Introduction to Concurrent Learning Adaptive Control

Recent Advances in Model Reference Adaptive Control: Theory and Applications

ORGANIZERS

ANTHONY CALISE, *Georgia Institute of Technology*
ERIC JOHNSON, *Georgia Institute of Technology*
TANSEL YUCELEN, *Georgia Institute of Technology*
GIRISH CHOWDHARY, *Massachusetts Institute of Technology*

AIAA Guidance Navigation and Control Conference,

11-12 Aug 2012, Minneapolis, MN

Motivation

- Most dynamic growth sector for Aerospace industry
- Most UAVs today require lots of human supervision
- Autonomy: the ability to take action for satisfying given higher level goals in presence of uncertainties
- UAVs must learn to fly themselves


Global Hawk RPV

Aurora Centaur OPV


AFI Hornet MAV

UAV Flight Control System Design

System

How do we deal with the uncertainties???

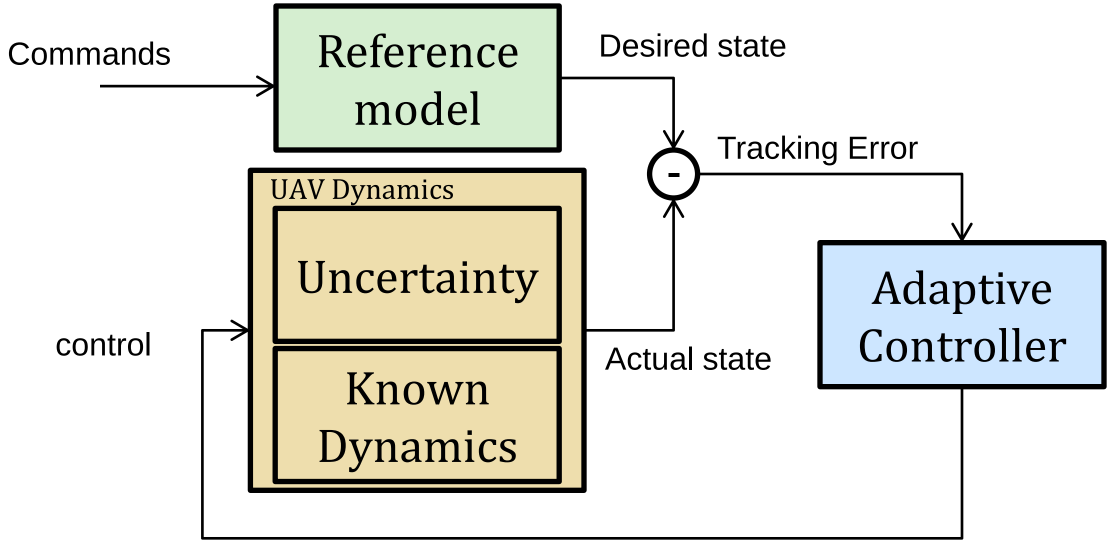
ematical
Modeling:

$f(x,u)$
$g(x,u)$

Sources of
uncertainties:
- Actuator
- Time dela
- Sensor N
- Disturbar

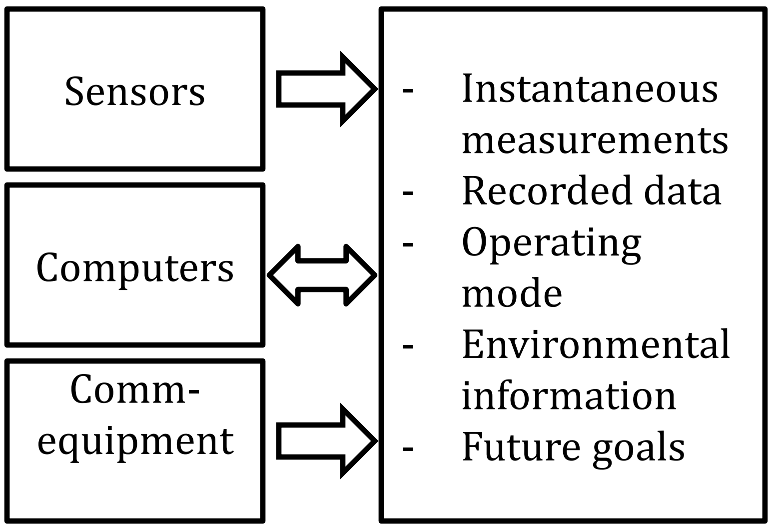
ces of
ertainties:
- Unmodelled dynamics
- Conservative controllers

Model Reference Adaptive Control (MRAC)



- Want to make the uncertain system behave like the reference model
- Need quantifiable metrics on performance and stability
- Need to enable agile and health-aware UAV operation
- Approach:
  - Simultaneously track the reference model and learn the uncertainty

Information enabled adaptive control



Online available Information

- Modern avionics can record and process significant data online
- How to extract information from data?
- How to make decisions using information?

- Overview of the main ideas:
  - Adaptive control methods that simultaneously stabilize the system and learn the uncertainty using *all* online available information: enable Health-aware operation
  - With provable and quantifiable guarantees of stability and performance
  - Flight test verified, capable of tolerating severe faults
  - A pathway to nonparametric adaptive control: towards adaptive controllers that require no prior knowledge

Online Parameter Estimation

Unknown dynamics

$$y(t) = {W^*}^T \Phi(x(t))$$

- $\phi(x) \in \Re^n$: basis vector (known)
- $y(t) \in \Re$ (known)
- W^* is an unknown constant vector (unknown)

- Estimation model: $\nu(t) = W^T(t)\Phi\big(x(t)\big)$
- Let $\widetilde{W}(t) = W(t) - W^*$
- Estimation error: $\epsilon = \nu - y = \widetilde{W}^T\Phi(x)$

Problem statement

Design an update law $\dot{W}$ such that $\lim_{t\to\infty} W(t) = W^*$

Traditional approach

Traditional gradient descent based approach

Updates weights in the direction of maximum reduction of instantaneous quadratic estimation cost

$$\dot{W} = -\gamma \frac{\partial(\epsilon^T \epsilon)}{\partial W} = -\gamma \Phi(x) \epsilon^T$$

- Issue: $W(t) \rightarrow W^*$ if an only if $\Phi(x)$ is Persistently Exciting

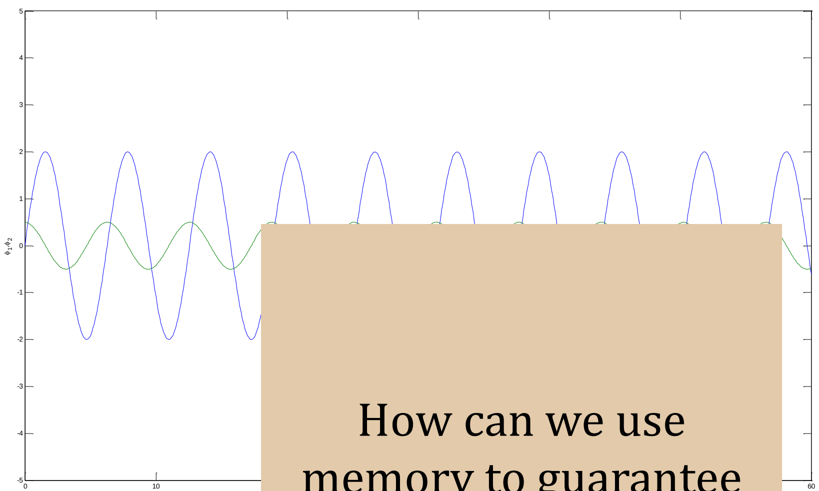
Persistence of Excitation

A bounded signal $\phi(t)$ is PE if for all $t > t_0$ there is a $T > 0$ s.t.

$$\int_t^{t+T} \phi(\tau)\phi^T(\tau)d\tau > \alpha I, \qquad \alpha \in \Re^+$$

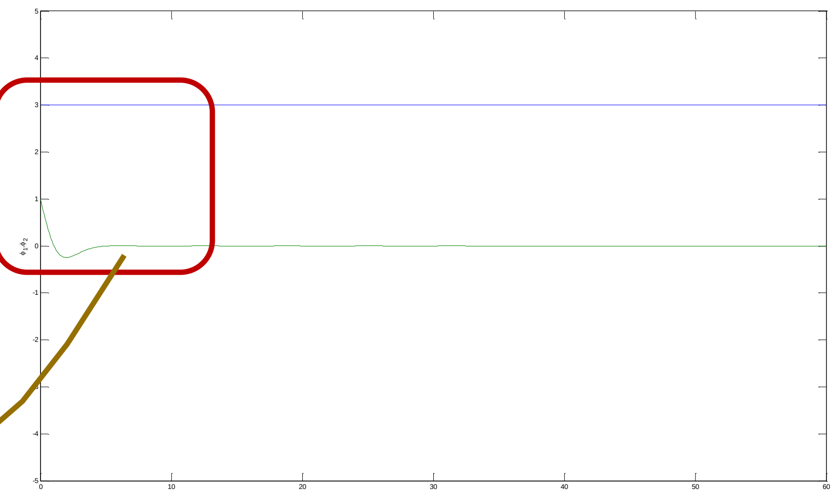
Motivation for using memory

- Traditional laws drive the weights only in the direction of instantaneous reduction of cost
- Hence, the training signal has to be rich enough to span the spectrum of the weight space



$\phi_1 =$ ...
PE ...

How can we use memory to guarantee convergence without *persistent* excitation?



$\phi_1 = 3, \phi_2 = e^{-t}$(2cos(t)−1)
Not PE, but exciting over a short interval

Concurrent Gradient Descent (CDC 10)

- Use recorded data concurrently with current data
- The concurrent learning law with p recorded data points is

Concurrent learning adaptive law

$$\dot{W}(t) = \underbrace{\boxed{-\gamma\Phi\big(x(t)\big)\epsilon^T(t)}}_{\text{Instantaneous update, } \dot{W}_t} - \underbrace{\boxed{\gamma\sum_{k=1}^{p}\Phi(x_k)\epsilon_k^T(t)}}_{\text{Update on recorded data, } \dot{W}_b}$$

- If the recorded data is sufficiently rich, convergence guaranteed

Richness of the recorded data

- Question: How do we determine if the recorded data is rich enough?

