Question 7:

a) Exercise 3.1.1, sections a - g

a) $A = \{x \in \mathbb{Z} : x$ is an integer multiple of $3\}$

$B = \{x \in \mathbb{Z} : x$ is a perfect square$\}$

$C = \{4, 5, 9, 10\}$

$D = \{2, 4, 11, 14\}$

$E = \{3, 6, 9\}$

$F = \{4, 6, 16\}$

An integer x is a perfect square if there is an integer y such that $x = y^2$

a) $27 \in A$ True 27 is multiple of 3.

b) $27 \in B$ False 27 can not be perfect square

c) $100 \in B$ True $100 = 10^2$ so it is true.

d) $E \subseteq C$ or $C \subseteq E$ False they are not subset of each other

e) $E \subseteq A$ True E is subset of A.

f) $A \subset E$ false A is infinite can not be subset of E

g) $E \in A$ False E can't be element of A.

Exercise 3.1.2. a - e

$A = \{x \in \mathbb{Z} : x$ is an integer multiple of $3\}$

$B = \{x \in \mathbb{Z} : x$ is a perfect square

$C = \{4, 5, 9, 10\}$

$E = \{3, 6, 9\}$

$F = \{4, 6, 16\}$

An integer x is a perfect square if there is an integer

a) $15 \subset A$ False 15 is not proper subset of A

b) $\{15\} \subset A$ True $\{15\}$ is a proper subset of A

c) $\emptyset \subset A$ True empty set a proper subset of A.

d) $A \subseteq A$ True Every subset is a subset of itself

e) $\emptyset \in B$ False empty set can not be element of B

f) A is an infinity set True

g) B is a finite set False

h) $|E| = 3$ True cardinality $|E|$ is 3.

i) $|E| = |F|$ True cardinality is 3 for $|E|$ and $|F|$

c) Exercise 3.1.5 b, d

Express each set using set builder. Then if the set is finite, give its cardinality. Otherwise, indicate that the set is infinite.

b) $\{3, 6, 9, 12, \ldots\}$
the cardinality is infinity because of $\{x \in N : x \text{ is an integer multiple of } 3\}$

d) $\{0, 10, 20, 30, \ldots, 1000\}$

the cardinality is 101

$\{x \in N : x \text{ is an integer multiple of } 10\}$

d) Exercise 3.2.1 a-k

Let $X = \{1, \{1\}, \{1,2\}, 2, \{3\}, 4\}$. Which statement are true?

a) $2 \in X$ True because we have element 2.

b) $\{2\} \subseteq X$ True $2 \in X$, so $\{2\} \subseteq X$

c) $\{2\} \in X$ False $2 \in X$, not $\{2\}$

d) $3 \in X$ False $\{3\} \in X$, not 3

e) $\{1,2\} \in X$ True we have elements 1, 2

f) $\{1,2\} \subseteq X$ True we have elements $\{1,2\} \subseteq X$ can be subset

g) $\{2,4\} \subseteq X$ True we have 2, 4 can be subset

h) $\{2,4\} \in X$ False $\{2,4\}$ is not subset of X

i) $\{2,3\} \subseteq X$ False can't be subset of X

j) $\{2,3\} \in X$ False no 2, 3 elements.

k) $|X| = 7$ False cardinality of $|X|$ is 6.

Question 8:

Exercise 3.2.4 b.

b) Let $A = \{1,2,3\}$. What is $\{X \in P(A) : 2 \in X\}$?

$P(A) = 2^3 = 8$

$A = \{1,2,3\}$

Size 0 $\{\emptyset,$

Size 1 $\{1\}, \{2\}, \{3\},$

Size 2 $\{1,2\}, \{1,3\}, \{2,3\},$

Size 3 $\{1,2,3\}\} = P(A)$

$\{X \in P(A) : 2 \in X\} = \{\{2\}, \{1,2\}, \{2,3\}, \{1,2,3\}\}$

Question 9:

a) Exercise 3.3.1 c-e.

