MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2007 SOLUTION KEY

Round 2

A) Since magic integers are 6^{th} powers of positive integers, we have

$1^6 = 1$, $2^6 = 64$, $3^6 = 729$ $4^6 = 2^{12} = 1024(4) = 4096$

$5^6 = 625(25) = \dfrac{625(100)}{4} = 15615$ $6^6 = 2^6 3^6 = 64(729) = 46656$

$1 + 64 + 729 + 4096 + 15625 + 46656 =$ **<u>67171</u>**

Note in the expansion of 5^6, instead of multiplying by 25, a two-digit number,
we multiply by 100 and divide by 4, i.e. append two zeros and divide by a 1-digit multiplier, 4.

B) Summing the entries in the first three rows we have: 10, 20 and 40
The sum of the entries is apparently doubling from one row to the next; or, looking from another perspective, is given by the formula $10(2^{\text{row}-1})$.
$\rightarrow S_{16} = 10(2^{15}) =$ **<u>327680</u>**

$2^{10} = 1024 \rightarrow 2048 \rightarrow 4096 \rightarrow 8192 \rightarrow 16384 \rightarrow 32768$

C) $\left(12^{x+1}\right)\cdot\left(18^{x-1}\right)\cdot\left(75^3\right) = \left(2^2\cdot 3\right)^{x+1}\cdot\left(2\cdot 3^2\right)^{x-1}\cdot\left(5^2\cdot 3\right)^3 = 2^{2x+2}\cdot 3^{x+1}\cdot 2^{x-1}\cdot 3^{2x-2}\cdot 5^6\cdot 3^3$

$= 2^{3x+1}\cdot 3^{3x+2}\cdot 5^6$

Since the factor must be even, there are $(3x + 1)$ choices for the exponent of 2, namely $1, 2, \ldots, 3x+ 1$.
However, since 0 is an allowable exponent for 3 and 5, there are $(3x + 3)$ and 7 choices for the exponents of 3 and 5 respectively.

Thus, the number of even factors is $\mathbf{(3x + 1)(3x + 3)(7)} =$ **<u>$3\cdot 7(x + 1)(3x + 1)$</u>** or **<u>$21(x + 1)(3x + 1)$</u>**