Rank-Condition

The recorded data have as many linearly independent elements as the dimension of the basis of the uncertainty. I.e. if $Z = [\Phi(x_1),\ \Phi(x_2), \dots, \Phi(x_p)]$, then

$$\mathrm{rank}(Z) = \dim\big(\Phi(x)\big)$$

- This condition applies to past data, whereas PE is also concerned with future system signals
- Condition applies only to a subset of the past data
- Easy to verify online
- Possible to meet this condition over normal course of operation

Guaranteed exponential convergence without PE

Theorem

If the Rank-Condition is satisfied, then $\dot{W}(t) = -\gamma\Phi\big(x(t)\big)\epsilon(t) - \gamma\sum_{k=1}^{p}\Phi(x_k)\epsilon_k^T(t)$ guarantees that $W(t) \rightarrow W^*$ exponentially

Proof sketch:

- Weight error dynamics

$$\dot{\tilde{W}} = -\left(\gamma\Phi\big(x(t)\big)\Phi^{\mathrm{T}}\big(x(t)\big) + \gamma\sum_{j=1}^{p}\Phi(x_j)\Phi^T(x_j)\right)\tilde{W}(t)$$

- Lyapunov candidate:

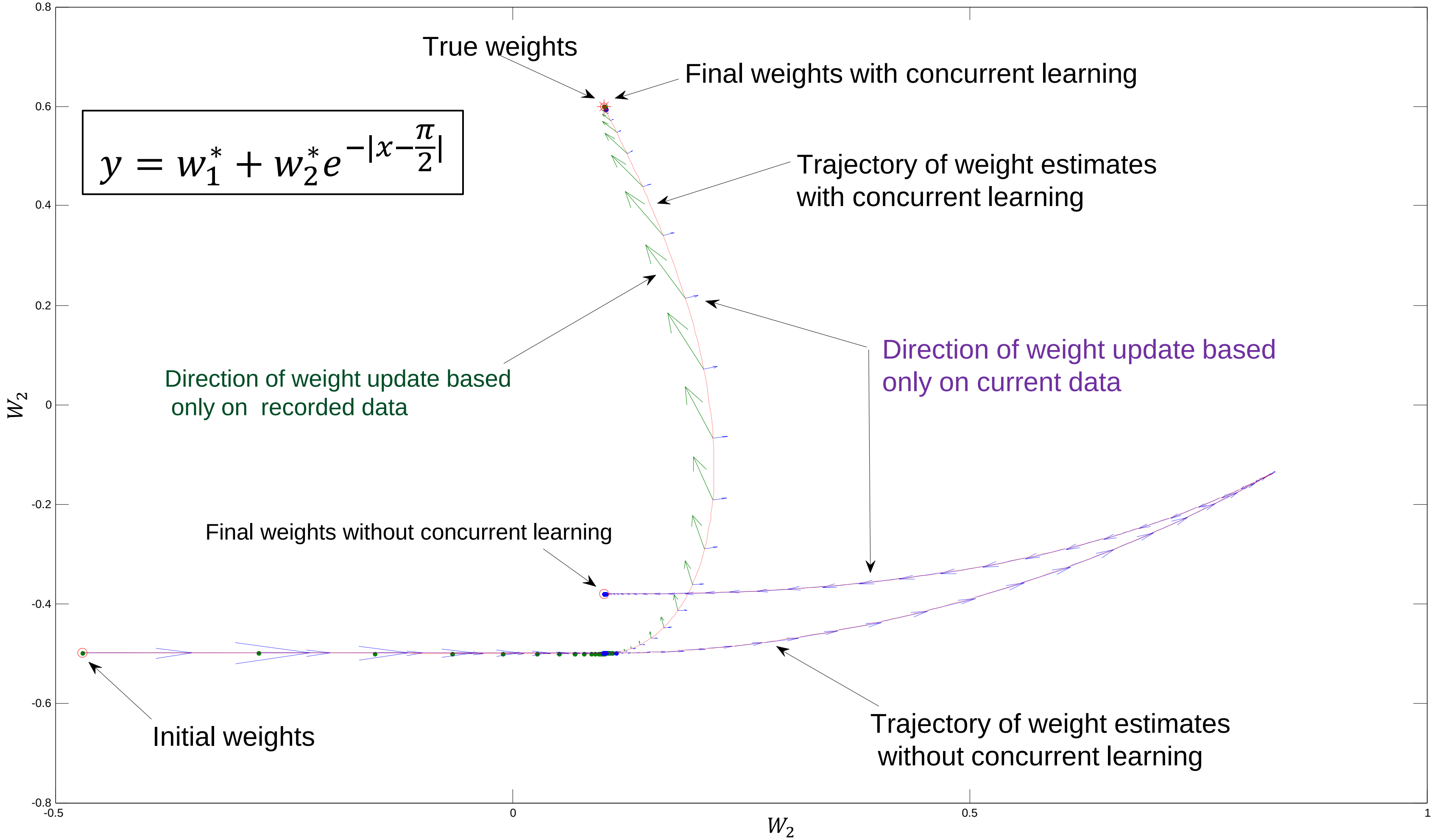
$$V(\tilde{W}) = \frac{1}{2}tr\big(\tilde{W}^T\gamma^{-1}\tilde{W}\big)$$

- Lie derivative of Lyapunov candidate:

$$\dot{V}(\tilde{W}) = -\tilde{W}^T(t)\left(\Phi\big(x(t)\big)\Phi^{\mathrm{T}}\big(x(t)\big) + \sum_{j=1}^{p}\Phi(x_j)\Phi^T(x_j)\right)\tilde{W}(t) < 0$$

- Due to the rank condition $\sum_{j=1}^{p}\Phi(\bar{x}_j)\Phi^T(\bar{x}_j) > 0$, hence don't need PE states

Example



Concurrent learning gradient descent combines two linearly independent directions to arrive at the true weights

Quick review of traditional MRAC

- $\dot{x} = Ax + B\big(u + \Delta(x)\big)$
- $\dot{x}_{rm} = A_{rm}x_{rm} + B_{rm}r$
- The goal is to design a control input u s.t. $x \rightarrow x_{rm}$
- Control action (subject to matching conditions): $u = ke + k_r[x_{rm}, r]^T - u_{ad}$
- Tracking error dynamics:

$$\dot{e} = A_m e + B_m(u_{ad} - \Delta)$$

Quick Review of AMI-MRAC

- Unknown system: $\ddot{x} = f(x, \dot{x}, u)$
- Designer selected reference model $\ddot{x}_{rm} = f_{rm}(x_{rm}, \dot{x}_{rm}, r)$
- The goal is to design a pseudo control input ν, such that $x \rightarrow x_{rm}$
- Approximate inversion model (invertible w.r.t to u): $u = \hat{f}^{-1}(x, \dot{x}, \nu)$
- $\nu = Ke + \ddot{x}_{rm} - \nu_{ad}$
- Modeling error: $\Delta(x, \dot{x}, u) = f(x, \dot{x}, u) - \hat{f}(x, \dot{x}, u)$
- Tracking error dynamics: $\dot{e} = Ae + B(\nu_{ad} - \Delta)$

Choice of adaptive element

Structured Uncertainty

Given that there exists a constant matrix W^* and known basis functions $\Phi(x,u) \in \Re^m$ such that

$$\Delta(x,u) = {W^*}^T \Phi(x,u)$$

Then, choose adaptive element:

$$\nu_{ad} = W^T \Phi(x,u)$$

Unstructured Uncertainty

Given $\Delta(x,u)$ is continuous and defined over a compact set, then :

$$\Delta(x,u) = {W^*}^T \sigma(x,u) + \tilde{\epsilon}$$

where $\sigma(x,u)$ are basis functions of a Neural Network adaptive element: $\nu_{ad} = W^T \sigma(x,u)$

$\sigma(x,u) = \phi(x,u)$ where ϕ is a Gaussian Kernel for RBF NN, or $g(x,u) = V\hat{}T\sigma(x,u)$, where σ is a logistic function for SHL NN

Case for weight convergence

- Let $\tilde{W}(t) = W(t) - W^*$

Tracking error dynamics for structured uncertainty

$$\dot{e} = Ae + B\tilde{W}^T\Phi(x)$$

- A_{m} is Hurwitz through design of the linear part

- If $\tilde{\mathrm{W}} = 0$ then we have $u_{ad} = \Delta$
  - The uncertainty is uniformly cancelled (Long Term Learning)
  - Linear, exponentially stable part of the tracking error dynamics dominates (Exponential tracking)
  - Hence, can possibly recover some of the desired transient response and stability characteristics of the reference model (Metrics for stability)

MRAC with Instantaneous data

Instantaneous Error Minimization

Updates weights in the direction of maximum reduction of quadratic cost of the instantaneous tracking error

$$\dot{W}(t) \cong -\Gamma \frac{\partial\left(e(t)^T e(t)\right)}{\partial W}$$

- Letting P be the solution of the closed loop Lyapunov equation and $\Gamma > 0$ the learning rate, this results in the following rank -1 adaptive law:

$$\dot{W} = -\Gamma\Phi(x)e^T PB$$

- This adaptive law requires PE to ensure exponential stability of $\left(e, \tilde{W} \equiv 0\right)$

Persistence of Excitation

Global Exponential Stability of MRAC with PE (Boyd and Sastry 1986)

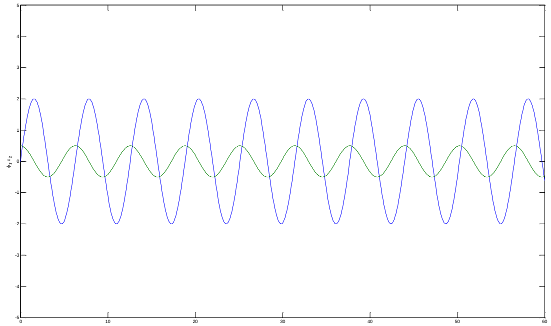
The adaptive law $\dot{W} = -\Gamma\Phi(x)e^T PB$ guarantees $\left(e(t), \tilde{W}(t)\right) \to 0$ exponentially fast if and only if $r(t)$ is persistently exciting. Furthermore if $r(t)$ is not PE, $\tilde{W} \nrightarrow 0$ in general.