Define the sets A, B, C and D as follows:

$A = \{-3, 0, 1, 4, 17\}$

$B = \{-12, -5, 1, 4, 6\}$

$C = \{x \in \mathbb{Z} : x \text{ is odd}\}$

$D = \{x \in \mathbb{Z} : x \text{ is positive}\}$

c) $A \cap C$

$\{-3, 1, 17\}$

d) $A \cup (B \cap C)$

$\{-5, -3, 0, 1, 4, 17\}$

e) $A \cap B \cap C$

$\{1\}$

b) Exercise 3.3.3 a, b, e, f.

Use the following definitions to express each union or intersection given. You can use roster or set builder notation in your responses, but no set operations.

- $A_i = \{i^0, i^1, i^2\}$ (Recall that for any number x, $x^0 = 1$)
- $B_i = \{x \in \mathbb{R} : -i < x \leq 1/i\}$
- $C_i = \{x \in \mathbb{R} : -1/i \leq x \leq 1/i$

a) $\bigcap_{i=2}^{5} A_i$

$= A_2 \cap A_3 \cap A_4 \cap A_5.$

$A_2 = \{2^0, 2^1, 2^2\} = \{1, 2, 4\}$

$A_3 = \{3^0, 3^1, 3^2\} = \{1, 3, 9\}$

$A_4 = \{4^0, 4^1, 4^2\} = \{1, 4, 16\}$

$A_5 = \{5^0, 5^1, 5^2\} = \{1, 5, 25\}$

$\bigcap_{i=2}^{5} A_i = \{1\}$

b) $\bigcup_{i=2}^{5} A_i$

$= A_2 \cup A_3 \cup A_4 \cup A_5$

$= \{1, 2, 3, 4, 5, 16, 25\}$

e) $\bigcap_{i=1}^{100} C_i$

$= C_1 \cap C_2 \cap C_3 \cap \dots \cap C_{99} \cap C_{100}$

$= \{x \in R : -\frac{1}{100} \leq x \leq \frac{1}{100}\}$

f) $\bigcup_{i=1}^{100} C_i$

$C_i \cup C_2 \cup C_3 \cup \dots C_{99} \cup C_{100}$

$= \{x \in R : -1 \leq x \leq 1\}$

c) Exercise 3.3.4 b, d

b) Use the set definition $A = \{a, b\}$ and $B = \{b, c\}$ to express each set below. Use roster notation in your solutions.

$P(A \cup B)$

$\{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{c, b\}, \{a, b, c\}\}$

d) $P(A) \cup P(B)$

$P(A) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

$P(B) = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$

$P(A) \cup P(B) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$.

Question 10:

a) Exercise 3.5.1 b, c

The sets A, B and C are defined as follows:

$A = \{tall, grande, venti\}$

$B = \{foam, no\text{-}foam\}$

$C = \{non\text{-}fat, whole\}$

Use the definitions for A, B, and C to answer the questions. Express the elements using tuple notation, not string notation.

b) Write an element from the set $B \times A \times C$

(foam, tall, non-fat)

c) write the set $B \times C$ using roster notation

$\{$(foam, non-fat), (foam, whole), (no-foam, non-fat), (no-foam, whole)$\}$.

b) Exercise 3.5.3 b, c, e

Indicate which of the following statements are true

b) $Z^2 \subseteq R^2$

True. The set of all pairs of integers would be a subset in the set of all pairs of real numbers.

c) $Z^2 \cap ^3 \neq \emptyset$

True. Set of pairs and set of triples does not have indentical elements.

e) For any three sets A, B and C, if $A \subseteq B$, then $A \times C \subseteq B \times C$

