MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2013
ROUND 7 TEAM QUESTIONS
ANSWERS

A) ____________________________ D) (_____ , _____ , _____ , _____)

B) ____________________________ E) ____________________________

C) ____________________________ F) ____________________________

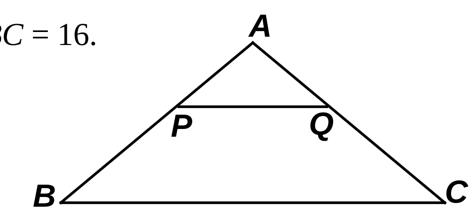
A) For all integer values of k, $\left(\frac{4+4i}{5}\right)^{4k}$ is a real number.

Compute the minimum value of k for which $\left(\frac{4+4i}{5}\right)^{4k} > 8$.

Using $\log_{10} 2 = 0.3$ as an approximation is adequate for this computation.

B) Find all positive two-digit integers N (in base 10) that satisfy both of the following statements:
1) The quotient of the integer divided by the positive difference of its digits is 21.
2) The sum of the product of the digits and the positive difference of the digits is 21.

C) Given: An isosceles triangle ABC with sides $AB = AC = 10$ and $BC = 16$.
The area of trapezoid $PQCB$ is 40.
Compute the height of the trapezoid.



D) If $x^{14} - x^8 - x^6 + 1$ is factored completely as a product of binomials and trinomials, where each lead coefficient is +1, the sum of these factors can be written in the form $Ax^4 + Bx^2 + Cx + D$.
Determine the ordered quadruple (A, B, C, D).

E) Given: For all x, $\tan 2x = \frac{2\tan x}{1-\tan^2 x}$ (provided $x \neq 45° + 180°n$ for any integer n).
Determine all values of x over $0° \le x < 360°$ which satisfy the following equation:

$$\frac{\cot 2x \cdot \cot x + 1}{\cot x - \cot 2x} = \tan 300°$$

F) Suppose p and q are positive integers.
In regular polygon A with n sides, the ratio of an interior angle to an exterior angle is $p : q$, which is not necessarily in simplest form. In regular polygon B with m sides, the ratio of an exterior angle to an interior angle is $1 : p$. If $p = 11$, determine all possible ordered pairs (n, q) for which the ratio of an interior angle of A to an exterior angle of B is also an integer.