

Problem 0 | Austin Ray | CS 221 | HW 6

P.O.a

Given

- n light bulbs
- All light bulbs initially off
- m buttons
- know subset of buttons that each button controls (T_j)
- Pressing button j toggles all lights in T_j set

Goals

- CSP w/ m vars + n constraints
- n -ary constraints allowed
- m variables, 1 for each button
 $\hookrightarrow X_j$ where $j \in \{1, \dots, m\}$
- Domain of each X_j is $\{0, 1, 2, \dots\}$
 where the number assigned is the number of times button j is pressed
- n constraints - 1 for each light bulb
 $\hookrightarrow f_i$ where $i \in \{1, \dots, n\}$

$$f_i(x) = \left[\sum_{j=1}^m [X_j \text{ if } (i \in T_j) \text{ else } 0] \right] = \text{odd \#}$$

Reasoning

- If each lightbulb toggled an odd # of times, its final state will be ON considering its initial state was OFF

P.O.b



Given

- $x_1, x_2, x_3 \in \{0, 1\}$
- $t_1(x) = x_1 \oplus x_2 = (x_1 \vee x_2) \wedge \neg(x_1 \wedge x_2)$
- $t_2(x) = x_2 \oplus x_3 = (x_2 \vee x_3) \wedge \neg(x_2 \wedge x_3)$

x_1	x_2	x_3	t_1	t_2
0	0	0	0	0
0	0	1	0	1
0	1	0	1	1
1	0	0	1	0
0	1	1	1	0
1	0	1	1	1
1	1	0	0	1
1	1	1	0	0

There are 2 consistent assignments (starred above)

PROBLEM 0 b (ii)

1. Root node
 - a. initial call to **backtrack()** with $x=\{\}$ and $w=1$
2. Parent = 1
 - a. $x = \{\}$
 - b. $w = 1$
 - c. $X1 = \{0, 1\}$
 - d. $X3 = \{0, 1\}$
 - e. $X2 = \{0, 1\}$
 - f. delta can't be calculated for any value assignments to $X1$ -> recurse **backtrack()** with $X1=0$ first then recurse **backtrack()** with $X1=1$
3. Parent = 2
 - a. $x = \{X1: 0\}$
 - b. $w = 1$
 - c. $X1 = \{0\}$
 - d. $X3 = \{0, 1\}$
 - e. $X2 = \{0, 1\}$
 - f. delta can't be calculated for any value assignments to $X3$ -> recurse **backtrack()** with $X3=0$ first then recurse **backtrack()** with $X3=1$
4. Parent = 3
 - a. $x = \{X1: 0, X3: 0\}$
 - b. $w = 1$
 - c. $X1 = \{0\}$
 - d. $X3 = \{0\}$
 - e. $X2 = \{0, 1\}$
 - f. delta is 0 for $X2=0$. delta is 1 for $X2=1$. -> recurse **backtrack()** with $X2=1$ since it is the only value where $\text{delta} \neq 0$
5. Parent = 4
 - a. $x = \{X1: 0, X3: 0, X2: 1\}$
 - b. $w = 1$
 - c. $X1 = \{0\}$
 - d. $X3 = \{0\}$
 - e. $X2 = \{1\}$
 - f. Complete assignment for x found. Update best and return answer.
6. Parent = 3
 - a. $x = \{X1: 0, X3: 1\}$
 - b. $w = 1$
 - c. $X1 = \{0\}$
 - d. $X3 = \{1\}$
 - e. $X2 = \{0, 1\}$
 - f. delta is 0 for both assignments to $X2$. Do not update best. Return.
7. Parent = 2
 - a. $x = \{X1: 1\}$

