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Master Thesis

Estimation of Underactuated Degrees of Freedom(DOF's) in Humanoid Robots

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INTRODUCTION

1.1 Motivation

The field of *Robotics* have seen a tremendous development since the introduction of the term by *Isaac Asimov* in 1940s. The fundamental components of robotic systems are mechanical structure, actuators, sensors and controller. Robotic system ranges from simple *Cartesian manipulator* to the complex *Humanoids*. *Industrial robots* are robots that are used in applications such as palletizing, material loading and unloading, part sorting, packaging etc. These robots usually operate in the structured environment whose geometrical or physical characteristics are known in priori. They are pre programmed to execute the set of tasks. These robots have largely aided the automation of manufacturing processes in the industries. *Mobile robots* that are used in the environments where human beings can hardly survive or be exposed to unsustainable risks are called *Field robots*. *Field robots* normally operate in the unstructured environments, where the geometry or physical characteristics are not know in priori. Mars rover *Curiosity* is one such example. Locomotion in these robots are achieved either by wheels or by mechanical legs. Operating in the unknown environments and dynamic balancing of mechanical structure demands advanced control schemes for *Field robots*.

Figure 1.1: Eine Beispiel-Abbildung.

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Grundlagen

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State Estimation

State estimation is the principle of estimating the internal state of the system from the measurement of inputs and outputs of the system. In general knowledge of the internal state of the system will make the system easy to control. Figure 3.1 shows the usage of state estimator in state feedback control loop.

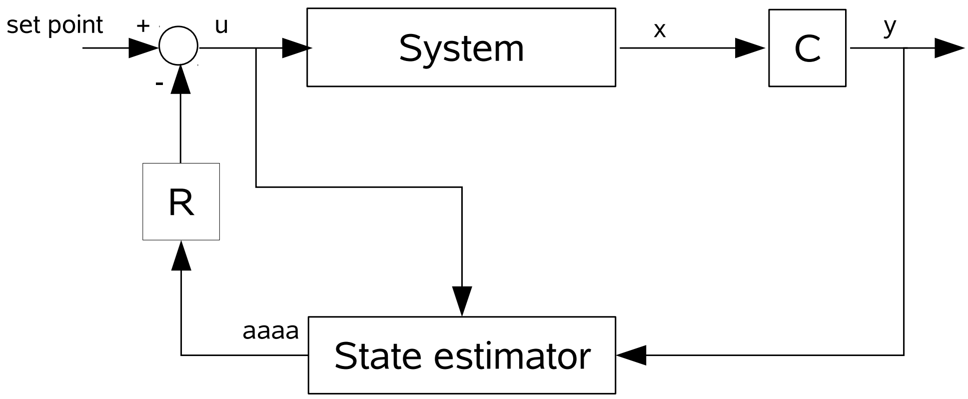

Figure 3.1: Structure of state feedback controller with state estimator

A general nonlinear system in state space form,

$$\begin{aligned} \dot{x}(t) &= f(x(t), u(t)), x(t=0) = x_0 \\ y(t) &= g(x(t), u(t)) \end{aligned} \tag{3.1}$$

In Equation 3.1, *x(t)* represents the vector of internal states, *u(t)* represents the vector of inputs and *y(t)* represents the vector of outputs of the system. x_0 is the initial state of the system which is usually unknown. The state estimator is described by the system equation with additional correction term

$$\begin{aligned} \dot{\hat{x}}(t) &= f(\hat{x}(t), u(t)) + K(y(t) - \hat{y}(t)), \hat{x}(t=0) = \hat{x}_0 \\ \hat{y}(t) &= g(\hat{x}(t), u(t)) \end{aligned} \tag{3.2}$$

$\hat{x}(t)$ is the state vector of estimator and K is the gain matrix. A state estimator should satisfy the following properties

- **Simulation property:** For the same initial condition $x(t_0) = \hat{x}_0$ of the estimator and the system to be observed, then it holds that $x(t) = \hat{x}(t) \forall t > 0$.
- **Convergence property:** If $x(t_0) \neq \hat{x}_0$, then $x(t) - \hat{x}(t)$ tends to zero as $t \to \infty$

The different approaches for state estimator design differs in the calculation of gain matrix K in Equation 3.2.

3.1 Kalman Filter

Kalman filter is a statistical state estimation algorithm which estimates the internal state of the system from the noisy measurements. It was designed by Rudolph E. Kalman in 1960 for discrete time linear systems. It is basically a predictor-corrector type estimator that is optimal in the sense that it minimizes the estimated error covariance. Since the measurements occur and the states are estimated at discrete points of time, it is easily implementable in digital computers. Kalman filters are extensively used in the area of autonomous and guided navigation.

3.1.1 Kalman gain

Given a discrete time linear system affected by random noise

$$\begin{aligned} x_k &= Ax_{k-1} + Bu_k + w_{k-1} \\ y_k &= Hx_k + v_k \end{aligned} \tag{3.3}$$

where the random variables w_k,v_k represent the process and measurement noise. Both the random variables are assumed to be zero mean Gaussian white noises. Let Q, R be the covariance of process and measurement noise. Let us assume,

$$e_k^- = x_k - \hat{x}_k^- \tag{3.4}$$

be the error between the actual and predicted value of the state. The error covariance is given by

$$P_k^- = E[e_k^- e_k^{-T}] \tag{3.5}$$

Kalman filter corrects it estimate based on the predicted state and measured output data by

$$\hat{x}_k = \hat{x}_k^{\,-} + K(y_k - H\hat{x}_k^{\,-}) \tag{3.6}$$

Kalman gain is computed by substituting Equation 3.6 in Equation 3.4 to compute the e_k^-. Computed e_k^- is substituted in Equation 3.5 and the expected values are computed to find the error covariance P_k^-. Finally K is computed by taking the derivative of trace of P_k^- and equating it to zero

$$\frac{\partial trace(P_k^-)}{\partial K} = 0$$

solving the above equation for K. One form of K that minimizes Equation 3.6

$$K_k = P_k^- H^T (H P_k^- H^T + R)^{-1} \tag{3.7}$$

From the Equation 3.7 as measurement covariance R approaches zero, Kalman gain K lays more trust on actual measurement y_k. On the other hand if P_k^- approaches zero, predicted measurement $H\hat{x}_k^{\,-}$ is trusted more.

3.1.2 Extended Kalman filter

Since most of the systems in practice are non linear in nature Kalman filter is non-linear systems by extending the actual algorithm. This type of kalman filter algorithms are called Extended kalman filters. There are also a new class of kalman filter algorithm which works on Bayasian principle, they are called unscented Kalman filters.

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Hiermit bestätige ich, die vorliegende Diplomarbeit selbständig und nur unter Zuhilfenahme der angegebenen Literatur verfasst zu haben.

Ich bin damit einverstanden, dass Exemplare dieser Arbeit in den Bibliotheken der Universität Dortmund ausgestellt werden.

Dortmund, den July 25, 2013

Name