UNIVERSITÀ DEGLI STUDI DI PADOVA
DIPARTIMENTO DI FISICA E ASTRONOMIA "GALILEO GALILEI"
MASTER DEGREE IN PHYSICS OF DATA
COURSE: STOCHASTIC METHODS FOR FINANCE

# Deduction of the discount factor and the implied dividend using option prices

GITHUB REPO: https://github.com/aidinattar/Financial-Mathematics

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March 30, 2022

## Abstract

In this paper, we will try to compute the implied dividend of a stock option. The procedure is in accord with the following: we compute the discount factor from a box spread price, which involves the price of two call and two put options; then we use the value to find the implied dividend using the Call-Put parity property. The results will be poor due to the usage, instead of European options, American ones. This should not be done: both the box-spread strategy and the put-call parity property need the use of European options.

## 1 Introduction

Spread trading is a business strategy that involves an investor simultaneously buying one security and selling related security. The securities being bought and sold often referred to as "legs", are typically executed with futures contracts or options, though there are other securities that can be used.

Some of the most popular types of spreads are bull spread, which involves taking a position on two Call options with different strikes, a bear spread which involves positions on two Put options and a box spread which is a combination of a bull call spread and a bear put spread. In Table 1 we report the payoff expected from a box spread using a bull spread with strike prices $K_1$ and $K_2$ and a bear spread with the same two strike prices. The payoff from a box spread is always $K_2 - K_1$ as can be seen in Figure 1. The value of a box spread is therefore always $(K_2 - K_1)e^{-rT}$. If it has a different value there is an arbitrage opportunity. If the market price of the box spread is too low, it is profitable to buy the box. This involves buying a call with strike price $K_1$, buying a put with strike price $K_2$, selling a call with strike price $K_2$, and selling a put with strike price $K_1$. If the market price of the box spread is too high, it is profitable to sell the box. This involves buying a call with strike price $K_2$, buying a put with strike price $K_1$, selling a call with strike price $K_1$, and selling a put with strike price $K_2$ [1].

Table 1: Payoff from a Box Spread.

| Stock price range | Payoff from bull call spread | Payoff from bear put spread | Total Payoff |
|---|---|---|---|
| $S_t \leq K_1$ | 0 | $K_2 - K_1$ | $K_2 - K_1$ |
| $K_1 < S_t < K_2$ | $S_T - K_1$ | $K_2 - S_T$ | $K_2 - K_1$ |
| $S_t \geq K_2$ | $K_2 - K_1$ | 0 | $K_2 - K_1$ |

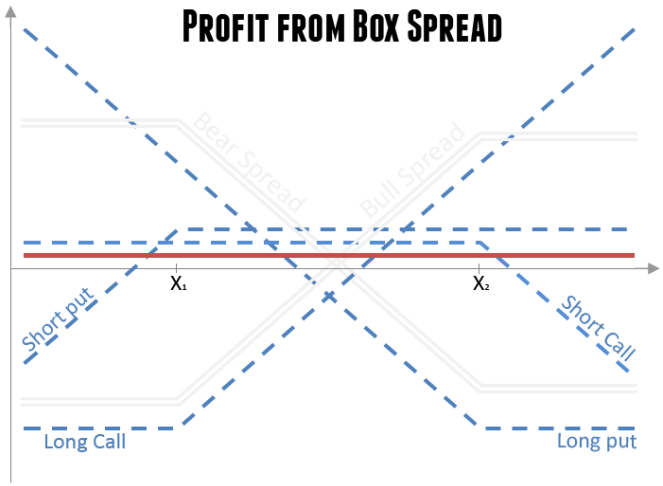


Figure 1: Profit diagram of a box spread. It is a combination of positions with a riskless payoff [2].

The Box Spread price can be used to deduce the discount factor as a function of the maturity, indeed

$$(S - K_1)^+ - (K_1 - S)^+ = S - K_1$$

$$-(S - K_2)^+ + (K_2 - S)^+ = K_2 - S$$

and then

$$(S-K_1)^+-(S-K_2)^++(K_2-S)^+-(K_1-S)^+ = K_2-K_1 \quad (1)$$

thus,

$$C(K_1)-C(K_2)+P(K_2)-P(K_1) = (K_2-K_1)\cdot D(0,T) \quad (2)$$

with $D(0,T)$ the discount factor. Since one can retrieve all the other quantities from the market, the only unknown is the discount factor for which we can make an estimation.

