St Aloysius' College

Year 9

Yearly Examination

2018

MATHEMATICS (5.3 course)

General Instructions

Reading time – 5 minutes

Working time – $1\frac{1}{2}$ hours

- Write using black pen only.
- Board approved calculators may be used
- All necessary working should be shown in every question in the spaces provided.
- Marks will be deducted for <u>careless</u> and <u>poorly arranged</u> work
- Examination papers must NOT be removed from the examination room.

Total marks – 80

Attempt all questions

Section I – Multiple Choice   **(25 Marks)**

- All questions are 1 mark
- Answer on the separate multiple choice answer sheet

Section II – Free Response   **(25 marks)**

Section III – Free Response   **(30 marks)**

Section I

25 marks
Attempt Questions 1 - 25

Use the multiple-choice answer sheet for Questions 1-25

1   What is 65.299 rounded correctly to two decimal places?
   (A)   65.20
   (B)   65.30
   (C)   65.300
   (D)   65.310

2   Simplify $2(x+5)-4$.
   (A) $8x$
   (B) $2x+1$
   (C) $2x+3$
   (D) $2x+6$

$2x+10-4$
$2x+6$

3   A bag contains 5 blue, 6 white and 9 black balls. If a ball is drawn at random, what is the probability that it is either black or white?
   (A) $\dfrac{7}{10}$
   (B) $\dfrac{11}{20}$
   (C) $\dfrac{3}{4}$
   (D) $\dfrac{3}{10}$

$\dfrac{9+6}{9+6+5} = \dfrac{3}{4}$

4   Eva invests $18 000 in an account paying 5.2% p.a. simple interest over a period of 3 years. What amount is Eva able to withdraw from her account at the end of 3 years?
   (A) $624
   (B) $2 808
   (C) $18 624
   (D) $20 808

$18\ 000 \times 5 \cdot 2\% \times 3$
$20\ 808$

5 The following figures are the amounts a family spent on food each week for 12 weeks.

$139 $141 $143 $143 $147 $148 $149 $150 $152 $152 $155 $158

What is the interquartile range of the data above?

(A) $9

(B) $12

(C) $10.50

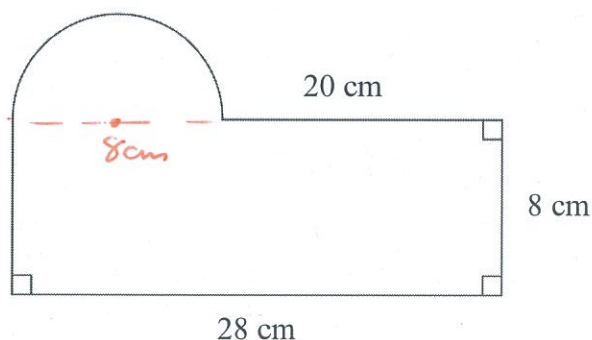(D) $11

$Q_3 = 152$

$Q_1 = 143$

$IQR = 152 - 143 = 9$

6 Scarlett earns a total of $572 during one week in which she works for 20 hours, including 4 hours paid at time-and-a-half. What is Scarlett's hourly rate of pay?

(A) $22

(B) $26

(C) $28.60

(D) $23.83

$16 + (4 \times 1.5) = 22$

$\therefore \dfrac{572}{22} = \$26$

7

20 cm

8 cm

8 cm

28 cm

NOT TO SCALE

$(28 \times 8) + \dfrac{\pi \times 4^2}{2}$

$249.13...$

What is the area of the above shape, correct to the nearest square centimetre?

