Neuroadaptive Control

Recent Advances in Model Reference Adaptive Control: Theory and Applications

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Introduction

- Neural Networks are universal function approximators
- That means, given any continuous function on a compact domain, we have a tool to form a good approximation without having to know more
- Single Hidden Layer, and Radial Basis Function NN have been widely used as adaptive elements

Model Reference Adaptive Control

Plant

$$\dot{x}(t) = \mathrm{A}x(t) + \mathrm{B}\left(u(t) + \Delta\big(x(t)\big)\right),$$

- $x \in \Re^n$: system states, $u \in \Re$: control inputs, $\Delta\big(x(t)\big)$: uncertainty
- $\mathrm{A} \in \Re^{n\times n}$, $\mathrm{B} = [0, \dots, 1]^T$, easily generalizes to multi-input cases

Problem Statement: Model Reference Adaptive Control

Design a control law $u(t)$ such that the plant tracks the reference model

$$\dot{x}_{rm}(t) = \mathrm{A_{rm}}x_{rm}(t) + \mathrm{B_{rm}}\mathrm{r}(\mathrm{t}),$$

- $x_{rm}(t) \in \Re^n$: system states, $r(t) \in \Re$: Exogenous inputs
- Assumed bounded input bounded output stable
- Chosen to characterize the desired response

Control Action

- The tracking control law has the following three parts:

- Linear feedback

$$u_{pd} = K(x_{rm} - x)$$

- Linear feedforward:

$$u_{rm} = K_r[x_{rm}, r]$$

- Adaptive part

$$u_{ad}$$

Tracking control law:

$$u = u_{pd} + u_{rm} - u_{ad}$$

- Tracking error dynamics, $(e = x_{rm} - x)$

$$\dot{e} = (A - BK)e + B(u_{ad} - \Delta)$$

Approximate Model Inversion (AMI) Based MRAC

Plant

$$\ddot{x} = f(x, \dot{x}, u),$$

- $x \in \Re^n$: system states
- $u \in \Re^m$: control inputs (multiple input), controllable
- f satisfies conditions for a unique solution, unknown

Problem Statement: Model Reference Adaptive Control

Design a control law $u(t)$ such that the plant tracks the reference model

$$\ddot{x}_{rm} = f_{rm}(x_{rm}, \dot{x}_{rm}, r)$$

- $x_{rm} \in \Re^n$: system states
- $r \in \Re^l$: Exogenous inputs
- f_{rm} bounded input bounded output stable and continuously differentiable

Approach

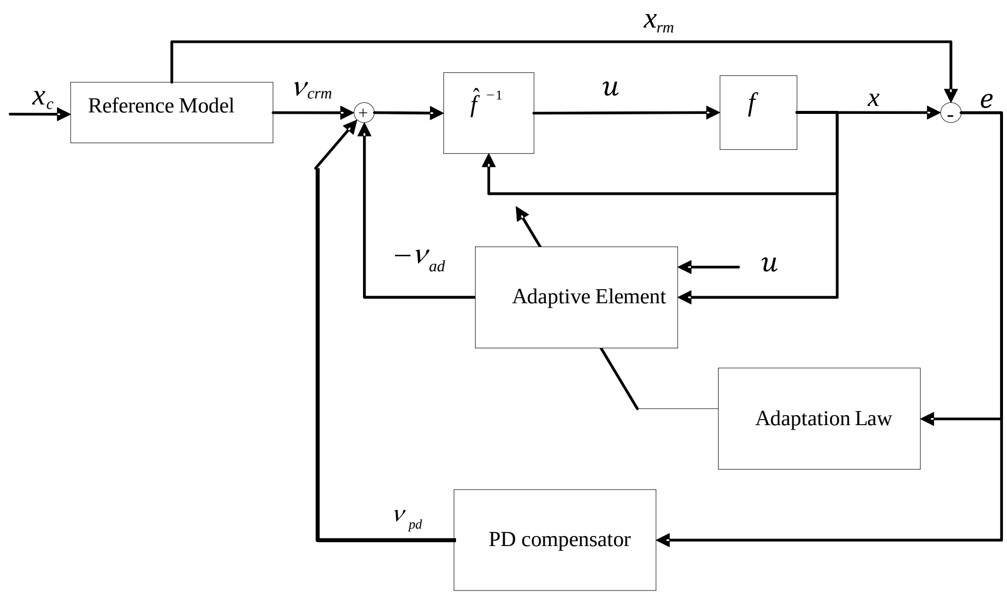
- Design a pseudo control $\nu \in \Re^n$ such that $x \rightarrow x_{rm}$
- If plant model were known and invertible, then we can simply set $u = f^{-1}(x, \nu)$
- However, this is usually not the case, so choose an inversion model $\hat{f}$ and let $u = \hat{f}^{-1}(x, \nu)$

