MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 – NOVEMBER 2014 SOLUTION KEY

Round 2

A) $|1-A|=5 \Leftrightarrow 1-A=\pm 5 \Leftrightarrow A=1\pm 5=$ ~~-4~~, 6

$|x-6|=5 \Leftrightarrow x-6=\pm 5 \Leftrightarrow x=1,$ **$\underline{11}$**

B) The sum of the solutions is $\frac{A}{2}+\frac{-B}{3}=\frac{3A-2B}{6}$.

Testing the 6 possible ordered pairs (6, 1), (5, 2), (4, 3), (3, 4), (2, 5) and (1, 6), only (4, 3) produces an integer solution sum $\left[\frac{3(4)-2(3)}{6}=1\right] \Rightarrow A^2-B^2=16-9=\underline{\mathbf{7}}$.

C) $\frac{x}{2}+\frac{y}{3}=k \Leftrightarrow$ (1) $3x+2y=6k$

(2) $2x+3y=k$

Adding the two equations and dividing by 5, we have $x+y=\frac{7k}{5}$.

Since we were given that $k>10$ and x and y must be integers, $k_{\min}=15$.

Substituting for y in (2), $2x+3(21-x)=15 \Rightarrow x=48 \Rightarrow \underline{(\mathbf{15, 48, -27})}$.