MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2013 SOLUTION KEY

Round 3

A) Since $\sin A = \sin C$, but $A \neq C$, A and C must be supplementary. Let $(A,C) = (\theta, 180-\theta)$.
Then: $\theta + 60 + (180-\theta) + m\angle D = 360 \Rightarrow m\angle D = \underline{\mathbf{120}}$.

B) Since $2\cos x \csc x = 2\dfrac{\cos x}{\sin x} = 2\cot x$, we have $\tan x - 3\cot x = 0$
Multiplying by $\tan x$, $\tan^2 x - 3 = 0 \Rightarrow \tan x = \pm\sqrt{3}$ and the result follows.

C) Using the identity $\sin A + \sin B = 2\sin\left(\dfrac{A+B}{2}\right)\cos\left(\dfrac{A-B}{2}\right)$, $\sin 3x + \sin 7x = 2\sin 5x \cos 2x$
Therefore, $\sin x + \sin 2x + \sin 3x = 0 \Rightarrow \sin 5x\left(2\cos 2x + 1\right) = 0$.
$\sin 5x = 0 \Rightarrow 5x = \begin{cases} 0 + 2n\pi \\ \pi + 2n\pi \end{cases} \Rightarrow 5x = 0 + n\pi \Rightarrow x = 0 + \dfrac{n\pi}{5}$ and as n assumes values from 0 to 9, we get 10 distinct solutions over the specified interval.
$2\cos 2x + 1 = 0 \Rightarrow \cos 2x = -\dfrac{1}{2} \Rightarrow x = \begin{cases} \dfrac{\pi}{6} + n\pi \\ \dfrac{5\pi}{6} + n\pi \end{cases}$ and as n assumes values of 0 and 1 we get 4 more solutions for a total of $\underline{\mathbf{14}}$.