MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2017 SOLUTION KEY

Round 2

A) For nonzero integer values of *k*, $\dfrac{k}{2k-1}$ produces an integer value only for $k = \underline{\mathbf{1}}$.

Now, we need to look at fractional exponents which will produce an integer result.

Since $8^{\frac{1}{3}} = 2$ and $8^{\frac{2}{3}} = 4$, we have
$$\frac{k}{2k-1} = \frac{1}{3} \Rightarrow 3k = 2k-1 \Rightarrow k = \underline{\mathbf{-1}}$$
$$\frac{k}{2k-1} = \frac{2}{3} \Rightarrow 3k = 4k-2 \Rightarrow k = \underline{\mathbf{2}}$$

B) <u>Remember</u>: $x+7 \ge 0$, since the square root on the left side of the equation must be non-negative.

Squaring both sides, $2x^2+21x-11 = x^2+14x+49 \Leftrightarrow x^2+7x-60 = (x-5)(x+12) = 0$

$\Rightarrow x = \underline{\mathbf{5}}, \cancel{-12}$.

C) $(x,y) = (7,1), (6,2), (5,3), (4,4)$

The units digits of positive integer powers of 2 are cyclic with a period of 4, i.e. they repeat in blocks of 4. $2^1, 2^2, 2^3, 2^4, 2^5, 2^6 \ldots \Rightarrow \underline{2,4,8,6,}2,4,\ldots$

The rightmost digit of $2^{71}+2^{17}$ is the same as that of $2^3+2^1 = 1\underline{0}$.

The rightmost digit of $2^{62}+2^{26}$ is the same as that of $2^2+2^2 = 0\underline{8}$.

The rightmost digit of $2^{53}+2^{35}$ is the same as that of $2^1+2^3 = 1\underline{0}$.

The rightmost digit of $2^{44}+2^{44}$ is the same as that of $2^0+2^0 = 0\underline{2}$.

Thus, $2^x+2^y = 2^7+2^1 = \underline{\mathbf{130}}$, $2^5+2^3 = \underline{\mathbf{40}}$.