C. Transversal DB gives $m\angle FDB = m\angle DBE$; transversal FB gives $m\angle DFB = m\angle CBE$, so $FB = DB$ (isos triangle) and DF is midline of ΔAEB, so $DE = AD$.
In ΔAEB, both BD and EF are medians so $BG = 2/3\ (BD)$.

Round Six:

A. The possible key sequences were: 4^2, 2^4, 4x4, 8x2, 8+8, and 2x8, 9+7 and 7+9, so prob =1/8.

B. Expand via binomial theorem or

$$\left(\sqrt{2}+\sqrt{3}\right)^{2(3)} = \left(5+2\sqrt{6}\right)^3 = 5^3 + 3(25)2\sqrt{6} + 3(5)4(6) + 8(6)\sqrt{6} = 485 + 198\sqrt{6}$$

C. If ${}_nC_6 - {}_nC_5 = {}_nC_5 - {}_nC_4$ then

$$\frac{n(n-1)(n-2)(n-3)(n-4)(n-5)}{6!} = \frac{2n(n-1)(n-2)(n-3)(n-4)}{5!} - \frac{n(n-1)(n-2)(n-3)}{4!}$$

so $\frac{(n-4)(n-5)}{6(5)} = \frac{2(n-4)}{5} - \frac{1}{1}$ so $n^2 - 9n + 20 = 12(n-4) - 30$ … n = 7 or 14

Team Round:

A. Find k so coeff. matrix has determinant = 0: $k^2(k^2-5)+4=0$ gives $k = \pm 2$ or ± 1
Substitute to find inconsistent when k = 1 or – 2; dependent when k = 2 or –1.
$A = B = -2$, so sum is – 4.

B. $A = 10a + b \rightarrow B = 10b + a$. $A^2 - B^2 = 99(a-b)^2 = 9[(11)(a-b)^2] =$
$9[(11)(a+b)(a-b)]$. Since $a > b$ and a and b represent base 10 digits, the latter factor can be a perfect square, if $a + b$ is a multiple of 11 and $a - b = 1$, which only happens for $(a, b) = (6, 5)$.

C. Let the roots of $y = g(x)$ be r, s and t. Then: $r+s+t = -1$, $rs+rt+st = -5$ and $rst = -2$
If $f(x)$ has zeros: $1 + 1/r$, $1 + 1/s$ and $1 + 1/t$:

$$(1+1/r) + (1+1/s) + (1+1/t) = \frac{3rst+rs+rt+st}{rst} = \frac{-6+(-5)}{-2} = \frac{11}{2}$$

$(1+1/r)(1+1/s) + (1+1/r)(1+1/t) + (1+1/s)(1+1/t) =$

$$\frac{3rst+2(rs+rt+st)+(r+s+t)}{rst} = \frac{-17}{-2} = \frac{17}{2}$$

$$(1+1/r)(1+1/s)(1+1/t) = 1 + \frac{1+(r+s+t)+(rs+rt+st)}{rst} = 1 + \frac{1+(-1)+(-5)}{-2} = \frac{7}{2}$$

$f(x) = k(x^3 - (11/2)x^2 + (17/2)x - 7/2 = -2x^3 + 11x^2 - 17x + 7$

D. The possible locations of the 4th vertex are: (13, 10), (-11, 4) and (5, -2). Note that A, B and C are midpoints of the triangle formed by connecting these three points. The one furthest from y = x is (-11, 4) which is $\sqrt{137}$ from the origin.