MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 – MARCH 2012 SOLUTION KEY

Round 1

A) $19 - 3x = 1 \Rightarrow x = 6$ which in turn implies $a = -6$, $b = -12$ and $c = -1$.
Therefore, (x, a, b, c) = **<u>(6, –6, –12, –1)</u>**.

B) $\begin{vmatrix} 1 & x & 2 \\ x & 1 & 2 \\ 1 & 2 & x \end{vmatrix} \begin{matrix} 1 & x \\ x & 1 \\ 1 & 2 \end{matrix} \Rightarrow (x+2x+4x)-(2+4+x^3) = -x^3+7x-6=0$

Clearly, $x = 1$ is a solution and by synthetic division
$-x^3+7x-6=-1(x^3-7x+6)=-1(x-1)(x^2+x-6)=-1(x-1)(x-2)(x+3)=0$
The roots are 1, 2 and –3 and $(n_1, n_2, n_3) = \underline{(\mathbf{-3,1,2})}$.

C) Adding the first two equations, $3x + (1 + a) = 0$ or $x = \dfrac{-(1+a)}{3}$.
Multiplying the first equation by 4 and subtracting the third, $x = b - 4$ (***).
Equating, $\dfrac{-(1+a)}{3} = b-4 \Leftrightarrow 3b+a=11$.
Over positive integers, (a, b) = (2, 3), (5, 2) and (8, 1). The maximum occurs for $a = 8$ and $b = 1$.
Substituting for b in (***), $x = -3$.
Substituting back in the first equation ($x + y + 1 = 0$), we have **(x, y) = <u>(–3, 2)</u>.**