(A) 237 cm^2

(B) 249 cm^2

(C) 274 cm^2

(D) 325 cm^2

8 What is the solution to $8 - x < 7$?

(A) $x < 1$

(B) $x > 1$

(C) $x > -1$

(D) $x < -1$

$-x < -8$

$x > 1$

3

9 If $E = mc^2$, which of the following statements is true?

(A) $c = \dfrac{E^2}{m}$

(B) $c = \sqrt{\dfrac{E}{m}}$

(C) $c = \pm\dfrac{\sqrt{E}}{m}$

(D) $c = \pm\sqrt{\dfrac{E}{m}}$

$\dfrac{E}{m} = c^2$

$\pm\sqrt{\dfrac{E}{m}} = c$

10 Xavier is quoted a package deal for the supply and installation of a solar panel system. He pays a deposit of \$325 followed by monthly repayments of \$256 for 2 years. What is the total amount he needs to pay for the solar panel system?
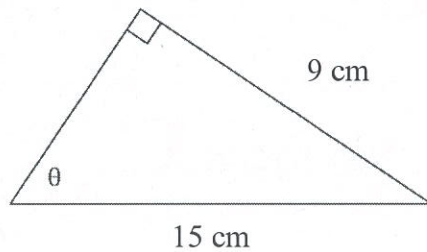
(A) \$3 990

(B) \$6 144

(C) \$6 469

(D) \$13 944

$325 + (256 \times 2 \times 12)$

6469

11

9 cm

NOT TO SCALE

θ

15 cm

What is the size of θ correct to the nearest degree?

(A) 31°

(B) 53°

(C) 37°

(D) 59°

$\sin\theta = \dfrac{9}{15}$

$\theta = \sin^{-1}\dfrac{9}{15}$

12 If three coins are tossed together, what is the probability of obtaining 3 heads?
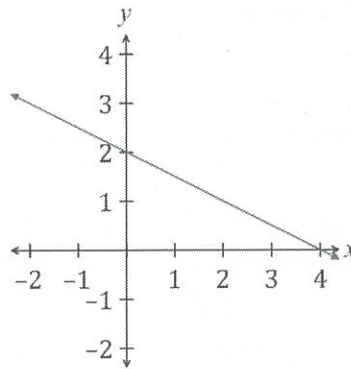
(A) $\dfrac{1}{8}$

(B) $\dfrac{1}{4}$

(C) $\dfrac{1}{3}$

(D) $\dfrac{1}{2}$

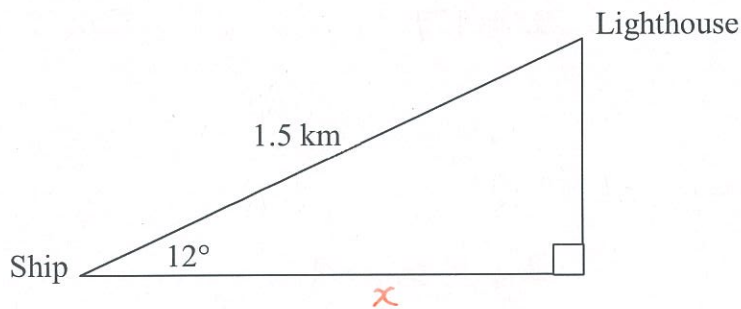$\dfrac{1}{2} \times \dfrac{1}{2} \times \dfrac{1}{2} = \dfrac{1}{8}$

13 What is the equation of the line drawn below?

(A) $y = -\dfrac{x}{2} + 2$

(B) $y = -2x + 2$

(C) $y = -\dfrac{x}{2} + 4$

(D) $y = -2x + 4$

y = mx + b

y = -½x + 2

14 A ship's captain calculates that the light beam from a lighthouse is 1.50 kilometres long and has an angle of elevation from the ship's deck of 12°.

Lighthouse

1.5 km

12°

Ship

x

NOT TO SCALE

What is the distance of the ship from the lighthouse?

