Home Work 2

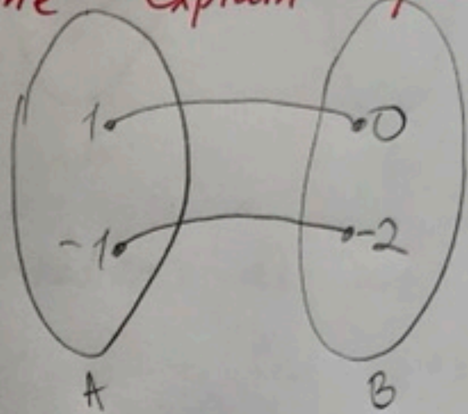
Question 5:

I) Determine whether each of these functions from $\mathbb{Z}$ to $\mathbb{Z}$ is one-to-one. Explain your answer:

a) $f(n) = n - 1$

$f(1) = 1 - 1 = 0$

$f(-1) = -1 - 1 = -2$

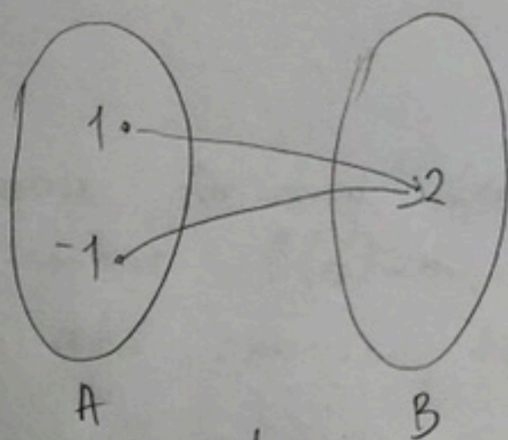


it's one-to-one function because we can see this is a strictly increasing and assign to different value.

b) $f(n) = n^2 + 1$

$f(1) = (1)^2 + 1 = 2$

$f(-1) = (-1)^2 + 1 = 2$

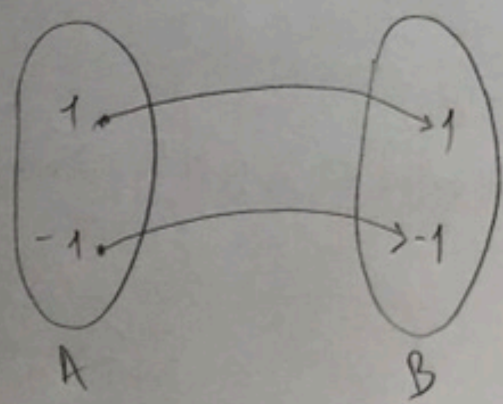


it's not one-to-one function because two functions assign to one value.

c) $f(n) = n^3$

$f(1) = 1$

$f(-1) = -1$

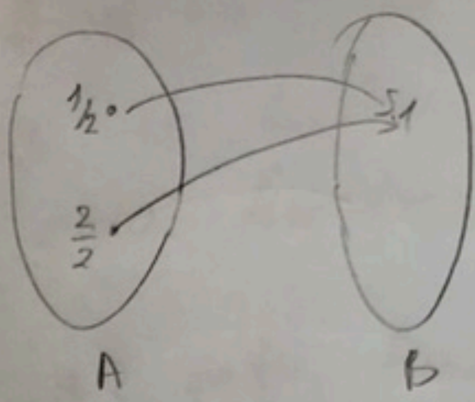


it's one-to-one function because we can see this is a strictly increasing and assign different value

d) $f(n) = \left[\frac{n}{2}\right]$

$f(1) = \left[\frac{1}{2}\right] = 1$

$f(2) = \left[\frac{2}{2}\right] = 1$



it's not one-to-one function, as we can see functions are assigned to the same value.

II) which functions in section (I) are onto?

a) $f(n) = n-1$

$f(n) = m$

$n-1 = m$

$n = m+1$

This is onto function because for any integer m there is integer n where $f(n) = m$. Where n is the domain and m is the codomain of the function.

b) $f(n) = n^2+1$

$f(n) = m$

$n^2+1 = m$

$n^2 = m-1$

$\pm n = m-1$

This is not an onto function.

c) $f(n) = n^3$ (3)

$f(1) = 1$

$f(2) = 8$

This function $f(n) = n^3$ is an <u>onto</u>.

d) $f(n) = \left\lceil \frac{n}{2} \right\rceil$

$f(1) = \left\lceil \frac{1}{2} \right\rceil = 1$

$f(3) = \left\lceil \frac{3}{2} \right\rceil = 2$

$f(n) = \left\lceil \frac{n}{2} \right\rceil$ is an onto function.

Question 6:

Determine whether each of these is a bijection function from R to R. Explain your answer:

a) $f(x) = -3x + 4$.

