Question 1:

A. Convert the following numbers to their decimal representation:

1) $(10011011)_2 = 155$

$\quad 2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0$

$= 1\cdot 2^0 + 1\cdot 2^1 + \cancel{0\cdot 2^2} + 1\cdot 2^3 + 1\cdot 2^4 + \cancel{0\cdot 2^5} + \cancel{0\cdot 2^6} + 1\cdot 2^7$

$= 1 + 2 + 8 + 16 + 128 = 155$

2) $(456)_7 = 335$

$6\cdot 7^0 + 5\cdot 7^1 + 6\cdot 7^2 = 6 + 35 + 294 = 335$

3) $(38A)_{16} = 394$

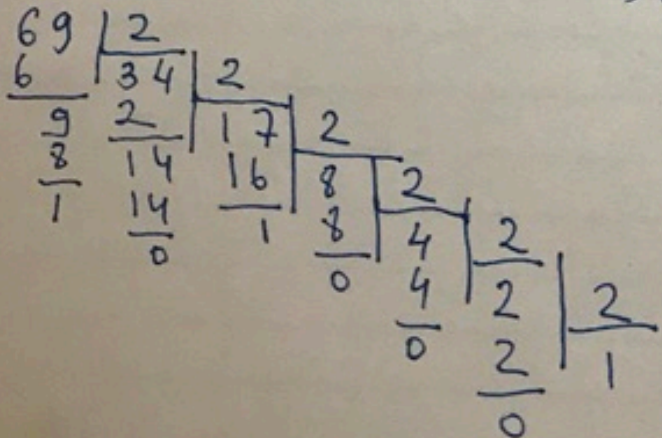
$A\cdot 16^0 + 8\cdot 16^1 + 3\cdot 16^2 = 10 + 128 + 256 = 394$

4) $(2214)_5 = 309$

$4\cdot 5^0 + 1\cdot 5^1 + 2\cdot 5^2 + 2\cdot 5^3 = 4 + 5 + 50 + 250 = 309.$

B. Convert the following numbers to their binary representation:

1) $(69)_{10} = (1000101)_2$



2) $(485)_{10} = (110111)_2$

$$
\begin{array}{l}
485 \mid 2 \\
4 \quad 242 \mid 2 \\
85 \quad 2 \quad 121 \mid 2 \\
8 \quad 42 \quad 12 \quad 6 \mid 2 \\
5 \quad 4 \quad 1 \quad 6 \quad 3 \mid 2 \\
4 \quad 2 \quad 0 \quad 2 \quad 1 \\
1 \quad 2 \quad 1 \\
1
\end{array}
$$

3) $(6\,D\,1\,A)_{16} = (0110\ 1101\ 0001\ 1010)_2$

$A = 10 \qquad 1\cdot 2^3 + 0\cdot 2^2 + 1\cdot 2^1 + 0\cdot 2^0 = 1010$

$1 = 0001$

$D = 13 \qquad 1\cdot 2^3 + 1\cdot 2^2 + 0\cdot 2^1 + 1\cdot 2^0 = 1101$

$6 = 0110$

c) Convert the following numbers to their hexadecimal representation:

1) $(\underbrace{110}\,\underbrace{1011})_2 = (6\,B)_{16}$

$(1011) = (1\cdot 2^0 + 1\cdot 2^1 + 0\cdot 2^2 + 1\cdot 2^3) = 1 + 2 + 8 = 11 \rightarrow B$

$(110) = (0\cdot 2^0 + 1\cdot 2^1 + 1\cdot 2^2) = 2 + 4 = 6$

2) $(895)_{10} = (37F)_{16}$

$$
\begin{array}{l}
895 \mid 16 \\
80 \quad 55 \mid 16 \\
95 \quad 48 \quad 3 \\
80 \quad 7 \\
15 \\
\rightarrow F
\end{array}
$$

Question 2:

Solve the following, do all calculation in the given base:

