

**MASSACHUSETTS MATHEMATICS LEAGUE  
CONTEST 3 - DECEMBER 2007 SOLUTION KEY**

**Round 6**

A)  $MD = 6$  and  $DN = 8 \rightarrow MN = \sqrt{6^2 + 8^2} = 10$ .

The area of  $\triangle MDN$  is given by  $\frac{1}{2} \cdot MD \cdot DN$  and  $\frac{1}{2} \cdot MN \cdot DP$ . Thus,  $\frac{1}{2} \cdot 6 \cdot 8 = \frac{1}{2} \cdot 10 \cdot DP$   
 $\rightarrow DP = \underline{4.8}$

B) Let  $n$  denote the number of sides in the original polygon.

Then  $\frac{(n+1)(n-2)}{2} - \frac{n(n-3)}{2} = 23 \rightarrow -n - 2 + 3n = 46 \rightarrow n = 24$  and  $\frac{180(22)}{24} = \underline{165}$ .

C) Let  $FE = x$  and  $DE = h$  Then:

$$x^2 + h^2 = 15^2 = 225 \text{ and } h^2 = 16x$$

$$\rightarrow x^2 + 16x = 225 \rightarrow x^2 + 16x - 225 = (x - 9)(x - 25) = 0$$

$$\rightarrow x = 9 \rightarrow AB = CE = 25$$

Applying the Pythagorean Theorem or using Pythagorean Triples,  $BE = \underline{65}$

$$BE^2 = 25^2 + 60^2 = 4225 \text{ and } \sqrt{4225} = \underline{65}$$

$$(25, 60, \underline{\quad}) = 5(5, 12, 13) \rightarrow \underline{65}$$

