

Thanks to Canton mathletes who, post facto, discovered that Houston we do have a problem, despite the fact that the above solution appears to solve the question nicely.

The segments on the intersecting chords have been color coded to remind us that the product of the lengths of the segments on each chord must be equal.

***For those for whom geometry is a distance memory, the proof is included below.

On chord \overline{QR} , the product is $(7.5) \cdot 1.5 = 11.25$, but

on chord \overline{ST} , the corresponding product is $(6.25) \cdot 1.75 = 10.9375$. Close, but no cigar!

In a circle of radius 10, perpendicular chords of length 8 and 9 can certainly be drawn, but if the short chord divides the long chord into a 5 : 1 ratio, the long chord can not divide the short chord into a 25 : 7 ratio. These conditions are inconsistent and no such circle can be constructed.

The author and the proofreaders missed this inconsistency and at least 4 teams (including Canton) submitted this **best possible wrong answer** to a problem which is not solvable as stated.

Only Canton lodged an appeal to the official answer. (D , $\frac{1}{4}$) or any answer which stated “no answer due to inconsistent conditions” receives credit.

Anyone able to replace these inconsistent conditions with conditions allowing a solution, without drastically changing the problem, should send his ideas to olson.re@gmail.com.

Here’s a reminder of why the product-chord theorem must be satisfied.

$m\angle A = m\angle D$ and $m\angle B = m\angle C$, since each pair are inscribed angles intercepting the same arc in the circle. Thus, $\triangle ACE \sim \triangle DBE$ by AA.

It follows that the lengths of corresponding sides are

proportional, namely $\frac{AE}{DE} = \frac{CE}{BE}$ and, cross multiplying,

$$AE \cdot BE = DE \cdot CE.$$

