MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2012 SOLUTION KEY

Round 4

A) Given: $x^2 - 3Ax + B = 0$ and A is a positive root.
Since the coefficient of x is $-3A$, the roots must be A and $2A$, to make a sum of $3A$.
Since B must be the product of the roots and A is positive, we have $2A^2 = 72 \Rightarrow A = \underline{\mathbf{6}}$.

Alternate Solution: (Brute Force)
Over integers, 72 factors as either 8·9, 6·12, 3·24, 2·36, or 1·72
Considering the possible factorizations and the coefficients of the middle term, we have
$3A$ = 17, 18, 27, 38 and 73. Only $3A$ = 18 produces a value of A which is also a root.

B) $5[(x+2)(x+3)-(x-2)(x-3)] = 7(x-2)(x+3)$
$5\left[\left(x^2+5x+6\right)-\left(x^2-5x+6\right)\right] = 7\left(x^2+x-6\right)$
$5(10x) = 7x^2+7x-42$
$7x^2-43x-42 = (7x+6)(x-7) = 0 \Rightarrow x = \underline{\mathbf{-\frac{6}{7}, 7}}$

C) $9+2mx = 4x - x^2 \Leftrightarrow x^2+(2m-4)x+9 = 0$
Using the quadratic formula, no real roots $\Leftrightarrow$ a negative discriminant
Therefore,
$(2m-4)^2-36<0 \Leftrightarrow \left[2(m-2)\right]^2-36<0 \Leftrightarrow (m-2)^2-9<0 \Leftrightarrow m^2-4m-5<0 \Leftrightarrow (m-5)(m+1)<0$

The critical values are +5 and –1 and the product is negative in between.
Thus, the integer values are 0, 1, 2, 3 and 4, resulting in a sum of $\underline{\mathbf{10}}$.

Note: $m = -1 \Rightarrow x^2-6x+9 = (x-3)^2 = 0$ which has one real root of +3.
$m = 5 \Rightarrow x^2+6x+9 = (x+3)^2 = 0$ which has one real root of –3.