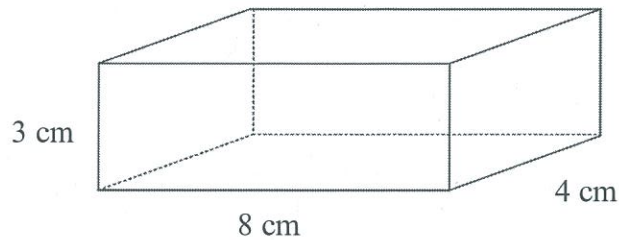(A) 0.31 km

(B) 0.32 km

(C) 1.46 km

(D) 1.47 km

cos 12 = $\dfrac{x}{1.5}$

x = 1.5 × cos 12

15 The line $3x - y - 2 = 0$ passes through which point?

(A) $(0, -2)$

(B) $(0, 2)$

(C) $(-2, 0)$

(D) $(2, 0)$

16 What is the surface area of this solid?

3 cm

4 cm

8 cm

(A)   15 cm^2

(B)   68 cm^2

(C)   96 cm^2

(D)   136 cm^2

$2(3 \times 8 + 8 \times 4 + 3 \times 4)$

17 What is the solution to $\dfrac{2x+1}{5} = 7$?

(A) $x = 17$

(B) $x = 18$

(C) $x = 68$

(D) $x = 72$

$2x + 1 = 35$
$2x = 34$
$x = 17$

18 Which line is not parallel to $x - 2y + 4 = 0$?

(A) $x - 2y + 5 = 0$

(B) $2x - 4y + 7 = 0$

(C) $2x - 5y + 1 = 0$

(D) $3x - 6y + 1 = 0$

$2y = x + 4$
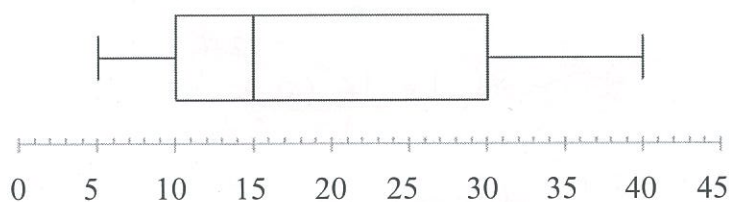$y = \dfrac{x}{2} + 2$

19 Simplify $\sqrt{20} + \sqrt{5}$

(A) $\sqrt{25}$

(B) $3\sqrt{5}$

(C) $\sqrt{5}$

(D)   5

$\sqrt{4}\sqrt{5} + \sqrt{5}$
$2\sqrt{5} + \sqrt{5}$
$3\sqrt{5}$

20 There were 120 students who completed an assessment task. The maximum mark was 40. The results are displayed in the box and whisker plot below.

Which of the following statements is **false**?

(A) 30 students scored a mark less than 10. ✓ 25% of 120 = 30

(B) The median score is 15. ✓

(C) The interquartile range is 15. ✗ 30 − 10 = 20

(D) 90 students achieved a score greater than the lower quartile. ✓ 75% of 120 = 90

21 Find the value of $\tan\theta$ in the diagram below.
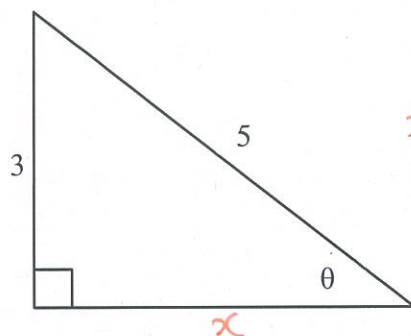
$x = \sqrt{5^2 - 3^2}$

$x = 4$

(A) $\dfrac{4}{5}$

(B) $\dfrac{5}{4}$

(C) $\dfrac{3}{4}$

(D) $\dfrac{4}{3}$

$\tan\theta = \dfrac{3}{4}$

22 Expand $(-2xy^{-2})^3$

(A) $-\dfrac{8x^3}{y^6}$

(B) $\dfrac{8x^3}{y^6}$

(C) $\dfrac{8}{x^3 y^6}$

(D) $-\dfrac{8}{x^3 y^6}$

$(-8x^3 y^{-6})$

$-\dfrac{8x^3}{y^6}$

7

23 A sum of \$4500 was invested for 2 years at 12% per annum, compounded monthly. What was amount available to withdraw at the end of 2 years?