Assumption

The approximate inversion model is continuous, and invertible w.r.t. u. I.e. the operator $\hat{f}^{-1}: \Re^{2n} \rightarrow \Re^m$ exists and assigns for every unique $(x, \nu) \in \Re^{2n}$ a unique $u \in \Re^m$

- Combined control action: $\nu = -Ke + \dot{x}_{rm} - \nu_{ad}$

Tracking Error Dynamics



$$\nu = \nu_{pd} + \nu_{rm} - \nu_{ad}$$

- Tracking error dynamics:

$$\dot{e} = Ae + B(\nu_{ad} - \Delta)$$

Choice of adaptive element

Structured Uncertainty

Given that there exists a constant matrix W^* and known basis functions $\Phi(x,u)$ such that

$$\Delta(x,u) = {W^*}^T \Phi(x,u)$$

Then, choose adaptive element:

$$\nu_{ad} = W^T \Phi(x,u)$$

Unstructured Uncertainty

Given $\Delta(x,u)$ is continuous and defined over a compact set, then :

$$\Delta(x,u) = g(x,u,W) + \tilde{\epsilon}$$

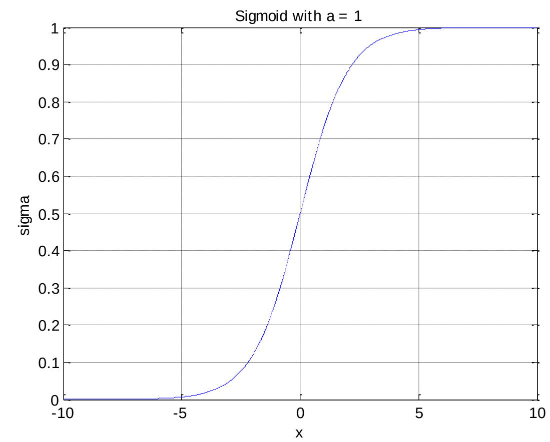
where g is some universal function approximator parameterized by W

Single Hidden Layer Neural Network

- Single Hidden Layer (SHL) feedforward NN are universal approximators

$$\nu_{ad} = W^T(t)\sigma(V^T\bar{x})$$

- $\bar{x} = [b_a \;\; x]$, where b_a is the input bias (constant)
- $\sigma = \begin{bmatrix} b_w \\ \frac{1}{1+e^{a_1 z_1}} \\ \vdots \\ \frac{1}{1+e^{a_{n_2} z_{n_2}}} \end{bmatrix} \in \Re^{n_2+1}$, sigmoidal activation function



$$V = \begin{bmatrix} \theta_{1,1} & \cdots & \theta_{1,n2} \\ v_{1,1} & \cdots & \theta_{1,n2} \\ \vdots & \ddots & \vdots \\ v_{n1,1} & \cdots & \theta_{n1,n2} \end{bmatrix} \in \Re^{(n_1+1)\times n_2} \qquad W = \begin{bmatrix} \theta_{1,1} & \cdots & \theta_{1,n2} \\ w_{1,1} & \cdots & w_{1,n2} \\ \vdots & \ddots & \vdots \\ w_{n2,1} & \cdots & w_{n2,n3} \end{bmatrix} \in \Re^{(n_{21}+1)\times n_3}$$