Persistence of Excitation (PE)

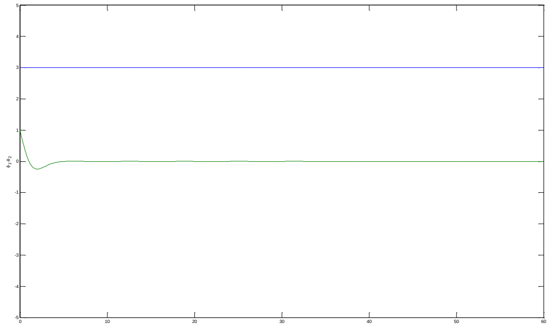
A bounded signal x(t) is PE if for all $t > t_0$ there is a $T > 0$ s.t.

$$\int_t^{t+T} x(\tau)x^T(\tau)d\tau > \alpha I, \qquad \alpha \in \Re^+$$

- Hard to use this definition to put a metric on PE of a signal
- Enforcing PE may lead to unnecessary/unwarranted effort



PE 2 dimensional signal



Not PE, but exciting over a short interval

Some relevant existing work

- Baseline adaptive law only guarantee $e(t) \to 0$ asymptotically, no performance guarantees
- Classic modifications for weight boundedness:
  - σ-mod (Ioannou 84), e-mod (Narendra 86), projection based adaptation
- Intelligent excitation (Cao 07):
  - Excitation enforced as a function of tracking error
  - Increased control cost due to added excitation
- L_1 adaptive control (Cao, Hovakimyan 08)
- Q-modification (Volyanksyy 09)
- Direct Recursive Least Squares Adaptation (Ngyuen 2006)

Concurrent Learning adaptive law

- Use recorded data concurrently with current data
- For the stored data point x_k assume that $\ddot{x}_k$ is available and let

$\epsilon_k(t) = W^T(t)\Phi(x_k) - \Delta(x_k) = W^T(t)\Phi(x_k) - (\ddot{x}_k - \nu_{\mathrm{k}})$,
since $\ddot{x} = \hat{f} + \Delta$

Concurrent learning adaptive law

$$\dot{W}(t) = \underbrace{\boxed{-\Gamma\Phi\big(x(t)\big)e^T(t)PB}}_{\text{Instantaneous update, } \dot{W}_t} \underbrace{\boxed{-\Gamma\sum_{k=1}^{p}\Phi(x_k)\epsilon_k^T(t)}}_{\text{Update on recorded data, } \dot{W}_b}$$

- If measurement of $\ddot{x}$ is not available, then use fixed point smoothing to estimate $\ddot{x}_k$ after a finite delay
- This delay does not affect tracking error as ϵ_k does not depend on $e(t)$

Weight error dynamics

- Key insight comes from analyzing the weight error dynamics:

$$\dot{W}(t) - \dot{W}^* = -\Gamma\Phi\big(x(t)\big)e^T(t)PB \quad -\Gamma\sum_{k=1}^{p}\Phi(x_k)\epsilon_k^T(t)$$

But, $\epsilon_k = \nu_{ad}(x_k) - \Delta(x_k) = W^T(t)\Phi(x_k) - \Delta(x_k)$

However, $\Delta(x_k) = {W^*}^T\Phi(x_k)$, and recall that $\tilde{W} = W - W^*$, so,

$$\epsilon_k = \tilde{W}^T\Phi(x_k)$$

- This yields:

$$\dot{\tilde{W}}(t) = -\Gamma\Phi\big(x(t)\big)e(t)^T P - \Gamma\sum_{k=1}^{p}\Phi(x_j)\left(\tilde{W}^T(t)\Phi(x_k)\right)^T$$

$$\dot{\tilde{W}}(t) = -\Gamma\Phi\big(x(t)\big)e(t)^T P - \Gamma\sum_{k=1}^{p}\Phi(x_k)\Phi^T(x_k)\,\tilde{W}(t)$$

Richness of the recorded data

- Question: How do we determine if the recorded data is rich enough?

Rank-Condition

The recorded data have as many linearly independent elements as the dimension of the basis of the uncertainty. I.e. if $Z = [\Phi(x_1),\ \Phi(x_2), \ldots, \Phi(x_p)]$, then

$$\mathrm{rank}(Z) = \dim\big(\Phi(x)\big)$$

- This condition applies to past data, whereas PE is also concerned with future system signals
- Condition applies only to a subset of the past data
- Algorithms available to verify Rank and SVD of a matrix
- Possible to meet this condition over normal course of operation

Guaranteed exponential stability without PE *(CDC 10)*

Theorem: GES for structured uncertainty

If the Rank-Condition is satisfied, then concurrent learning weight update law guarantees that the zero solution $\left(e(t), \tilde{W}(t)\right) \equiv 0$ is globally exponentially stable.

Proof sketch:

- Closed loop error dynamics

$$\dot{e} = Ae + B(\nu_{Ad} - \Delta)$$

$$\dot{\tilde{W}}(t) = -\Gamma\Phi\big(x(t)\big)e(t)^T P - \Gamma \sum_{j=1}^{p} \Phi(x_j)\Phi^T(x_j)\,\tilde{W}(t)$$

- Lyapunov candidate:

$$V(e, \tilde{W}) = \frac{1}{2}e^T Pe + \frac{1}{2}(\tilde{W}^T\Gamma^{-1}\tilde{W})$$

- Lie derivative of Lyapunov candidate:

$$\dot{V}(e, \tilde{W}) \leq -\frac{1}{2}\lambda_{min}(Q)||e(t)||^2 - \lambda_{min}\left(\sum_{j=1}^{p}\Phi(x_j)\Phi^T(x_j)\right)||\tilde{W}(t)||^2$$

- Due to the rank condition $\sum_{j=1}^{p}\Phi(\bar{x}_j)\Phi^T(\bar{x}_j) > 0$, hence don't need PE states

Methods for recording data points

- Rate of convergence is directly proportional to the minimum singular value of the history stack $Z = [\Phi_1, \ldots, \Phi_p]$
- Want to record data such that:
  - Rank-Condition is met
  - $\sigma_\min Z$is maximized
- A simple method: record points sufficiently different from last point recorded, i.e. record $x(t)$ if

$$\frac{\|x_p - x(t)\|}{\|x_p\|} \geq \bar{\epsilon}$$

- Can use with
  - Static history stack
  - Cyclic History stack
- Does not ensure $\sigma_{\min} Z$ is maximized, or rank-condition satisfied (except for Neuro-adaptive control due to Michelli's theorem)

A singular Value Maximizing Approach

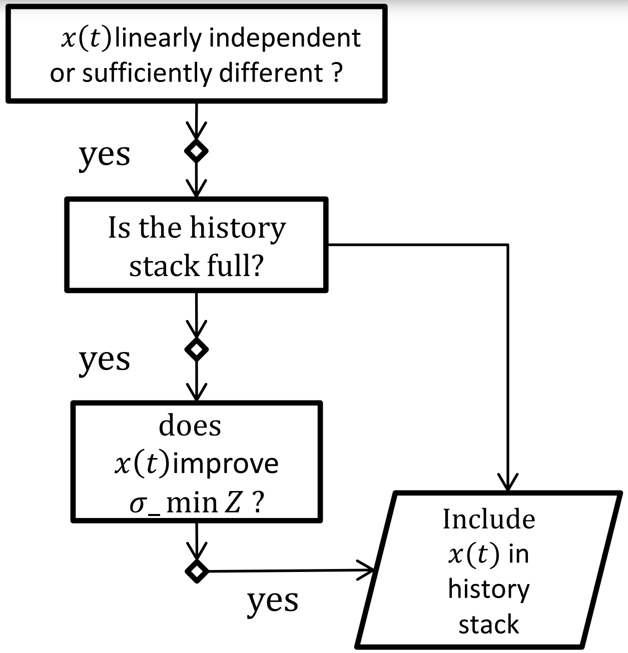
- Size of history stack may be limited
- How to fit in rich information in a limited history-stack?
- A Singular Value Maximizing Approach
  - Record data points by replacing older data points only if it results in an increase in $\sigma_{min}\, Z$
  - Guarantees rank condition is met when $\sigma_{min}\, Z$ >0

Key result: Guaranteed exponential stability without PE (CDC 2010)

Theorem: GES for structured uncertainty without PE (CDC 2010)

Let the data be recorded in a history stack matrix, $Z = [\Phi(x_1),\ \Phi(x_2), \ldots, \Phi(x_p)]$, if $rank(Z) = m$, the Concurrent learning weight update law guarantees that $\left(e(t), \tilde{W}(t)\right) \rightarrow 0$ exponentially fast. Furthermore, the rate of convergence is proportional to the minimum singular value of Z ($\sigma_{min} Z$).

- This theorem answers :
  - How to select *online* the best data to store in a limited history-stack?
- Need at-least m linearly independent points



Lyapunov based proof of integration *(GNC 11)*

Theorem: GES for structured uncertainty

If the singular value maximizing data recording algorithm is used for selecting data online for concurrent learning, then, the zero solution $\left(e(t), \tilde{W}(t)\right) \equiv 0$ is globally exponentially stable if the reference signal $r(t)$ is exciting over a finite interval.

Proof sketch:

- Lie derivative of Lyapunov candidate:

$$\dot{V}\left(e, \tilde{W}\right) \le -\frac{1}{2}\lambda_{min}(Q)||e(t)||^2 - \lambda_{min}\left(\sum_{j=1}^{p}\Phi\left(x_j\right)\Phi^T\left(x_j\right)\right)||\tilde{W}(t)||^2$$

- Before the rank condition is satsified, $\dot{V}\left(e, \tilde{W}\right) \le 0$, so boundedness guaranteed
- Suppose rank condition satisfied at T, $\Omega(t) = \sum_{j=1}^{p}\Phi\left(\bar{x}_j\right)\Phi^T\left(\bar{x}_j\right) > 0$, and

$$\dot{V}\left(e, \tilde{W}\right) \le \frac{\min\left(\lambda_{min}(Q), \lambda_{min}\left(\Omega(T)\right)\right)}{\max(\lambda_{max}(P), \lambda_{max}(\Gamma_W^{-1}))}V\left(e, \tilde{W}\right)$$

So, $V\left(e, \tilde{W}\right) = \frac{1}{2}e^T P e + \frac{1}{2}\left(\tilde{W}^T \Gamma^{-1} \tilde{W}\right)$ is common Lyapunov candidate, and exponential closed loop stability is guaranteed as the algorithm guarantees monotonically increasing $\lambda_{min}\left(\Omega(t)\right)$

Extension to Unstructured uncertainty *(IJC submitted)*

- Recall that for the case of unstructured uncertainty, $\Delta(x,u) = {W^*}^T\sigma(x,u) + \tilde{\epsilon}$
- The following concurrent learning adaptive law guarantees $e(t)$ bounded and $W(t)$ approaches and remains bounded in a compact neighborhood of W^*

$$\dot{W}(t) = \begin{cases} -\Gamma\sigma\big(x(t)\big)e^T(t)PB \;\; -\Gamma\sum_{k=1}^{p}\sigma(x_k)\epsilon_k^T(t) - \kappa W(t) & if\ rank\big(Z(t)\big) < m \\ -\Gamma\sigma\big(x(t)\big)e^T(t)PB \;\; -\Gamma\sum_{k=1}^{p}\sigma(x_k)\epsilon_k^T(t) & if\ rank\big(Z(t)\big) = m \end{cases}$$

- The term $\kappa W(t)$ is $\sigma - mod$ required to guarantee boundedness before the rank condition is met
- It can be shown that the bounds with concurrent learning can be smaller than those with $\sigma - mod$

Key Contributions of CL-MRAC

- THM: Globally Exponentially Stable adaptive controllers without PE (CDC 2010) (need only finite excitation)
- A singular value maximizing algorithm for recording data online (GNC 2011)
- THM: Exponentially square bounded linearly parameterized neuro-adaptive controllers (Automatica 2012, submitted)
- THM: Guaranteed robustness to parameter switching of CL-MRAC (Automatica 2012, submitted)
- THM: Robustness to errors in estimating $\dot{x}$ (Automatica 2012 submitted, IJACSP 2012 accepted)
- THM: Globally exponentially stable CL-MRAC for linear uncertain dynamical systems without requiring PE (IJACSP 2012 accepted)
- THM: Ultimately bounded nonlinearly parameterized neuro-adaptive controllers (JGCD 2010)

What does this all mean to adaptive control?

