**MASSACHUSETTS MATHEMATICS LEAGUE**
**CONTEST 4 - JANUARY 2012 SOLUTION KEY**

**Round 2**

A) Note: 210 = 210(1) (sum: 211), 105(2) (sum: 107), 3(70) (sum: 73) The value of the sum $a + b$ decreases as the difference between the factors decreases.
Thus, we are looking for the pair of factors of 210 with the minimum difference.
The prime factorization of 210 is 2·3·5·7. Pairing the outer and the inner factors, we have 14 and 15. Clearly, 1 is the minimum possible difference (unless 210 were a perfect square which it is not). Thus, the minimum value of $a + b$ = **29**.

B) $$\frac{x^3y - xy^3}{x^3y^2 - x^2y^3} = \frac{xy\left(x^2 - y^2\right)}{x^2y^2(x-y)} = \frac{xy(x+y)\cancel{(x-y)}}{x^2y^2\cancel{(x-y)}} = \frac{x+y}{xy} = \frac{1}{y} + \frac{1}{x} = \frac{91}{105} = \frac{\mathbf{13}}{\mathbf{15}}$$

C) Certainly, $(x - 4)(4x - 1)$ gives the proper lead coefficient and coefficient of the middle term, so $A = 4$ works. But there are many possible factorizations to examine.
How do we approach the search systematically? How can we limit the search?
First, notice that the lead coefficient either factors as 4·1 or 2·2.
Since filling in the blanks in (2$x$ + ___ )(2$x$ – ___ ) with integers would always produce an even coefficient in the middle term, our search is limited to products of the form (4$x$ – ___ )($x$ – ___ ).
One of the solutions will always be a positive *integer*.
Our first factorization has established an upper limit for integer solutions, since $(x - 5)$ requires a second factor of $(4x + 3)$ and $d = -3$ violates the condition of positive constants.

Thus, we try products ($x$ – 1) (4$x$ – ___ ), ($x$ – 2)(4$x$ – ___ ) and ($x$ – 3)(4$x$ – ___ ).
FOILing out these products, each can produce a middle coefficient of 17, since the outer product simply makes up the difference. The fill-ins are 13, 9 and 5 respectively.
Therefore, there are 4 possibilities for the product $A$, namely $A$ = **13, 18** and **15**, plus $A$ = **4** that we had found originally (Answers allowed in any order).