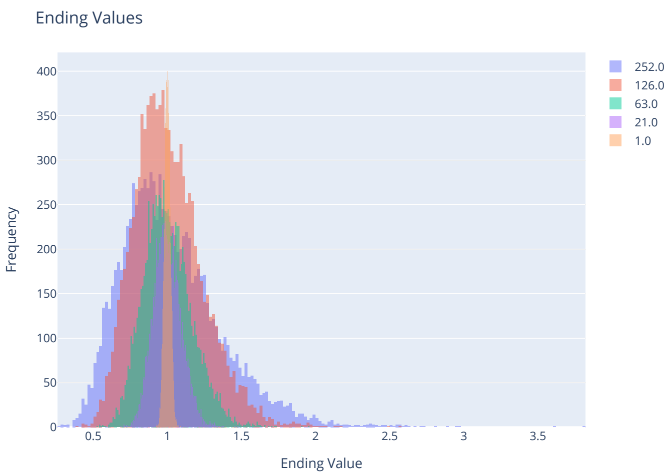

Figure 9: Ending values for each MonteCarlo simulation. Each value represent the multiplication coefficient of the initial value at the end of the period of time considered. Each period is represented as the number of day. The periods considered are 1day, 1 month, 3 months, 6 months, and 1 year.

Conclusions

We implemented the computation of the Value at Risk according to several models and we observed the behaviour of the VaR as a function of the time window and the confidence level. The results for the several methods used are generally compatible, anyway the historical simulation and the MonteCarlo simulation give the greatest difference from the average. While the former gives the smallest values, the latter gives the largest ones.

The future development of the work done should be the implementation of more sophisticated models such as the conditional value at risk (CVaR), the entropic value at risk (EVaR) and the range value at risk (RVaR).

References

[1] HULL, J. *Options, futures, and other derivatives*, 6. ed., pearson internat. ed ed. Pearson Prentice Hall, Upper Saddle River, NJ [u.a.], 2006.