MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 - FEBRUARY 2013
ROUND 7 TEAM QUESTIONS

ANSWERS

A) _____ D) _____ ¢

B) _____ E) (_____ , _____)

C) _____ F) _____

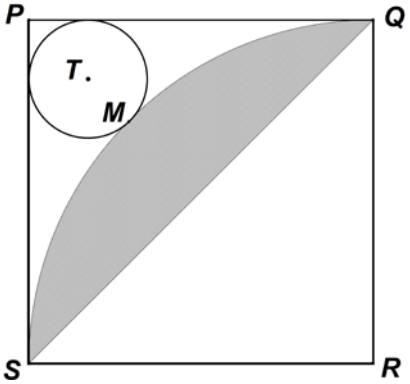
A) Given: $f(x) = \dfrac{Ax^3 + Bx^2 - 6x + 3}{4x^2 - 1}$, $f(5) = 27$, $f(-1) = -1$
It has a linear asymptote $y = mx + b$ as $x \to \pm\infty$.
Compute the ordered pair (*m*, *b*).

B) Let *N* be a 4-digit integer consisting exclusively of prime base 10 digits, but not necessarily distinct. How many of these integers are divisible by 11?

C) Compute *x* such that $Arc\cos\left(\dfrac{25}{32}\right) + Arc\cos(x) = Arc\cos\left(\dfrac{1}{20}\right)$.

D) Suppose a sheet of first-class FOREVER stamps costs \$6.72 in 2021.
Suppose that due to a 4¢ increase in the rate, a sheet of FOREVER stamps with 8 more stamps cost \$12.00 in 2022. No sheet ever contains more than 50 stamps.
If a FOREVER stamp costs 46¢ in 2013, <u>*how much more*</u> will a FOREVER stamp cost in 2022?

E) *PQRS* is a square, *PQ* = 6.
$\overset{\frown}{QMS}$ is an arc of a circle with center at *R* and radius *RQ*.
Circle *T* is tangent to 2 sides of the square and to the arc at point *M*. The ratio of the area of circle *T* to the area of the segment on $\overline{SQ}$ (i.e. the shaded region) may be expressed as $\dfrac{A\pi}{\pi - B}$, where *B* is an integer. Compute the ordered pair (*A*, *B*).



F) Suppose a sequence is defined by the recursive relation $a_n - 2a_{n+1} = 1$.
If the first five terms are all <u>positive integers</u>, compute the <u>minimum sum</u> of these five terms.