Question 1:

A Convert the following numbers to their decimal representation. Show your work.

1. $(10011011)_2$

$1 \cdot 2^0 + 1 \cdot 2^1 + 0 \cdot 2^2 + 1 \cdot 2^3 + 1 \cdot 2^4 + 0 \cdot 2^5 + 0 \cdot 2^6 + 1 \cdot 2^7 =$

$= 1 + 2 + 8 + 16 + 128 = 155$

2 $(456)_7$

$6 \cdot 7^0 + 5 \cdot 7^1 + 4 \cdot 7^2 = 6 + 35 + 196 = 237$

3. $(38A)_{16}$

$A \cdot 16^0 + 8 \cdot 16^1 + 3 \cdot 16^2 = 10 + 128 + 768 = 906$

4 $(2214)_5$

$4 \cdot 5^0 + 1 \cdot 5^1 + 2 \cdot 5^2 + 2 \cdot 5^3 = 4 + 5 + 50 + 250 = 309$

B. Convert the following numbers to their binary representation:

1. $69_{10} = (1000101)_2$

69 % 2 = 34 R 1

34 % 2 = 17 R 0

17 % 2 = 8 R 1

8 % 2 = 4 R 0

4 % 2 = 2 R 0

2 % 2 = 1 R 0

1 % 2 = 0 R 1

2. $485_{10} = 111100101_2$

485 % 2 = 242 R 1
242 % 2 = 121 R 0
121 % 2 = 60 R 1
60 % 2 = 30 R 0
30 % 2 = 15 R 0
15 % 2 = 7 R 1
7 % 2 = 3 R 1
3 % 2 = 2 R 1
1 % 2 = 0 R 1

3. $6D1A_{16} = (0110\ 1101\ 0001\ 1010)_2$

A = 10 1010
1 0001
D = 13 1101
6 0110

C. Convert the following numbers to their hexadecimal representation:

1. $\underline{1101}\underline{1011}_2 = 6B_{16}$

2. $895_{10} = 37F$

895 % 16 = 55 R 15 = F
55 % 16 = 3 R 7
3 % 16 = 0 R 3

Question 2:

Solve the following, do all calculation in the given base. Show your work.

1. $7566_8 + 4515_8 = (14303)_8$

1) $\begin{array}{r} 7566 \\ 4515 \\ \hline 14303 \end{array}$

2) $1011011_2 + 1101_2 = (11000000)_2$

$\begin{array}{r} 1011011_2 \\ 1101_2 \\ \hline 11000000_2 \end{array}$

3) $7A66_{16} + 45C5_{16} = (C02B)_{16}$

$\begin{array}{r} 7A66 \\ +\ 45C5 \\ \hline C02B_{16} \end{array}$

4) $3022_5 - 2433_5 = (34)_5$

$\begin{array}{r} 3022 \\ -\ 2433 \\ \hline 34 \end{array}$

Question 3:

A. Convert the following numbers to their 8 bits two's complement representation.

1 $124_{10} = (0\,1111100)_{8 \text{ bits}}$ two's complement

124	% 2	= 62	R 0
62	% 2	= 31	R 0
31	% 2	= 15	R 1
15	% 2	= 7	R 1
7	% 2	= 3	R 1
3	% 2	= 1	R 1
1	% 2	= 0	R 1

2. $-124_{10} = 10000100$ 8 bits two's complement

1) 01111100
   10000100

   100000000

3. $109_{10} = (0110\,1101)_{8 \text{ bits}}$ two's complement

109	% 2	= 54	R 1
54	% 2	= 27	R 0
27	% 2	= 13	R 1
13	% 2	= 6	R 1
6	% 2	= 3	R 0
3	% 2	= 1	R 1
1	% 2	= 0	R 1

4. $-79_{10} = (10110001)$

79	% 2	= 39	R 1
39	% 2	= 19	R 1
19	% 2	= 9	R 1
9	% 2	= 4	R 1
4	% 2	= 2	R 0
2	% 2	= 1	R 0
1	% 2	= 0	R 1

1) 1 1 1 1 1 1 1
   01001111
   10110001
   100000000

B. Convert the following numbers (represented as 8-bit two's complement) to their decimal representation. Show your work.

