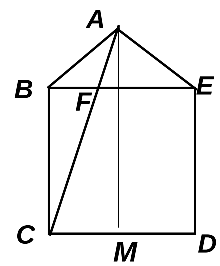
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 – NOVEMBER 2014 SOLUTION KEY

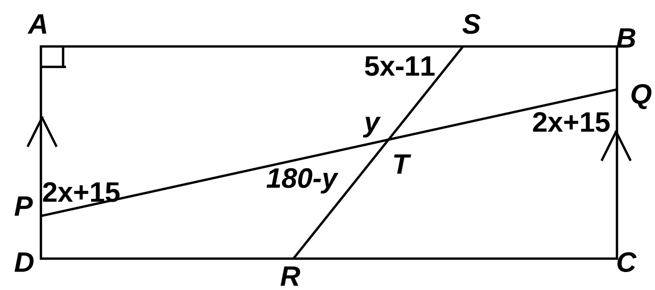
Round 6



A) Drop a perpendicular from A to $\overline{CD}$.
$m\measuredangle EAM = m\measuredangle BAM = 57°$.
$m\measuredangle BAF = \frac{1}{3}\cdot 114 = 38° \Rightarrow m\measuredangle CAM = 19°$
$\Rightarrow m\measuredangle ACD = m\measuredangle ACM = 90 - 19 = \underline{\mathbf{71}}°$

B) Since the measure of the exterior angle equals the sum of the measures of the two remote interior angles, we have $(6x+7)+(8x-9) = x^2+46 \Leftrightarrow x^2-14x+48 = (x-6)(x-8) = 0$.

For x = 8, $m\angle A = m\angle B = 55°$ and ABC is isosceles. This solution is rejected.
For x = 6, $m\angle A = 43$, $m\angle B = 39°$, and $m\angle C = 180-(43+39) = 98$ and ABC is scalene.
Thus, x = **6** only.



C) In quadrilateral $PAST$,
$7x+4+90+y = 360 \Rightarrow y = 266-7x > 90$
$\Rightarrow 7x < 176 \Rightarrow x \le 25$
But we also know that $y < 180$
$\Rightarrow 7x > 86 \Rightarrow x \ge 13$
Thus, $13 \le x \le 25$ generates all possible values of $m\angle PTR$, a total of **13** different values.
Check: For $x = 13, \ldots, 25$,
$m\angle PTR = 180 - y = 7x - 86° \Rightarrow 5°, 12°\ (5+7\cdot 1), \ldots, 89°\ \left(5+7\cdot 12\right)$ - 13 distinct values.