Question 7:

a) Exercise 6.1.5 sections b - d

b) $$\frac{13 \times C_4^3 \times 12\ C_4^1 \times 11 \times C_4^1}{C_{52}^5}$$

c) $$\frac{C_4^1 \times C_{13}^5}{C_{52}^5}$$

d) $$\frac{13 \times C_4^2 \times 12 \times C_4^1 \times 11 \cdot C_4^1 \times 10 \times C_4^1}{C_{52}^5} = 0.0081$$

b) Exercise 6.2.4 sections a - d

a) $$\frac{1 - \binom{39}{5}}{\binom{52}{5}}$$

b) $$1 - \left(\frac{\binom{13}{5} \cdot 4^5}{\binom{52}{5}}\right)$$

c) 13 ways select club
13 ways select spade

$$P(C) = P(S) = \frac{13 \cdot C\binom{4}{39}}{C\binom{5}{52}}$$

$$P(C \cup S) = \frac{2 \cdot 13\ C_{39}^4 - 13 \cdot 13 \cdot C_{26}^3}{C\binom{5}{52}}$$

d) $$\frac{1 - C\binom{5}{26}}{C\binom{5}{52}}$$

Question 8:

a) Exercise 6.3.2, sections a - e

a) P(A) b falls in the middle

$$\frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} = \frac{1}{7}$$

P(B) c appears to the right of b.

$$P(B) = \frac{1}{2}$$

P(C) "def" occur together in that order

$$\frac{5 \times 4 \times 3 \times 2 \times 1}{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} = \frac{1}{42}$$

b) Exercise 6.3.6, sections b, c

b) $$\left(\frac{1}{3}\right)^5 \times \left(\frac{2}{3}\right)^5$$

c) $\left(\frac{1}{\ }\right)^1$

Question 8:

a) Exercise 6.3.2, section a-e:

a) P(A) b in the middle

$$\frac{6\times5\times4\times3\times2\times1}{7\times6\times5\times4\times3\times2\times1} = \frac{1}{7}$$

P(B) c appears to the righ of b

P(c appears to the right of b) =

= P(c appears to the left of b)

it is symetry.

$P(B) = \frac{1}{2}$

P(C) The letter "def"

$$\frac{5\times4\times3\times2\times1}{7\times6\times5\times4\times3\times2\times1} = \frac{1}{42}$$

b) $|A \cap C| = 3\times2\times1 + 3\times2\times1 = 12$

$\_\ \_\ \_\ \underline{b}\ \underline{d}\ \underline{e}\ \underline{f}$ $\quad 3\times2\times1$

$\underline{d}\ \underline{e}\ \underline{f}\ \underline{b}\ \_\ \_\ \_$ $\quad 3\times\times2\times1$

$|C| = 5\times4\times3\times2\times1$

$$P(A|C) = \frac{|A \cap C|}{|C|} = \frac{12}{5\times4\times3\times2\times1} = \frac{1}{10}$$

c) $|B \cap C| = \frac{1}{2}|C|$

$P(B \cap C) = \frac{1}{2}$

d) $|A \cap B| = \frac{1}{2}(6 \times 5 \times 4 \times 3 \times 2 \times 1)$

$|B| = \frac{1}{2}(7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1)$

$P(A|B) = \frac{|A \cap B|}{|B|} = \frac{1}{7}$

e) $P(B|C) = P(B) = \frac{1}{2}$

B and C are independent

$P(A|B) = P(A) = \frac{1}{9}$

A and B are independent

$P(A|C) = \frac{1}{10} \neq P(A) = \frac{1}{9}$

A and C are not independent

$7\times6\times5\times4\times3\times2\times1$

P(B) c appears to the right of b

$P(B) = \frac{1}{2}$

P(c) "def" occur together in that ord

$$\frac{5\times4\times3\times2\times1}{7\times6\times5\times4\times3\times2\times1} = \frac{1}{42}$$

b) Exercise 6.3.6, sections b, c

b) $\left(\frac{1}{3}\right)^5 \times \left(\frac{2}{3}\right)^5$

c) $\left(\frac{1}{3}\right)^1 \times \left(\frac{2}{3}\right)^9$

c) Exercise 6.4.2, section a

a) $$\frac{\left(\frac{1}{6}\right)^{6} \cdot \left(\frac{1}{2}\right)}{\left(\frac{1}{6}\right)^{6} \cdot \left(\frac{1}{2}\right) + (.15)^{4} \cdot (.25)^{2} \cdot \left(\frac{1}{2}\right)}$$

