MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2011 SOLUTION KEY

Team Round – continued

E) Solution #4 (Tuan Lee)

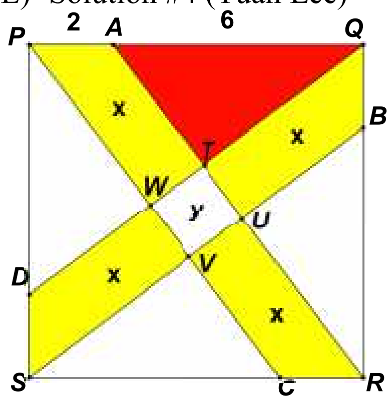

Diagram #1

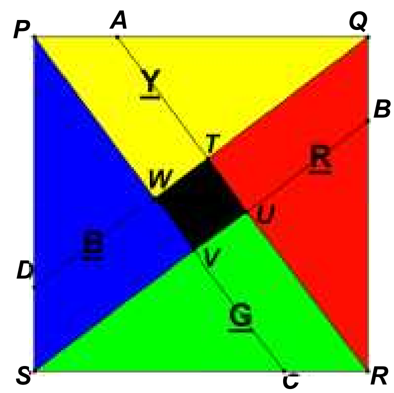

Diagram #2

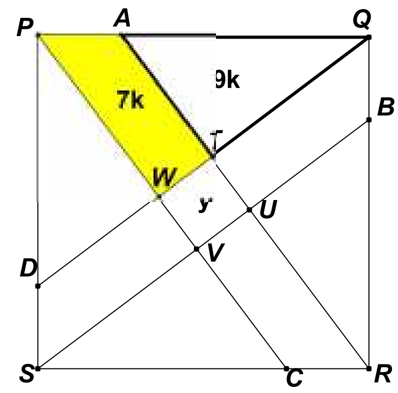

Diagram #3

From diagram #1:
Convince yourself that $ATWP$, $QBUT$, $RCVU$ and $DWVS$ have equal areas (yellow regions, labeled x).

area(ΔPQD) = 24, $PQBS$ is a right-angled trapezoid and its area is $\frac{1}{2}8(2+8)=40$

➔ area($QBSD$) = 16 ➔ equation #1: $\boxed{2x + y = 16}$

Diagram #2:
Clearly, the area of the large square is equal to the sum of the areas of the five shaded regions.

We aim to show that the area of ΔPQW (yellow region, labeled Y) can be expressed entirely in terms of the area of the trapezoidal region $ATWP$ (which was called x in diagram #1)

$$\Delta QAT \sim \Delta QPW \rightarrow \frac{QT}{QW}=\frac{QA}{QP}=\frac{3}{4} \rightarrow \frac{area(\Delta QAT)}{area(\Delta QPW)}=\frac{9}{16}$$

Diagram #3:
If the area(ΔQAT) = $9k$ and the area(ΔQWP) = $16k$, then the area($ATWP$) = $7k$.

$$16k=\frac{16}{7}\cdot 7k \rightarrow area(\Delta PQW)=\frac{16}{7}area(ATWP)$$

Let absolute value notation be shorthand for "the area of".

Similarly, $|QRT| = \frac{16}{7}|QBUT|$ (red-R), $|RSU| = \frac{16}{7}|RCVU|$ (green-G) and $|SPV| = \frac{16}{7}|DWVS|$ (blue-B).

$|PQRS| = |PQW| + |QRT| + |RSU| + |SPV| + |TUVW|$ = (Y + R + G + B + y)

$$=\frac{16}{7}\left(|ATWP|+|QBUT|+|RCVU|+|DWVS|\right)+TUVW=\frac{16}{7}(4x)+y$$

➔ Equation #2: $\boxed{64=\frac{64}{7}x+y}$. Solving, $(x, y)=\left(\frac{168}{25}, \mathbf{\frac{64}{25}}\right)$.