- b. $w = 1$
 - c. $X1 = \{1\}$
 - d. $X3 = \{0, 1\}$
 - e. $X2 = \{0, 1\}$
 - f. delta can't be calculated for any value assignments to $X3$ -> recurse **backtrack()** with $X3=0$ first then recurse **backtrack()** with $X3=1$
8. Parent = 7
- a. $x = \{X1: 1, X3: 0\}$
 - b. $w = 1$
 - c. $X1 = \{1\}$
 - d. $X3 = \{0\}$
 - e. $X2 = \{0, 1\}$
 - f. delta is 0 for both $X2=0$ and $X2=1$. Do not update best. Return.
9. Parent = 7
- a. $x = \{X1: 1, X3: 1\}$
 - b. $w = 1$
 - c. $X1 = \{1\}$
 - d. $X3 = \{1\}$
 - e. $X2 = \{0, 1\}$
 - f. delta is 0 for $X2=1$ and 1 for $X2=0$. -> recurse **backtrack()** with $X2=0$
10. Parent = 9
- a. $x = \{X1: 1, X3: 1, X2: 0\}$
 - b. $w = 1$
 - c. $X1 = \{1\}$
 - d. $X3 = \{1\}$
 - e. $X2 = \{0\}$
 - f. Complete assignment for x found. Update best and return answer.

backtrack() is called a total of **9 times**.

Note: If **backtrack()** was designed so that it stopped once it found one consistent assignment to the CSP, **backtrack()** would only be called 5 times.

PROBLEM 0 b (iii)

1. Root node
 - a. initial call to **backtrack()** with $x=\{\}$ and $w=1$
2. Parent = 1
 - a. $x = \{\}$
 - b. $w = 1$
 - c. $X1 = \{0, 1\}$
 - d. $X3 = \{0, 1\}$
 - e. $X2 = \{0, 1\}$
 - f. Try $X1=0$
 - i. $\Delta = 1$
 - ii. After AC-3...
 1. $\text{Domains}' = \{X1: \{0\}, X2: \{1\}, X3: \{0\}\}$
 - iii. Recurse **backtrack()** with $X1=0$
 - g. Try $X1=1$
 - i. $\Delta = 1$
 - ii. After AC-3...
 1. $\text{Domains}' = \{X1: \{1\}, X2: \{0\}, X3: \{1\}\}$
 - iii. Recurse **backtrack()** with $X1=1$
3. Parent = 2
 - a. $x = \{X1: 0\}$
 - b. $w = 1$
 - c. $X1 = \{0\}$
 - d. $X3 = \{0\}$
 - e. $X2 = \{1\}$
 - f. Try $X3=0$
 - i. $\Delta=1$
 - ii. After AC-3...
 1. Domains don't change
 - iii. Recurse **backtrack()** with $X3=0$
4. Parent = 3
 - a. $x = \{X1: 0, X3: 0\}$
 - b. $w = 1$
 - c. $X1 = \{0\}$
 - d. $X3 = \{0\}$
 - e. $X2 = \{1\}$
 - f. Try $X2=1$
 - i. $\Delta=1$
 - ii. After AC-3...
 1. Domains don't change
 - iii. Recurse **backtrack()** with $X2=1$
5. Parent = 4
 - a. $x = \{X1: 0, X3: 0, X2: 1\}$

- b. $w = 1$
 - c. $X1 = \{0\}$
 - d. $X3 = \{0\}$
 - e. $X2 = \{1\}$
 - f. Complete assignment for x found. Update best and return answer.
- 6. Parent = 2
 - a. $x = \{X1: 1\}$
 - b. $w = 1$
 - c. $X1 = \{1\}$
 - d. $X3 = \{1\}$
 - e. $X2 = \{0\}$
 - f. Try $X3=1$
 - i. $\Delta=1$
 - ii. After AC-3...
 - 1. Domains don't change
 - iii. Recurse **backtrack()** with $X3=1$
- 7. Parent = 3
 - a. $x = \{X1: 1, X3: 1\}$
 - b. $w = 1$
 - c. $X1 = \{1\}$
 - d. $X3 = \{1\}$
 - e. $X2 = \{0\}$
 - f. Try $X2=0$
 - i. $\Delta=1$
 - ii. After AC-3...
 - 1. Domains don't change
 - iii. Recurse **backtrack()** with $X2=0$
- 8. Parent = 4
 - a. $x = \{X1: 1, X3: 1, X2: 0\}$
 - b. $w = 1$
 - c. $X1 = \{1\}$
 - d. $X3 = \{1\}$
 - e. $X2 = \{0\}$
 - f. Complete assignment for x found. Update best and return answer.

backtrack() is called a total of **7 times**.