    (A) \$5 644.80

    (B) \$4 590.45

    (C) \$5 713.81

    (D) \$5 580.00

$$4500\left(1+\frac{12\%}{12}\right)^{24}$$

$$\$5713.81$$

24 Simplify $\dfrac{1}{2}+\dfrac{1}{\sqrt{2}}$

    (A) $\dfrac{1+\sqrt{2}}{2}$

    (B) $\dfrac{2+\sqrt{2}}{\sqrt{2}}$

    (C) $\dfrac{2}{2\sqrt{2}}$

    (D) $\dfrac{1}{2\sqrt{2}}$

$$\frac{1}{2}+\frac{\sqrt{2}}{2}=\frac{1+\sqrt{2}}{2}$$

25 Fully factorise $4x^4-64y^4$

    (A) $4\left(x^4-16y^4\right)$

    (B) $4\left(x^2+4y^2\right)\left(x+2y\right)^2$

    (C) $4\left(x^2+4y^2\right)\left(x^2-4y^2\right)$

    (D) $4\left(x^2+4y^2\right)\left(x-2y\right)\left(x+2y\right)$

$$4\left(x^4-16y^4\right)$$
$$4\left(x^2-4y^2\right)\left(x^2+4y^2\right)$$
$$4\left(x-2y\right)\left(x+2y\right)\left(x^2+4y^2\right)$$

End of Section I

NAME: .... SOLUTIONS ....

TEACHER: **ADA**   **SRO/JWL**   **GON**   **IMO**

Section II   **25 Marks**

Answer the questions in the spaces provided.
All necessary working should be shown in every question.

(a)   Simplify $4x^2 - 3x + 2x^2 + 5x$ **1**

$$6x^2 + 2x$$

(b)   Simplify $25a^4 \div 5a^3$ **1**

$$5a$$

(c)   Round $\dfrac{1-\sqrt{3}}{4.5^2}$ to three significant figures.   **1**

$$-0.0362$$

(d)   Evaluate $ab^2 - b$ where $a = -1$ and $b = 2$ **1**

$$(-1)(2)^2 - (2) = -6$$

(e)   Jessica earns \$4 890 per month. How much does she earn per fortnight?   **1**

$$\frac{4890 \times 12}{26} = \$2256.92$$

(f) Factorise $4x^2y - 8xy^2$.   **1**

$$4xy(x - 2y)$$

(g) Solve $\dfrac{3x}{8} = \dfrac{3}{4}$ **1**

$$12x = 24$$
$$x = 2$$

(h) Expand and simplify $2(x^2 - x) + x(3x + 1)$ **1**

$$2x^2 - 2x + 3x^2 + x$$
$$5x^2 - x$$

(i) A square has perimeter 40cm. What is the area of the square?   **1**

$$A = 100 \text{cm}^2$$

(j) Find the **interest earned** on $2 000 invested at 7.2% p.a. compounded annually   **2**
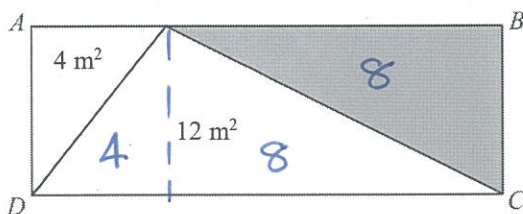for two years .

$$A = 2000(1 + 7.2\%)^2 = 2298.368$$
$$\therefore 2298.368 - 2000$$
$$\$298.37$$

(k) Find the area of the shaded region in rectangle ABCD.   **1**
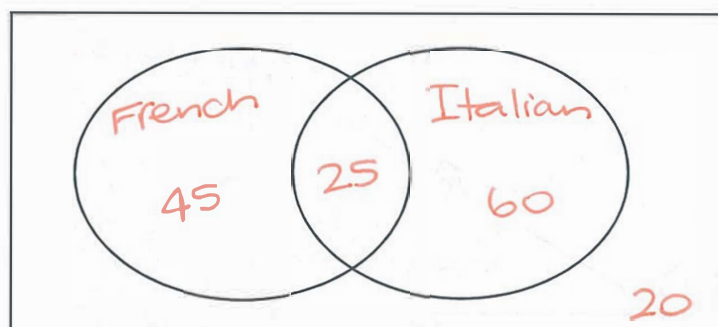

