technische universität
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Master Thesis

Estimation of Underactuated Degrees of
Freedom(DOF's) in Humanoid Robots

Rajesh Rajendran
rajesh.rajendran@tu-dortmund.de

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Examiner:

Name of first examiner	Name of second examiner
Abteilung Informationstechnik	Lehrstuhl des Zweitgutachters
Institut für Roboterforschung	Fakultät für Informatik

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Introduction

1.1 Motivation

The field of *Robotics* have seen a tremendous development since the introduction of the term by *Isaac Asimov* in 1940s. The fundamental components of robotic systems are mechanical structure, actuators, sensors and controller. Robotic system ranges from simple *Cartesian manipulator* to the complex *Humanoids*. *Industrial robots* are robots that are used in applications such as palletizing, material loading and unloading, part sorting, packaging etc. These robots usually operate in the structured environment whose geometrical or physical characteristics are known in priori. They are pre programmed to execute the set of tasks. These robots have largely aided the automation of manufacturing processes in the industries. *Mobile robots* that are used in the environments where human beings can hardly survive or be exposed to unsustainable risks are called *Field robots*. *Field robots* normally operate in the unstructured environments, where the geometry or physical characteristics are not know in priori. Mars rover *Curiosity* is one such example. Locomotion in these robots are achieved either by wheels or by mechanical legs. Operating in the unknown environments and dynamic balancing of mechanical structure demands advanced control schemes for *Field robots*.

Figure 1.1: Eine Beispiel-Abbildung.

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State Estimation

State estimation is the principle of estimating the internal state of the system from the measurement of inputs and outputs of the system. In general knowledge of the internal state of the system will make the system easy to control. Figure 2.1 shows the usage of state estimator in state feedback control loop.

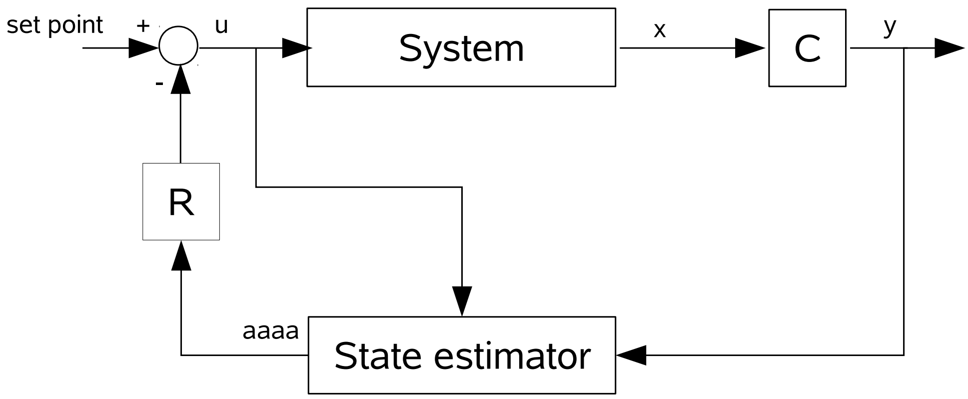

Figure 2.1: Structure of state feedback controller with state estimator

A general nonlinear system in state space form,

$$
\begin{aligned}
\dot{x}(t) &= f(x(t), u(t)), x(t=0) = x_0 \\
y(t) &= g(x(t), u(t))
\end{aligned}
\tag{2.1}
$$

In Equation 2.1, *x(t)* represents the vector of internal states, *u(t)* represents the vector of inputs and *y(t)* represents the vector of outputs of the system. x_0 is the initial state of the system which is usually unknown. The state estimator is described by the system equation with additional correction term

$$\begin{aligned}\dot{\hat{x}}(t) &= f(\hat{x}(t), u(t)) + K(y(t) - \hat{y}(t)), \hat{x}(t=0) = \hat{x}_0 \\ \hat{y}(t) &= g(\hat{x}(t), u(t))\end{aligned} \tag{2.2}$$

$\hat{x}(t)$ is the state vector of estimator and K is the gain matrix. A state estimator should satisfy the following properties

- **Simulation property:** For the same initial condition $x(t_0) = \hat{x}_0$ of the estimator and the system to be observed, then it holds that $x(t) = \hat{x}(t) \forall t > 0$.
- **Convergence property:** If $x(t_0) \neq \hat{x}_0$, then $x(t) - \hat{x}(t)$ tends to zero as $t \to \infty$

The different approaches for state estimator design differs in the calculation of gain matrix K in Equation 2.2.