1) 00011110 8 bit 2's comp = 30_{10}
$1 \cdot 2^1 + 1 \cdot 2^2 + 1 \cdot 2^3 + 1 \cdot 2^4 = 30$

2) 11100110 8 bit 2's comp = -26_{10}
$1 \cdot 2^1 + 1 \cdot 2^2 + 1 \cdot 2^5 + 1 \cdot 2^6 + 1 \cdot 2^7 = 230$

a) 1 1 1 1 1 1 1
   00011010
   11100110
   100000000

b) 00011010
$2^1 \cdot 1 + 2^3 \cdot 1 + 2^4 \cdot 1 = 26$

3) 00101101 8 bit 2's comp $= (45)_{10}$
$1 \cdot 2^0 + 1 \cdot 2^2 + 1 \cdot 2^3 + 1 \cdot 2^5 = 45$

4) 10011110 8 bit 2's comp $= (-98)_{10}$

a) 1 1 1 1 1 1 1
   01100010
   10011110
   100000000

b) 01100010
$1 \cdot 2^1 + 1 \cdot 2^5 + 1 \cdot 2^6 = 98$

Question 4

1 Exercise 1.2.4 section b, c

b) $\neg(p \vee q)$

p	q	$\neg(p \vee q)$
T	T	F
T	F	F
F	T	F
F	F	T

c) $r \vee (p \wedge \neg q)$

p	q	r	$r \vee (p \wedge \neg q)$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	F
F	F	T	T
F	F	F	F

2. Exercise 1.3.4 section b, d

b) $(p \to q) \to (q \to p)$

p	q	$p \to q$	$q \to p$	$(p \to q) \to (q \to p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

d) $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

p	q	$p \leftrightarrow q$	$p \leftrightarrow \neg q$	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	T	T	F	F
T	F	T	T	T
F	T	T	T	T
F	F	F	T	F

Question 5:

1. Exercise 1.2.7 section b, c.

b) The applicant must present at least two of the following forms of indentification: birth certificate, driver's license, marriage license

$$(B \wedge M) \vee (B \wedge D) \vee (D \wedge M)$$

c) Applicant must present either a birth certificate or both a driver's license and a marriage license

$$B \vee (D \wedge M)$$

2. Exercise 1.3.7 section b - e

b) A person can park in the school parking lot if they are a senior or at least seventeen years old

$$(S \vee Y) \rightarrow P$$

c) Being 17 years of age is a necessary condition for being able to park in the school parking lot

$$P \rightarrow Y$$

d) A person can park in the school parking lot if and only if the person is a senior and at least 17 years of age

$$P \longleftrightarrow (S \wedge Y)$$

e) Being able to park in the school parking lot implies that the person is either a senior or at least 17 years old

$$P \rightarrow (S \vee Y)$$

3. Exersise 1.3.9 section c, d

c) The applicant can enroll in the course only if the applicant has parental permission

$$C \longrightarrow P$$

d) Having parental permission is a necessary condition for enrolling in the course

$$P \longrightarrow C$$

Question 6:

1. Exercise: 1.3.6 section b-d

b) Maintaining a B average is necessary for Joe to be eligible for the honors program

If maintaining a B average then Joe be eligible for the honors program.

c) Rajiv can go on the roller coaster only if he is at least four feet tall

If Rajiv can go on the roller coaster then he is at least four feet tall

d) Rajiv can go on the roller coaster if he is at least four feet tall

If he is at least four feet tall then Rajiv can go on the roller coaster

Exercise 1.3.10 sections c–f

c) $(p \vee r) \leftrightarrow (q \wedge r)$

p	q	r	$(p \vee r) \leftrightarrow (q \wedge r)$
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	T
F	F	F	T

Unknown

d) $(p \wedge r) \leftrightarrow (q \wedge r)$

p	r	q	$(p \wedge r) \leftrightarrow (q \wedge r)$
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	T
F	T	T	F
F	T	F	T
F	F	T	T
F	F	F	T

Unknown

e) $p \rightarrow (r \vee q)$

p	q	r	p → (r ∨ q)
T	T	T	T → T
T	T	F	T → T
T	F	T	T → T
T	F	F	T → F
F	T	T	F → T
F	T	F	F → T
F	F	T	F → T
F	F	F	F → F

T
T
T
F
T
T
T
T

Unknown

f) $(p \wedge q) \rightarrow r$

p	q	r	(p ∧ q) → r
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

Unknown

Question 7

Exercise 1.4.5 b-d:

b) If Sally did not get the job she has late for interview or did not update her resume.

