ST ALOYSIUS' COLLEGE

Year 10 Mathematics Stage 5.3 Term One Assessment

Thursday 4th April 2019

Time allowed: 45 minutes

Instructions:

- Approved calculators may be used.
- Show working where necessary
- Circle the correct letter for multiple choice questions

Name: SOLUTIONS

Teacher: Mort O'Neill Luchi

TOTAL MARKS: 50

SECTION I: Measurement

20 Marks

1. The formula for the surface area (S) of a **closed** cylinder with radius r and height h is given by:

A. $S = \pi r^2 + \pi rh$

B. $S = 2\pi r^2 + \pi rh$

C. $S = \pi r^2 + 2\pi rh$

(D.) $S = 2(\pi r^2 + \pi rh)$

$SA = 2\pi r^2 + 2\pi rh$

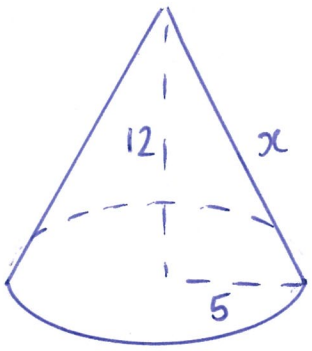
2. A cone with a base diameter of 10 m and vertical height of 12 m will have a slant height (in metres) of:

(A.) 13

B. 14

C. 15

D. $\sqrt{244}$



$x^2 = 12^2 + 5^2$

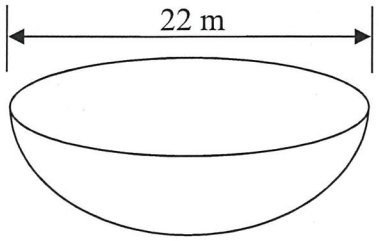
$x = 13$

3. Convert the following:

(i) 50713 m = 50.713 km ÷ 1000 **1**

(ii) 0.345 m^2 = 3 450 cm^2 × 100^2 **1**

4. Find the volume of the closed hemisphere, to the nearest cubic metre. **3**



$V = \frac{4\pi r^3}{3} \times \frac{1}{2}$

$= \frac{2}{3} \times \pi \times 11^3$

$= 2787.6...$

$\doteq 2788\ m^3$

5. Calculate the volume, to the nearest m^3, of a cone with height 11.2 m and diameter 8 m. **2**

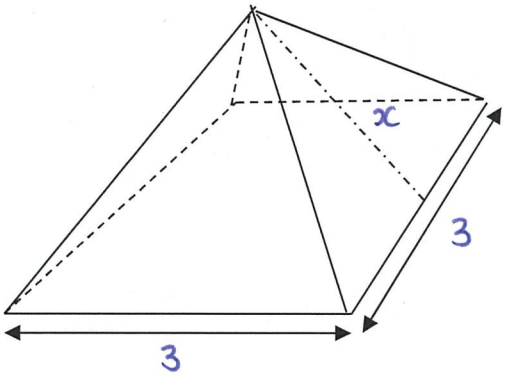
$V = \frac{\pi r^2 h}{3}$

$= \frac{\pi}{3} \times 4^2 \times 11.2$

$= 187.657...$

$= 188\ m^3$

6. A square pyramid has a total surface area of 27 cm^2 and a base length of 3 cm. The slant height of the pyramid is x cm.



(i) Form an equation, in terms of x, for the total surface area of the pyramid. **1**

