MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2010 SOLUTION KEY

Team Round - continued

D) $x^{14k} - x^{8k} - x^{6k} + 1 = \left(x^{6k} - 1\right)\left(x^{8k} - 1\right) = \left(x^{3k} + 1\right)\left(x^{3k} - 1\right)\left(x^{4k} + 1\right)\left(x^{4k} - 1\right)$

$= \left(x^{k} + 1\right)\left(x^{2k} - x^{k} + 1\right)\left(x^{k} - 1\right)\left(x^{2k} + x^{k} + 1\right)\left(x^{4k} + 1\right)\left(x^{2k} + 1\right)\left(x^{k} + 1\right)\left(x^{k} - 1\right)$

Thus, the sum of the factors is $x^{4k} + 3x^{2k} + 4x^{k} + 4$ ➔ **(1, 3, 4, 4)**.

E) $\sin 54° = \dfrac{\sqrt{5}+1}{4}$ ➔ $\cos 36° = \dfrac{\sqrt{5}+1}{4}$. Utilizing basic identities,

$\sin 2\theta \sin\theta = 2\sin^2\theta\cos\theta = 2\cos\theta\left(1-\cos^2\theta\right)$ (***)

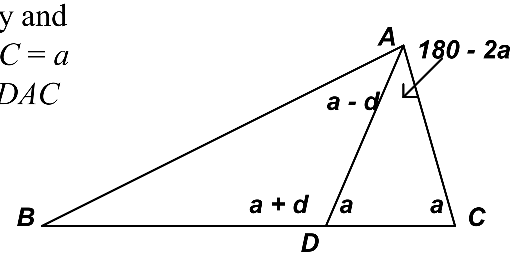
➔ $\sin 144° \sin 72° = \sin(180° - 36°)\sin 72° = \sin 36° \sin\left(2(36)°\right)$

Let θ = 36. Then (***) ➔ $\sin 144° \sin 72° = 2\cos 36°(1-\cos^2 36°)$

$$= 2\left(\frac{\sqrt{5}+1}{4}\right)\left(1-\left(\frac{\sqrt{5}+1}{4}\right)^2\right) = \left(\frac{\sqrt{5}+1}{2}\right)\left(\frac{16-6-2\sqrt{5}}{16}\right) = \left(\frac{\sqrt{5}+1}{2}\right)\left(\frac{5-\sqrt{5}}{8}\right) = \frac{4\sqrt{5}}{16} = \frac{\sqrt{5}}{4}$$

➔ (A, B) = **(5, 4)**.

F) Let m$\angle BAD$, m$\angle ADC$, m$\angle ADB$ = $a - d$, a and $a + d$ respectively and $d^2 = a + 60$. Since $BA = BC$, m$\angle C$ = m$\angle BAC$, so $a - d$ + m$\angle DAC = a$ ➔ m$\angle DAC = d$. But notice also that in ΔDAC, the vertex angle DAC has measure $180 - 2a$.



Equating and solving for a, $d = 180 - 2a$ ➔ $a = \dfrac{180-d}{2}$.

Thus, $d^2 = a + 60$ becomes $d^2 = \dfrac{180-d}{2} + 60$

➔ $2d^2 = 180 - d + 120$ ➔ $2d^2 + d - 300 = (2d+25)(d-12) = 0$ ➔ $d = 12$ only, $a = 84$.
($d = -12.5$ ➔ $a = 96.25$ which is impossible for the base angle in an isosceles triangle.)
Finally, m$\angle BAD$ = 84 – 12 = **72**.

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