If Sally updated her resume and was not late for her interview, then she got the job

$\neg j \rightarrow (l \vee \neg r)$

$(r \wedge \neg L) \rightarrow j$

r	l	j	$\neg j \rightarrow (l \vee \neg r)$	$(r \wedge \neg L) \rightarrow j$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	F	F
F	F	F	T	T

c) If Sally got the job then she was not late for her interview

If Sally did not get the job, then she was late for her interview

$j \rightarrow \neg L$

$\neg j \rightarrow L$

J	l	$J \rightarrow \neg L$	$\neg j \rightarrow L$
T	T	F	T
T	F	T	T
F	T	T	T
F	F	T	F

d) If Sally updated her resume or she was not late for her interview, then she got the job
If Sally got the job, then she updated her resume and was not late for her interview.

$(r \vee \neg l) \rightarrow j$

$j \rightarrow (r \wedge \neg l)$

r	j	l	$(r \vee \neg l) \rightarrow j$	$j \rightarrow (r \wedge \neg l)$
T	T	T	T	F
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	F	T
F	T	F	T	T
F	F	T	F	T
F	F	F	F	T

Question 8

1. Exercise 1.5.2 section c, f, i

c) $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$

$(\neg p \vee q) \wedge (p \rightarrow r)$ Conditional identity

$(\neg p \vee q) \wedge (\neg p \vee r)$ Conditional identity

$\neg p \vee (q \wedge r)$ Distributive law

$p \rightarrow (q \wedge r)$ Conditional identity

f) $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$

$= \neg((F) \wedge (p \vee q))$

$= \neg((p \vee q) \wedge (F))$

$= \neg(p \vee q)$

$= \neg p \wedge \neg q$

i) $(p \wedge q) \longrightarrow r \equiv (p \wedge \neg r) \longrightarrow \neg q$

$= \neg(p \wedge q) \vee r$

$= (\neg p) \vee (\neg q) \vee r$

$= (\neg p) \vee r \vee (\neg q)$

$= \neg(p \wedge \neg r) \vee (\neg q)$

$= (p \wedge \neg r) \longrightarrow (\neg q)$

Exercise 1.5.3 sections c, d

g) $\neg r \vee (\neg r \rightarrow p)$

$= \neg r \vee (\neg r \vee p)$

$= \neg r \vee (r \vee p)$

$= (r \vee p) \vee (\neg r)$

$= r \vee p \vee (\neg r)$

$= r \vee (\neg r) \vee p$

$= T \vee D$

$= T$

d) $\neg(p \longrightarrow q) \longrightarrow \neg q$

$= \neg(\neg p \vee q) \longrightarrow \neg q$

$= (\neg\neg p) \wedge (\neg q) \longrightarrow \neg q$

$= (p \wedge (\neg q)) \longrightarrow \neg q$

$= \neg(p \wedge (\neg q)) \vee \neg q$

$= (\neg p \vee \neg\neg q) \vee \neg q$

$= (\neg p \vee q) \vee \neg q$

$= (q \vee \neg q) \vee \neg p$

$= T \vee \neg p \Rightarrow T$

Question 9:

Exercise 1.6.3 sections c,d

c) There is a number that is equal to its square

$\exists x \ (x = x^2)$

d) Every number is less than or equal to its square

$\forall x \ (x <= x^2)$

Exercise 1.7.4 sections b-d.

b) Everyone was well and went to work yesterday

$\forall x \ (\neg S(x) \wedge W(x))$

c) Everyone who was sick yesteday did not go to work

$\forall x \ (S(x) \rightarrow \neg W(x))$

d) Yesterday someone was sick and went to work

$\exists x \ (S(x) \wedge W(x))$

Question 10

Exercise 1.7.9 sections c–i

c) $\exists x ((x = c) \rightarrow P(x)$

False. $P(c)$ is false

d) $\exists x (Q(x) \wedge R(x))$

True. When $x = e$, $Q(e) \wedge R(e) = \text{true}$

e) $Q(a) \wedge P(d)$

True. $Q(a)$ is true, $P(d)$ is true

f) $\forall x ((x \neq b) \rightarrow Q(x))$

True $Q(a)$, $Q(c)$, $Q(d)$, $Q(e)$ are true

g) $\forall x (P(x) \vee R(x))$

False $P(c) \vee R(c)$ is false

h) $\forall x (R(x) \rightarrow P(x))$

True. $R(a)$ is false, $P(a)$ is true, so $R(a) \rightarrow P(a)$ tru
etc true.

i) $\exists x (Q(x) \vee R(x))$

True $Q(a) \vee R(a)$ is true.