$SA = 3^2 + 4 \times \frac{1}{2} \times 3 \times x$

$= 9 + 6x$

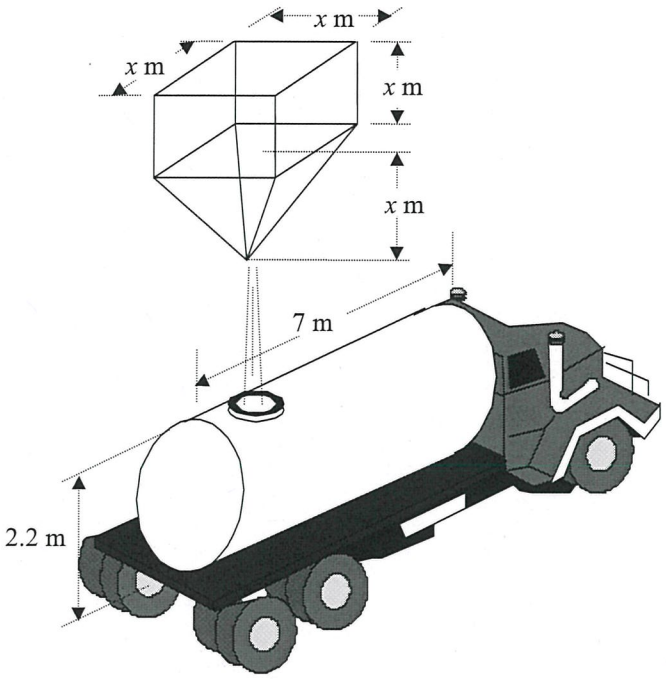
(ii) Given that the total surface area of the pyramid is 27 cm^2, solve the equation to find the value of x. **1**

$27 = 9 + 6x$

$6x = 18$

$x = 3$

7.



A `hopper' is a hollow storage container. This hopper is made by joining a cube and square pyramid, each of height x m. It is used to fill a cylindrical tank of diameter 2.2 metres and length 7 metres.

(i) Find the volume of the cylindrical tank in exact form. **2**

$V = \pi r^2 h$

$= \pi \times 1.1^2 \times 7$

$= 8.47\pi \ m^3$

(ii) Find a formula for the volume (V) of the hopper in terms of x. **2**

$V = x^3 + \frac{1}{3} \times x^2 \times x$

$= \frac{4x^3}{3}$

(iii) A full load in the hopper exactly fills the tank. Find the value of x correct to one decimal place. **2**

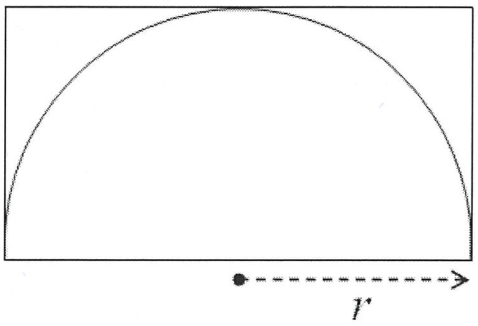
$\frac{4x^3}{3} = 8.47\pi$

$x^3 = 6.3525\pi$

$x = 2.712...$

$x \doteq 2.7 m$

8. A semi-circle of radius r is drawn in a rectangle as show below.



(i) What percentage of the area of the rectangle is the semi-circle? Answer to the nearest percentage. **2**

$A_{\triangle} = \frac{\pi r^2}{2}$

$A_{\square} = 2r \times r$

$= 2r^2$

$\therefore \left(\frac{\pi r^2}{2} \div 2r^2\right) \times 100$

$= \frac{\pi r^2}{2} \times \frac{1}{2r^2} \times 100$

$= \frac{\pi}{4} \times 100$

$\div 79\%$

(ii) How will the area of the semi-circle change if the radius is doubled? **1**

$A_{(\text{small semi circle})} = \frac{\pi r^2}{2}$

$A_{(\text{large semi circle})} = \frac{\pi (2r)^2}{2}$

$= \frac{\pi \times 4r^2}{2}$

$\therefore$ Area is multiplied by 4.

END OF SECTION I

Name:

Teacher: Mort O'Neill Luchi

SECTION II: Surds and Indices

30 Marks

1. Simplify $9^3 \times 3^2$ $= 3^6 \times 3^2 = 3^8$

A. 27^5 B. 27^6 C. 3^7 D. 3^8

2. Which of the following is **NOT** equal to $16a^6$?

A. $2a^3 \times 8a^3$ $= 16a^6$

B. $48a^7 \div 3a$ $= 16a^6$

C. $2a^3 \times 8a^2$ $= 16a^5$

D. $(4a^3)^2$ $= 16a^6$

3. Simplify the following:

(i) $\sqrt{48}$ $= \sqrt{16} \times \sqrt{3}$ $= 4\sqrt{3}$ **1**

(ii) $4\sqrt{3} + 2\sqrt{12} - \sqrt{3}$ $= 4\sqrt{3} + 2\sqrt{4}\sqrt{3} - \sqrt{3}$ $= 4\sqrt{3} + 4\sqrt{3} - \sqrt{3}$ $= 7\sqrt{3}$ **1**