- Use of recorded information concurrently with current data enable simultaneous learning of the system uncertainty and stabilization of the system, without requiring PE
- Lifted an assumption held in adaptive control since 1960s
- First and the only adaptive control method to guarantee exponential stability without PE
- Rate of convergence is exponential and quantifiable: good transient response
- Weight convergence guarantees long term learning
- The online learned model has many uses
- New analysis methods introduced to incorporate discrete switching in adaptive laws

Wing Rock dynamics simulation

- Highly swept-back aircraft are susceptible to Wing Rock: lightly damped oscillations

ϕ	Roll angle
p	Roll rate
δ_a	Aileron input

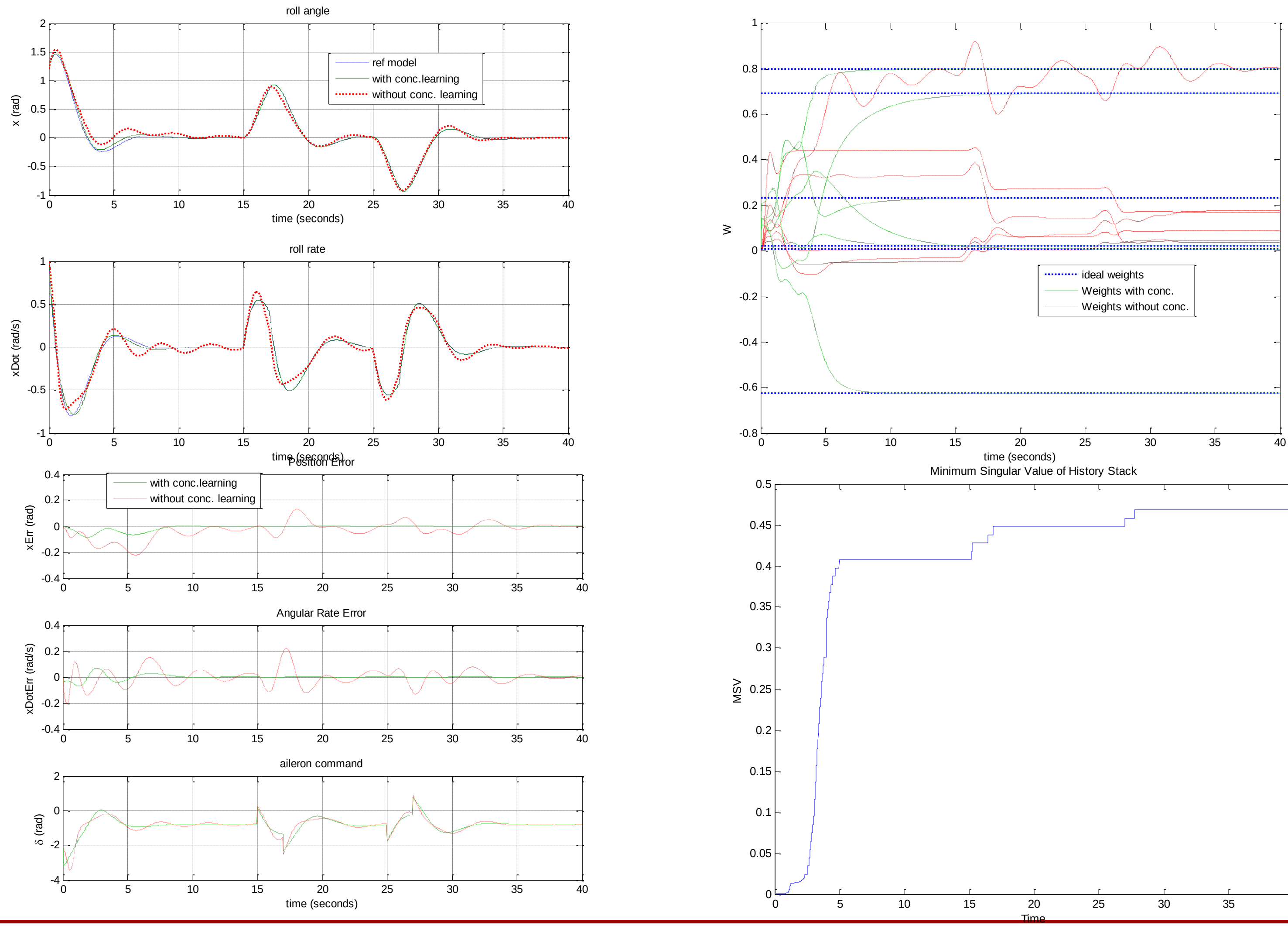
A model for Wing Rock dynamics (Monahemi 96)

$$\dot{\phi} = p$$
$$\dot{p} = \delta_a + \Delta(\phi, p)$$

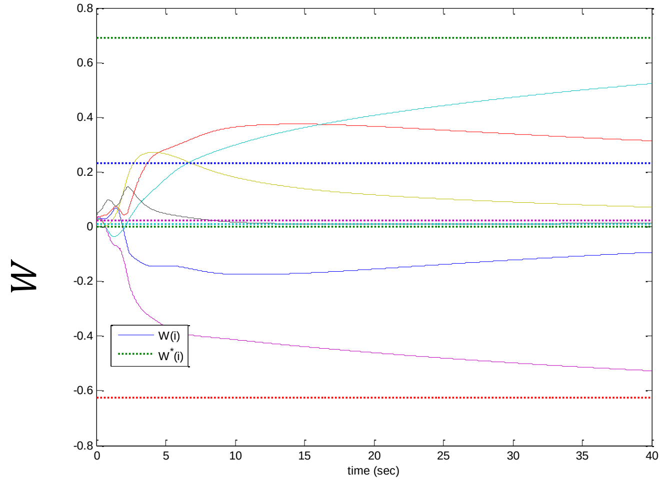
- Task: track roll commands in presence of wing rock dynamics:

$$\Delta(\phi, p) = w_1^* + 0.23\phi + 0.69p - 0.62|\phi| + 0.01|p|p + 0.02p^3$$

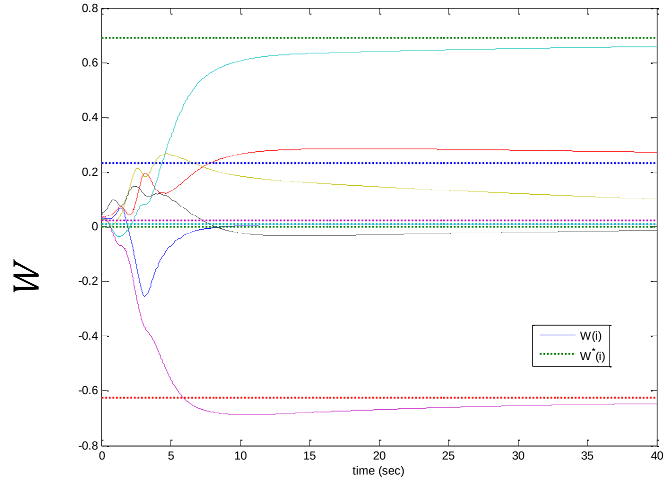
Wing Rock dynamics with concurrent learning



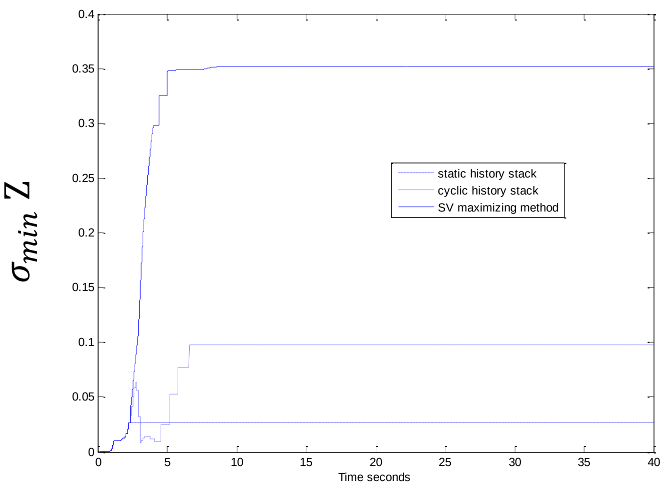
Online data point recording algorithm performance

