Question 3:

a) Solve Exercise 8.2.2 section b.

b) $f(n) = n^3 + 3n^2 + 4$. Prove that $f = \Theta(n^3)$

Lower bound

$n^3 + 3n^2 + 4 \geq n^3$ for any n

lower bound is $c_1 = 1$

$n^3 + 3n^2 + 4 \leq 3n^3$

$2n^3 - 3n^2 - 4 \leq 0$

$(n-2)(2n^2+n+2) \leq 0$

We have $2n^2 + n + 2 > 0$

So upper bound is 3 for $n \leq 2$

We have $n^3 + 3n^2 + 4 = \Theta(n^3)$

for any $n \leq 2$. Upper bound is 3
lower bound is 1

b) Solve Exercise 8.3.5, sections a-e.

a) Start from begenning to find the number greater or equal to p, then start from end to this number to find the number smaller than p.
If the index of larger number is smaller then the index of smaller number, change their location. Otherwise stop the process
Then start from their location to do the same searching.

b) the total nummer depends on the length of sequence.
total count = n-1

c) The number of swaps depend on inputs.
The number of swaps is minimized where the negative numbers are already in the beginning of the input sequence. The number of swaps is maximized where negative numbers are at the end of the sequence and when the number of inputs is even.

d) For any inputs, the number of increments of i and/or j is $n-1$. Worst case, the a_j and a_i values are swapped each time i or j are iterated. Therefore the number of sweps is $n-1$
The lower bound of this equation, therefore, is $\Omega(n)$.

e) During one iteration of the outer loop, the inner iterates at most $n-1$ times. The total number of times the outer loop iterates is one, operations of the inner loops will satisfy outer loop
There fore, upper bound is $\Theta(n)$

Question 4:

a) Exercise 5.1.1, sections b, c

b) Strings of length 7, 8 or 9.
Characters can be special characters, digits, or letters

Let D be the set of digits,
L the set of letters,
and S the set of special characters.
The three sets are mutually disjoint,
so the total number of characters is

$$|D \cup L \cup S| = |D| + |L| + |S| = 10 + 26 + 4 = 40$$

Total of length 7, 8 or 9.

$$40^7 + 40^8 + 40^9$$

c) String of length 7, 8, 9. Characters can be special characters, digits, or letters.
The first character cannot be a Letter.

1. first character is not a letter

2. first character is a Letter

$$26 \times 40^6 + 26 \times 40^7 + 26 \times 40^8 -$$

$$- 40^7 + 40^8 + 40^9 =$$

$$= 14 \times (40^6 + 40^7 + 40^8) = 14 \times 40^6 (1 + 40 + 40^2)$$

b) Exercise 5.3.2 sections a

$\{a, b, c\}$

1st 3 choice $\underline{3}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}$

Ans: 3×2^9

c) Exercise 5.3.3 sections b, c.

b) How many license plate numbers are possible if no digit appears more than once?

D - L - L - L - L - D - D

10 - 26 - 26 - 26 - 26 - 9 - 8

$26^4 \times 10 \times 9 \times 8.$

c) How many license plate numbers are possible if no digit or letter appears more than once?

D - L - L - L - L - D - D

$10 \times 26 \times 25 \times 24 \times 23 \times 9 \times 8$

d) Exercise 5.2.3, sections a, b

Exercise 5.2.3.

a) We can one-to-one map element in B_9 to E_{10}. For $x \in B^9$ with odd number of digits of 1. We add 1 to the end for new number y. Then $y \in E_{10}$.

For $x \in B^9$ with even number of digits of 1 we add for new number y.

Then $y \in E_{10}$

We can also one-to-one map element is E_{10} to B_9

b) Since there is a bijection from b^9 to E^{10}, $|E_{10}| = |B^9| = 2^9$

Question 5:

a) Exercise 5.4.2, sections a, b.

a) How many different phone numbers are possible?

