Derivation of the classifier weight α_m Considering the optimization problem

$$\alpha_m = \arg\min_{\alpha \in \mathbb{R}} \sum_{i=1}^{n} w_i^{(m)} \exp(-2\alpha\, \mathbb{I}[h(x_i) \neq y_i]) \,.$$

Step 1: Weighted error Define the weighted error of the weak classifier at iteration m as

$$\varepsilon_m = \sum_{i=1}^{n} w_i^{(m)} \mathbb{I}[h(x_i) \neq y_i].$$

Assuming the weights are normalized,

$$\sum_{i=1}^{n} w_i^{(m)} = 1.$$

Step 2: Split the objective For correctly classified samples, $\mathbb{I}[h(x_i) = y_i] = 0$, and for misclassified samples, $\mathbb{I}[h(x_i) \neq y_i] = 1$. The objective function becomes

$$\begin{aligned} J(\alpha) &= \sum_{i:h(x_i)=y_i} w_i^{(m)} + \sum_{i:h(x_i)\neq y_i} w_i^{(m)} e^{-2\alpha} \\ &= (1 - \varepsilon_m) + \varepsilon_m e^{-2\alpha}. \end{aligned}$$

Step 3: Equivalent exponential form An equivalent form of the objective (up to a constant factor) is

$$\tilde{J}(\alpha) = (1 - \varepsilon_m)e^{\alpha} + \varepsilon_m e^{-\alpha}.$$

Step 4: Optimization Taking the derivative with respect to α and setting it to zero,

$$\frac{d\tilde{J}}{d\alpha} = (1 - \varepsilon_m)e^{\alpha} - \varepsilon_m e^{-\alpha} = 0.$$

This yields

$$(1 - \varepsilon_m)e^{\alpha} = \varepsilon_m e^{-\alpha}.$$

Step 5: Solve for α_m

$$e^{2\alpha_m} = \frac{1 - \varepsilon_m}{\varepsilon_m},$$

and therefore

$$\boxed{\alpha_m = \frac{1}{2} \ln\left(\frac{1 - \varepsilon_m}{\varepsilon_m}\right)}.$$