Deep Learning
Ex:01

1. Weight parameters

Step:01

$$z_{in} = w^T u + b$$
$$z_{out} = 6(z_{in}) = \frac{1}{1+e^{-z_{in}}}$$
$\rightarrow$ first Neuron

Second Neuron

$$T(z_{in}) = \frac{1}{1+e^{-2z_{in}}}$$

$$z_{in}' = w'^T x + b'$$

Therefore sigmoid steeper by factor [2]
$e^{-z_{in}}$ to change $e^{-2z_{in}}$

$$6(z_{in}') = \frac{1}{1+e^{-z_{in}'}}$$

$$T(z_{in}) = 6(2 \cdot z_{in})$$

$$\frac{1}{1+e^{-2z_{in}}} = 6(2 \cdot z_{in})$$

$$= 6(z_{in}')$$

$$z_{in} = w^T u + b$$

$$z_{in}' = 2 z_{in}$$
$$= 2(w^T u + b)$$

equating $z_{in}' = w'^T u + b'$

Therefore

$$w'^T u + b' = 2w^T u + 2b$$

$$\boxed{w' = 2w} \quad \boxed{b' = 2b}$$

steeper by factor 2 equivalent
multiplying weight and bias by 2

2. XOR Problem

Given

First layer weights
$W = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, bias $b = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

2nd layer weight and bias
$U = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ $c = -0.5$

Hidden layer

$$z_1 = w_{11} \cdot x_1 + w_{12} \cdot x_2 + b_1$$
$$z_2 = w_{21} \cdot x_1 + w_{22} \cdot x_2 + b_2$$

$$\overline{z_1} = 1 \cdot x_1 + 1 \cdot x_2 + 0 = \boxed{x_1 + x_2} \text{①}$$

Put the value $W = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, $b = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

$$z_2 = 1 \cdot x_1 + 1 \cdot x_2 - 1$$
$$= \boxed{x_1 + x_2 - 1} \text{②}$$

ReLU, $h_1 = \max(0, z_1)$ ③
$h_2 = \max(0, z_2)$ ④

XOR Table		
x_1	x_2	XOR
0	0	0
0	1	1
1	0	1
1	1	0

Output layer

$$f_{in} = U_1 \cdot h_1 + U_2 \cdot h_2 + c$$

$$\boxed{f_{in} = 1 \cdot h_1 + (-2) \cdot h_2 - 0.5}$$

⑤

Now make the calculation

XOR

input	
$x_1 = 0$	$z_1 = 0$
$x_2 = 0$	$z_2 = -1$

Fill the i/p value to all our equation

$I - V$

$f(fin) = \begin{cases} 1, & fin \geq 0 \\ 0 & othn \end{cases}$

$h_1 = 0$
$h_2 = 0$

$fin = -0.5$

output = 0

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i/p	
$x_1 = 0$	$z_1 = 1$
$x_2 = 1$	$z_2 = 0$
	$h_1 = 1$
	$h_2 = 0$

$f(fin) = \begin{cases} 1, & fin \geq 0 \\ 0 & othn \end{cases}$

$fin = 0.5$

Output = 1

---

i/p
$x_1 = 1$
$x_2 = 0$

$T \, fin = \begin{cases} 1, & fin \geq 0 \\ 0 & oth \end{cases}$

$z_1 = 1$
$z_2 = 0$
$h_1 = 1$
$h_2 = 0$

$fin = 0.5$

Output = 1

---

i/p
$x_1 = 1$
$x_2 = 1$

$z_1 = 2$
$z_2 = 1$
$h_1 = 2$
$h_2 = 1$

$fin = -0.5$

Output = 0

Matches XOR Logic

OR

choose parameters

$W = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} ; \quad b = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$U = \begin{bmatrix} 1 \\ 1 \end{bmatrix} ; \quad C = -0.5$

Touth Pable

x_1	x_2	OR
0	0	0
0	1	1
1	0	1
1	1	1

hidden Layer

$z_1 = x_1 , \quad z_2 = x_2$

$h_1 = max(0, x_1), \quad h_2 = max(0, x_2)$

output Layer

$fin = h_1 + h_2 - 0.5$

i/p		
$x_1 = 0$	$h_1 = 0$	output = 0
$x_2 = 0$	$h_2 = 0$	
	$fin = -0.5$	

$J(fin) = \begin{cases} 1, & fin \geq 0 \\ 0 & othn \end{cases}$

i/p	
$x_1 = 0$	$h_1 = 0; \quad h_2 = 1$
$x_2 = 1$	$fin = 0.5 ; \quad$ output = 1

i/p	
$x_1 = 1$	$h_1 = 1; \quad h_2 = 0$
$x_2 = 0$	$fin = 0.5 ; \quad$ output = 1

i/p	
$x_1 = 1$	$h_1 = 1; \quad h_2 = 1$
$x_2 = 1$	$fin = 1.5 ; \quad$ output = 1

Matches OR logic

AND

Choose parameters

$$W = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} ; \quad b = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 \\ 1 \end{bmatrix} ; \quad c = -1.5$$

Truth Table

x_1	x_2	AND
0	0	0
0	1	0
1	0	0
1	1	1

hidden Layer

Same as OR

$h_1 = x_1$, $h_2 = x_2$

Output Layer

$fin = h_1 + h_2 - 1.5$

i/P
$x_1 = 0$
$x_2 = 0$

$h_1 = 0 ; h_2 = 0$
$fin = -1.5 ;$ output $= 0$

$$T(fin) = \begin{cases} 1 , & fin \geq 0 \\ 0 , & other \end{cases}$$

i/P
$x_1 = 0$
$x_2 = 1$

$h_1 = 0 ; h_2 = 1$
$fin = -0.5 ;$ output $= 0$

i/P $x_1 = 1$
$x_2 = 0$

$h_1 = 1 ; h_2 = 0$
$fin = -0.5 ;$ output $= 0$

i/P $x_1 = 1$
$x_2 = 1$

$h_1 = 1 ; h_2 = 1$
$fin = 0.5 ;$ output $= 1$

Matches AND Logic