Course Scheduling
Stanford CS 221 Fall 2015-2016

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1 a. $X = (x_1, \ldots, x_m)$, where $x_j \in \mathbb{Z}_{\geq 0}$ for each button $j = 1, \ldots, m$

$f_i = \left[\sum_{j=1}^{m} (x_j \, [i \in T_j]) \text{ is odd} \right]$

b. i. $\{x_1: 0, x_2: 1, x_3: 0\}$, $\{x_1: 1, x_2: 0, x_3: 1\}$ $\boxed{2}$

ii. backtrack $(\emptyset, 1, (\{0,1\}, \{0,1\}, \{0,1\}))$
choose x_1
- $v = 0$
  $\delta = 1$
  backtrack $(\{x_1: 0\}, 1, (\{0\}, \{0,1\}, \{0,1\}))$
  choose x_3
  - $v = 0$
    $\delta = 1$
    backtrack $(\{x_1: 0, x_3: 0\}, 1, (\{0\}, \{0,1\}, \{0\}))$
    choose x_2
    - $v = 0$
      $\delta = 0$
    - $v = 1$
      $\delta = 1$
      backtrack $(\{x_1: 0, x_2: 1, x_3: 0\}, 1, (\{0\}, \{1\}, \{0\}))$
  - $v = 1$
    $\delta = 1$
    backtrack $(\{x_1: 0, x_3: 1\}, 1, (\{0\}, \{0,1\}, \{1\}))$
    choose x_2
    - $v = 0$
      $\delta = 0$
    - $v = 1$
      $\delta = 0$

→

- $v = 1$
  $\delta = 1$
  backtrack$(\{x_1 : 1\}, 1, (\{1\}, \{0,1\}, \{0,1\}))$
  choose x_3
    - $v = 0$
      $\delta = 1$
      backtrack$(\{x_1 : 1, x_3 : 0\}, 1, (\{1\}, \{0,1\}, \{0\}))$
      choose x_2
        - $v = 0$
          $\delta = 0$
        - $v = 1$
          $\delta = 0$
    - $v = 1$
      $\delta = 1$
      backtrack$(\{x_1 : 1, x_3 : 1\}, 1, (\{1\}, \{0,1\}, \{1\}))$
      choose x_2
        - $v = 0$
          $\delta = 1$
          backtrack$(\{x_1 : 1, x_2 : 0, x_3 : 1\}, 1, (\{1\}, \{0\}, \{1\}))$
        - $v = 1$
          $\delta = 0$

$\boxed{9}$

iii. backtrack$(\emptyset, 1, (\{0,1\}, \{0,1\}, \{0,1\}))$
choose x_1
- $v = 0$
  $\delta = 1$
  backtrack$(\{x_1 : 0\}, 1, (\{0\}, \{1\}, \{0\}))$
  choose x_3
    - $v = 0$
      $\delta = 1$

→

backtrack($\{x_1: 0, x_3: 0\}$, 1, ($\{0\}, \{1\}, \{0\}$))
choose X_2
• $v = 1$
$\delta = 1$
backtrack($\{x_1: 0, x_2: 1, x_3: 0\}$, 1, ($\{0\}, \{1\}, \{0\}$))
• $v = 1$
$\delta = 1$
backtrack($\{X_1: 1\}$, 1, ($\{1\}, \{0\}, \{1\}$))
choose X_3
• $v = 1$
$\delta = 1$
backtrack($\{x_1: 1, x_3: 1\}$, 1, ($\{1\}, \{0\}, \{1\}$))
choose X_2
• $v = 0$
$\delta = 1$
backtrack($\{x_1: 1, x_2: 0, x_3: 1\}$, 1, ($\{1\}, \{0\}, \{1\}$))

$\boxed{7}$

2. a. ~~Initialization: $[B_1[1] = 0]$~~
Processing: $[B_i[2] = B_i[1] + X_i]$
Final Output: $[B_3[2] \leq K]$
Consistency: $[B_{i-1}[2] = B_i[1]]$

We introduce auxiliary variables
- B_1 with domain $\{(0,0), (0,1), (0,2)\}$
- B_2 with domain $\{(0,0), (0,0), (0,2), (1,1), (1,2), (1,3), (2,2), (2,3), (2,4)\}$
- B_3 with domain $\{(0,0), (0,1), (0,2), (1,1), (1,2), (1,3), (2,2), (2,3), (2,4), (3,3), (3,4), (3,5), (4,4), (4,5), (4,6)\}$

The initialization constraint can be eliminated because →

B_1's domain already constrains $B[1]$.

My scheme works similar to the OR example from lecture. First, we break down the computation of the ternary constraint into three simple steps. As a first attempt, we introduce an auxiliary variable A_i for $i = 1, 2, 3$ which represents the sum of variables $X_1, \ldots, X_i$. Then we can write

$[A_0 = 0]$

constraints enforcing a simple recurrence that updates A_i with A_{i-1}: $[A_i = A_{i-1} + X_i]$. The constraint $[A_3 \leq K]$ enforces that the sum of all the variables is less than or equal to K. To turn the ternary constraint $[A_i = A_{i-1} + X_i]$ into a binary constraint, we merge A_{i-1} and A_i into one variable, represented as one variable B_i. The variable B_i represents a pair of booleans, where $B_i[1]$ represents A_{i-1} and $B_i[2]$ represents A_i.

(Processing)

A_{i-1} is represented twice, so we need to add another binary constraint to enforce that the two are equal (Consistency). The Initialization and Final Output factors are the same as before.

3.c. The schedule produced is correct: CS347 is picked over CS224N (not offered), and DANCE46 is taken in Spr over Win to maximize units. Here's the best schedule:

Win2016	3	CS245
Win2016	4	CS246
Win2016	3	STATS315A
Spr2016	3	STATS315B
Spr2016	3	CS347
Spr2016	1	DANCE46