MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2015
ROUND 7 TEAM QUESTIONS

D) A projectile is fired at a target 1 mile away (i.e. 5280 feet), but it never reaches the target. Every second it moves to a point which is half the distance to the target. (After 1 second, it has moved 2640 feet.) After a minimum of k seconds, it is less than 88 feet from the target and, to the nearest integer, its average speed over k seconds must be at least S ft/sec. Compute the ordered pair (k, S).

E) A highway department truck, traveling at a constant speed, is spraying the center line on a newly paved highway, where passing is permitted. The spray gun is supposed to cycle on for 1 second and off for 3/10 of a second. Due to a timing malfunction, the time for the off cycles is uniformly increasing 3/10, 4/10, 5/10, … . Instead of the expected uniform striping

we get

as the gaps between strips get increasingly longer. The distance between points A and B contains 4 complete stripes. If AB = 104 feet, compute how many complete or partial stripes will be painted in the first mile (5280 feet) of this new road.

F) Lattice points in the interior of the rectangular region in quadrant 1 with a pair of opposite vertices at the origin and $(N, 3)$ determine 450 segments whose lengths are greater than 1 unit. Compute N.