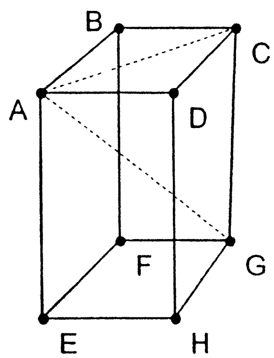
Team Round:

A. Each vertex (example A) is the vertex of three such angles ($\angle$GAC, $\angle$GAF, and $\angle$GAH) Since AH > AD = GH tan($\angle$GAH) = GH/AH < 1 and the angle with



tangent 2 must be $\angle$GAC. Let AD = x, then AC = $x\sqrt{2}$ so tan($\angle$GAC) gives CG = $2x\sqrt{2}$ thus hypotenuse of ΔAGH is AG=$x\sqrt{10}$ so AH =3 and the other two angles at A have tangents of 1/3. Sum at A is 8/3 so total from all vertices is 8(8/3) = 64/3

B. Brute force: 3,5 N = 4 = 2^2 = 3 factors. 5,7 N = 6 = 2(3) = 4 factors. 11,13 N = 12 = 2^2 3= 6 factors. 17,19 N = 18 = $(2)3^2$ = 6 factors. 29, 31 N = 30 = 2(3)5= 8 factors- **smallest found**. Working from top 149, 151 N = 150 = $2(3)5^2$ = 12 factors. 137, 139 N = 138 = 2(3)23 = 8 factors- **largest found.** Alternatively, if N has <u>eight</u> factors (one of which must be 2) its factorization must be p^7 (try 2^7 =128 but 129 not prime) or p^3q (if p=2 we can show one of 8q-1, 8q, 8q+1 divisible by 3 so q=3 but N=24 doesn't work. So q= 2, if p=3 no good and otherwise one of $2p^3\pm1$ a multiple of 3) or pqr (so we systematically try products of 2 with 2 other primes. to locate N= 2(3)5 and eventually 2(3)23)

C. $x = \dfrac{23y + 2}{17} = \dfrac{17y + 2(3y + 1)}{17}$ so 3y+1 is a multiple of 17 (and 1 more than a multiple of 3) so 3y+1 =34 y=11 x=15. Enclose Δ in rectangle with sides y=0, y=11, x=4, x=26 area is 22(11)- 3 rt Δs = 242 – 11 – 55 – 121/2 = 115.5

D. $\dfrac{\log AB}{\log C} - \dfrac{\log A}{\log C} = \dfrac{\log A}{\log C} + 2$ so $\dfrac{\log A + \log B - \log A - \log A}{\log C} = 2$ so

$\log B - \log A = 2 \log C$ so $\log(\dfrac{B}{A}) = \log(C^2)$ thus AC^2 = B

E. Start with (27A, 9B, 3C) $\Rightarrow$(9A, 9A+9B, 9A+3C) $\Rightarrow$ (12A+3B, 3A+3B, 12A+3B+3C) $\Rightarrow$(16A+4B+C, 7A+4B+C, 4A+B+C) so if Al has 45 more 9A=45 A=5; Bob has 45 more 3A+3B=45 B=10; 9A+3C=78 C=11. 16A+4B+C=131.

F. Maximum is with pentagon m$\angle$ABP=144 . Thereafter two interior angles exceeds 180 so $\angle$ABP decreases in size. If n<25 minimum is 24-gon of 30.