Data Application Lab

Answer for Machine Learning Basis Quiz (SVM)

1 Support Vector Machine

Let the separating line be

$$\omega x + b = 0$$

let γ^i be the distance of point x^i to the line, we have

$$\gamma^i = \frac{y_i(\omega x_i + b)}{\|\omega\|}$$

let

$$\gamma = \min_i \gamma^i$$

the SVM maximization problem is

$$\begin{aligned} & max_{\gamma,\omega,b} \quad \gamma \\ \text{s.t.} \quad & \frac{y_1(\omega x_1 + b)}{\|\omega\|} \geq \gamma \\ & \frac{y_2(\omega x_2 + b)}{\|\omega\|} \geq \gamma \end{aligned}$$

which is equivalent to

$$\begin{aligned} & min_{\omega,b} \quad \frac{1}{2}\|\omega\|^2 \\ \text{s.t.} \quad & y_1(\omega x_1 + b) \geq 1 \\ & y_2(\omega x_2 + b) \geq 1 \end{aligned}$$

given $x_1 = 1, x_2 = 0, y_1 = 1, y_2 = -1$

$$\begin{aligned} & min_{\omega,b} \quad \frac{1}{2}\|\omega\|^2 \\ \text{s.t.} \quad & \omega + b \geq 1 \\ & -b \geq 1 \end{aligned}$$

the solution to the problem is $b = -1, \omega = 2$
The separating plane is $x = \frac{1}{2}$

2 Lagrange Multiplier

Given the constraint optimization problem

$$\begin{aligned} &\min_{\omega} f(\omega) \\ \text{s.t.} \quad &g_i(\omega) \leq 0 \\ &h_i(\omega) = 0 \end{aligned}$$

Let α_i, β_i be the Lagrange Multiplier and the Lagrangian is

$$\mathcal{L}(\omega, \alpha, \beta) = f(\omega) + \sum_{i=1}^{m} \alpha_i g_i(\omega) + \sum_{i=1}^{m} \beta_i h_i(\omega), \quad \alpha_i \geq 0$$

Let

$$p(\omega) = \max_{\alpha \geq 0, \beta} \mathcal{L}(\omega, \alpha, \beta)$$

Then we can set α, β to 0 if the constraint hold, and to ∞ if not hold. So we have

$$p(\omega) = \begin{cases} f(\omega) & \text{if the constraint is satisfied} \\ \infty & \text{if otherwise} \end{cases}$$

So the primal problem is equivalent to the original problem

$$p^* = \min_{\omega} max_{\alpha \geq 0, \beta} \mathcal{L}(\omega, \alpha, \beta)$$