NOT TO SCALE

$$8\text{m}^2$$

10

(l)   In a year group of 150 students, 70 study French and 85 study Italian. 25 students study both languages. Complete the Venn diagram below to show the students study preferences.

1

In the diagram: French 45, overlap 25, Italian 60, outside 20.

i. What is the probability of choosing a student who studies French but not Italian?

1

$$\frac{45}{150} = \frac{3}{10}$$

ii. If a student studies Italian, what is the probability they study French?

1

$$\frac{25}{85} = \frac{5}{17}$$

(m)   Simplify $\left(8x^6\right)^{\frac{2}{3}}$

2

$$4x^4$$

(n)   The tyrannosaurus Rex lived 66 million years ago. Write this number in scientific notation.

1

$$6.6 \times 10^7$$

(o)   The sum of two numbers is 56. Twice the first number minus the second number is equal to 25. Form a pair of simultaneous equations and solve to find the two numbers.

2

$$x + y = 56 \cdots ① \quad \rightarrow \quad y = 56 - x \cdots ③$$
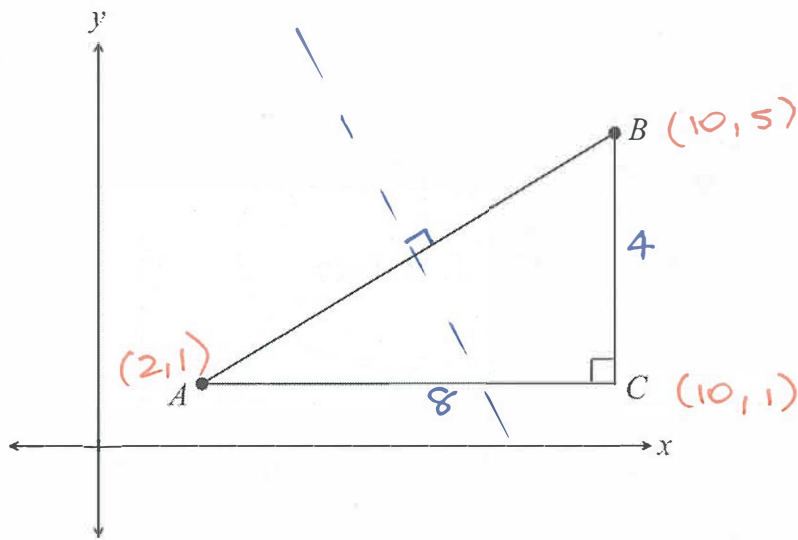$$2x - y = 25 \cdots ②$$

Sub ③ into ②

$$2x - (56 - x) = 25$$
$$2x - 56 + x = 25$$
$$3x = 81$$
$$x = 27 \qquad \therefore 27, 29$$

(p)   Two vertices of the right angled triangle are $A(2, 1)$ and $B(10, 5)$.

i. Find the coordinates of C, the third vertex of the triangle.   **1**

$$(10, 1)$$

ii. Find the coordinates of the midpoint M of the hypotenuse AB.   **1**

$$(6, 3)$$

iii. What is the gradient of line AB?   **1**

$$\frac{4}{8} = \frac{1}{2}$$

iv. Find the equation of a line perpendicular to AB passing through M.   **2**

$$m_{AB} = \frac{1}{2} \qquad \therefore \quad y = -2x + b$$

$$\text{Sub } (6,3) \qquad 3 = -2(6) + b$$

$$3 = -12 + b$$

$$b = 15$$

$$\therefore \quad y = -2x + 15$$

End of Section II

12

NAME: ......SOLUTIONS..........