Note: If **backtrack()** was designed so that it stopped once it found one consistent assignment to the CSP, **backtrack()** would only be called 4 times.

2a | old vars: X_1, X_2, X_3 New vars: B_1, B_2, B_3, B_4 Domains:

$$X_1: \{0, 1, 2\}$$

$$X_2: \{0, 1, 2\}$$

$$X_3: \{0, 1, 2\}$$

$$B_1: \{ \text{all } [x, y] \text{ where } 0 \leq x \leq K \text{ and } 0 \leq y \leq K \}$$

 B_2 : same as B_1 's B_3 : same as B_1 's

$$B_4: \{0, \dots, K\}$$

Constraints/Factors

$$B_1[0] = 0$$

$$B_1[1] = B_1[0] + X_1$$

$$B_2[0] = B_1[1]$$

$$B_2[1] = B_2[0] + X_2$$

$$B_3[0] = B_2[1]$$

$$B_3[1] = B_3[0] + X_3$$

$$B_4 = B_3[1]$$

$$B_4 \leq K$$

{8 factors}

After constraints applied:

$$B_1 = [0, X_1]$$

$$B_2 = [X_1, X_1 + X_2]$$

$$B_3 = [X_1 + X_2, X_1 + X_2 + X_3]$$

$$B_4 = X_1 + X_2 + X_3$$

Problem 3c

I ran run_p3.py on 'profile.txt'

Final output:

Here's the best schedule:

Quarter	Units	Course
Win2016	4	CS228
Win2016	3	CS223A
Win2016	3	CS140
Spr2016	3	CS155
Spr2016	3	CS225A
Spr2016	4	CS161
Aut2016	4	CS145
Aut2016	3	CS144
Aut2016	3	CS229

Analysis:

Yup. That's exactly what I decided to take. Cool stuff! ☺

Program output:

Units: 0-10

Quarter: ['Win2016', 'Spr2016', 'Aut2016']

Taken: set(['CS221', 'CS103', 'STATS116', 'CS110', 'CS107', 'CS106B', 'CME104', 'CME106', 'CS109', 'CME100', 'CME102'])

Requests:

Request(['CS145'] [] [] 4.0)

Request(['CS228'] [] [] 4.0)

Request(['CS223A'] [] [] 4.0)

Request(['CS144'] [] [] 6.0)

Request(['CS140'] [] [] 2.0)

Request(['CS155'] [] ['CS140'] 6.0)

Request(['CS225A'] [] ['CS223A'] 3.0)

Request(['CS161'] ['Spr2016', 'Aut2016'] [] 10.0)

Request(['CS229'] ['Aut2016'] [] 6.0)

Request(['CS246'] [] ['CS145'] 10.0)

