CS1231/CS1231S Assignment #2

AY2019/20 Semester 1

Deadline: 4:30pm on Friday, 8 November 2019

Please read the instructions below

This is a graded assignment worth 10% of your final grade. Please work on it by yourself, not in a group or in collaboration with anybody.

Write your answers on A4-size papers (foolscap papers acceptable too), properly stapled if you use multiple sheets. A handwritten submission is fine; there is no need to use Word or Latex to typeset. Please write legibly and neatly, or marks may be deducted for untidiness or illegible writing.

At the top of the first page, please write your **Name**, **Student Number** and **Tutorial Group** prominently. You can check out which tutorial group you are in by referring to the class list on LumiNUS or the Midterm test seating arrangement document posted on LumiNUS Files. One mark will be deduced if your tutorial group is missing or wrongly written.

We are **VERY** strict about deadlines, so please submit on time. Late submissions will not be accepted.

Please hand in your answers to the SoC Undergraduate Studies office (COM1-02-19). **There are two drop boxes, one for CS1231 and one for CS1231S. Drop your submission into the correct box. Two marks will be deducted if you submit into the wrong box.**

You may hand in your work directly to your tutor before the deadline.

You do not need to submit this question paper. Also, do keep a copy of your submission with you just in case your submission gets lost in transition. Also, we may not be able to return your graded assignment to you this time as it is too close to the end of the semester.

If you have any queries about this assignment, please raise them on the LumiNUS forum.

Question 1. (5 marks)

Let $a, b \in \mathbb{Z}^+$, let $c = \frac{ab}{\gcd(a,b)}$. Show that:

(a) $a \mid c$ and $b \mid c$; [1 mark]

(b) If $a \mid d$ and $b \mid d$, then $c \mid d$; [2 marks]

(c) $c = \min\{d \in \mathbb{Z}^+ : a \mid d \ \wedge b \mid d\}$. [2 marks]

(We use ":" for "such that" here to avoid confusing with the divisibility symbol "|".)

Question 2. (7 marks)

Let R be a relation on a non-empty set A. Define, explicitly in terms of R, the relation which is the *symmetric closure* of R. You should prove that your relation is indeed the smallest symmetric relation on A containing R as a subset.

Question 3. (5 marks)

A language L is defined over a set of three letters $\{a, b, c\}$. A string is a sequence of letters where ordering matters (i.e. "abc" is not equal to "bca"). The length of a string is the number of letters the string contains. For example, "abc" has length 3.

(a) How many strings of length 4 can you create? [1 mark]

(b) How many strings of length 4 do not have consecutive characters being the same? [1 mark]

(c) How many strings of length 4 contain neither "cab" nor "bac", or do not have consecutive characters being the same? [3 marks]

Do not list out the answers exhaustively.

Question 4. (3 marks)

Consider the equation:

$$x_1 + x_2 + x_3 + x_4 = 30$$

where each x_i is an integer ≥ 2.

(a) How many solutions are there? [1 mark]

(b) Suppose now there is an additional constraint: $x_1 \neq 5$. How many solutions are there? [2 marks]

Reminders:

- Did you write your tutorial group number at the top of the first page? 1 mark will be deducted if the tutorial group is missing or wrongly written.
- 2 marks will be deducted if you drop your submission into the wrong box.