MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2016
ROUND 7 TEAM QUESTIONS

ANSWERS

A) _____ D) _____

B) { x | _____ } E) _____

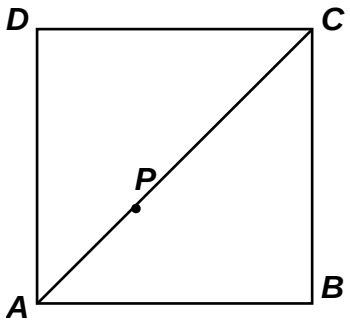
C) _____ F) _____

A) Given a hyperbola defined by $3(y+3)^2-(x-4)^2=12$.
The major axis of an ellipse is the transverse (or major) axis of this hyperbola.
(The vertices on the major axis of the ellipse coincide with the vertices of the hyperbola.)
The major axis of the ellipse is twice as long as its minor axis. Compute the length of the line segment on $y=-2$ whose endpoints are on the ellipse.

B) $\{(x,y)\,|\,xy(y-2)=x^2-6,\text{ where } y\geq 1\}$ defines a real-valued function $y=f(x)$ whose domain is a subset of the real numbers. Specify the domain of *f*.
For disjoint intervals, the use of the proper connector ("and" or "or") is required.
Alternatively, the symbols "$\wedge$" and "$\vee$" may be used.

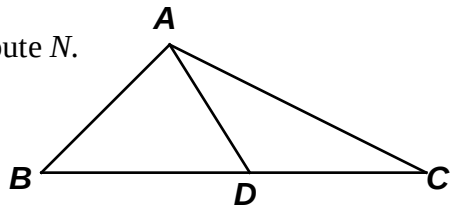
C) Compute all possible solutions *x*, where $x\in[0,\pi]$, to the equation $\tan^2 x+\cot^2 x=14$.

D) Given: *ABCD*, a square with side 6.
Beetle Bailey received these marching orders:
- Start at *A*.
- Proceed along $\overline{AC}$.
- Stop at point *P* which is twice as far from *B* as it is from *A*.



These instructions are way above Beetle's pay grade, but with your help he can prove how smart he is to his commanding officer. In simplified form, $AP=\sqrt{2}\left(\sqrt{N}-1\right)$, where *N* is an integer. Compute *N*.

E) Given: $m\angle DAC=m\angle DCA$, $\Delta BAC\sim\Delta BDA$, $AB=1$, $AC=2$.
Compute *AD*.



F) My little brother JJ and I are very competitive. JJ's best mile time to date is 4:30.
JJ's goal is to be able to beat my time. Assuming he improves his time *k*% each year for the next two years, he will beat my current best time of 4:03.
Determine the smallest integer value of *k* which allows him to reach his goal.
[A helpful fact: $\sqrt{10}\approx 3.162$]