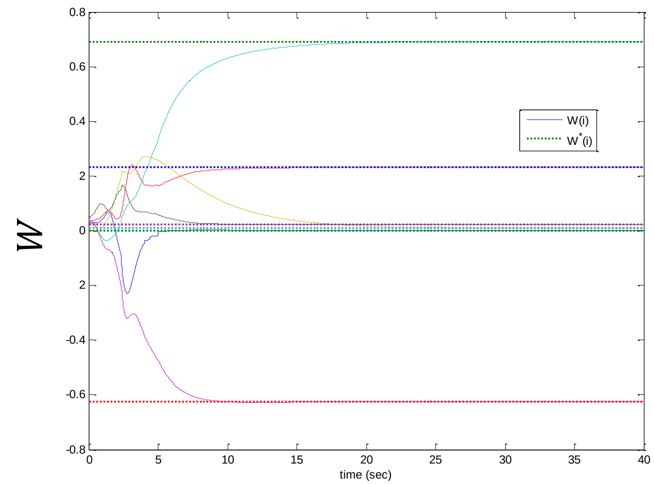


Static history stack



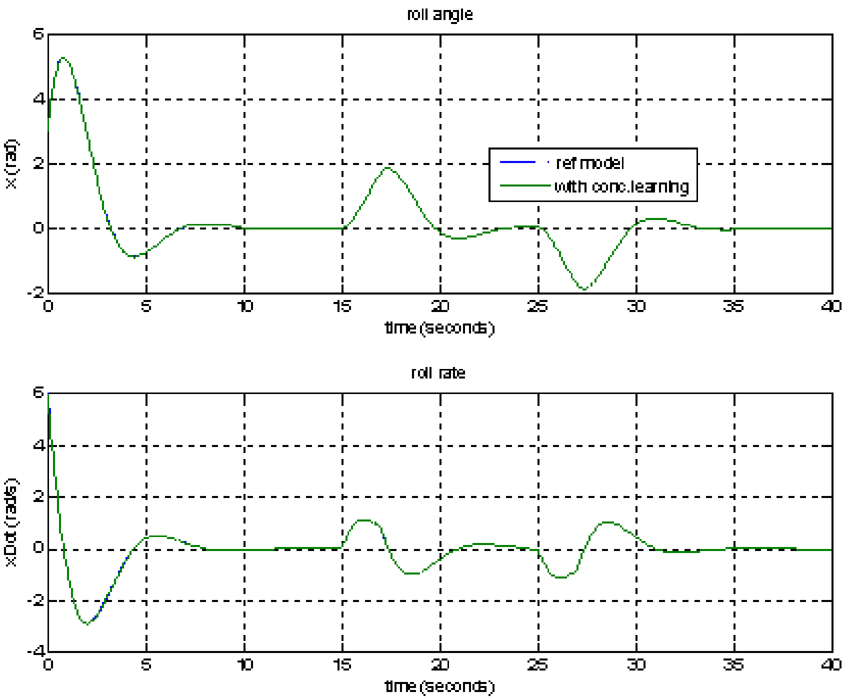
cyclic history stack

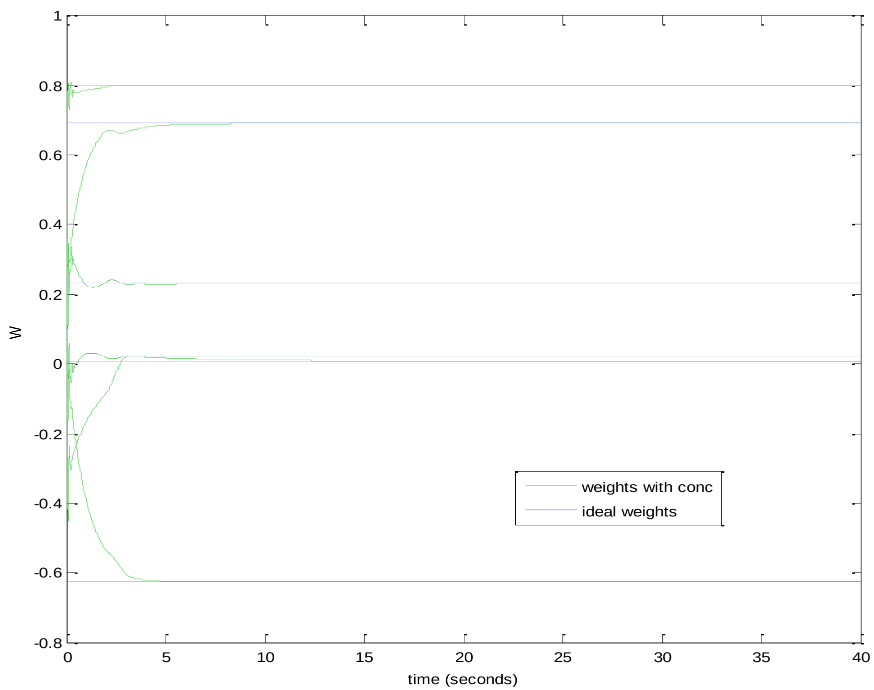




Singular Value Maximizing approach

Performance with a Pre-Recorded History Stack





- Ideal scenario: e.g. history-stack selected from experimental data
- Notice ref-model and plant states are almost identical
- Fast convergence of weights

Neuroadaptive control with SHL NN (JGCD 10)

- Single Hidden Layer NN as adaptive element: $\nu_{ad} = W^T\sigma(V^T\bar{x})$
- Concurrent learning adaptive law guarantees UUB:

$$\dot{W} = -\Gamma_{\mathrm{W}}(\sigma(V^T\bar{x}) - \sigma'(V^T\bar{x})V^T\bar{x})\mathrm{e^TPB} - \kappa||e||W - W_C\sum_{i=1}^{p}\Gamma_{\mathrm{W}}(\sigma(V^T\bar{x}_i) - \sigma'(V^T\bar{x}_i)V^T\bar{x}_i))$$

$$\dot{V} = -\Gamma_V\bar{x}e^TPBW^T\sigma'(V^T\bar{x}) - \kappa||e||V - V\_c\sum_{i=1}^{p}\Gamma_V\bar{x}_i r_{b_i}^T W\hat{}T\sigma'(V^T\bar{x})$$

- Where

$$r_{b_i} = W^T\sigma(V^T\bar{x}_i) - \Delta(x_i, \dot{x}_i, u)$$

- And the orthogonal projection operators prioritize instantaneous weight updates

$$W_C = I - \frac{(\sigma(V^T\bar{x} - \sigma'(V^T\bar{x})V^T\bar{x})(\sigma(V^T\bar{x} - \sigma'(V^T\bar{x})V^T\bar{x})^T}{(\sigma(V^T\bar{x} - \sigma'(V^T\bar{x})V^T\bar{x})^T(\sigma(V^T\bar{x} - \sigma'(V^T\bar{x})V^T\bar{x})}$$

$$V_c = I - \frac{\Gamma_V\bar{x}\bar{x}^T\Gamma_V}{\bar{x}^T\Gamma_V\Gamma_V\bar{x}}$$

AMI-MRAC with Concurrent Learning for unstructured uncertainty

Theorem: UUB with unstructured uncertainty

If Rank-Condition is satisfied, then concurrent learning weight update law guarantees that $e \equiv 0$, and $\tilde{W} \equiv 0$ is UUB for bounded initial conditions

- Nonlinear mechanical system dynamics:

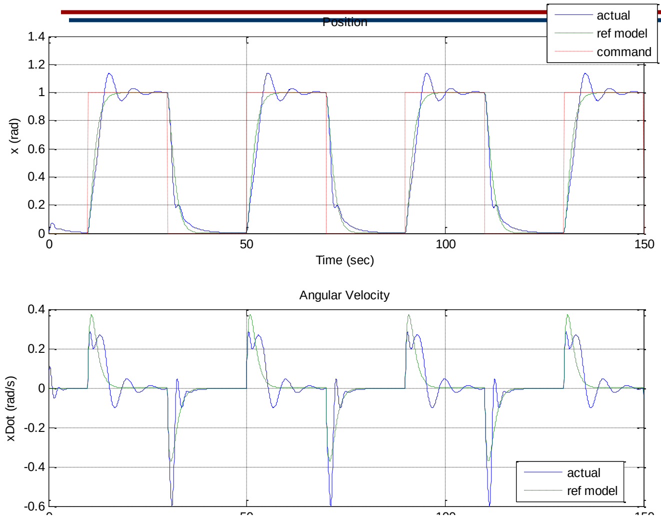
$$\dot{\theta} = u + \sin(\theta) - |\dot{\theta}|\theta + 0.5e^{\theta\dot{\theta}}$$

- Approximate inversion mode:

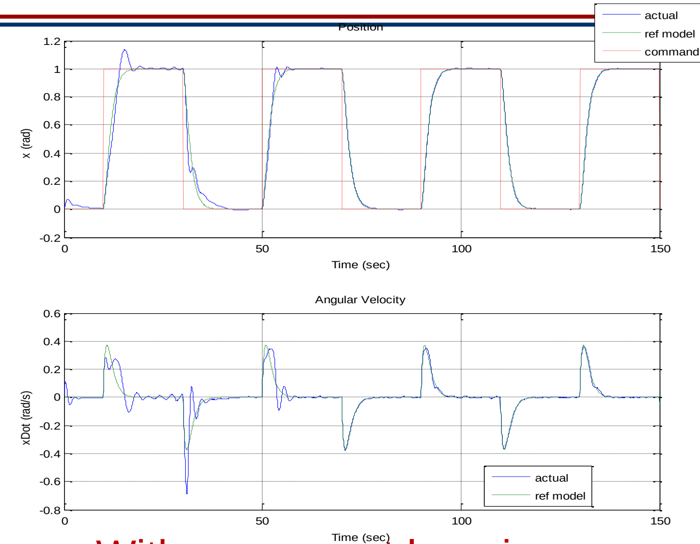
$$u = \nu$$

- Second order linear reference model used
- Single Hidden Layer NN as adaptive element: $\nu_{ad} = W^T \sigma(V^T \bar{x})$