TEACHER: **ADA** **SRO/JWL** **GON** **IMO**

Section III   **30 marks**

Answer the questions in the spaces provided.
All necessary working should be shown in every question.

(a) Simplify

$$\left(2 - 3\sqrt{3}\right)^2$$ 2

$$4 - 12\sqrt{3} + 27 \quad = \quad 31 - 12\sqrt{3}$$

(b) Factorise

  i. $x^2 + x - 6$ 1

$$(x+3)(x-2)$$

  ii. $2x^2 - 7x - 4$ 2

$$(2x+1)(x-4)$$

(c) Solve

  i. $x^2 - 16x + 28 = 0$ 2

$$(x-2)(x-14) = 0$$
$$\therefore x = 2, \quad x = 14$$

  ii. $x^2 = 4x$ 2

$$x^2 - 4x = 0$$
$$x(x-4) = 0 \qquad \therefore x = 0, \quad x = 4$$

  iii. $x - 3 = \dfrac{10}{x}$ 3

$$x^2 - 3x = 10$$
$$x^2 - 3x - 10 = 0$$
$$(x+2)(x-5) = 0$$
$$\therefore x = -2, \quad x = 5$$

(d) The height of a cylinder is the same as the circumference of its base. What is the **exact volume** of the cylinder if the height is 18cm?   **3**

$h = 18$ $h = 2\pi r$

$18 = 2\pi r$

$\dfrac{18}{2\pi} = r$

$r = \dfrac{9}{\pi}$
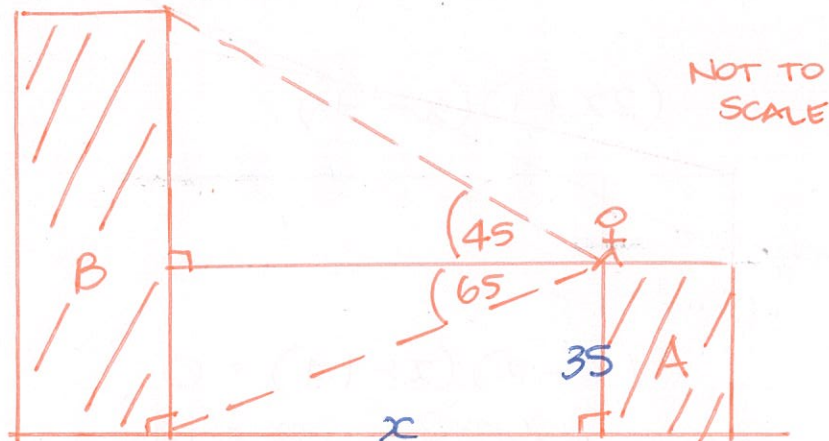
$\therefore V = \pi r^2 h$

$= \pi \left(\dfrac{9}{\pi}\right)^2 \cdot 18$ $\therefore V = \dfrac{1458}{\pi} \ cm^3$

(e) Lily is standing on the top of a Building *A* and measures the angle of depression to the base of another Building *B* to be 65°. She also measures the angle of elevation to the top of the Building *B* to be 45°. Building *A* is 35 metres tall.

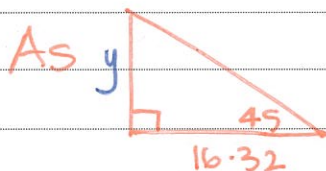i. Draw a diagram showing the above information.   **2**

NOT TO SCALE

ii. Find the distance between the buildings correct to 2 decimal places.   **1**

$\tan 65 = \dfrac{35}{x}$

$x = 35 \div \tan 65$ $16.32m$

iii. Calculate the height of Building *B* correct to 2 decimal places.   **1**

As   Isosceles triangle.