It is important also to realize that a box-spread arbitrage only works with European options, and this can lead to a potential danger[1] because using an American option hides the risk of early exercise. American style options, such as those options listed on most U.S. stocks, may be exercised early (*i.e.*, before expiration), and so it can happen that a short option becoming deep-in-the-money can be assigned. In the normal construction of a box, this is unlikely, since you would own the deep call

[1]which is what happened in this paper

and put, but the stock price can move significantly and then find yourself in a situation where you might be assigned.

Knowing the discount factor, one can compute the implied dividend as a function of the maturity using the put-call parity and using the price of an option *At The Money* (ATM) and Forward contracts.

Table 2: Values of the portfolio at time $t = 0$ and $t = T$.

| | $t = 0$ | $t = T$ |
|---|---|---|
| Long-Forward | 0 | $S - F(0,T)$ |
| Short-S | S | $value - S - div$ |
| Riskless | $long(F + div) \cdot D(0,T)$ | $F + div$ |
| TOT | $S - (F + div) \cdot D(0,T)$ | 0 |

Indeed, as can be seen in Table 2 one can compute the value of the portfolio considered, and deduce the dividend, knowing that $F(0,T) = S \cdot e^{rT} - div$. In the case of proportional dividends, where $div = Se^{\delta T}$, we can write the following

$$\begin{aligned} Call - Put &= S - K * D(0,T) - S \cdot e^{\delta T} * D(0,T) \\ &= D(0,T) \cdot (S \cdot e^{rT} - S \cdot e^{\delta T} - K) \\ &= D(0,T) \cdot (F(0,T) - K) \end{aligned} \tag{3}$$

In this way, knowing the Call and Put option prices ATM and the discount factor we can retrieve the dividend.

# 2 Materials & Methods

In this paper, we try to compute the discount factor and the implied dividend of an asset, using the prices of call and put options and the stock prices of the underlying. The underlying chosen is the company Johnson&Johnson (JNJ on the NYSE - Nasdaq), a biotechnology company, which researches and develops, manufactures, and sells various products in the healthcare field worldwide. It operates in three segments: Consumer Health, Pharmaceutical, and Medical Devices. The Consumer Health segment offers baby care products under the JOHNSON'S and AVEENO Baby brands; oral care products under the LISTERINE brand; skin health/beauty products under the AVEENO, CLEAN & CLEAR, DR. CI:LABO, NEUTROGENA, and OGX brands; acetaminophen products under the TYLENOL brand; cold, flu, and allergy products under the SUDAFED brand; allergy products under the BENADRYL and ZYRTEC brands; ibuprofen products under the MOTRIN IB brand; smoking cessation products under the NICORETTE brand; and acid reflux products under the PEPCID brand. This segment also provides women's health products, such as sanitary pads and tampons under the STAYFREE, CAREFREE, and o.b. brands; wound care products comprising adhesive bandages under the BAND-AID brand; and first aid products under the NEOSPORIN brand. The Pharmaceutical segment offers products in various therapeutic areas, including immunology, infectious diseases, neuroscience, oncology, pulmonary hypertension, and cardiovascular and metabolic diseases. The Medical Devices segment provides electrophysiology products to treat cardiovascular diseases; neurovascular care products to treat hemorrhagic and ischemic stroke; orthopaedics products in support of hips, knees, trauma, spine, sports, and others; advanced and general surgery solutions that focus on breast aesthetics, ear, nose, and throat procedures; and disposable contact lenses and ophthalmic products related to cataract and laser refractive surgery under the ACUVUE brand. The company markets its products to the general public, retail outlets and distributors, as well as distributes directly to wholesalers, hospitals, and healthcare professionals for prescription use. Johnson & Johnson was founded in 1886 and is based in New Brunswick, New Jersey. The company is ranked No. 36 on the 2021 Fortune 500 list of the largest United States corporations by total revenue. Johnson & Johnson is one of the world's most valuable companies and is one of only two U.S.-based companies that have a prime credit rating of AAA, higher than that of the United States government.