2.1 Kalman Filter

Kalman filter is a statistical state estimation algorithm which estimates the internal state of the system from the noisy measurements. It was designed by Rudolph E. Kalman in 1960 for discrete time linear systems. It is basically a predictor-corrector type estimator that is optimal in the sense that it minimizes the estimated error covariance. Since the measurements occur and the states are estimated at discrete points of time, it is easily implementable in digital computers. Kalman filters are extensively used in the area of autonomous and guided navigation.

2.1.1 Kalman gain

Given a discrete time linear system affected by random noise

$$\begin{aligned}x_k &= Ax_{k-1} + Bu_k + w_{k-1} \\ y_k &= Hx_k + v_k\end{aligned} \tag{2.3}$$

where the random variables w_k,v_k represent the process and measurement noise. Both the random variables are assumed to be zero mean Gaussian white noises. Let Q, R be the covariance of process and measurement noise. Let us assume,

$$e_k^- = x_k - \hat{x}_k^- \tag{2.4}$$

be the error between the actual and predicted value of the state. The error covariance is given by

$$P_k^- = E[e_k^- {e_k^-}^T] \tag{2.5}$$

Kalman filter corrects it estimate based on the predicted state and measured output data by

$$\hat{x_k} = \hat{x_k}^- + K(y_k - H\hat{x_k}^-) \tag{2.6}$$

Kalman gain is computed by substituting Equation 2.6 in Equation 2.4 to compute the e_k^-. Computed e_k^- is substituted in Equation 2.5 and the expected values are computed to find the error covariance P_k^-. Finally K is computed by taking the derivative of trace of P_k^- and equating it to zero

$$\frac{\partial trace(P_k^-)}{\partial K} = 0$$

solving the above equation for K. One form of K that minimizes Equation 2.6

$$K_k = P_k^- H^T (HP_k^- H^T + R)^{-1} \tag{2.7}$$

From the Equation 2.7 as measurement covariance R approaches zero, Kalman gain K lays more trust on actual measurement y_k. On the other hand if P_k^- approaches zero, predicted measurement $H\hat{x_k}^-$ is trusted more.

2.1.2 Extended Kalman filter

Most of the real world estimation scenarios are non linear in nature. Kalman filter algorithm cannot be applied to the non linear systems. *NASA Ames* devised a method to apply Kalman filter for non linear systems which is called the Extended Kalman filter(EKF). In EKF the non linear system is linearised by multivariate Taylor series expansion of the non linear function.

Given a discrete time non linear system,

$$\begin{aligned} x_k &= f(x_{k-1}, u_k, w_{k-1}) \\ y_k &= h(x_k, u_k, v_k) \end{aligned} \tag{2.8}$$

x,y denotes the vector of system's state and output. *w,v* represents the process and measurement covariance noise. f is the non linear function that relates the previous state to the current state and h is the non linear function that relates the output and state.

In practice the individual values of noise w_k and v_k at each time step k is not known. So one can compute the approximated state and measurement vector without them as

$$\begin{aligned} \hat{x}_k^- &= f(\hat{x}_{k-1}, u_k, 0) \\ \hat{y}_k^- &= h(\hat{x}_k^-, u_k, 0) \end{aligned} \tag{2.9}$$

$\hat{x}_k^-$ and $\hat{y}_k^-$ are the *priori* estimates of state and measurements at time step k computed from *posteriori* estimate of state $\hat{x}_{k-1}$ from previous time step *k-1*.

A_k and H_k be the Jacobian matrices that results taking partial derivative of f and h with respect to x at time instant k. W_k and V_k be the Jacobian matrices that results taking partial derivative of f with respect to w and h with respect to v at time step k.

$$\begin{aligned}
A_k(i,j) &= \frac{\partial f_i}{\partial x_j}(\hat{x}_{k-1}, u_k, 0) \\
C_k(i,j) &= \frac{\partial h_i}{\partial x_j}(\hat{x}_k^-, u_k, 0) \\
W_k(i,j) &= \frac{\partial f_i}{\partial w_j}(\hat{x}_{k-1}, u_k, 0) \\
V_k(i,j) &= \frac{\partial h_i}{\partial v_j}(\hat{x}_k^-, u_k, 0)
\end{aligned} \tag{2.10}$$

At each time step these Jacobian matrices are evaluated with current predicted states $\hat{x}_k^-$.