1)
$$\begin{array}{r} 7566_8 \\ +\ 4515_8 \\ \hline 14303_8 \end{array}$$

2)
$$\begin{array}{r} 10110011_2 \\ +\ 1101_2 \\ \hline 11000000_2 \end{array}$$

3)
$$\begin{array}{r} 7A66_{16} \\ +\ 45C5_{16} \\ \hline C02B_{16} \end{array}$$

4)
$$\begin{array}{r} 3022_5 \\ -\ 2433_5 \\ \hline 0034_5 \end{array}$$

Question 3:

A. Convert the following numbers to their 8-bits two's complement representation:

1. $(124)_{10} = (01111100)_{8\text{ bits}}$ two's complement

$124 \div 2 = 62$ R 0 ; $62 \div 2 = 31$ R 0 ; $31 \div 2 = 15$ R 1 ; $15 \div 2 = 7$ R 1 ; $7 \div 2 = 3$ R 1 ; $3 \div 2 = 1$ R 1 ; 1

2. $(-124)_{10} = (10000100)_{8\text{ bit}}$ 2's comp.

$$\begin{array}{r} 01111100 \\ 10000100 \\ \hline 100000000 \end{array}$$

3. $(109)_{10} = (01101101)_{8\text{ bit}}$ 2's compl

$109 \div 2 = 54$ R 1
$54 \div 2 = 27$ R 0
$27 \div 2 = 13$ R 1
$13 \div 2 = 6$ R 1
$6 \div 2 = 3$ R 0
$3 \div 2 = 1$ R 1
$1 \div 2 = 0$ R 1

Question 3

N4: $(-79)_{10} = (10110001)_{8 \text{ bits } 2\text{'s compl.}}$

a)
$79 \div 2 = 39$ R 1
$39 \div 2 = 19$ R 1
$19 \div 2 = 9$ R 1
$9 \div 2 = 4$ R 1
$4 \div 2 = 2$ R 0
$2 \div 2 = 1$ R 0
$1 \div 2 = 0$ R 1

b)
$$\begin{array}{r} (01001111) \\ + \quad 10110001 \\ \hline 100000000 \end{array}$$

B. Convert the following numbers (represented as 8-bits two's complement) to their decimal representation:

1) $(00011110)_{8 \text{ bit } 2\text{'s compl.}} = (30)_{10}$

64 32 16 8 4 2 1

a) $2+4+8+16 = 30$

2) $(11100110)_{8 \text{ bit } 2\text{'s compl.}} = (-26)_{10}$

a)
$$\begin{array}{r} 00011010 \\ + \quad 11100110 \\ \hline 100000000 \end{array}$$

b) (00011010)

64 32 16 8 4 2 1

c) $2+8+16 = 26$

3) $(00101101)_{8 \text{ bit } 2\text{'s comp}} = (45)_{10}$

64 32 16 8 4 2 1

a) $1+4+8+32 = 45$

4) $(10011110)_{8 \text{ bit } 2\text{'s comp}} = (-98)_{10}$

a)
$$\begin{array}{r} 01100010 \\ + \quad 10011110 \\ \hline 100000000 \end{array}$$

b) 01100010

64 32 16 8 4 2 1

c) $64+32+2 = 98$

Question N4

A. For each of the following sets, determine 2 is a member of that set.

a) $\{x \in R \mid x$ is an integer greater than $1\}$

$\mathbb{Z}$ is $\{\dots, -2, -1, 0, 1, 2, \dots\}$ the set of integer,
so X is an integer greater than 1 $(X > 1)$,
means X is positive and start of 2.
This set contains $\{2, 3, 4, 5, 6 \dots\}$ so first member of its is 2.

b) $\{2, \{2\}\}$

Yes, we have 2

c) $\{x \in R \mid x$ is the square of an integer$\}$

No, when we have $X = a^2$, so in our set we have $\{1, 4, 9, 16, \dots\}$ that set does't contain 2.

d) $\{\{2\}, \{\{2\}\}\}$

No, two is not a member of that set
We have $\{2\}$, $\{\{2\}\}$ which is not equal 2.

e) $\{\{2\}, \{2, \{2\}\}\}$

No, two is not a member of that set.
We have $\{2\}$, $\{2, \{2\}\}$. which is not equal 2.

f) $\{\{\{2\}\}\}$

No, we have $\{\{\{2\}\}\}$ which is not equal 2.

