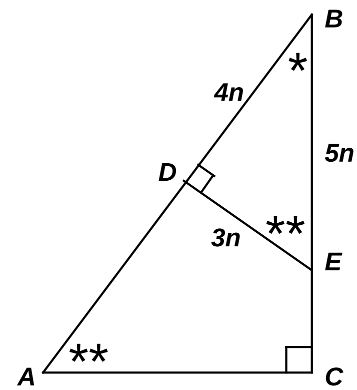
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 – NOVEMBER 2012 SOLUTION KEY

Round 5

A) $x+\frac{x}{5}+\frac{3x}{10}=180 \Leftrightarrow 15x=180(10) \Rightarrow x=120$

Recognizing the double angle formula, or plugging directly into $2\cos^2(x)-1$, we have

$2\cos^2(120)-1=2\left(-\frac{1}{2}\right)^2-1=\underline{\boldsymbol{-\frac{1}{2}}}$.



B) Since $\angle A \cong \angle BED$ and $\cos(\angle BED)=\frac{3}{5}$, $\cot A=\frac{3}{4}$.

Since $\angle DEC$ is the supplement of $\angle BED$, $\cos(\angle DEC)=-\frac{3}{5}$.

$\Rightarrow \cot^2 A+\cos(\angle DEC) = \frac{9}{16}-\frac{3}{5}=\frac{9\cdot 5-3\cdot 16}{16\cdot 5}=\underline{\boldsymbol{-\frac{3}{80}}}$

($\frac{-3}{80}$ and $\frac{3}{-80}$ are also acceptable.)

C) Substituting for A, $k\left(\frac{\sqrt{3}}{3}\right)=\frac{\sqrt{3}}{2}+B\left(\frac{\sqrt{3}}{2}\right) \Rightarrow \frac{k}{3}=\frac{1+B}{2} \Rightarrow 2k=3(1+B)$

B must be odd, since the product on the left side is even.
$B = 1 \Rightarrow k = 3$, but $3 + 1 = 4$ is not prime
$B = 3 \Rightarrow k = 6$, but $6 + 3 = 9$ is not prime
$B = 5 \Rightarrow k = 9$, but $5 + 9 = 14$ is not prime
$B = 7 \Rightarrow k = 12$ and $7 + 12 = 19$ is prime and $gcf(12,7)=1$
(i.e. 12 and 7 are relatively prime integers). Thus, $(k, B) = (12, 7) \Rightarrow$ **<u>19</u>**.