2.1.2.1 Algorithm

Predict

$$\begin{aligned}
\hat{x}_k^- &= f(\hat{x}_{k-1}, u_k, 0) \\
P_k^- &= A_k P_{k-1} A_k^T + W_k Q_{k-1} W_k^T
\end{aligned} \tag{2.11}$$

Correct

$$\begin{aligned}
K_k &= P_k^- H^T (H_k P_k^- H_k^T + V_k R_k V_k^T)^{-1} \\
\hat{x}_k &= \hat{x}_k^- + K_k(y_k - h(x_k^-, u_k, 0)) \\
P_k &= (I - K_k H_k) P_k^-
\end{aligned} \tag{2.12}$$

2.1.3 Unscented Kalman filter

3

MODELLING

This chapter discusses two modelling approaches used in the Kalman filter. In the first approach, the robot is modelled as multi body dynamic system in which the motion of the robot is caused by application of torques at the respective joints. In the second approach a simplified model of a inertial navigation system is considered to estimate the motion of the robot.

3.1 Multi body system

Humanoid robots consists of group of rigid bodies that are joined together to resemble the human skeletal structure. As human beings are driven by the muscular system, humanoids are driven by electric, hydraulic or pneumatic actuators, which apply torques at the joints of the robot. The dynamics of robots describes how the robots moves in response to the actuator forces or torques. The dynamics of the robot is defined by the equations of motion of the rigid body system. In general there are several approaches in deriving equations of motion of rigid body dynamic system. One such approach is by deriving Lagrange equation. Lagrangian analysis relies on the energy properties of the mechanical system to derive the equations of motion. The *Lagrangian*, L, is defined as the difference between kinetic and potential energy of the system.

$$L(q, \dot{q}) = T(q, \dot{q}) - V(q)$$

q represents the vector of joint angles T and V represents the kinetic and potential energies of the system. q is called the generalized coordinates of the system as the specification of joint angles uniquely determines position of all the rigid bodies.

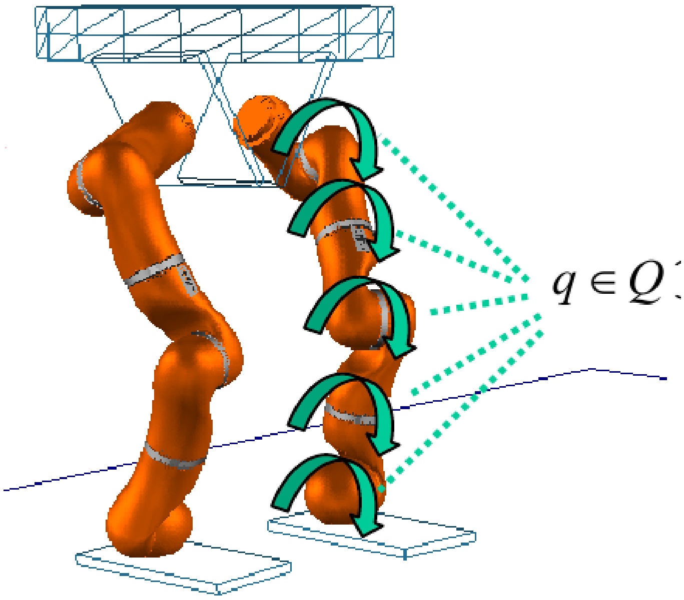

Figure 3.1: Simplified lower body model of biped

The equations of motion for a rigid body system with generalized coordinates $q \in \Re^m$ is given by,

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q_i} = \Gamma_i i = 1, ..., m \tag{3.1}$$

Γ_i is the external force acting on the ith generalized coordinate. The general formulation of equation of motion of biped shown in Figure 3.1 with configuration space Q is

$$M(q)\ddot{q} + C(q, \dot{q}) + g(q) = \tau \tag{3.2}$$

M(q) is the generalized inertia matrix, $C(q, \dot{q})$ is the matrix representing Coriolis and centrifugal forces, *g(q)* is the gravity vector acting on the system and τ is the actuator torque applied to the joints.
Toro is modelled as floating base dynamic model.

