Proof for Problem 3

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For environment ρ, we have the following axioms:

$$\frac{\rho(x) = v}{\rho \vdash x \ \mathbf{b} \ v} \qquad \frac{x \notin \rho}{\rho \vdash x \ \mathbf{err} \ Err_x} \qquad \frac{}{\rho \vdash i \ \mathbf{b} \ i} \qquad \frac{}{\rho \vdash b \ \mathbf{b} \ b}$$

$$\frac{}{\rho \vdash Err_s \ \mathbf{b} \ Err_s} \qquad \frac{e \ \mathbf{b} \ e'}{e \to_{\mathbf{b}} e'}$$

Then we divide the reduction relations for language B into several cases, which can be found as follows:

Equal operator reduction rules :

$$\frac{i \in \mathbb{Z}, j \in \mathbb{Z}}{\rho \vdash Eq(i,j) \ \to_{\mathbf{b}} \ i = j} \qquad \frac{v_1 \notin \mathbb{Z}}{\rho \vdash Eq(v_1, v_2) \ \mathbf{err} \ Err_{\mathsf{EQ1}}}$$

$$\frac{v_2 \notin \mathbb{Z}}{\rho \vdash Eq(v_1, v_2) \ \mathbf{err} \ Err_{\mathsf{EQ2}}} \qquad \frac{}{\rho \vdash Eq(r, e) \ \mathbf{prop} \ r}$$

$$\frac{}{\rho \vdash Eq(v, r) \ \mathbf{prop} \ r} \qquad \frac{e_2 \to_{\mathbf{b}} e_2'}{\rho \vdash Eq(v, e_2) \ \to_{\mathbf{b}} \ Eq(v, e_2')}$$

$$\frac{e_1 \to_{\mathbf{b}} e_1'}{\rho \vdash Eq(e_1, e_2) \ \to_{\mathbf{b}} \ Eq(e_1', e_2)}$$

Pred operator reduction rules :

$$\frac{i \in \mathbb{Z}}{\rho \vdash Pred(i) \ \mathbf{b} \ i - 1} \qquad \frac{}{\rho \vdash Pred(r) \ \mathbf{prop} \ r}$$

$$\frac{v \notin Z}{\rho \vdash Pred(v) \ \mathbf{err} \ Err_{\mathsf{PRED}}} \qquad \frac{e \to_{\mathbf{b}} e'}{\rho \vdash Pred(e) \ \to_{\mathbf{b}} \ Pred(e')}$$

Succ operator reduction rules :

$$\frac{i \in \mathbb{Z}}{\rho \vdash Succ(i) \ \mathbf{b} \ i-1} \qquad \frac{}{\rho \vdash Succ(r) \ \mathbf{prop} \ r}$$

$$\frac{v \notin Z}{\rho \vdash Succ(v) \ \mathbf{err} \ Err_{\mathsf{SUCC}}} \qquad \frac{e \rightarrow_{\mathbf{b}} e'}{\rho \vdash Succ(e) \ \rightarrow_{\mathbf{b}} \ Succ(e')}$$

Plus reduction rules :

$$\frac{v \notin \mathbb{Z}}{\rho \vdash Plus(v,e) \ \mathbf{err} \ Err_{\mathsf{PLUS1}}} \qquad \frac{v \notin \mathbb{Z}}{\rho \vdash Plus(i,v) \ \mathbf{err} \ Err_{\mathsf{PLUS2}}}$$

$$\frac{}{\rho \vdash Plus(r,e) \ \mathbf{prop} \ r} \qquad \frac{}{\rho \vdash Plus(i,r) \ \mathbf{prop} \ r}$$

$$\frac{e_1 \rightarrow_{\mathbf{b}} e_1'}{\rho \vdash Plus(e_1,e_2) \ \rightarrow_{\mathbf{b}} \ Plus(e_1',e_2)}$$

$$\frac{e_2 \rightarrow_{\mathbf{b}} e_2'}{\rho \vdash Plus(i,e_2) \ \rightarrow_{\mathbf{b}} \ Plus(i,e_2')} \qquad \frac{}{\rho \vdash Plus(i,j) \ \mathbf{b} \ i+j}$$

Div reduction rules :

$$\frac{v \notin \mathbb{Z}}{\rho \vdash Div(v,e) \ \mathbf{err} \ Err_{\mathsf{DIV1}}} \qquad \frac{v \notin \mathbb{Z}}{\rho \vdash Div(i,v) \ \mathbf{err} \ Err_{\mathsf{DIV2}}}$$

$$\frac{}{\rho \vdash Div(r,e) \ \mathbf{prop} \ r} \qquad \frac{}{\rho \vdash Div(i,r) \ \mathbf{prop} \ r}$$

$$\frac{j = 0}{\rho \vdash Div(i,j) \ \mathbf{err} \ Err_{\mathsf{DIV0}}} \qquad \frac{e_1 \rightarrow_{\mathbf{b}} e_1'}{\rho \vdash Div(e_1,e_2) \ \rightarrow_{\mathbf{b}} \ Div(e_1',e_2)}$$

$$\frac{e_2 \rightarrow_{\mathbf{b}} e_2'}{\rho \vdash Div(i,e_2) \ \rightarrow_{\mathbf{b}} \ Div(i,e_2')} \qquad \frac{}{\rho \vdash Div(i,j) \ \mathbf{b} \ i/j}$$

Mul reduction rules :

$$\frac{v \notin \mathbb{Z}}{\rho \vdash Mul(v, e) \textbf{ err } Err_{\mathsf{MUL1}}} \qquad \frac{v \notin \mathbb{Z}}{\rho \vdash Mul(i, v) \textbf{ err } Err_{\mathsf{MUL2}}}$$

$$\frac{}{\rho \vdash Mul(r, e) \textbf{ prop } r} \qquad \frac{}{\rho \vdash Mul(i, r) \textbf{ prop } r}$$

$$\frac{e_1 \rightarrow_{\mathbf{b}} e_1'}{\rho \vdash Mul(e_1, e_2) \ \rightarrow_{\mathbf{b}} \ Mul(e_1', e_2)} \qquad \frac{e_2 \rightarrow_{\mathbf{b}} e_2'}{\rho \vdash Mul(i, e_2) \ \rightarrow_{\mathbf{b}} \ Mul(i, e_2')}$$

$$\frac{}{\rho \vdash Mul(i, j) \textbf{ b } i * j}$$

If reduction rules :

$$\frac{v \notin T, F}{\rho \vdash If(v, e_1, e_2) \textbf{ err } Err_{\mathsf{IF}}} \qquad \frac{}{\rho \vdash If(True, e_1, e_2) \textbf{ b } e_1}$$

$$\frac{}{\rho \vdash If(False, e_1, e_2) \textbf{ b } e_2} \qquad \frac{}{\rho \vdash If(r, e_1, e_2) \textbf{ prop } r}$$

$$\frac{c \rightarrow_{\mathbf{b}} c'}{\rho \vdash If(c, e_1, e_2) \ \rightarrow_{\mathbf{b}} \ If(c', e_1, e_2)}$$

According to the above reduction rules, the answers could consists of three different types, i.e. integer, boolean and error (string). Therefore, if the expression is not an answer, the reduction rules defined above should continue to reduce until it reaches the status of answers.