MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2009 SOLUTION KEY

Round 4

A) Since $R \cdot T = D$, i.e. (Rate)(Time) = Distance, the <u>average</u> rate is the total distance traveled divided by the total time it took to travel that distance. $r_{ave} = \dfrac{2+10+40}{3+4+1.5} = \dfrac{52}{8.5} = \dfrac{104}{17}$.

$6 < \dfrac{104}{17} = 6\dfrac{2}{17} < 7$ ➔ (A, C, B) = **<u>(6, 6, 7)</u>**

B) $\begin{cases} x = \dfrac{4}{5}y \\ y = \dfrac{1}{2}w \end{cases}$ ➔ $x = \dfrac{4}{15}w$ ➔ $x^2 = \dfrac{16}{225}w^2$ Since $k\% = \dfrac{k}{100}$, we have $\dfrac{k}{100} = \dfrac{16}{225}$.

$\dfrac{k}{4} = \dfrac{16}{9}$ ➔ $k = \dfrac{64}{9} = 7\dfrac{1}{9} = 7.111\cdots$ ➔ **<u>7.1</u>**

C) $\left.\begin{matrix} (1)\ a+b=2c \\ (2)\ a+c=3b \end{matrix}\right\}$ ➔ $b - c = 2c - 3b$ ➔ $4b = 3c$

It's reasonable to assume that the value of $\dfrac{b+c}{a}$ is <u>unique</u>, i.e. is the same for all values of a, b and c that satisfy the initial conditions. Thus, take $b = 3$, $c = 4$ ➔ $a = 5$ ➔ $\dfrac{3+4}{5} = \underline{\mathbf{\dfrac{7}{5}}}$

Of course <u>two</u> equations in <u>three</u> unknowns do not nail down a unique solution.
We need to also show that the value of the required fraction is invariant, i.e. the same for all choices of a, b and c.

Substituting $b = \dfrac{3}{4}c$ in (1), we have $a + \dfrac{3}{4}c = 2c$ ➔ $c = \dfrac{5}{4}c$

Now, $\dfrac{b+c}{a} = \dfrac{\frac{3}{4}c + c}{\frac{5}{4}c} = \dfrac{3c+4c}{5c} = \dfrac{7}{5}$