$$\begin{aligned} \ddot{q} &= M(q)^{-1}(-C(q, \dot{q})\dot{q} - g(q) + (J^b_{bl})^T F^b_l + (J^b_{br})^T F^b_r + \tau) \\ &\text{where,} \\ q_{base} &= (p^T_f, \theta^T_f)^T \in \Re^6 \\ q_j &= (q_1, q_2, \dots, q_{25})^T \in \Re^{25} \\ q &= (q^T_f, q^T_j)^T \in \Re^{31} \end{aligned} \tag{3.3}$$

q is the generalized coordinates. The coordinates of the floating base q_f are defined with reference to spatial frame, whereas q_j are defined with reference to body frame. q_f =

$(p_f^T, \theta_f^T)^T$, where $p_f = (x, y, z)^T$ is a vector representing the position of origin of floating base in spatial frame and $\theta_f = (\theta_x, \theta_y, \theta_z)^T$ is a vector of Euler angles that describes the rotation of base frame with respect to spatial frame.$q_j \in \Re^{25}$ is the vector of joint angles. $\dot{q} = (\hat{V}_f^b, \dot{q}_j)^T \in \Re^{31}$ is the vector of generalized velocities. $\hat{V}_f^b = (v_f^b, \omega_f^b)^T \in \Re^6$ is the body twist. $\dot{q}_j \in \Re^{25}$ is the vector of joint velocities. $\ddot{q} \in \Re^{31}$ is the vector of generalized accelerations. $M(q) \in \Re^{31\times31}$ is the inertia matrix, $C(q, \dot{q}) \in \Re^{31\times31}$ is the matrix accounting for centrifugal and Coriolis forces. $g(q) \in \Re^{31}$ is the gravity vector. $\tau \in \Re^{31}$ is the vector of actuating torques acting on q_j, where the first six components are zero because those degrees of freedom corresponding to q_f are not actuated.$(J_{br}^{fb})^T, (J_{bl}^{fb})^T \in \Re^{31\times6}$ are the Jacobian matrices transforming the wrenches $F_r^{fb}, F_l^{fb} \in \Re^6$ applied in the right and left foot to joint torques.

State Space representation: General state space representation of a non linear system

$$\begin{aligned} \dot{x} &= f(x, u) \\ y &= h(x, u) \end{aligned} \tag{3.4}$$

where, $x \in \Re^n$ is the vector representing the states of the system. $u \in \Re^p$ is the vector of inputs acting on the system. $y \in \Re^m$ is the vector of outputs of the system.
State space representation of *Toro*

$$\dot{x} = \begin{pmatrix} R_{sb}\dot{p}_f \\ T(\theta_f)^{-1}R_{sb}\omega_f^b \\ \dot{q}_j \\ M(q)^{-1}(-C(q,\dot{q})\dot{q} - g(q) + (J_{bl}^b)^T F_l^b + (J_{br}^b)^T F_r^b + \tau) \end{pmatrix}$$

$$y = \begin{pmatrix} q_j \\ \dot{q}_j \\ \ddot{p}_f^b \\ \omega_f^b \\ p_c^b \\ \hat{V}_c^b \end{pmatrix}$$

where,

$$x = (q^T, \dot{q}^T)^T \in \Re^{62} \tag{3.5}$$

$$T(\theta_f) = \begin{pmatrix} 1 & 0 & \sin(\theta_y) \\ 0 & \cos(\theta_x) & -\sin(\theta_x)\cos(\theta_y) \\ 0 & \sin(\theta_x) & \cos(\theta_x)\cos(\theta_y) \end{pmatrix}$$

$$\ddot{p}_f^b = \ddot{q}(1,2,3)^T + R_{sb}^T \begin{pmatrix} 0 \\ 0 \\ 9.81 \end{pmatrix}$$

$$p_c^b = (p_{a,r}^b, p_{b,r}^b, p_{a,l}^b, p_{b,l}^b)^T$$

i.e $p_{a,r}^b = H_{sr}^{-1} p_{a,r}^s, p_{b,r}^b = H_{sr}^{-1} p_{b,r}^s, p_{a,l}^b = H_{sl}^{-1} p_{a,l}^s, p_{b,l}^b = H_{sb}^{-1} p_{b,l}^s$