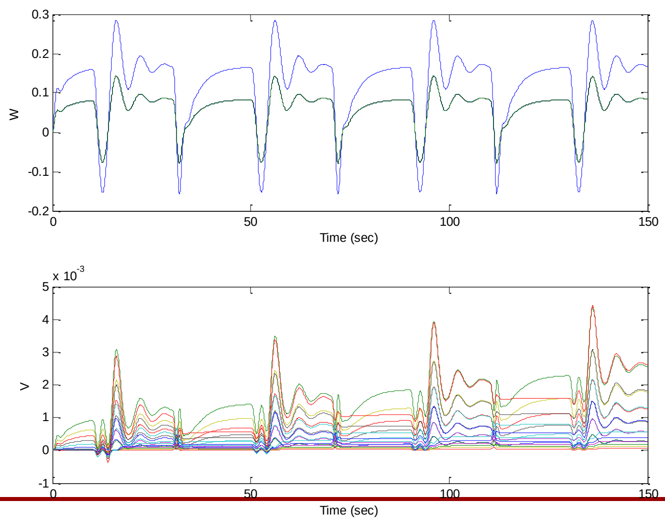
Command Tracking Performance

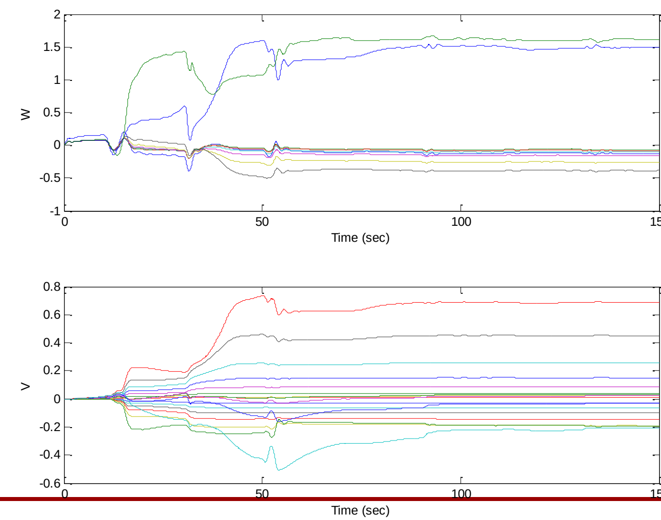


Without concurrent learning

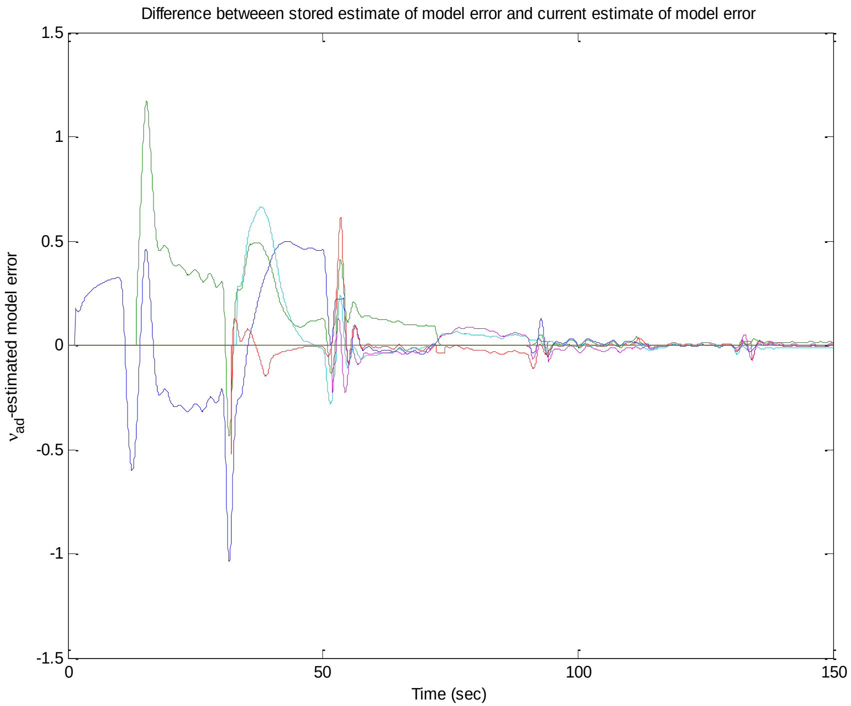


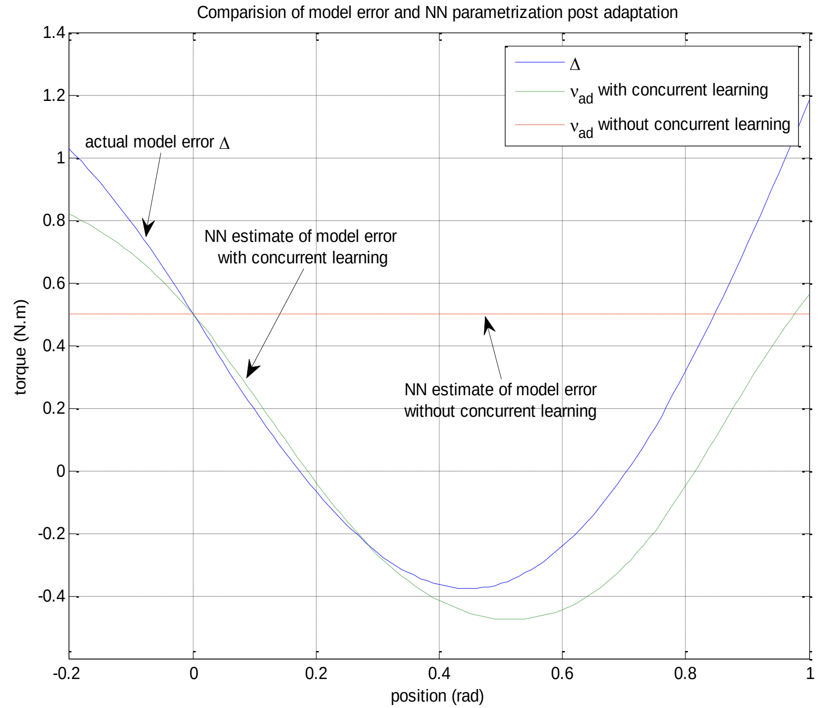
With concurrent learning





Semi-Global Model approximation with concurrent learning





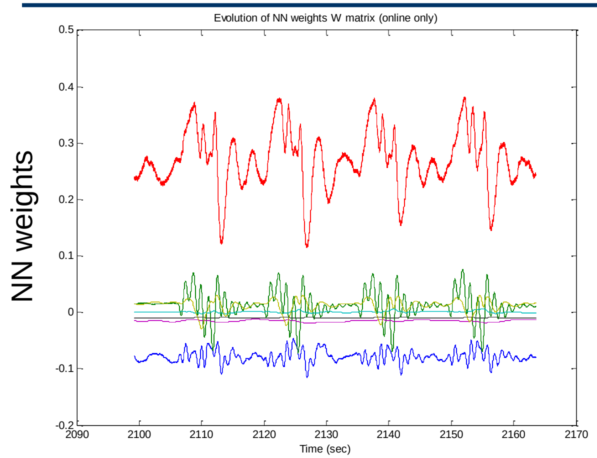
Function approximation error reduces for all stored point concurrently

Flight test Results: GTMax *(JGCD 10)*

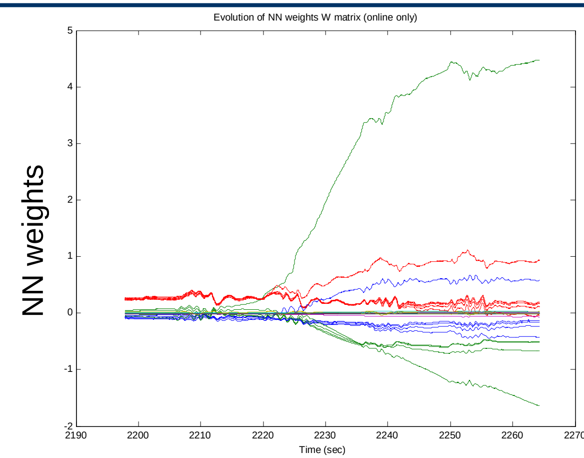
See "Theory and flight Test validation of a concurrent learning adaptive controller", Chowdhary, Johnson, JGCD March-April 2011

- Yamaha RMAX,
  - 66kg
  - 3m Rotor
- More than 450 research test flights since March 2002
- Multiple concurrent learning approaches tested

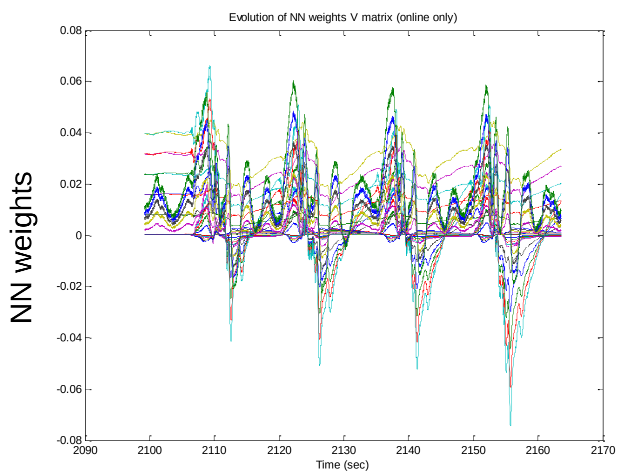
Evolution of NN weights with Only Instantaneous learning, W matrix



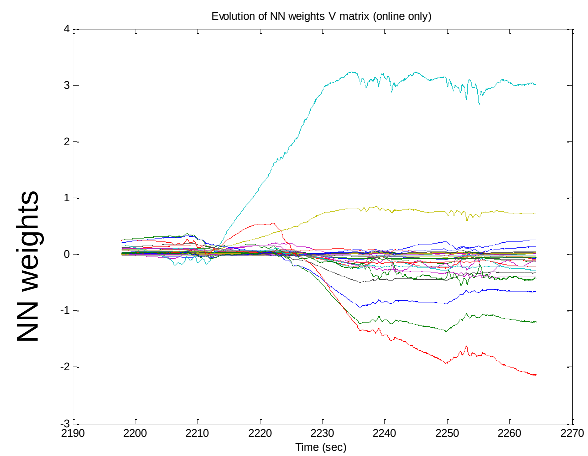
Evolution of W weights without concurrent learning



Evolution of W weights with concurrent learning



Evolution of V weights without concurrent learning



Evolution of V weights with concurrent learning

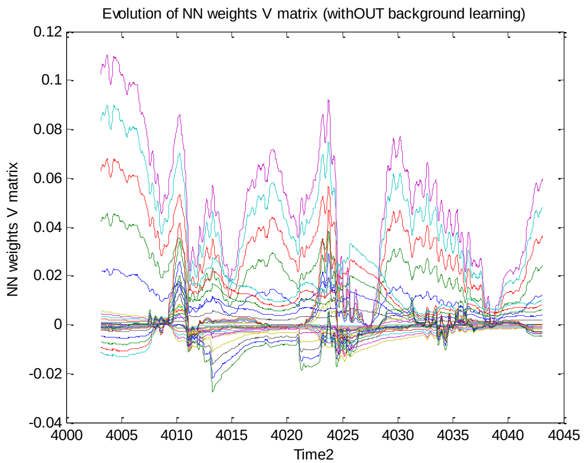
Flight Test Results: Repeated Steps

Recorded Inner and Outerloop tracking errors

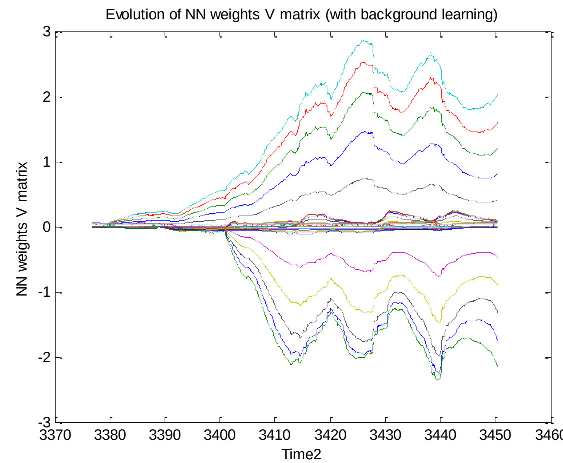
- Improvement in performance as the rotorcraft performs repeated steps maneuvers

F.T. Comparison of Evolution of NN weights

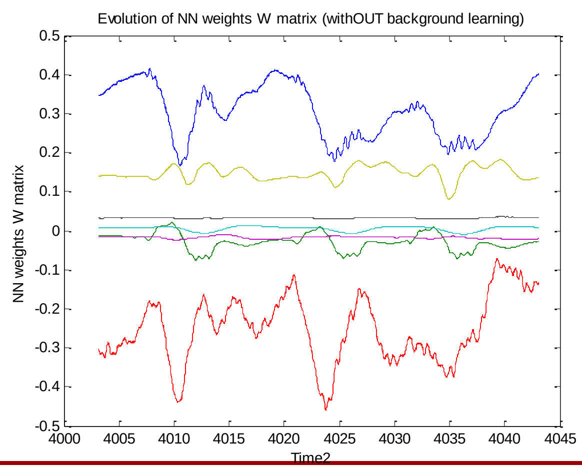
Evolution NN weights V matrix without concurrent learning



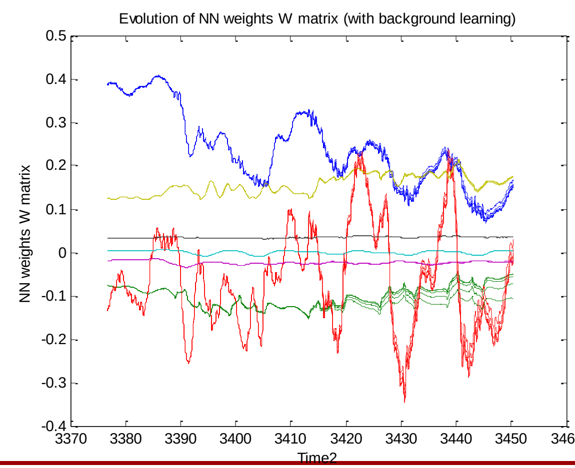
Evolution NN weights V matrix with concurrent learning



