# What We Do (not?) Know About Quantization

Sergey Salishev, Jan Akhremchik

# How Engineers See Quantization

$$f = q + r,$$

then

$$q = f - r$$

- Here $f$ is analog value, $q$ is quantized value, $r$ is quantization noise (residue, error)
- In some systems while error is small everything works OK
- If error is critically large the system breaks down
- How large?
  - It is empirically checked

# The Big Question

- Consider discrete feedforward networks
    - Digital Artificial NN
    - Digital Circuits, e.g. multiplier
- Artificial NN usually tolerate some noise, while Digital Circuits do not
- What is the difference?

# Information Theory View on NN

- Latent space → External Encoder → NN Decoder → Latent Space → NN Encoder → NN Decoder → Latent Space
  - E.g. Meaning → Human → English text → NN Decoder → Meaning → NN Encoder → French text
- NN Decoder and Encoder are Shannon Channels
  - Assume External Encoder is lossless
  - As long as $R < C$ there is the code (Weights?) with ~0% probability of data decoding error
    - Here $R$ is transfer rate, $C$ is channel capacity
  - Moreover, almost any random code is good enough (2nd Shannon Theorem)
- $R = H(L)$, $L$ is latent space, $H$ is Entropy (1st Shannon Theorem)
- Why no one uses random codes?
  - Latent space is unknown
  - Overfitting to training data
  - Exponential worst case training complexity
- Still lots of randomness
  - Random weight initialization
  - Random training permutation

# Why Does Gradient Descent Even Work on NN?

$$y = f_q(\Theta, x)$$
$$\Theta = g(x_0, y_0)$$

From information theory $f_q$ is a purely discrete object, BUT we also expect robustness of training and stability
We ignore generalization for now

$$y + \Delta y = f(\Theta + \Delta\Theta, x + \Delta x)$$
$$\Theta + \Delta\Theta = g(x_0 + \Delta x_0, y_0 + \Delta y_0)$$

If $\Delta x_0, \Delta y_0, \Delta \mathrm{x} \ll 1$ then $\Delta \mathrm{y} \ll 1$

With limit conditions this means Limited Variations
Can be replaced with Differentiability almost everywhere (stronger)

$$f_a(x) = f_q(x) + r(x)$$

Here $r(x)$is continuous differentiable regularization term

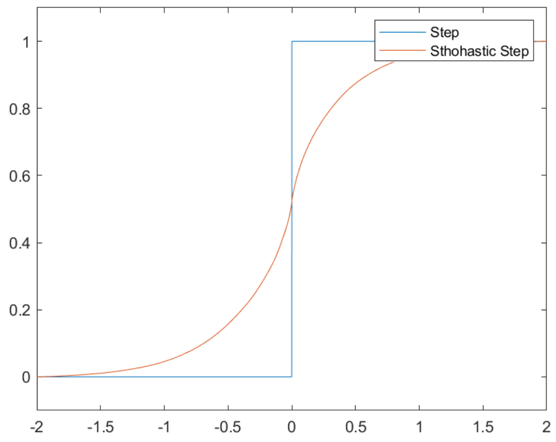
# Stochastic Regularization

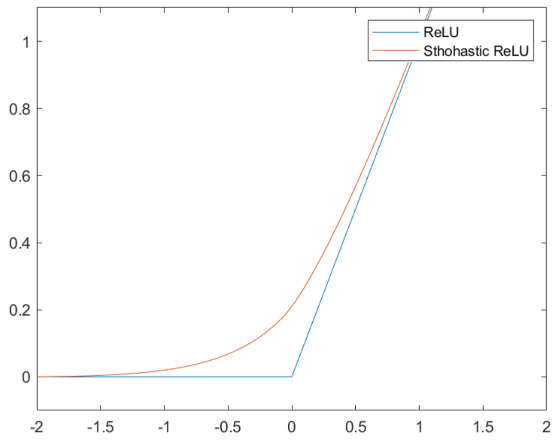
- What $r(x)$ term might be?
- An example

$$f_1(x) = c\int_{-\infty}^{x} E\left(\frac{f_0(t+n) - f_0(t)}{n}\right) dt\,, \qquad n \sim N(0,1)$$

Here $f_0$ is discrete nonlinearity (step, ReLU, etc)

- Replace $f_0 \rightarrow f_1$
- Noisy training + Computation errors + Mini Batches + SGD = Some Stochastic Regularization



# Analog Meets Digital

- Analog NN have infinite capacity (good)
  - Infinitely complex digital implementation (bad)
- True quantization problem
  - Select discrete NN weights to make $C = R + \varepsilon$, $\varepsilon$ is small
  - Means lossless quantization as Latent space is not changed
- How to change analog channel capacity?
  - Just add some noise (3rd Shannon Theorem)

# Back to Engineering

$$q = f \; - \alpha r, \qquad E|r|^2 = 1$$

- Just optimize $\alpha$ with SGD along with other NN parameters

$$I_\alpha = \log_2 \frac{E|f|^2}{\alpha|r|^2} = \log_2 E|f|^2 - \log_2 \alpha$$

$I_\alpha$ is Shannon SNR in bits

- If $I_\alpha > 0$ (some signal left) it is quantization
- If $I_\alpha = 0$ (no signal) it is pruning

# Open Questions

- Can we deduce differentiability almost everywhere by formalizing generalization?
- Is there a closed form for stochastic regularization depending on the noise distribution?
- What are sufficient conditions of convergence of differentiable stochastic quantization?
- What are sufficient conditions of convergence of stochastic approximation of stochastic quantization with quantization noise only?