$\hat{V}_c^b = (\hat{V}_r^b, \hat{V}_l^b)^T$ i.e $\hat{V}_r^b = J_r\dot{q}$ and $\hat{V}_l^b = J_l\dot{q}$

- $T(\theta_f)$ is the matrix that transforms the angular velocity ω_f^s to the time derivative of Euler angles $\dot{\theta}_f^s$. i.e $\omega_f^s = T(\theta_f)\dot{\theta}_f^s$.
- $Ad_{sb} \in \Re^{6\times 6}$ is the adjoint transformation matrix that transforms the body velocity to spatial velocity.
- $\ddot{p}_f^b$ is the vector of Cartesian accelerations of the floating base in body frame (measured by IMU). $\ddot{q}(1,2,3)$ is the first three elements of $\ddot{q}$ computed by Eq. 3.3. $R_{sb}^T(0,0,9.81)^T$ is the term added to compensate for the gravity measured by IMU.
- ω_f^b is the vector of angular rates of floating base in body frame (measured by Gyroscope).

- p_c^b is the vector of position constraints of right and left foot. These positions constraints are the static points on the foot of the robot. H_{sx} is the homogeneous transformation matrix from frame x to spatial frame. $p_{a,r}^s, p_{b,r}^s, p_{a,l}^s, p_{b,l}^s$ are known points which are constant with respect to spatial frame.

- $\hat{V}_c^b$ are velocity constraints on right and left foot.

Observability: State space representation of a linear system is,

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du. \end{aligned} \tag{3.6}$$

where, $x \in \Re^n$ is the vector representing the states of the system. $u \in \Re^p$ is the vector of inputs, $y \in \Re^m$ is the vector of outputs of the system. $A \in \Re^{n\times n}$ is the system matrix. $B \in \Re^{n\times p}$ is the matrix relating state and input, $C \in \Re^{m\times n}$ is the measurement matrix relating output and state, $D \in \Re^{m\times p}$ is the matrix relating input and output of the system. Linearising a non linear system in Eq. 3.4 at some operating point will lead to linear system of form Eq. 3.6. For a linear system to be observable, it should satisfy

$$obs = \begin{pmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{pmatrix}, rank(obs) = n \tag{3.7}$$

Let $g_{sb} = T_A(\theta_f)^{-1} Ad_{sb}$, then from Eq. 3.5,

$$\alpha = g_{sb}\hat{V}_f^b$$

$$\frac{\partial \alpha}{\partial x} = \left(\frac{\partial \alpha}{\partial x_1}, \frac{\partial \alpha}{\partial x_2}, \cdots, \frac{\partial \alpha}{\partial x_{62}}\right) \in \Re^{6\times 62} \tag{3.8}$$

$$\frac{\partial \alpha}{\partial x_i} = \begin{cases} g_{sb}adj_{J_f^i}\hat{V}_f^b & \text{if } i \leq 3 \\ -T(\theta)^{-1}\frac{\partial T}{\partial x_i}\alpha + g_{sb}adj_{J_f^i}\hat{V}_f^b & \text{if } i > 3 \text{ or } i \leq 6 \\ g_{sb}adj_{J_f^i}\hat{V}_f^b & \text{if } i > 6 \text{ or } i \leq 31 \\ col(g_{sb}, i-31) & \text{if } i > 31 \text{ or } i \leq 37 \end{cases}$$

where,

- $col(X, i)$ - represents the column of matrix X.