Table 3: Major Shareholders, *i.e.* a shareholder who directly or indirectly holds 10% or more of the voting rights.

| Major Holders | |
|---|---|
| **0.09%** | % of Shares Held by All Insider |
| **70.30%** | % of Shares Held by Institutions |
| **70.36%** | % of Float Held by Institutions |
| **4489** | Number of Institutions Holding Shares |

Table 4: Valuation Measures, provided by Morningstar, Inc.

| Valuation Measures | |
|---|---|
| Market Cap (intraday) | 458.45B |
| Enterprise Value | 460.59B |
| Trailing P/E | 22.32 |
| Forward P/E | 16.61 |
| PEG Ratio (5 yr expected) | 2.86 |
| Price/Sales (ttm) | 4.97 |
| Price/Book (mrq) | 6.19 |
| Enterprise Value/Revenue | 4.91 |
| Enterprise Value/EBITDA | 15.18 |

The pharmaceutical branch of the company achieved new success in 2021 after developing a successful monodose candidate for the COVID-19 vaccine. It is a viral vector vaccine based on a human adenovirus that has been modified to contain the gene for making the spike protein of the SARS-CoV-2 virus that causes COVID-19. The body's immune system responds to this spike protein to produce antibodies. The vaccine requires only one dose and does not need to be stored frozen.

Clinical trials for the vaccine were started in June 2020, with Phase III involving around 43,000 people. On 29 January 2021, Janssen announced that 28 days after a completed vaccination, the vaccine was 66% effective in a one-dose regimen in preventing symptomatic COVID-19, with an 85% efficacy in preventing severe COVID-19 and 100% efficacy in preventing hospitalization or death caused by the disease.

The vaccine has been granted an emergency use authorization (EUA) by the US Food and Drug Administration (FDA) and a conditional marketing authorization by the European Medicines Agency (EMA) and the UK Medicines and Healthcare products Regulatory Agency.

Because cases of thrombosis with thrombocytopenia syndrome and Guillain-Barré syndrome have been reported after receipt


Figure 2: Top (a): Stock price trend for the last 6 months. The boxes represent the spread between the open and close values and the lines represent the spread between the low and high values. Increasing candles are drawn in green whereas decreasing are drawn in red. Bottom (b) Call and Put option prices as a function of the strike price for different maturities.

of the Janssen COVID-19 vaccine, the US Centers for Disease Control and Prevention (CDC) recommends "preferential use of mRNA COVID-19 vaccines over the Janssen COVID-19 vaccine, including both primary and booster doses administered to prevent COVID-19, for all persons aged 18 years of age and older. The Janssen COVID-19 vaccine may be considered in some situations, including for persons with a contraindication to receipt of mRNA COVID-19 vaccines." On 9 February 2022, Johnson & Johnson announced it has temporarily suspended production of the vaccine though they also noted that it will likely resume at some point in the future and that it will honour all pre-existing contracts that oblige Janssen to supply its vaccine by using the millions of already existing vaccine doses in its inventory were requested [4] [3] [5].

To find the discount with the methods described above, we need to use the data about the option prices. In particular the strike prices $K_1$ and $K_2$ should be chosen in accord with a rule of thumb as $K_{1,2} = K_{ATM} \pm 20\%$. Unfortunately, if we want to use the same $K_1$ and $K_2$ for all the maturities we can not use the rule of thumb since their options with those strikes are not available for every maturity. We try in any case to use, for each maturity, the closer strikes to the ones retrieved using the rule of thumb described.

As the price of the option, we use the mid-price, obtained as a simple average between ask and bid prices. Here, there exists the main problem: the options used are not European but American. As said earlier, American style options are subject to the risk that an option could be exercised early. If this is the case, the remaining option positions would no longer be considered a box spread and the risk profile of the strategy would have changed.