Universal approximation property

- Given sufficient number of hidden layer neurons (n_2), and a corresponding set of ideal weights (W^*, V^*) there exists $\bar{\epsilon}$ such that

$$\bar{\epsilon} = \sup_{\bar{x} \in D} ||{W^*}^T \sigma\left({V^*}^T \bar{x}\right) - \Delta(z)||$$

- Ideally we would like to update W, V such that they approach a compact neighborhood of W^*, V^*
- Direct adaptive control is happy with just updating W, V such that tracking error e stays bounded

NN training law

- The "Classical" Error backpropagation based learning Law:

$$\dot{W} = -(\sigma - \sigma' V^T \bar{x}) r^T \Gamma_w$$

$$\dot{V} = -\Gamma_V \bar{x} r W^T \sigma'$$

where $\quad r^T = e^T PB$

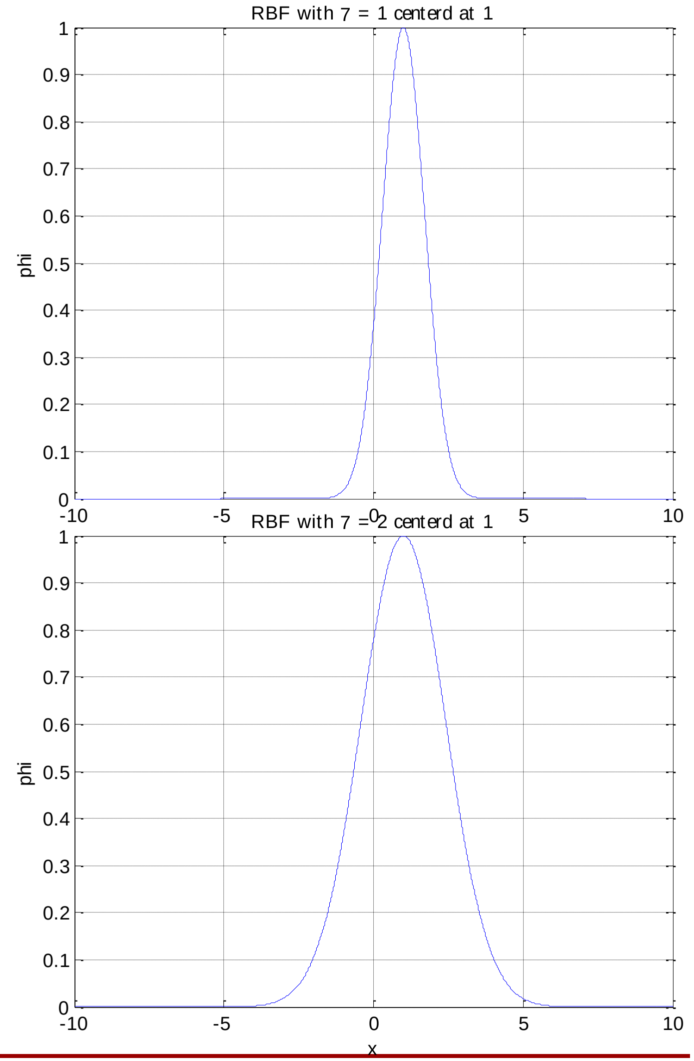
$$\sigma' = diag\left(\sigma(V^T\bar{x})\left(I - diag\left(\sigma(V^T\bar{x})\right)\right)\right)$$

- This adaptive law guarantees uniform ultimate boundedness of $(e, \widetilde{W})$ (Lewis et al.)

