Team Round - continued

F) $a_n - 2a_{n+1} = 1 \Rightarrow a_{n+1} = \dfrac{a_n - 1}{2}$

Using this form of the recursive definition, we can find expressions for the first five terms, all in terms of a_1.

$$\begin{cases} a_2 = \dfrac{a_1 - 1}{2} \\ a_3 = \dfrac{a_2 - 1}{2} = \dfrac{\frac{a_1 - 1}{2} - 1}{2} = \dfrac{a_1 - 3}{4} \\ a_4 = \dfrac{a_1 - 7}{8} \\ a_5 = \dfrac{a_1 - 15}{16} \end{cases}$$

All of these expressions must generate integers. The smallest value of a_1 for which a_5 is an integer is 31, producing $a_5 = 1$

Substituting back up the chain, $(a_5,\ a_4,\ a_3,\ a_2,\ a_1) = (1,\ 3,\ 7,\ 15,\ 31)$ and the sum is **57**.