Evolution of NN weights W matrix without concurrent learning



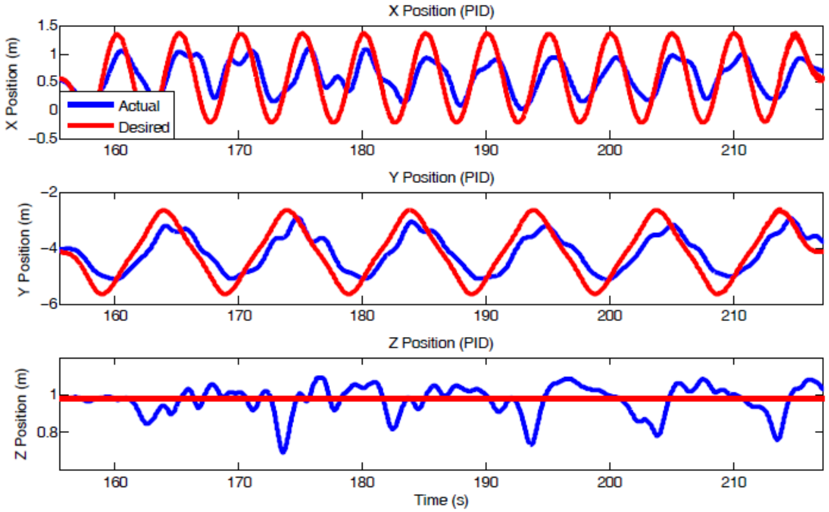
Evolution NN weights W matrix with concurrent learning

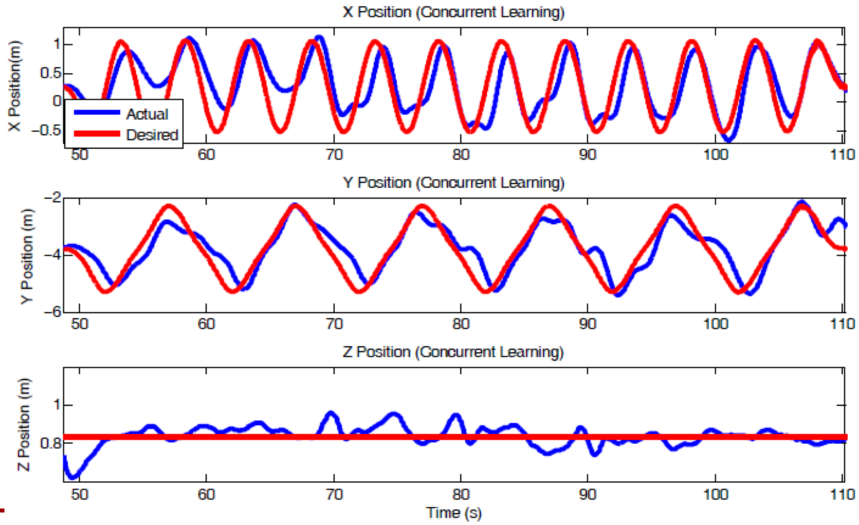


Flight test: quads at MIT (untuned) GNC 12

- PID controllers often need a lot of tuning
- Question: How to minimize time/effort required in getting a UAV to fly autonomously
- Outerloop performance is often poor if PID not tuned well
- CL adaptive controller improves performance over the long-term

- ACL quads: Small quad uses gains from the bigger quad
- Throttle mapping is incorrect



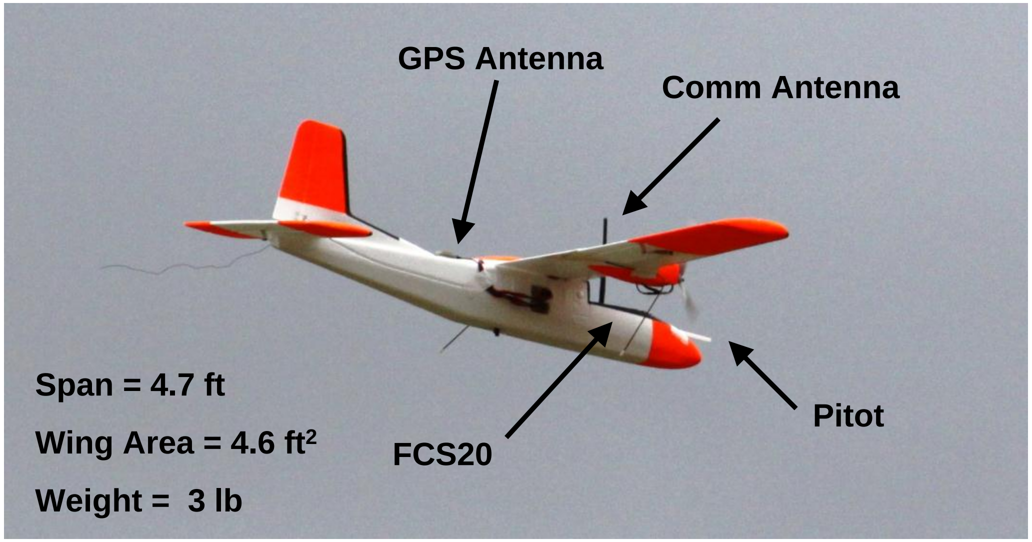


MIT flight test video: Transferring controllers

Twin-Engine Electric Airplane: TwinStar

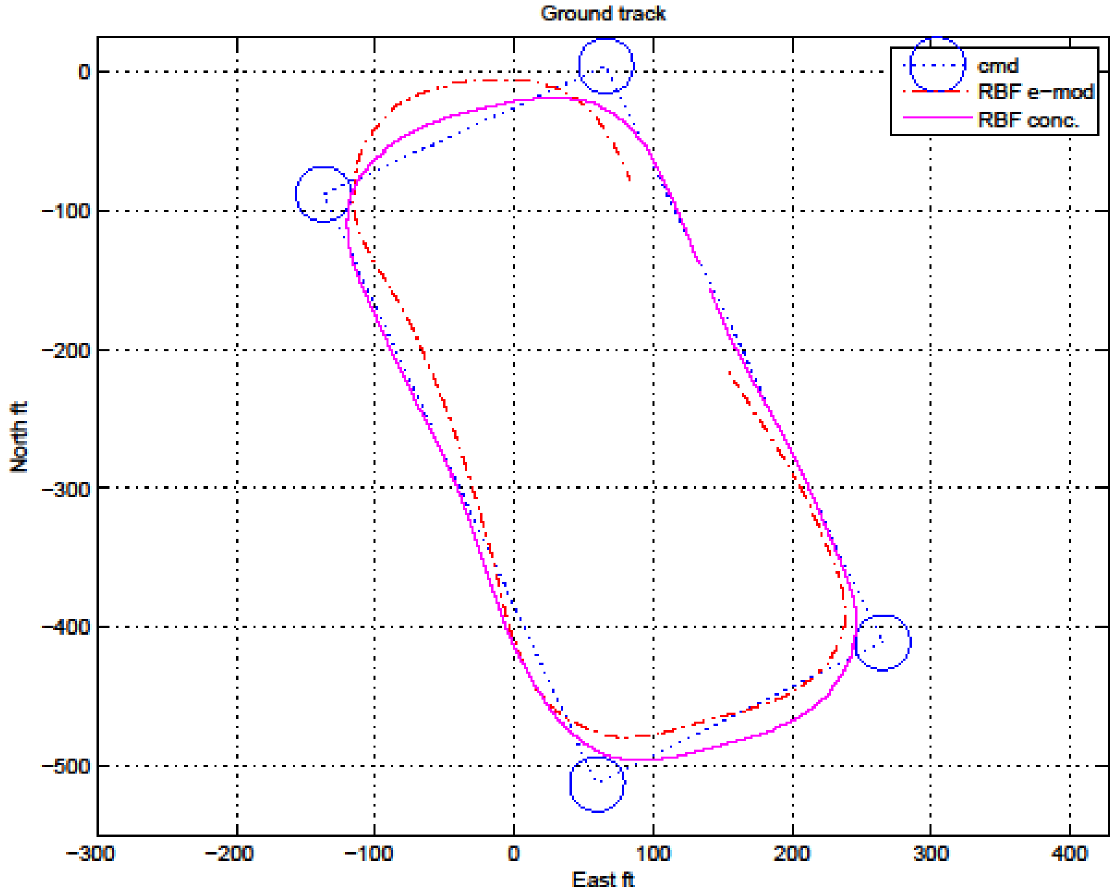
- Relatively inexpensive almost-ready-to-fly COTS airplane
- Complements existing aircraft
  - Multi-engine
  - Simulate damage

About $100 for Airplane
(without engines/batteries/avionics)



http://uav.ae.gatech.edu/videos/ta1_090415c1_tightTurns.wmv

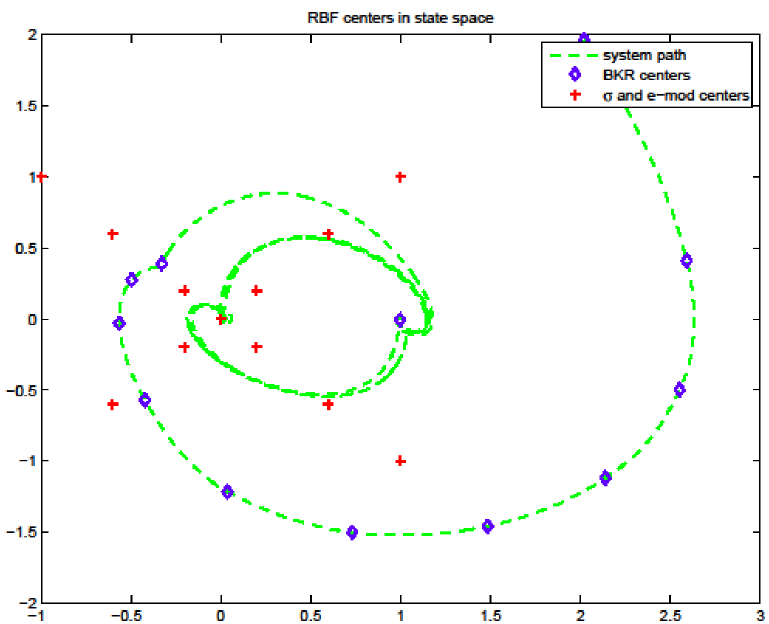
Flight Test Results: Ground Track



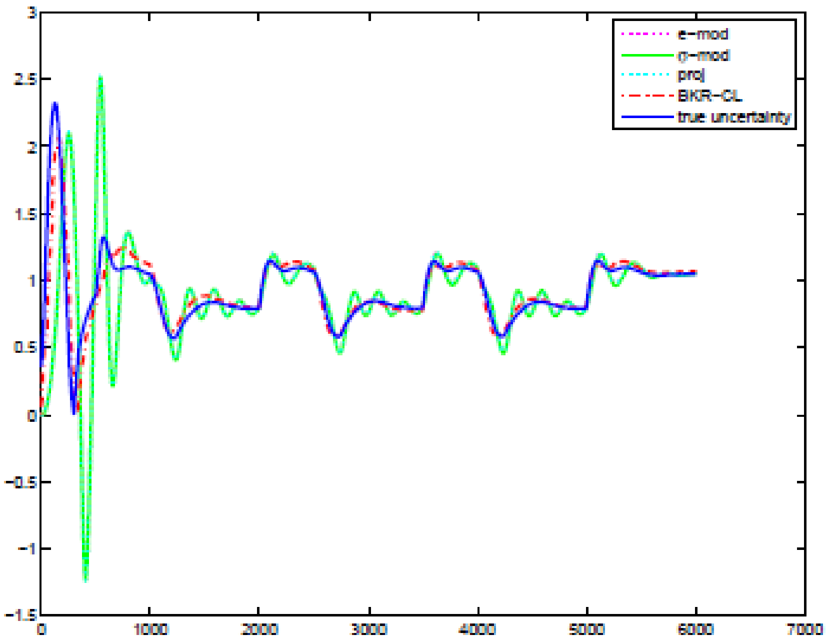
- Concurrent learning controller has better cross-tracking performance than e-mod enabled baseline learning law
- Relates to better steady-state trim estimation: improved parameter convergence

