MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2016 SOLUTION KEY

Round 1

A) Multiplying the second equation by 6, we have $\begin{cases} 3x-4y=10 \\ \dfrac{x}{2}+ky=c \end{cases} \Leftrightarrow \begin{cases} 3x-4y=10 \\ 3x+6ky=6c \end{cases}$

If $6k=-4$, the lines have the same slope and the lines are either parallel or coincident (identical).
If $6c \neq 10$, the equations would not be identical and there would be no solution.

Since inconsistent linear equations have no solution, we have $(k,c)=\underline{\left(-\mathbf{\frac{2}{3}},\mathbf{\frac{5}{3}}\right)}$.

B) Since the 2 x 2 identity matrix is $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, we have $\begin{bmatrix} A+B & B+k \\ C+k+3 & C+D \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Leftrightarrow$

$$\begin{cases} A+B=C+D=1 \\ B+k=0 \\ C+k+3=0 \end{cases} \Rightarrow \begin{cases} B=-k \\ A=1+k \\ C=-k-3 \\ D=1-C=4+k \end{cases}$$

Thus, $(A-B)+(C-D)=(1+2k)+(-7-2k)=\underline{\mathbf{-6}}$.

C) Let *x*, *y* *z* and *w* represent the 4 numbers. Then:

$$\begin{cases} x+\dfrac{1}{3}(y+z+w)=26, \\ \text{etc.} \end{cases} \Leftrightarrow \begin{cases} (1)\ \ 3x+y+z+w=78 \\ (2)\ \ x+3y+z+w=90 \\ (3)\ \ x+y+3z+w=114 \\ (4)\ \ x+y+z+3w=126 \end{cases} \Rightarrow \begin{cases} (2)-(1)\ \ -x+y=6 \\ (3)-(1)\ \ -x+z=18 \\ (4)-(1)\ \ -x+w=24 \end{cases} \Rightarrow$$

$$\begin{cases} y=x+6 \\ z=x+18 \\ w=x+24 \end{cases}$$

Substituting in (1), $3x+(x+6)+(x+18)+(x+24)=78 \Rightarrow 6x=30 \Rightarrow x=5$
$\Rightarrow (x, y, z, w) =$ **<u>(5, 11, 23, 29)</u>**. Numbers may be listed in any order.

Alternatively, **add all 4 equations.**
$6(x+y+z+w)=408 \Rightarrow x+y+z+w=68$
Subtracting this equation from equations (1), …, (4) above, we have
$2x=10,\ 2y=22,\ 2z=46,\ 2w=58$ and the same results follow.