Q9:

There are 4 possible values of remainder:
0, 1, 2, 3

There are 5 intregers available
Therefore, by pigeon hole principle, at least 2 of the integers must give the same remainder when decided by 4.

Q10:

Consider the 6 computers to be the pigeons and the number of possible direct connections to be the pigeonholes. As every computer is directly connected to at least one other computer, there are five pigeonholes, {1}, {2}, {3}, {4}, and {5}. Thus, by the pigeonhole principle, at least $\lceil \frac{6}{5} \rceil = 2$ computers are directly connected to the same number of other computers.

Q 11:

let $A_1 = \{1, 100\}$ Each of these pair shows adds up to 101.

$A_2 = \{2, 99\}$ According to the pigeon principle, since there more dojeels placed into 'k' boxes then

$\vdots$

$A_{50} = \{50, 51\}$ there is at least one box will contain two or more pairs that add up to 101.

Q12:

Proof by contradiction:
There are 100 possible addresses, and 51 houses. In order for there to be no houses with consecutive addresses, each house must have at least one address in btw it.

This can be done by only assigning even number to houses

$\frac{100}{2} = 50$ useable houses.

assigning even numbers to houses. We now have $\frac{100}{2} = 50$ useable addresses.

Q13:

Any integer is either odd or even. The possible combinations of parity for points with integer coordinates in the xy-planes are then:

(odd, odd)
(odd, even)
(even, odd)
(even, even)

$$\text{Mid point} = \left(\frac{x_j + x_k}{2}, \frac{y_j + y_k}{2}\right)$$