MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2014 SOLUTION KEY

Round 4

A) As the product of primes, $42 = 2(3)(7)$ and $90 = 2(3)^2 5$.
Taking the smallest exponents of the common factors,
The numerical component is $2(3) = 6$ and the literal component is x^2z^3.
Therefore, the GCF is $\underline{\mathbf{6x^2z^3}}$.

B) Since the product is not equal to zero, the factorization does not help. Multiplying out the left side and combining like terms, we have
$(3x+4)(8x-5) = -23 \Leftrightarrow 24x^2 + 17x + 3 = 0$. Since the coefficient of the middle term is odd, the factors of 24 cannot both be even, leaving only possibilities of $24 \cdot 1$ or $8 \cdot 3$. If these fail, we would have to use the quadratic formula and none of the solutions would be rational.
Since $24x^2 + 17x + 3 = (8x+3)(3x+1) = 0$ and we have rational solutions, namely,
$x = \underline{\mathbf{-\frac{3}{8}, -\frac{1}{3}}}$.

C) $x(x-2A) + A(A+5) - 4 = 5(x+4) \Leftrightarrow (x^2 - 2Ax + A^2) + 5A - 5x - 24 = 0$
$\Leftrightarrow (x-A)^2 - 5(x-A) - 24 = 0$
$\Leftrightarrow (x-A-8)(x-A+3) = 0$
$\Rightarrow x = \underline{\mathbf{A+8}}$ or $x = \underline{\mathbf{A-3}}$