MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2013 SOLUTION KEY

Team Round

B) Let (t, u) denote the tens' and units' digits of the two-digit number (in base 10).
The original two-digit integer is $10t + u$.
The positive difference of the digits is either $t - u$ or $u - t$.
According to the first condition, if $t > u$, $\dfrac{10t+u}{t-u} = 21$ or if $t < u$, $\dfrac{10t+u}{u-t} = 21$.
$t > u \Rightarrow 10t + u = 21t - 21u \Rightarrow 22u = 11t \Rightarrow t = 2u$
$t < u \Rightarrow 10t + u = 21u - 21t \Rightarrow 31t = 20u$ has no solutions for base 10 digits.
According to the second condition, $tu + (t - u) = 21$.
Substituting, $2u^2 + u - 21 = (2u + 7)(u - 3) = 0 \Rightarrow u = 3$ only $\Rightarrow N =$ **<u>63</u>** only.

C) $AS = 6 \Rightarrow$ area of ΔABC is 48
$\Rightarrow$ the area of $\Delta APQ = 8$.
$\overline{PQ} \parallel \overline{BC} \Rightarrow \Delta APR \sim \Delta ABS \Rightarrow \dfrac{x}{y} = \dfrac{6}{8} \Rightarrow 4x = 3y$.
$\dfrac{1}{2}x(2y) = xy = 8$
$12xy = (4x)(3y) = 12 \cdot 8 = 96$
Substituting, $16x^2 = 96 \Rightarrow x = \sqrt{6} \Rightarrow h =$ **<u>$6 - \sqrt{6}$</u>**.