B. Determine whether each of these statements is true or false.

a) $x \in \{x\}$ True

b) $\{x\} \subseteq \{x\}$ True

c) $\{x\} \in \{x\}$ False

d) $\{x\} \in \{\{x\}\}$ True

e) $\emptyset \subseteq \{x\}$ True

f) $\emptyset \in \{x\}$ False

C. Find two sets A and B such that $A \in B$ and $A \subseteq B$.

$A \in B$ = A $\{1, 2, 3\}$
B $\{1, 1, 2, 2, 3, 3\}$

$\cancel{A \in B} \neq A \subseteq B$; A is subset of B

D. For each of these pairs of sets, determine whether the first is a subset of the second, the second of the first, or neither is a subset of the other.

(i) the set of airline flight from NY to New Delhi, the set of nonstop airline flights from New York to New Delhi.

$B \subseteq A$. Every element in the set of nonstop airline flights will also be an element in the set of all flights.

(ii) the set of people who speak English; the set of people who speak Chinese.

(b) Neither of these are subsets of the other. $A \not\subseteq B$
$B \not\subseteq A$.

Q4

N (C) iii the set of flying squirres, the set of living creatures that can fly.

flying squirrel is the subset of creature that can fly. Not vice versa.

$A \subseteq B$.

Question 5:

Let $A = \{a, b, c, d, e\}$ and $\{a, b, c, d, e, f, g, h\}$

Find:

a) $(A \vee B) = \{a, b, c, d, e, f, g, h\}$

b) $(A \cap B) = \{a, b, c, d, e\}$

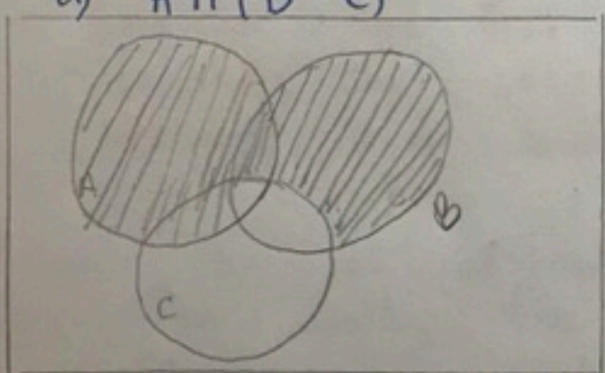
c) $(A - B) = \emptyset$

d) $(B - A) = \{f, g, h\}$

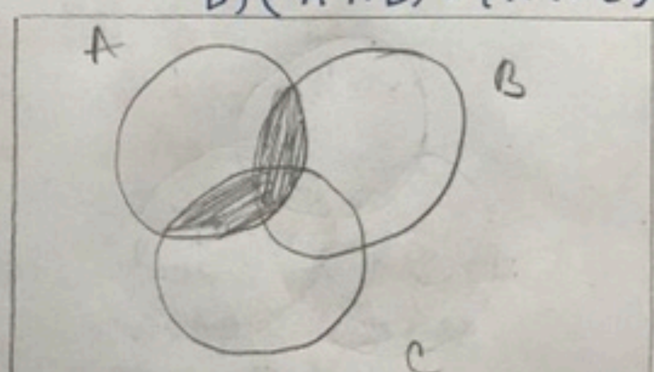
Question 6:

Draw the Venn diagrams for each of these combinations of the sets A, B, and C.