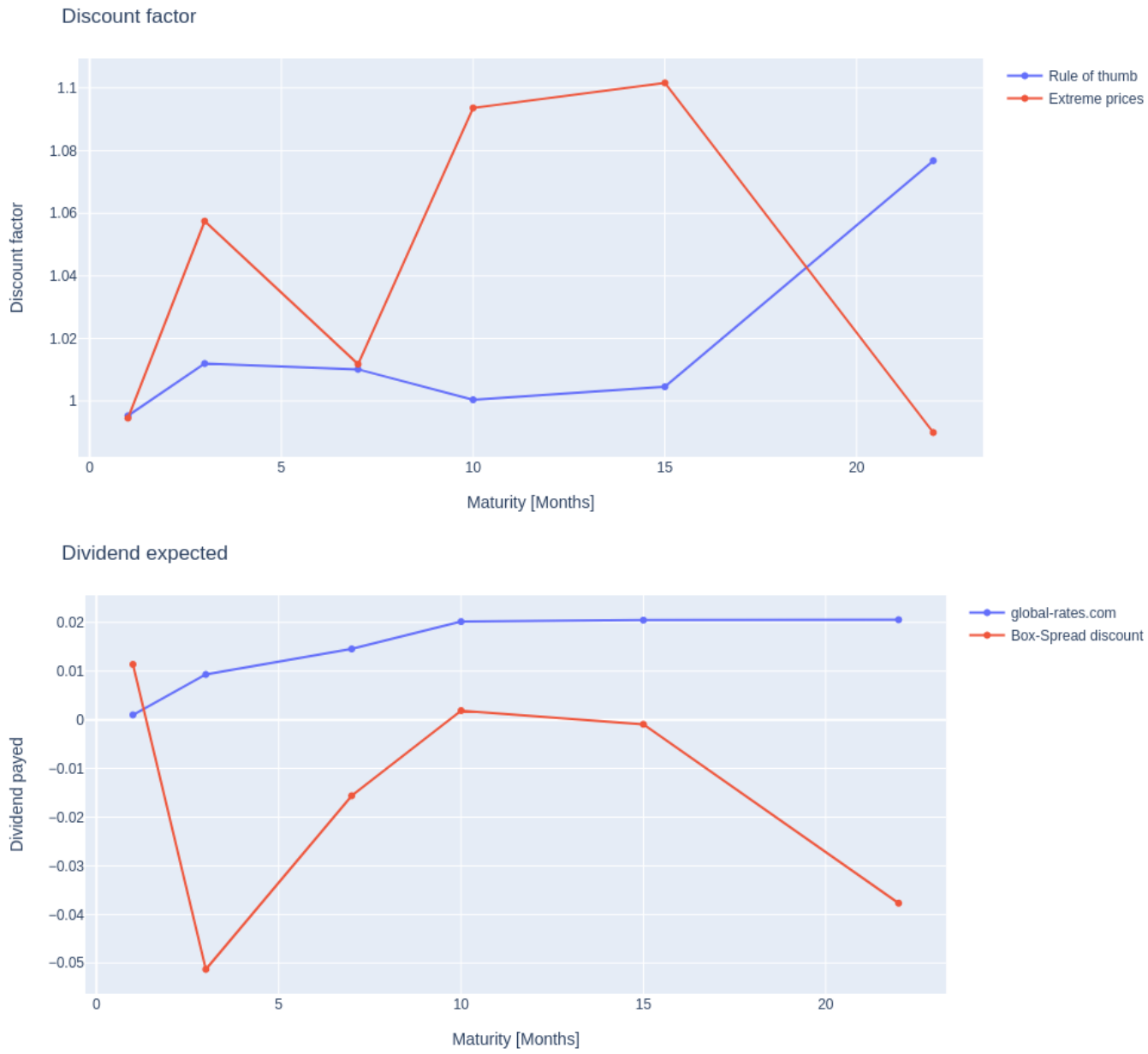


Figure 3: Top (a): Discount factor computed using the rule of thumb and the extreme prices. Bottom (b): Implied dividend computed using the discount factor obtained and the real interest rates.

This problem will lead to discount factor values $> 1$, which means that we would be dealing with a non-positive interest rate, which is not the case for the USD LIBOR.

We can then compute the implied dividend and compare it with the historical real values.

The data used are taken from the Yahoo Finance site and the analysis involved the use of `Excel-VBA` and `Julia 1.7.0` using a `Jupyter` framework and can be found in the `GitHub` repository.

# 3 Results and Discussion

Here we describe the results of the task. We divide the section into a first part in which we describe the outcome of the computation of the discount factor as a function of the maturity and the latter is the computation of the implied dividend.

Both the analysis are made using the historical prices of the stock and the options of the underlying JNJ described earlier, whose values can be found in Figures 2(a) and 2(b).