- $adj_{J_f^i}$ - Skew symmetric matrix formed by $col(J_f, i)$

- $\frac{\partial Ad_{sb}}{\partial x} = Ad_{sb} adj_{J_f^i}$ (Gianluca)

$$\frac{\partial \dot{q}}{\partial x} = \left(\frac{\partial \dot{q}}{\partial x_1}, \frac{\partial \dot{q}}{\partial x_2}, \cdots, \frac{\partial \dot{q}}{\partial x_{62}}\right) \in \Re^{25\times 62} \tag{3.9}$$

$$\frac{\partial \dot{q}}{\partial x_i} = \begin{cases} 1 & \text{if } i > 37 \\ 0 & \text{if } i \leq 37 \end{cases}$$

Let $B = -C(q,\dot{q})\dot{q} - g(q) + J_l^T F_l + J_r^T F_r + \tau$, then from Eq. 3.5,

$$\Lambda = M(q)^{-1} B$$

$$\frac{\partial \Lambda}{\partial x} = \left(\frac{\partial \Lambda}{\partial x_1}, \frac{\partial \Lambda}{\partial x_2}, \cdots, \frac{\partial \Lambda}{\partial x_{62}}\right) \in \Re^{31\times 62} \tag{3.10}$$

where,

$$\frac{\partial \Lambda}{\partial x_i} = \begin{cases} \left.\begin{matrix} -M^{-1}\dfrac{\partial M}{\partial x_i}\Lambda + M^{-1}\left(-\dfrac{\partial C}{\partial x_i}\dot{q} - \dfrac{\partial g}{\partial x_i} + \left(\dfrac{\partial J_r^b}{\partial x_i}\right)^T F_r\right) \\ + M^{-1}\left(\dfrac{\partial J_l^b}{\partial x_i}\right)^T F_l \end{matrix}\right\} & \text{if } i \leq 31 \\ -M^{-1}\dfrac{\partial M}{\partial x_i}\Lambda + M^{-1}\left(-\dfrac{\partial C}{\partial x_i}\dot{q} - col(C, i-31)\right) & \text{if } i > 31 \end{cases}$$

System matrix is given by Eq. 3.8,3.9 and 3.10

$$A = \begin{pmatrix} \dfrac{\partial \alpha}{\partial x} \\ \dfrac{\partial \dot{q}}{\partial x} \\ \dfrac{\partial \lambda}{\partial x} \end{pmatrix} \in \Re^{62\times 62} \tag{3.11}$$

For computation of measurement matrix C the derivative of y in Eq. 3.5 with respect to system state is computed.

$$\frac{\partial q}{\partial x} = \left(\frac{\partial q}{\partial x_1}, \frac{\partial q}{\partial x_2}, \cdots, \frac{\partial q}{\partial x_{62}}\right) \in \Re^{25\times 62} \tag{3.12}$$

$$\frac{\partial q}{\partial x_i} = \begin{cases} 1 & \text{if } 7 \leq i \leq 31 \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{\partial \dot{q}}{\partial x} = \left(\frac{\partial \dot{q}}{\partial x_1}, \frac{\partial \dot{q}}{\partial x_2}, \cdots, \frac{\partial \dot{q}}{\partial x_{62}}\right) \in \Re^{25\times 62} \tag{3.13}$$

$$\frac{\partial \dot{q}}{\partial x_i} = \begin{cases} 1 & \text{if } 38 \leq i \leq 62 \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{\partial \ddot{p}_f^b}{\partial x} = \left(\frac{\partial \ddot{p}_f^b}{\partial x_1}, \frac{\partial \ddot{p}_f^b}{\partial x_2}, \cdots, \frac{\partial \ddot{p}_f^b}{\partial x_{62}} \right) \in \Re^{3\times 62} \tag{3.14}$$

$$\frac{\partial \ddot{p}_f^b}{\partial x_i} = \frac{\partial \Lambda}{\partial x}(1:3,:) + \left(\frac{\partial R_{sb}}{\partial x_i} \right)^T \begin{pmatrix} 0 \\ 0 \\ 9.81 \end{pmatrix}$$

where,
$\frac{\partial \Lambda}{\partial x}(1:3,:)$ represents the first three rows of $\frac{\partial \Lambda}{\partial x}$ in Eq. 3.10.