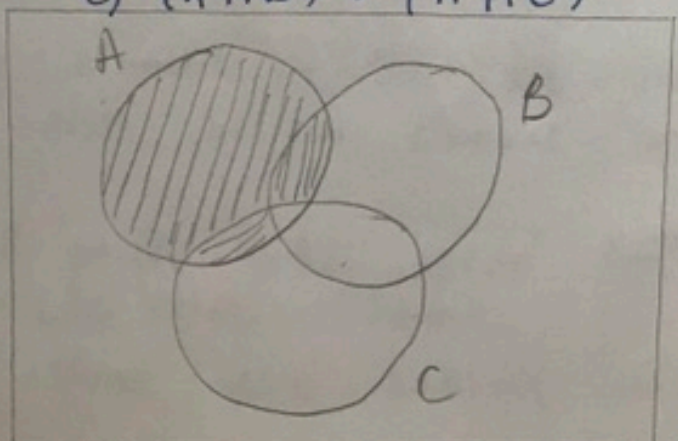
a) $A \cap (B - C)$



b) $(A \cap B) \cup (A \cap C)$



c) $(A \cap \bar{B}) \cup (A \cap \bar{C})$



Question 7:

Let A, B and C be sets. Use a member table to show that:

a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

A	B	C	$B \cap C$	$A \cup (B \cap C)$	$A \cup B$	$A \cup C$	$(A \cup B) \cap (A \cup C)$
1	1	1	1	1	1	1	1
1	1	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	0	0	0	1	1	1	1
0	1	1	1	1	1	1	1
0	1	0	0	0	1	0	0
0	0	1	0	0	0	1	0
0	0	0	0	0	0	0	0

$A \cup (B \cap C)$ and $(A \cup B) \cap (A \cup C)$ are the same, so identities are valid.

b) $(B - A) \cup (C - A) = (B \cup C) - A$

A	B	C	$B - A$	$C - A$	$(B-A) \cup (C-A)$	$(B \cup C)$	$(B \cup C) - A$
1	1	1	0	0	0	1	0
1	1	0	0	0	0	1	0
1	0	1	0	0	0	1	0
1	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1
0	1	0	1	0	1	1	1
0	0	1	0	1	1	1	1
0	0	0	0	0	0	0	0

$(B - A) \cup (C - A)$ and $(B \cup C) - A$ are the same, so ~~inthe~~ identity is valid

Q7 (cont).

c) $(\overline{A \cap B \cap C}) = (\bar{A} \cup \bar{B} \cup \bar{C})$

A	B	C	$A \cap B \cap C$	$\overline{A \cap B \cap C}$	$\bar{A}$	$\bar{B}$	$\bar{C}$	$\bar{A} \cup \bar{B} \cup \bar{C}$
1	1	1	1	0	0	0	0	0
1	1	0	0	1	0	0	1	1
1	0	1	0	1	0	1	0	1
1	0	0	0	1	0	1	1	1
0	1	1	0	1	1	0	0	1
0	1	0	0	1	1	0	1	1
0	0	1	0	1	1	1	0	1
0	0	0	0	1	1	1	1	1

$\overline{A \cap B \cap C}$ and $\bar{A} \cup \bar{B} \cup \bar{C}$ are the same, so it is valid identity.

d) $(A - C) \cap (C - B) = \emptyset$

A	B	C	$(A - C)$	$(C - B)$	$(A - C) \cap (C - B)$	$\emptyset$
1	1	1	0	0	0	0
1	1	0	1	0	0	0
1	0	1	0	1	0	0
1	0	0	1	0	0	0
0	1	1	0	0	0	0
0	1	0	0	0	0	0
0	0	1	0	1	0	0
0	0	0	0	0	0	0

$(A - C) \cap (C - B)$ and $\emptyset$ are the same, so it is valid identity.

Question 7 (e)

Let A, B and C be sets. Use a membership table to show that:

e) $(A - B) - C \subseteq (A - C)$

A	B	C	$(A - B)$	$(A - B) - C$	$(A - C)$
1	1	1	0	0	0
1	1	0	0	0	1
1	0	1	1	0	0
1	0	0	1	1	1
0	1	1	0	0	0
0	1	0	0	0	0
0	0	1	0	0	0
0	0	0	0	0	0

All of the elements in $(A - B) - C$ are in $(A - C)$, so it is true.