(iii) $\frac{3\sqrt{60}}{\sqrt{15}}$ $= \frac{3\sqrt{4}\sqrt{15}}{\sqrt{15}}$ $= 3 \times 2$ $= 6$ **1**

4. Expand leaving your answer in simplest form

(i) $\sqrt{3}\left(2\sqrt{3}-4\right)$ **1**

$= 2\times 3 - 4\sqrt{3}$

$= 6 - 4\sqrt{3}$

(ii) $\left(4-2\sqrt{5}\right)\left(4+2\sqrt{5}\right)$ **2**

$= 4^2 - (2\sqrt{5})^2$

$= 16 - 20$

$= -4$

5. By rationalising the denominator, express $\frac{6}{3-\sqrt{7}}$ in the form $a+b\sqrt{7}$. **3**

$\frac{6}{3-\sqrt{7}} \times \frac{3+\sqrt{7}}{3+\sqrt{7}} = \frac{18+6\sqrt{7}}{9-7}$

$= \frac{18+6\sqrt{7}}{2}$

$= 9+3\sqrt{7}$

6. Find the value of k^2+4k-1 if $k=\sqrt{2}-5$ **3**

$(\sqrt{2}-5)^2 + 4(\sqrt{2}-5) - 1$

$= 2 - 10\sqrt{2} + 25 + 4\sqrt{2} - 20 - 1$

$= 6 - 6\sqrt{2}$

7. Find a pair of values for x and y such that $(x+y)(x-y)=1$ **2**

$x^2 - y^2 = 1$

$\therefore$ $x = \sqrt{3}$ $y = \sqrt{2}$ or various other combinations such as $\sqrt{4}, \sqrt{3}$ etc.

8. Fully simplify the following:

(i) $m^4 \times 2m \times 3m^5$ $= 6m^{10}$ **1**

(ii) $(3ab^2)^3$ $= 27a^3b^6$ **1**

(iii) $m^3 \div m^{-4}$ $= m^7$ **1**

(iv) $\dfrac{a^{a+2} \times a^{a+4}}{a^{2a+2}}$ $= \dfrac{a^{2a+6}}{a^{2a+2}}$ $= a^4$ **2**

9. Fully simplify $\dfrac{3\left(m^4p^{-2}\right)^{-2} \times \left(2m^{-1}p\right)^3}{16m^{-5}p^3}$ **3**

$= \dfrac{3m^{-8}p^4 \times 8m^{-3}p^3}{16m^{-5}p^3}$

$= \dfrac{3m^{-11}p^7}{2m^{-5}p^3}$ $= \dfrac{3p^4}{2m^6}$

10. Simplify $\dfrac{1-a^{-1}}{1+a^{-1}}$ the following, expressing your answers with positive powers of a. **3**

$$\left(1-\frac{1}{a}\right)\div\left(1+\frac{1}{a}\right)$$

$$=\left(\frac{a}{a}-\frac{1}{a}\right)\div\left(\frac{a}{a}+\frac{1}{a}\right)$$

$$=\frac{a-1}{a}\div\frac{a+1}{a}$$

$$=\frac{a-1}{a}\times\frac{a}{a+1}$$

$$=\frac{a-1}{a+1}$$

11. Show $\dfrac{a^2x^m-b^2x^{m+4}}{a-bx^2}$ can be simplified to $x^m\left(a+bx^2\right)$ **3**

$$=\frac{a^2x^m-b^2x^mx^4}{a-bx^2}$$

$$=\frac{x^m\left(a^2-b^2x^4\right)}{a-bx^2}$$

$$=\frac{x^m\cancel{(a-bx^2)}(a+bx^2)}{\cancel{a-bx^2}}$$

$$=x^m(a+bx^2)$$

END OF ASSESSMENT