$$\frac{\partial \dot{\theta}_f^b}{\partial x} = \left(\frac{\partial \dot{\theta}_f^b}{\partial x_1}, \frac{\partial \dot{\theta}_f^b}{\partial x_2}, \cdots, \frac{\partial \dot{\theta}_f^b}{\partial x_{62}} \right) \in \Re^{3\times 62} \tag{3.15}$$

$$\frac{\partial \dot{\theta}_f^b}{\partial x_i} = \begin{cases} \left(\frac{\partial R_{sb}}{\partial x_i} \right)^T T^{-1} R_{sb} \omega_f^b + R_{sb}^T T^{-1} \left(\frac{\partial R_{sb}}{\partial x_i} \right) \omega_f^b & \text{if } 1 \le i \le 3 \\ \left(\frac{\partial R_{sb}}{\partial x_i} \right)^T T^{-1} R_{sb} \omega_f^b - R_{sb}^T T^{-1} \left(\frac{\partial T}{\partial x_i} \right) T^{-1} R_{sb} \omega_f^b \\ \qquad + R_{sb}^T T^{-1} \left(\frac{\partial R_{sb}}{\partial x_i} \right) \omega_f^b & \text{if } 4 \le i \le 6 \\ \left(\frac{\partial R_{sb}}{\partial x_i} \right)^T T^{-1} R_{sb} \omega_f^b + R_{sb}^T T^{-1} \left(\frac{\partial R_{sb}}{\partial x_i} \right) \omega_f^b & \text{if } 7 \le i \le 31 \\ col(R_{sb}^T T^{-1} R_{sb}, i) & \text{if } 35 \le i \le 37 \\ 0 & \text{otherwise} \end{cases}$$

$$\left. \begin{aligned} B' &= -\partial \times E, \\ E' &= \partial \times B - 4\pi j, \end{aligned} \right\} \qquad \text{Maxwell's equations}$$

$$\frac{\partial p_c^b}{\partial x} = \left(\frac{\partial p_c^b}{\partial x_1}, \frac{\partial p_c^b}{\partial x_2}, \cdots, \frac{\partial p_c^b}{\partial x_{62}} \right) \in \Re^{12\times 62} \tag{3.16}$$

$$\frac{\partial p_c^b}{\partial x_i} = \begin{cases} \begin{pmatrix} -H_{sr}^{-1} \frac{\partial H_{sr}}{\partial x_i} H_{sr}^{-1} p_{a,r}^s \\ -H_{sr}^{-1} \frac{\partial H_{sr}}{\partial x_i} H_{sr}^{-1} p_{b,r}^s \\ -H_{sl}^{-1} \frac{\partial H_{sl}}{\partial x_i} H_{sl}^{-1} p_{a,l}^s \\ -H_{sl}^{-1} \frac{\partial H_{sl}}{\partial x_i} H_{sl}^{-1} p_{b,l}^s \end{pmatrix} & \text{if } 1 \le i \le 31 \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{\partial \hat{V}_c^b}{\partial x} = \left(\frac{\partial \hat{V}_c^b}{\partial x_1}, \frac{\partial \hat{V}_c^b}{\partial x_2}, \cdots, \frac{\partial \hat{V}_c^b}{\partial x_{62}} \right) \in \Re^{12\times 62} \tag{3.17}$$

$$\frac{\partial \hat{V}_c^b}{\partial x_i} = \begin{cases} \begin{pmatrix} \frac{\partial J_r^b}{\partial x_i}\dot{q} \\ \frac{\partial J_l^b}{\partial x_i}\dot{q} \end{pmatrix} & \text{if } 1 \leq i \leq 31 \\ \begin{pmatrix} col(J_r^b, i) \\ col(J_l^b, i) \end{pmatrix} & \text{if } 32 \leq i \leq 62 \end{cases}$$

The measurement matrix of the system is given by Eq. 3.12, 3.13, 3.14, 3.15, 3.16, 3.17

$$C = \begin{pmatrix} \frac{\partial q}{\partial x} \\ \frac{\partial \dot{q}}{\partial x} \\ \frac{\partial \ddot{p}_f^b}{\partial x} \\ \frac{\partial \dot{\theta}_f^b}{\partial x} \\ \frac{\partial p_c^b}{\partial x} \\ \frac{\partial \hat{V}_c^b}{\partial x} \end{pmatrix} \in \Re^{80 \times 62} \tag{3.18}$$

For our system to be observable $rank(obs) = 62$.

List of Figures

Hiermit bestätige ich, die vorliegende Diplomarbeit selbständig und nur unter Zuhilfenahme der angegebenen Literatur verfasst zu haben.

Ich bin damit einverstanden, dass Exemplare dieser Arbeit in den Bibliotheken der Universität Dortmund ausgestellt werden.

Dortmund, den July 29, 2013

Name