Question 8:

Let A, B, and C be sets. Use a set indentities to show that:

a) $A - B = \bar{B} - \bar{A}$

Solution:

$$A - B = A \cap \bar{B}$$
$$= x \notin A \wedge x \in B$$
$$= x \in A \wedge x \in B$$
$$= x \in B \wedge A$$
$$= \bar{B} - \bar{A}$$

b) $(A \cap B) \cup (A \cap \bar{B}) = A$

$A = A \cap U =$

$= A \cap (B \cup \bar{B})$

$= (A \cap B) \cup (A \cap \bar{B})$

c) $A - (B - C) = (A - B) \cup (A - \bar{C})$

$A - (B - C) \Rightarrow A \cap (\overline{B \cap C}) =$

$= (A \cap \bar{B}) \cup (A \cap \bar{C})$

$= (A - B) \cup (A - \bar{C})$

Question 9:

Can you conclude that $A = B$, if A, B and C are sets, such that:

a) $A \cup C = B \cup C$?

You can't conclude that $A = B$ if $A \cup C = B \cup C$

It will work if A and B are subset of C.

Ex: $A = \{0\}$

$B = \{1\}$

$C = \{0, 1\}$

$A \cup C = \{0, 1\}$

$A \cup B = \{0, 1\}$, so $A \neq B$.

b) $A \cap C = B \cap C$?

b) $A \cap C = B \cap C$?

You can't conclude that $A = B$ if $A \cap C = B \cap C$ we need to consider that C could be an empty set.

Ex: $A = \{0, 1\}$ $\quad$ $A \cap C = \{0\}$

$B = \{0, 2\}$ $\quad$ $A \cap B = \{0\}$

$C = \{0\}$

we can see that $A \neq B$.

c) $(A \cup C = B \cup C)$ and $(A \cap C = B \cap C)$?

we can conclude that $A = B$

we can take A, B, C randomly and we will find $A \cup C = B \cup C$ and $A \cap C = B \cap C$ only if $A = B$.

Question 10:

Find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$, if for every positive integer i:

a) $A_i = \{-i, -i+1, \dots, -1, 0, 1, \dots, i-1, i\}$

$A_1 = \{-1, 0, 1\}$

$A_2 = \{-2, -1, 0, 1, 2\}$

$A_3 = \{-3, -2, -1, 0, 1, 2, 3\}$

$\bigcup_{i=1}^{\infty} A_i = A_1 \cup A_2 \cup A_3 = \{\dots -3, -2, -1, 0, 1, 2, 3 \dots\} = \mathbb{Z}$

$\bigcap_{i=1}^{\infty} A_i = A_1 \cap A_2 \cap A_3 \dots = \{-1, 0, 1\}$

b) $A_i = \{-i, i\}$

$A_1 = \{-1, 1\}$

$A_2 = \{-2, 2\}$

$A_3 = \{-3, 3\}$

$\bigcup^{\infty} A_i = A_i \cup A_2 \cup A_3 \dots = \{-3, -2, -1, 1, 2, 3 \dots\}$
$= \mathbb{Z}$

$\bigcap_{i=1}^{\infty} A_i = A_1 \cap A_2 \cap A_3 = \emptyset$

c) $A_i = [-i, i] \qquad -i \leq x \leq i$

$A_i = [-1, 1]$

$A_2 = [-2, 2]$

$A_3 = [-3, 3]$

$\bigcup_{i=1}^{\infty} A_i = A_1 \cup A_2 \cup A_3 \dots [\dots -3, -2, -1, 1, 2, 3] =$ 13
$= [\mathbb{R}^-, \mathbb{R}^+]$ no 0 (zero)

$$\bigcap_{i=1}^{\infty} A_i = A_1 \cap A_2 \cap A_3 \ldots = [-1, 1]$$

d) $A_i = [-i, \infty)$ $\quad x \geq -i$

$A_1 = [-1, \infty)$

$A_2 = [-2, \infty)$

$A_3 = [-3, \infty)$

$$\bigcup_{i=1}^{\infty} A_i = A_1 \cup A_2 \cup A_3 \ldots = [\bar{R}, \infty)$$

$$\bigcap_{i=1}^{\infty} A_i = A_1 \cap A_2 \cap A_3 \ldots = [-1, \infty)$$