Suppose D. $D \cup A = B$

$D \cap A = \emptyset$

$B \times C$

$= (A \times C) \cup (D \times C)$

$A = \{1, 2\}$

$B = \{1, 2, 3\}$

$C = \{4\}$

$A \times C = \{(1,4), (2,4)\}$

$B \times C = \{(1,4), (2,4), (3,4)\}$

c) Exercise 3.5.6, sections d, e

d) $\{xy : \text{where } x \in \{0\} \cup \{0\}^2 \text{ and } y \in \{1\} \cup \{1\}^2\}$

$x \in \{0, 00\}$

$y \in \{1, 11\}$

$x \cdot y = \{01, 001, 011, 0011\}$

e) $\{xy : x \in \{aa, ab\} \text{ and } y \in \{a\} \cup \{a\}^2\}$

$x \in \{aa, ab\}$

$y \in \{a, aa\}$

$xy = \{aaa, aaaa, aba, abaa\}$

d) Exercise 3.5.7 sections c, f, g

Use the following set definions to specify each set in roster notation. Except were noted, express elements as Cartesian products as strings.

- $A = \{a\}$
- $B = \{b, c\}$
- $C = \{a, b, d\}$.

c) $(A \times B) \cup (A + C) = \{(a, b), (a, c) \cup (a, a), (a, b), (a, d)$
$= \{(a, b), (a, c), (a, a), (a, b), (a, d)\}$

f) $P(A \times B)$

$A = \{a\}$ $\quad A \times B = \{(a, b), (a, c)\}$

$B = \{b, c\}$ $\quad P(A \times B) = \{\emptyset, \{a, b\}, \{a, c\}, \{ab, ac\}$

g) $P(A) \times P(B)$. Use ordered pair notation for elements of the Cartesian product.

$P(A) = \{\emptyset, \{a\}\}$

$P(B) = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$

$P(A) \times P(B) = \{(\emptyset, \emptyset), (\emptyset, b), (\emptyset\ c), (\emptyset\ cb),$
$(a, \emptyset), (a, b), (a, c), (a, bc)\}$

Question 11

a) Exercise 3.6.2 b, c

b)

$(B \cup A) \cap (\bar{B} \cup A) = A$	Hypothesis
$(A \cup B) \cap (\bar{B} \cup A)$	Commutative law, 1
$(A \cup B) \cap (A \cup \bar{B})$	Commutative law, 2
$A \cup (B \cap \bar{B})$	Distributive law, 3
$A \cup \emptyset$	Complement law
A	

c)

$\overline{A \cap \bar{B}} = \bar{A} \cup B$	
$\overline{A \cap \bar{B}}$	Hypothesis
$\bar{A} \cup \bar{\bar{B}}$	De Morgan's law 1
$\bar{A} \cup B$	Double complement law, 2

b) Exercise 3.6.3 b, d

b) $A - (B \cap A) - A$

Suppose $A = \{a, b\}$

$B = \{a\}$, then

$A - (B \cap A) = \{a, b\} - \{a\} = \{b\}$ is not A,

so it is false

d) $(B - A) \cup A = A$

Suppose $A = \{a, b\}$

$B = \{b, c\}$, then

$(B - A) \cup A = \{c\} \cup \{a, b\} = \{c, a, b\}$

is not A, so it is false

c) Exercise 3.6.4 b, d

b) $A \cap (B - A) = \emptyset$

$A \cap (B - A)$	Hypothesis
$A \cap (B \cap \bar{A})$	Subtraction law, 1
$A \cap (\bar{A} \cap B)$	Commutative law 2
$(A \cap \bar{A}) \cap B$	Associative law, 3
$\emptyset \cap B$	Complement law, 4
$B \cap \emptyset$	Commutative law
$\emptyset$	Domination law, 6

c) $A \cup (B - A) = A \cup B$

$A \cup (B - A)$	Hypothesis
$A \cup (B \cap \bar{A})$	Subtraction law, 1
$(A \cup B) \cap (A \cup \bar{A})$	Distributive law, 2
$(A \cup B) \cap \emptyset$	Complement law, 3
$A \cup B$	Domination law, 4