2. Exercise 1.9.2 b-i

b) $\exists x \forall y \; Q(x,y)$

True. when $Q(2,1)$, $Q(2,2)$ and $Q(2,3)$ are all true

c) $\exists x \forall y \; P(y,x)$

True. when $P(1,1)$, $P(2,1)$, $P(3,1)$

d) $\exists x \exists y \; S(x,y)$

False. There is no $S(x,y)$ is true

e) $\forall x \exists y \; Q(x,y)$

False. when $x=1$ there is no $Q(1,y)$ is true

f) $\forall x \exists y \; P(x,y)$

True. when $x=1$, $P(1,1)$ and $P(1,3)$; $x=3$ $P(3,1)$, $P(3,2)$;
$x=2$, $P(2,1)$, $P(2,3)$;

g) $\forall x \forall y \; P(x,y)$

False when $x=1$ $P(1,2)$ is false;
$x=2$ $P(2,2)$ is false;
$x=3$ $P(3,3)$ is false;

h) $\exists x \exists y \; Q(x,y)$

True when $Q(2,1)$, $Q(2,2)$, $Q(2,3)$

i) $\forall x \forall y \; \neg S(x,y)$

True There are $S(x,y)$ is true.

Question 11:

1 Exercise 1.10.4 sections c-h

c) $\exists x \exists y \quad (x+y = xy)$

d) $\forall x \ (x>0) \ \forall y \ (y>0) \rightarrow (\frac{x}{y} > 0)$

e) $\forall x (0 < x < 1) \rightarrow (\frac{1}{x} > 1)$

f) $\forall x \exists y \ (y < x)$

g) $\forall x \ (x \neq 0) \ \exists y \ (y = \frac{1}{x})$

h) $\forall x \ (x \neq 0) \ \exists y \ (y = \frac{1}{x})$

2. Exercise 1.10.7, sections c-f

c) $\exists x \quad D(x)$

d) $\exists x \forall y P(x, y)$

e) $\exists x \ \forall x \quad N(x) \ D(x)$

f) $\exists x \ D(x)$

3. Exercise 1.10.10, sections c-f

c) $\forall x \exists y \ (y \neq \text{Math 101}) \ T(x, y)$

d) $\exists x \forall y \ (y \neq \text{Math 101}) \ T(x, y)$

e) $\exists y \exists z \ (z \neq y) \forall x \ (x \neq SAM) \ (T(x, y) \wedge \neg(x, z))$

f) $\exists y_1, \exists y_2 \ (y_2 \neq y_1) \ ((T(sam, y) \wedge T(SAM, y_2) \wedge$

$\wedge \forall z \ ((z \neq y_1) \wedge (z \neq y_2)) \rightarrow \neg T(sam, z)$

Question 12

1) Exercise 1.8.2 section b-e

b) $\forall x \quad D(x) \vee P(x)$

$\forall x \quad \neg P(x) \vee \neg P(x)$

$\exists x \quad (\neg P(x) \wedge \neg P(x))$

There is a patient who not given the medication and not given placebo

c) $\exists x \quad D(x) \wedge M(x)$

$\neg \exists x \quad D(x) \wedge M(x)$

$\forall x \quad (\neg D(x) \vee \neg M(x)$

Every patient did not get the medication or did not get have migrains or both

d) $\forall x \quad P(x) \rightarrow M(x)$

$\forall x \quad \neg P(x) \rightarrow M(x)$

$\exists x \quad P(x) \wedge \neg M(x)$

Some patient took the placebo and did not have migraines

e) $\exists x \; M(x) \wedge P(x)$

$\neg \exists x \; M(x) \wedge P(x)$

$\forall x \; \neg M(x) \vee \neg P(x)$

Every patient didn't have migraines or wasn't given placebo

Exercise 1.3.4 sections c-e

c) $\exists x \forall y \; (P(x,y) \rightarrow Q(x,y))$

$\forall x \; \exists y \; (P(x,y) \wedge \neg Q(x,y))$

d) $\exists x \forall y \; (P(x,y) \leftrightarrow P(y,x))$

$\exists x \forall y \; (P(x,y) \rightarrow P(y,x)) \wedge (P(y,x) \rightarrow P(x,y))$

$= \exists x \forall y \; (\neg P(x,y) \vee P(y,x)) \wedge \neg P(y,x) \vee P(x,y))$

$\forall x \; \exists x \; \neg [\neg P(x,y) \vee P(y,x)] \vee \neg [\neg P(y,x) \vee P(x,y)]$

$\forall x \; \exists x \; (P(x,y) \wedge \neg P(y,x)) \vee (P(y,x) \wedge \neg P(x,y))$

e) $\exists x \; \exists y \; P(x,y) \wedge \forall x \; \forall y \; Q(x,y)$

$\forall x \; \forall x \; P(x,y) \vee \exists x \; \exists y \; Q(x,y)$