

MASSACHUSETTS MATHEMATICS LEAGUE  
NOVEMBER 2003  
ROUND 1: COMPLEX NUMBERS

ANSWERS

A)  $\underline{-12/25}$

B)  $\underline{2\sqrt{3}}$

C)  $\underline{-1 \pm i}$

A) Simplify and leave in  $a + bi$  form.  $\frac{5}{4+3i}$ , Calculate  $ab$ ,

$$\frac{5}{4+3i} \cdot \frac{4-3i}{4-3i} = \frac{20-15i}{25} = \frac{4}{5} - \frac{3}{5}i,$$

$$ab = -12/25$$

B) Calculate the absolute value of:  $\frac{-2\sqrt{3}+6i}{-\sqrt{2}-i\sqrt{2}}$  in simple radical form.

$$\frac{|-2\sqrt{3}+6i|}{|-\sqrt{2}-i\sqrt{2}|} = \frac{\sqrt{12+36}}{\sqrt{2+2}} = \frac{\sqrt{48}}{\sqrt{4}} = \sqrt{12} = 2\sqrt{3}$$

C) Find the imaginary roots of the equation  $x^4 + x^3 - 6x^2 - 14x - 12 = 0$

$$\begin{array}{r|rrrrr} & 1 & 1 & -6 & -14 & -12 \\ 3 & 1 & 4 & 6 & 4 & 0 \\ -2 & 1 & 2 & 2 & 0 & \end{array}$$

$$x^2 + 2x + 2 = 0$$

$$x^2 + 2x + 1 = -2 + 1$$

$$(x+1)^2 = -1$$

$$x = -1 \pm i$$