$\underline{8}\ \underline{2}\ \underline{4}\ \underset{10}{\_}\ \underset{10}{\_}\ \underset{10}{\_}\ \underset{10}{\_}$

Ans: $2 \times 10^4 =$
$= 10^4 + 10^4 = 20{,}000$

$\underline{8}\ \underline{2}\ \underline{5}\ \underset{10}{\_}\ \underset{10}{\_}\ \underset{10}{\_}\ \underset{10}{\_}$

b) How many different phone numbers are there in which the last four digits are all different?

8 2 4

$\underline{8}\ \underline{2}\ \underline{5}\ \underset{10}{\_}\ \underset{9}{\_}\ \underset{8}{\_}\ \underset{7}{\_}$ Ans: $2 \times 10 \times 9 \times 8 \times 7 = 10{,}800$

b) Exercise 5.5.3, sections a-g

a) No restrictions

10-bit $\underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}$

Ans: $2^{10} = 1024$

b) The string starts with 001

$\underline{0}\ \underline{0}\ \underline{1}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}$

Ans: $2^7 = 128$

c) The string starts with 001 or 10

$$\underline{0}\ \underline{0}\ \underline{1}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}$$

$$2^7$$

$$\underline{1}\ \underline{0}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}$$

$$2^8$$

Ans: $2^7 + 2^8$

d) The first two bits are the same as the last two bits

$$\overbrace{\underline{\ }\ \underline{\ }}^{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \overbrace{\underline{\ }\ \underline{\ }}^{2}$$

Ans: $2^8 = 256$

e) The string has exactly six 0's.

$$\binom{10}{6} = \frac{10!}{6!\,(10-6)!} = 210$$

f) The string has exactly six 0's and the first bit is 1.

$$\binom{9}{6} = \frac{9!}{6!\,(9-6)!} = 84$$

g) There is exactly one 1 in the first half and exactly three 1's in the second half

First half $\binom{5}{1} = 5$ possibilities

Second half $\binom{5}{3} = 10$ possibilities

So total possibilities $5 \times 10 = 50$

c) Exercise 5.5.5, section a.

a) We have 30 b and 35 g.

girls $C\binom{35}{10}$ possibilities

boys $C\binom{30}{10}$ possibilities

$$\binom{35}{10} \times \binom{30}{10}$$

d) Exercise 5.5.8, sections c - f.

c) How many five-card hands are made entirely of hearts and diamonds?

$$C\binom{26}{5} = \frac{26!}{(5!(26-5)!)}$$

d) How many five-card hands have four cards of the same rank?

$$13 \times 48 = 624$$

e) Ans: $\binom{13}{1} \times \binom{4}{2} \times \binom{12}{1} \times \binom{4}{3} = 13 \times 6 \times 12 \times 4 = 3744$

f) First five rank from 13 ranks $\binom{13}{5}$ within each rank there are 4 possibilities

so, $4^5 \times \binom{13}{5}$

Exercise 5.6.6, sections a, b

a) We have 100 members

44 - Demonstrators

56 - Rupudiators.

$C\binom{44}{5} \cdot C\binom{56}{5}$

b) $P(44, 2) \times P(56, 2)$

Question 6:

a) Exercise 5.7.2, sections a, b

a) opposite: we have no club

$\binom{39}{5}$

So, answer is $C\binom{52}{5} - C\binom{39}{5}$

b) opposite: no same rank

$\binom{13}{5} \times 4^5$

So, answer is $\binom{52}{5} - \binom{13}{5} \times 4^5$

b) Exercise 5.3.4, sections a, b.

a) $20 \times 19 \times 18 \times 17 \times 16$

b) $\binom{20}{4} \cdot \binom{16}{4} \cdot \binom{12}{4} \binom{8}{4} \cdot \binom{4}{4}$

Question 7:

How many one-to-one functions are there from a set with five elements to sets with the following number of elements.

a) 4 — 0 one-to-one function

b) 5 — $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$ One-to-one

c) 6 — $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 = 720$ one-to-one

d) 7 — $7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 = 2520$ one-to-one