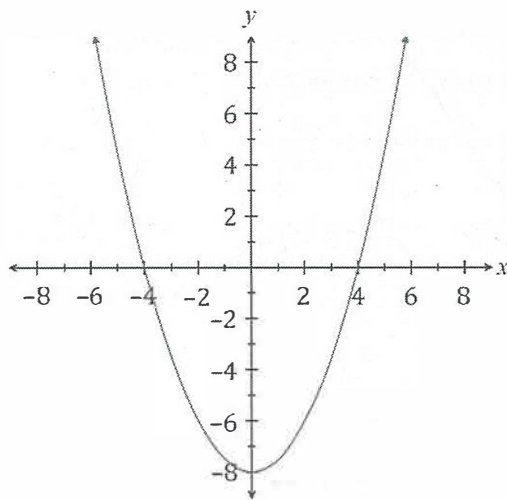$\therefore y = 16.32$

$\therefore 16.32 + 35 = 51.32m$

14

(f) James could not remember his scores from 5 Mathematics tests that were all **2**
marked out of 100. He knows his mean was 80, his median was 83 and his
mode was 84. If all his scores were whole numbers, what is the lowest mark
he could have received on any one test?

$$\frac{x + 82 + 83 + 84 + 84}{5} = 80$$

$$x + 333 = 400$$

$$x = 67$$

(g) The graph below shows the curve $y = ax^2 + c$.

$$y = ax^2 - 8$$

$$y = a(x - 4)(x + 4)$$

$$y = a(x^2 - 16)$$

∴ As y intercept is -8

$$16a = 8$$

$$a = \frac{1}{2}$$

i. Find the values of a and c. **2**

$$c = -8 \qquad a = \frac{1}{2}$$

$$∴ y = \frac{x^2 - 8}{2}$$

ii. Find the exact values of x when $y = 1$ **2**

$$1 = \frac{x^2 - 8}{2} \qquad \rightarrow \qquad x^2 = 18$$

$$9 = \frac{x^2}{2} \qquad \qquad x = \pm\sqrt{18}$$

$$x = \pm 3\sqrt{2}$$

15

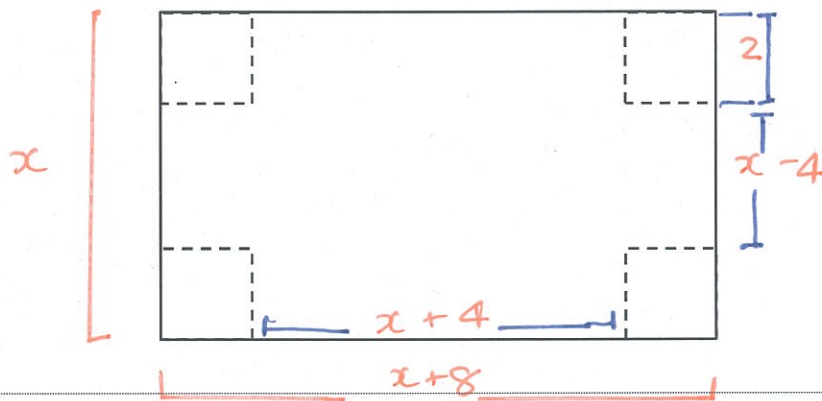(h)   Simplify $\dfrac{12}{x^2+2x} \div \dfrac{4}{x^2-4}$ **2**

$$\frac{12}{x(x+2)} \times \frac{(x-2)(x+2)}{4}$$

$$\frac{3(x-2)}{x}$$

(i)   An open box is to be formed out of a rectangular piece of cardboard whose   **3**
length is 8cm longer than its width. To form the box, squares of 2 cm are
removed from the four corners of the cardboard. If the volume of the box is
168 cm^3, what was the area of the original piece of cardboard?

$V = 168$

$168 = 2(x+4)(x-4)$

$84 = x^2 - 16$

$0 = x^2 - 100$

$0 = (x-10)(x+10)$

$\therefore x = 10 \qquad x = -10$

$(x > 0)$

$\therefore x = 10$

$\therefore 18 \times 10 = 180 cm^2$

End of Examination