Question 9:

a) Exercise 6.5.2, sections a, b

a) $\{0, 1, 2, 3, 4\}$

b) $\left(0, \binom{48}{5} / \binom{52}{5}\right), \left(1, 4\binom{48}{4} / \binom{52}{5}\right), \left(2, \binom{4}{2}\binom{48}{3} / \binom{52}{5}\right), \left(3, 4\binom{48}{2} / \binom{52}{5}\right), \left(4, 48 / \binom{52}{5}\right)$

b) Exercise 6.6.1 section a

a) $G = C\binom{2}{2} = 21$

$$P(G=2) = \frac{21}{45} = \frac{7}{15}$$

$$E[G] = 2 \cdot \left(\frac{7}{15}\right) + 1 \cdot \left(\frac{7}{15}\right) + 0 \times \left(\frac{1}{15}\right) =$$

$$= \frac{21}{15} = \frac{7}{5}$$

c) Exercise 6.6.4, sections a, b

a) $E(x) = \frac{1}{6} \times 1^2 + \frac{1}{6} \times 2^2 + \frac{1}{6} \times 3^2 + \frac{1}{6} \times 4^2 + \frac{1}{6} \times 5^2 + $

$+ \frac{1}{6} \times 6^2$

$= \frac{1}{6}(1 + 4 + 9 + 16 + 25 + 36) = \frac{91}{6}$

b) $E(x) = \binom{3}{1} \times \left(\frac{1}{2}\right)^3 \times 1^2 + \binom{3}{2} \times \left(\frac{1}{2}\right)^3 \times 2 +$

$+ \binom{3}{3} \times \left(\frac{1}{2}\right)^3 \times 3^2$

$= \frac{1}{8}(3 + 3 \times 4 + 3^2)$

$= \frac{1}{8}(24) = \frac{24}{8}$

d) Exercise 6.7.4, section a

a) One children get his own coat is probability is $\frac{9!}{10!} = \frac{1}{10}$

Expected number of children get his coat:

$$\frac{1}{10} + \frac{1}{10} + \cdots + \frac{1}{10} = \frac{1}{10} \times 10 = 1$$

Question 10:

a) Exercise 6.8.1, sections a - d

a) $\binom{100}{2}(1\%) \cdot (99\%)^{98}$

b) $1 - \binom{100}{1}(1\%)(99\%) - (99\%)^{100}$

c) expected number of circuit board with defect 1 mode:

$$\frac{1}{100} \times 1 = \frac{1}{100}$$

Because of linearity of expectation:
Expected number of circuit boards with defects with 100 mode:

$$100 \times \frac{1}{100} = 1$$

d) At least 2 have defects

$$1 - \binom{50}{1}(1\%)(99\%)^{49} - (99\%)^{50}$$

expected number:

$$50 \times \frac{1}{100} = \frac{1}{2}$$

the expected number drops because the probability of circuit boards in a batch are free of defects is 99% which is greater than the first case

b) Exercise 6.8.3, section b.

$$(0.3)^{10} + 10 \times 0.7 \times (0.3)^{9} + \binom{10}{2} \times 0.7^{2} \times (0.3)^{8} +$$

$$+ \binom{10}{3} \times 0.7^{3} \times (0.3)^{7} + \binom{10}{4} \times 0.7^{4} \times (0.3)^{6}$$

$$+ \binom{10}{5} \times 0.7^{5} \times (0.3)^{5} + \binom{10}{6} \times 0.7^{6} \times (0.3)^{4}$$