Radial Basis Function NN

- Radial Basis Functions (RBFs) are Gaussian Kernels
- Select n_2 centers x_{c_j}
- Select a width parameter
- Add a bias term, then $\Phi(\bar{x}) \in \Re^{n_2+1}$

$$\Phi(\bar{x}) = \begin{bmatrix} b_w \\ e^{\frac{-||x_{c_1} - \bar{x}||}{\mu}} \\ \vdots \\ e^{\frac{-||x_{c_{n_2}} - \bar{x}||}{\mu}} \end{bmatrix}$$



Universal Approximation with RBFs

- Given fixed number of RBFs with a fixed width μ there exists an ideal set of weights W^* such that

$$\bar{\epsilon} = \sup_{\bar{x} \in \mathrm{D}} ||W^{*^T}\Phi(\bar{x}) - \Delta(z)||$$

- $\bar{\epsilon}$ can be made arbitrarily small given sufficient number of RBFs
- RBF NN $\nu_{ad} = W^T(t)\Phi(\bar{x})$
- Ideally, we would like $W(t) \rightarrow \mathrm{W}^*$
- Traditionally, adaptive control is happy with keeping e bounded

RBF adaptive law

- The following adaptive law guarantees that the tracking error stays bounded in a compact neighborhood

$$\dot{W} = -\Gamma\Phi(\bar{x})e^T PB$$

- Can use $\sigma -$ mod, or $e -$ mod to guarantee boundedness in presence of noise:

$$\dot{W} = -\Gamma\Phi(\bar{x})e^T PB - \kappa W$$

Which NN to choose?

- Benefits of SHL NN:
  - Don’t need to select centers
  - More general parameterization
- Issues:
  - Nonlinearly parameterized, which makes analysis hard
  - Rank-1 updates do not leverage the power of this NN
- Benefits of RBF NN
  - Linearly parameterized, relatively straight forward to analyze
  - Lends to the theory of Gaussian Kernels and Reproducing Kernel Hilbert Spaces
- Issues:
  - How to select the centers (see Concurrent Learning Talk)
  - How many RBFs to use?
  - Can run into issues with bias estimation if the centers are far away

Example: Inverted pendulum (SHL NN)

- A nonlinear system which resembles an inverted pendulum

$$\ddot{x} = \delta + \sin(x) - |\dot{x}|\dot{x} + 0.5e^{x\dot{x}}$$

- Approximate inversion model: $\nu = \delta$
- Model error $\Delta = \begin{bmatrix} 1 & -1 & 0.5 \end{bmatrix} \begin{bmatrix} \sin(x) \\ |\dot{x}|\dot{x} \\ e^{x\dot{x}} \end{bmatrix}$
- Second order reference model

Example Inverted Pendulum

Software:NN_InvertedPendulum_simple

Try the following

- Control without NN: set
  - gammaW=0; (line 61)
  - gammaV=0; (line 62)

Wing Rock dynamics(RBF NN)

- Highly swept-back aircraft are susceptible to Wing Rock: lightly damped oscillations
- ϕ: roll angle, p roll rate, δ_a aileron

A model for Wing Rock dynamics (Monahemi 96)

$$\dot{\phi} = p$$
$$p = \delta_a + \Delta(\phi, p)$$

- Inversion model: $\nu = \delta$
- Task: track roll commands in presence of wing rock dynamics:

$$\Delta(\phi, p) = 0.8 + 0.23\phi + 0.69p - 0.62|\phi| + 0.01|p|p + 0.02p^3$$

- Second order reference model used

Example wingrock with RBF

Software: wingrock_rbf_simple

Things to try

- Turn off adaptive control by setting gammaW=0
- Add sigma mod: by setting kappa=0.1
- Add e-mod by setting zeta=0.1
- What effect does the bias term ($W^*(1)$) have (this simulates trim)

Issues

- Boundedness of $\tilde{W}$ in presence of noise etc
- Transient response
- Actuator saturation
- In case of RBFs, how do we distribute the centers?
  - See paper *"A Reprodcing Kernel Hilbert Space Approach to Adaptive Control", Kingravi, Chowdhary, Vela, Johnson, CDC 2011*
- In general, traditional adaptive laws don't guarantee that the weights approach their ideal values (as dictated by the universal approximation property)
- Persistency of excitation is needed to guarantee this