Found 24 optimal assignments with weight 829440.000000 in 568611 operations

First assignment took 112 operations

829440.0

('CS161', 'Spr2016') = 4

(Request(['CS246'] [] ['CS145'] 10.0), 'Spr2016') = None

('CS145', 'Win2016') = 0


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(Request{['CS228'] [] [] 4.0}, 'Aut2016') = None
('CS161', 'Win2016') = 0
(Request{['CS161'] ['Spr2016', 'Aut2016'] [] 10.0}, 'Spr2016') = CS161
('CS223A', 'Win2016') = 3
('sum', 'Win2016_units', 1) = (0, 4)
(Request{['CS145'] [] [] 4.0}, 'Win2016') = None
('CS161', 'Aut2016') = 0
('sum', 'Win2016_units', 3) = (7, 7)
('sum', 'Spr2016_units', 10) = 10
('sum', 'Aut2016_units', 4) = (7, 7)
('sum', 'Win2016_units', 5) = (10, 10)
('or', ((Request{['CS246'] [] ['CS145'] 10.0}, 'Spr2016'), 'CS145'), 'aggregated') =
False
('CS246', 'Spr2016') = 0
('sum', 'Aut2016_units', 6) = (7, 7)
('sum', 'Win2016_units', 7) = (10, 10)
('or', ((Request{['CS155'] [] ['CS140'] 6.0}, 'Win2016'), 'CS140'), 'aggregated') =
False
('sum', 'Aut2016_units', 0) = (0, 4)
('or', ((Request{['CS246'] [] ['CS145'] 10.0}, 'Aut2016'), 'CS145'), 1) = no
('sum', 'Win2016_units', 9) = (10, 10)
('CS223A', 'Spr2016') = 0
('CS140', 'Win2016') = 3
('sum', 'Aut2016_units', 2) = (4, 4)
('or', ((Request{['CS155'] [] ['CS140'] 6.0}, 'Aut2016'), 'CS140'), 'aggregated') = True
('CS155', 'Aut2016') = 0
('CS144', 'Spr2016') = 0
(Request{['CS155'] [] ['CS140'] 6.0}, 'Win2016') = None
('sum', 'Spr2016_units', 8) = (10, 10)
(Request{['CS246'] [] ['CS145'] 10.0}, 'Win2016') = None
('or', ((Request{['CS225A'] [] ['CS223A'] 3.0}, 'Win2016'), 'CS223A'), 'aggregated') =
False
('or', ((Request{['CS155'] [] ['CS140'] 6.0}, 'Aut2016'), 'CS140'), 1) = prev
('or', ((Request{['CS246'] [] ['CS145'] 10.0}, 'Aut2016'), 'CS145'), 'aggregated') =
False
('sum', 'Aut2016_units', 8) = (7, 10)
('CS140', 'Spr2016') = 0
(Request{['CS140'] [] [] 2.0}, 'Aut2016') = None
('sum', 'Spr2016_units', 4) = (0, 0)
(Request{['CS161'] ['Spr2016', 'Aut2016'] [] 10.0}, 'Aut2016') = None
('CS228', 'Aut2016') = 0
('sum', 'Spr2016_units', 6) = (3, 6)
('or', ((Request{['CS155'] [] ['CS140'] 6.0}, 'Spr2016'), 'CS140'), 0) = equals
('or', ((Request{['CS225A'] [] ['CS223A'] 3.0}, 'Spr2016'), 'CS223A'), 0) = equals
(Request{['CS144'] [] [] 6.0}, 'Win2016') = None
('sum', 'Spr2016_units', 0) = (0, 0)

