MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2011 SOLUTION KEY

Round 4

A) $\frac{1}{8}+\frac{5}{6}=\frac{3+20}{24}=\frac{23}{24}$ Thus, Gottfried (or for those who aren't on a first name basis – Mr. Leibnitz) represents $\frac{1}{24}$ th of the people in line and the minimum number of people is **24**.

FYI: An alternate solution (He's a real intellectual and some would say egotistical and long-winded.) Courtesy of Mr. Leibnitz himself with thanks to his great grandson for translating from the original German – they both love algebra and all forms of higher math.)
Suppose there were *x* people behind me and *y* people in front of me.

It follows that $\frac{1}{8}\left(x+y+\overset{\int ME}{\boxed{1}}\right)+\overset{\int ME}{\boxed{1}}+\frac{5}{6}\left(x+y+\overset{\int ME}{\boxed{1}}\right)=x+y+\overset{\int ME}{\boxed{1}}$. Multiplying through by 24,

$3(x + y + 1) + 24 + 20(x + y + 1) = 24(x + y + 1)$. Combining terms, $(x + y + 1) =$ **24**.

BUT $x + y + 1 = 24 \Leftrightarrow x + y = 23$, so why aren't there more possibilities?
We wanted the length of the line, not the number of people in front of me and behind me.
$x = 3$ and $y = 20$ is the only possible *x*, *y*-combination. [3/24 = 1/8, 20/24 = 5/6]
For example, 4 + 19 = 23 (This would be fine with ME, I'd be closer to the ticket window.), but this would have 4/24 = 1/6 of the people behind me and 19/24 of the people in front of me, clearly violating the initial conditions and similarly for any other *x*, *y*-values.
Q.E.D – That's all I've got to say about that, but what about the law of diminishing returns?

The sequence of individual terms of the harmonic series $\sum_{k=1}^{\infty}\frac{1}{k}$ approach 0,

but I think the series itself diverges. What do you think?

B) $\frac{W}{W+L}=\frac{46+x}{82}=\frac{4}{5}\Rightarrow 328 = 230 + 5x \Rightarrow x = 98/5 = 19.6 \Rightarrow$ **20** games
Check: 65/82 = 0.793   66/82 = 0.805

C) With respect to the *x*-coordinate, $(4t – 3)$ must be a multiple of 9 for *x* to be an integer. Multiples of 9 are 9 apart, so we look at *t*-values which are 9 apart, starting at –15, –6, 3, 12, 21, ….
With respect to the *y*-coordinate, $(13 – 5t)$ must be a multiple of 8 for *y* to be an integer. Multiples of 8 are 8 apart, so we look at *t*-values which are 8 apart, starting at –15, –7, 1, 9, 17, ….
What is the next number that these lists will have in common?
We could continue the lists until the common number appeared or simply note that the least common multiple of 8 and 9 is 72. –15 + 72 = **57**.
It's left to you to check that for $x = 57$, both *x* and *y* are integers.

Alternative Solutions
Solve for *y* in terms of *x*: $(45x – 32y = –14)$ A (reduced) slope of 45/32 $\Rightarrow x = –6 + 32 = 26$ and substituting for *x*, $26 = 1 + (4t – 3)/9 \Rightarrow t = (25\cdot 9 + 3)/4 = 228/4 =$ **57**.

The calculus student would have found the slope this way: $m=\frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}=\frac{5}{8}\div\frac{4}{9}=\frac{45}{32}$
and proceeded as above. The derivative – a powerful tool indeed!