## 3.1 Discount factor

in accord with equation 2 we can estimate the discount factor as

$$D(0,T) = \frac{C(K_1) - C(K_2) + P(K_2) - P(K_1)}{K_2 - K_1} \tag{4}$$

As said earlier we want to use the rule of thumb such that $K_{1,2} = K_{ATM} \pm 20\%$ but this is problematic, since, as can be seen in Figure 2(b) not all the strike prices have available options. Thus, we try to select when it is possible, those strike

prices, and when it is not the closer prices. Using extreme strike prices can lead to problems linked to the fact that those options are not much traded and can have irregular prices (since we use mid-prices which are not real prices, and the trading price can significantly differ from that). On the other hand, using prices too close to the strike *at the money* can lead to degeneration.

In any case, we try to compute the results using both the rule of thumb and taking the extremes. In Figure 3 and Table 5 one can see the visual and numerical results. It is immediately evident that there is a problem: the majority of the values obtained are greater than 1. At the cost of being redundant, we want to stress the fact that this procedure of deduction of the discount factor using the cost of the box spread works only with European options, and not for American options which carry a risk of early exercise[2].

Table 5: Discount factor computed using the rule of thumb and the extreme prices.

| | Rule of thumb | | Extreme prices | |
|---|---|---|---|---|
| Maturity | $K_1 - K_2$ $[USD]$ | Discount factor | $K_1 - K_2$ $[USD]$ | Discount factor |
| 1-Month | $150 - 185$ | 0.9946 | $150 - 185$ | 0.9946 |
| 3-Months | $140 - 210$ | 1.0120 | $70 - 265$ | 1.0575 |
| 7-Months | $140 - 210$ | 1.0101 | $115 - 210$ | 1.0118 |
| 10-Months | $140 - 210$ | 1.0004 | $70 - 260$ | 1.0936 |
| 15-Months | $140 - 200$ | 1.0046 | $80 - 260$ | 1.1016 |
| 22-Months | $140 - 210$ | 1.0768 | $80 - 250$ | 0.9899 |

In Table 6 is reported a comparison between the discount factor computed and the expected one, obtained from the USD LIBOR interest rates.

Table 6: Comparison between discount factors computed from the price of a box spread and the ones computed using USD LIBOR.

| | Consultation date | Box spread | USD LIBOR |
|---|---|---|---|
| 1-Month | 03-18-2022 | 0.9946 | 0.9996 |
| 3-Months | 03-18-2022 | 1.0120 | 0.9977 |
| 7-Months | 03-18-2022 | 1.0101 | 0.9936[3] |
| 10-Months | 03-18-2022 | 1.0004 | 0.9985[4] |
| 15-Months | 03-18-2022 | 1.0046 | 0.9985[4] |
| 22-Months | 03-18-2022 | 1.0768 | 0.9985[4] |

## 3.2 Implied dividend

in accord with equation 3, we can estimate the implied dividend as

$$\delta = -\frac{1}{T}\log\frac{Call - Put + KD(0,T)}{S_0} \tag{5}$$

In Table 7 and in Figure 3(b) one can see the comparison between the implied dividend computed using the previous discount factors and using the ones retrieved from `global-rates.com`. It is evident the existence of a systematic problem: we used an american option instead of an european one.

[2] cfr. RobinHood Box Spread incident
[3] Used USD LIBOR for 6 months
[4] Used USD LIBOR for 12 months

Table 7: Comparison between the implied dividend computed from the discount factor retrieved from the box spread and the USD LIBOR interest rate.

| | USD LIBOR | Box spread |
|---|---|---|
| 1-Month | .00102 | .01139 |
| 3-Months | .00933 | $-.01324$ |
| 7-Months | .01456 | $-.01560$ |
| 10-Months | .02017 | .00189 |
| 15-Months | .02047 | $-.00092$ |
| 22-Months | .0205 | $-.03767$ |

The results are incompatible with the payed dividend, which for the underlying we are considering is $1.06\%$.

# Conclusions

We computed the discount factor using the price of a box spread. We obtain poor results due to the usage, instead of the standard European options, American ones. The latter brings an additional risk linked to the possibility of early exercise. The results of the dividends obtained are not encouraging either. There is the obvious problem inherited from the discount factor.

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