Question 8:

What is the probability that a fair die never comes up an even number when its rolled six times?

A die has six possible outcomes: 1, 2, 3, 4, 5, 6.

1st roll : 6 ways
2nd roll : 6 ways
3rd roll: 6 ways
4th roll : 6 ways
5th roll : 6 ways
6th roll : 6 ways

Use the product rule:

$$6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 = 6^6 = 46656$$

even outcomes: 2, 4, 6.

1st roll: 3 ways
2nd roll: 3 ways
3rd roll: 3 ways
4rd roll: 3 ways
5th roll: 3 ways
6th roll: 3 ways

$$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 3^6 = 729$$

$$P = \frac{729}{46656} = \frac{1}{64} = 0.015.$$

Question 9:

What is the conditional probability that a randomly generated bit string of length four contains at least two consecutive 0s, given that the first bit is a 1?

total number of length 4 = 16

let A be the event that a randomly generated string has 1 in the first position.

let B be the event that a randomly generated string has two consecutive 0s.

$$P(A) = \frac{8}{16} = \frac{1}{2}$$

$$P(A \cap B) = \frac{3}{16}$$

. conditional probability that a randomly generated bit string of length four contains at least two consecutive 0s $= \frac{P(A \cap B)}{PA}$

$$= \frac{\frac{3}{16}}{\frac{1}{2}} = \frac{3}{8}.$$

Question 10:

Assume that the probability a child is a boy is 0.51 and that the sexes of children born into a family are independent. What is the probability that a family of five children has.

a) exactly three boys?

b) at least one boy?

c) at least one girl?

d) all children of the same sex?

e) the first child is a boy or that the last two children of the family are girls.

a) $(5, 3)$ independent born children

Chance of being a girl $1 - 0.51 = 0.49$

$C(5,3) = (0.51)^3 \, (0.49)^2 =$

$P_1 = 0.318.$

b) $1 - (0.49)^5 = 0.97$

c) $1 - (0.51)^5 = 0.965$

d) all 5 are boys $= (0.51)^5$

or

all 5 are girls $= (0.49)^5$

$(0.51)^5 + (0.49)^5 = 0.06275.$

e) $P(E \cup F) = (0.51)(0.49)^2 = 0.122$; $P(E \cup F) = 0.51 + 0.49 + 3$[illegible] $= 0.627$[illegible]

$P(E \cup F) = 0.51 + 0.49^2 + 0.122 = 0.6276$

Question 11:

a) boy and girl equally likely

boy = girl $= \frac{1}{2}$

$C(5,0)\left(\frac{1}{5}\right)^5 \left(\frac{1}{2}\right)^0$

$$\frac{5!}{0!\,(5-0)!} = \frac{5!}{5!} = 1$$

$1 \cdot \left(\frac{1}{32}\right) = \frac{1}{32}$

b) the probability of a boy is 0.51.

$C(5,0)(0.51)^0(0.49)^5 = 0.02824$

c) the probability that the i-child is a boy is $0.51 - \frac{i}{100}$.

$1 - \left(0.51 - \frac{i}{100}\right) \Rightarrow 49 + \frac{i}{100}$

$49 + \frac{i}{100}$ = i th child is a girl

For all 5 ith child to be girls.

$\left(0.49 + \frac{1}{100}\right)\left(0.49 + \frac{2}{100}\right)\left(0.49 + \frac{3}{100}\right)$

$\left(0.49 + \frac{4}{100}\right)\left(0.49 + \frac{5}{100}\right) = (0.5)\cdot(0.51)\cdot(0.52)\cdot(0.53)\cdot(0.54)$

$= 0.038$

Q 12:

a) the probability of no failures.

$= p^n$

b) probability of at least one fallure

$1-p^n \Rightarrow$ probability of at least 1 faillure

c) the probability of at most one failure

$C(n,1)(1-p)(p^{n-1})$

$p^n + C(n,1)(1-p)(p^{n-1})$

$= p^n + np^{n-1}(1-p)$

d) probability of at least two faillures.

$1-p^n - np^{n-1}(1-p)$