$f(x) = f(y)$

$-3x + 4 = -3y + 4$

$-3x = -3y$

$x = y$

one-to-one

$f(x) = y$

$y = -3x + 4$

$x = \frac{y - 4}{-3}$

$x = \frac{4 - y}{3}$

Onto

b) $f(x) = -3x^2 + 7$

$f(2) = -5$

$f(-2) = -5$

This means the function is not injective, and therefore cannot be a bijection.

c) $f(x) = \dfrac{x+1}{x+2}$

$x = -2$, we get division by 0, which is undefined. Therefore, this is not a function from R to R and cannot be a bijection

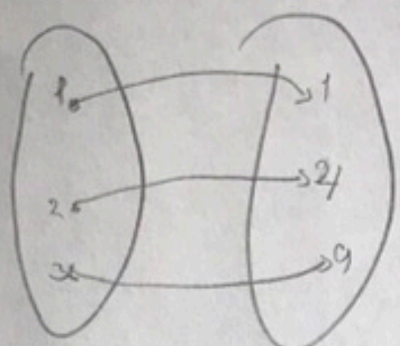
d) $f(x) = x^5 + 1$

There exists an inverse of this function therefore it must be a bijection.

Question 7:

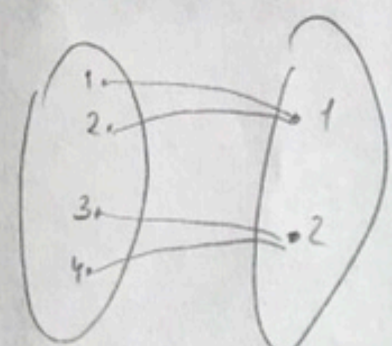
Give an example of a function the set of integers to the set of positive integers that is:

a) one-to-one, but not onto.



$f(n) = n^2$

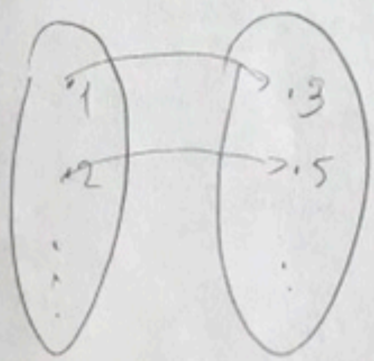
b) onto, but not one-to-one:



intreger

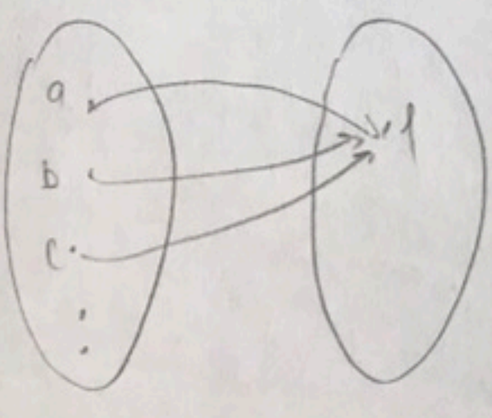
$f(n) = \left[\frac{n}{2}\right]$

c) one-to-one and onto:



$f(n) = 2 \cdot n + 1$

d) neither one-to-one nor onto



$f(n) = 1$

Question 8:

Let $f(x) = ax + b$, and $g(x) = cx + d$, where a, b, c and d are constants.

Determine ~~or~~ necessary and sufficient conditions on the constants a, b, c and d so that $f \circ g = g \circ f$.

$$f \circ g = g \circ f$$

$$f(x) = ax + b$$

$$g(x) = cx + d$$

$$a(cx + d) + b = c(ax + b) + d$$

$$\cancel{acx} + ad + b = \cancel{acx} + cb + d$$

$$ad + b = cb + d$$

$$ad - d = cb - b$$

$$d(a-1) = b(c-1)$$

Since a, b, c, d are constants

$$d(a-1) = b(c-1)$$

the sufficient and necessary condition is that $d(a-1) = b(c-1)$

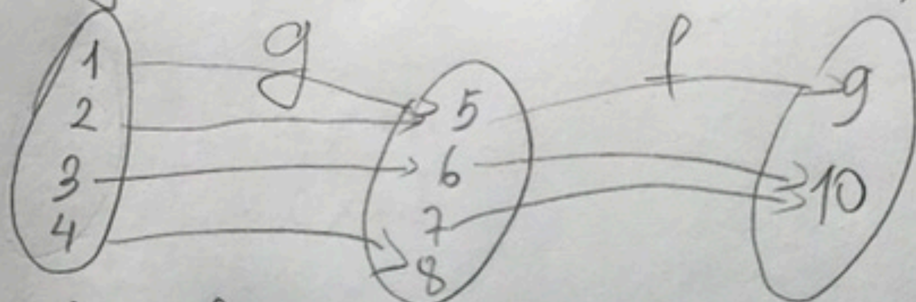
Question 9:

Let $g: A \to B$ and $f: B \to C$ be functions

a) If $f \circ g$ is onto, does it follow that both f and g are onto functions? Explain

b) If both f and g are onto functions, does it follow that $f \circ g$ is onto? Explain

$f \circ g$ is an onto function.



a) If $f \circ g$ is onto

$f \circ g\,(1) = 9$

$f \circ g\,(2) = 9$

$f \circ g\,(3) = 10$

$f \circ g\,(4) = 10$

b) $f \circ g(x) = f(g(x))$

suppose $y = g(x)$

all y can be mapped to x

suppose $z = f(x)$

all z can be mapped to y

so all z can be mapped to x

$f \circ g$ is onto