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('CS246', 'Aut2016') = 0
('or', ((Request[['CS225A'] [] ['CS223A'] 3.0], 'Aut2016'), 'CS223A'), 1) = prev
('CS144', 'Aut2016') = 3
('sum', 'Spr2016_units', 2) = (0, 0)
(Request[['CS145'] [] [] 4.0], 'Spr2016') = None
(Request[['CS225A'] [] ['CS223A'] 3.0], 'Win2016') = None
('CS145', 'Spr2016') = 0
(Request[['CS140'] [] [] 2.0], 'Win2016') = CS140
('CS223A', 'Aut2016') = 0
('or', ((Request[['CS246'] [] ['CS145'] 10.0], 'Aut2016'), 'CS145'), 0) = no
(Request[['CS228'] [] [] 4.0], 'Spr2016') = None
('sum', 'Aut2016_units', 10) = 10
('CS228', 'Spr2016') = 0
(Request[['CS225A'] [] ['CS223A'] 3.0], 'Spr2016') = CS225A
(Request[['CS155'] [] ['CS140'] 6.0], 'Spr2016') = CS155
('CS144', 'Win2016') = 0
(Request[['CS223A'] [] [] 4.0], 'Aut2016') = None
('CS225A', 'Win2016') = 0
(Request[['CS144'] [] [] 6.0], 'Spr2016') = None
(Request[['CS161'] ['Spr2016', 'Aut2016'] [] 10.0], 'Win2016') = None
(Request[['CS140'] [] [] 2.0], 'Spr2016') = None
('or', ((Request[['CS155'] [] ['CS140'] 6.0], 'Spr2016'), 'CS140'), 'aggregated') = True
(Request[['CS223A'] [] [] 4.0], 'Win2016') = CS223A
('sum', 'Win2016_units', 0) = (0, 0)
('sum', 'Aut2016_units', 5) = (7, 7)
('sum', 'Win2016_units', 2) = (4, 7)
('or', ((Request[['CS225A'] [] ['CS223A'] 3.0], 'Spr2016'), 'CS223A'), 'aggregated') =
True
('sum', 'Aut2016_units', 7) = (7, 7)
('sum', 'Win2016_units', 4) = (7, 10)
(Request[['CS145'] [] [] 4.0], 'Aut2016') = CS145
('sum', 'Aut2016_units', 1) = (4, 4)
('sum', 'Win2016_units', 6) = (10, 10)
('CS246', 'Win2016') = 0
(Request[['CS229'] ['Aut2016'] [] 6.0], 'Spr2016') = None
('sum', 'Aut2016_units', 3) = (4, 7)
('CS229', 'Win2016') = 0
('sum', 'Win2016_units', 8) = (10, 10)
('CS155', 'Win2016') = 0
('sum', 'Win2016_units', 10) = 10
('CS140', 'Aut2016') = 0
(Request[['CS144'] [] [] 6.0], 'Aut2016') = CS144
(Request[['CS223A'] [] [] 4.0], 'Spr2016') = None
('CS228', 'Win2016') = 4
('or', ((Request[['CS155'] [] ['CS140'] 6.0], 'Aut2016'), 'CS140'), 0) = equals
('sum', 'Spr2016_units', 9) = (10, 10)

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('sum', 'Aut2016_units', 9) = (10, 10)
('CS155', 'Spr2016') = 3
('CS229', 'Spr2016') = 0
('or', ((Request[['CS246'] [] ['CS145'] 10.0], 'Win2016'), 'CS145'), 'aggregated') =
False
(Request[['CS229'] ['Aut2016'] [] 6.0], 'Win2016') = None
('CS229', 'Aut2016') = 3
('sum', 'Spr2016_units', 5) = (0, 3)
(Request[['CS246'] [] ['CS145'] 10.0], 'Aut2016') = None
(Request[['CS225A'] [] ['CS223A'] 3.0], 'Aut2016') = None
('sum', 'Spr2016_units', 7) = (6, 10)
(Request[['CS155'] [] ['CS140'] 6.0], 'Aut2016') = None
(Request[['CS228'] [] [] 4.0], 'Win2016') = CS228
('sum', 'Spr2016_units', 1) = (0, 0)
('CS225A', 'Spr2016') = 3
('or', ((Request[['CS225A'] [] ['CS223A'] 3.0], 'Aut2016'), 'CS223A'), 0) = equals
('CS225A', 'Aut2016') = 0
('CS145', 'Aut2016') = 4
('sum', 'Spr2016_units', 3) = (0, 0)
('or', ((Request[['CS225A'] [] ['CS223A'] 3.0], 'Aut2016'), 'CS223A'), 'aggregated') =
True
(Request[['CS229'] ['Aut2016'] [] 6.0], 'Aut2016') = CS229
('or', ((Request[['CS246'] [] ['CS145'] 10.0], 'Spr2016'), 'CS145'), 0) = no
Here's the best schedule:

```

Quarter	Units	Course
Win2016	4	CS228
Win2016	3	CS223A
Win2016	3	CS140
Spr2016	3	CS155
Spr2016	3	CS225A
Spr2016	4	CS161
Aut2016	4	CS145
Aut2016	3	CS144
Aut2016	3	CS229