BKR-CL: a non-parametric approach on a budget (CDC 11)

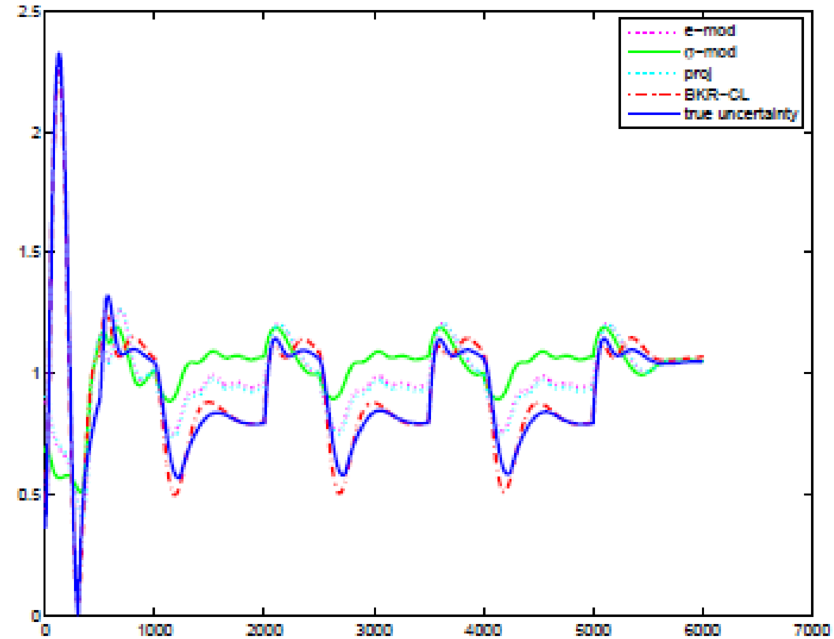
- Leverage insights from Reproducing Kernel Hilbert Spaces to *"optimally"* pick and place RBF centers online *"on a budget"*
- "add" or "remove and add" $x(t)$ as a center based on a linear independence test in the feature space, (based on work by Nguyen-Tong et al.)



Long-Term learning *(TNN 11)*



Online estimate of uncertainty



Estimate of uncertainty with weights frozen post-simulation

- Important insight: Good online tracking of uncertainty doesn't mean you are actually *learning* the uncertainty
- BKR-CL tracks the uncertainty and learns it over the long term
- Reference: *Kingravi, Chowdhary, Vela, Johnson CDC 2011 (Dec), and TNN 2011 (submitted)*

Summary

- A Concurrent learning adaptive control law with the SVM algorithm **guarantees global exponential stability** of the closed loop adaptive system **without requiring persistency of excitation**
- A singular value maximizing algorithm guarantees the **richest possible history stack without exceeding memory limitation** for a given time history
- No free lunch:
  - need $\ddot{x}_j$ after a finite delay: can use optimal fixed point smoothing: more computational effort
  - Need to know what mode the system is in to ensure relevant recorded data is used (ongoing work ACC 13)

Still to come

Metrics based adaptive fault tolerant control GNC 09, 10; ACC 10; Infotech 10,11

- What happens when the ideal parameters vary? (beyond ultimate boundedness?): Need ways to monitor history stack applicability
- The ever elusive self-tuning-regulator problem
- How can we extend stability guarantees globally: Nonparameteric adaptive elements

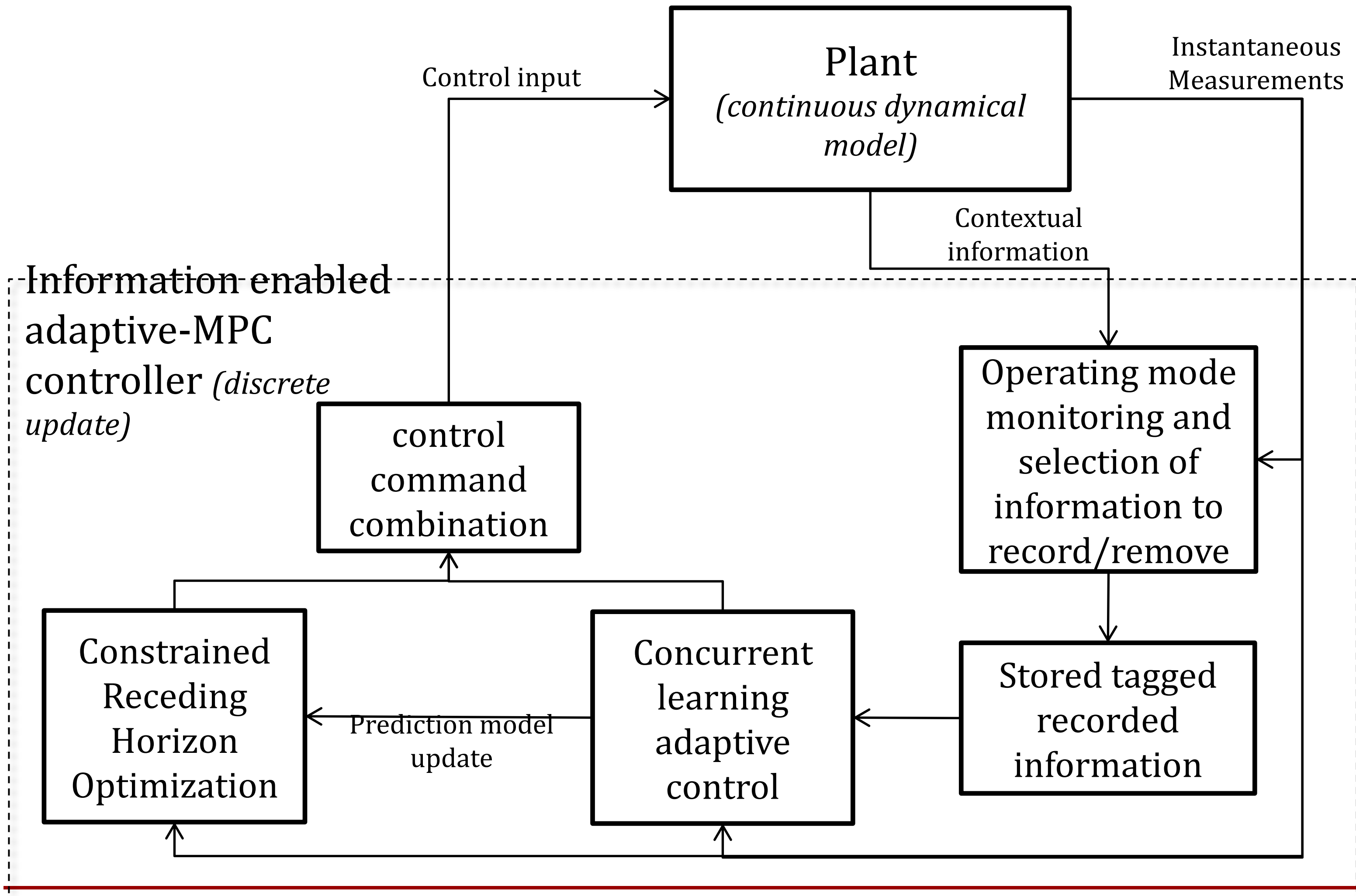
Example: Wingrock dynamics

- Main file: wingrock_concurrent

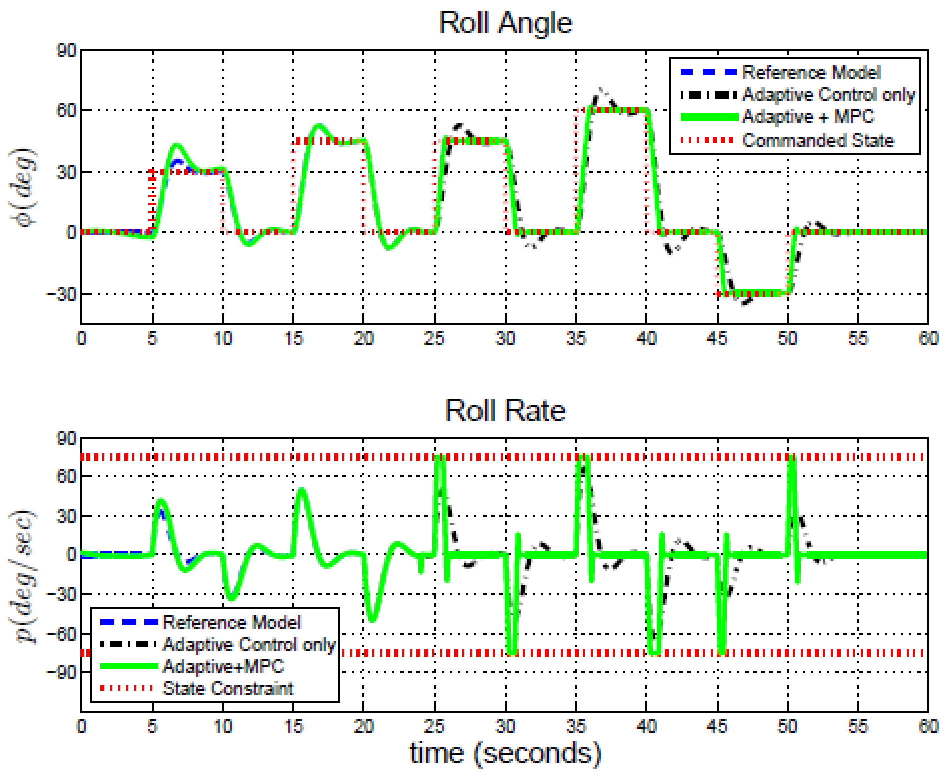
Example: Inverted Pendulum

- SHL NN: Main file: NN_InvertedPendulum_SHL
- RBF NN: invpendu_conc_lip_dmrac

Extension to MPC



Exemplary simulations



Combined MRAC-MPC approach guarantees stability and optimality in presence of state and actuator constraints