

**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2016 SOLUTION KEY**

Round 2

A) $x = \left(\frac{x-1}{x+3} \right)^{-1} \Leftrightarrow x = \frac{x+3}{x-1}$, provided $x \neq -3$.

Cross multiplying, $x^2 - x = x + 3 \Leftrightarrow x^2 - 2x - 3 = (x-3)(x+1) = 0 \Rightarrow x = \underline{\mathbf{3, -1}}$.

B) Squaring both sides, $\sqrt{4x-2} - \sqrt{2x} = 1 \Rightarrow (4x-2) - 2\sqrt{4x-2}\sqrt{2x} + (2x) = 1$

$\Leftrightarrow 6x - 3 = 2\sqrt{4x-2}\sqrt{2x}$

Squaring both sides again, $36x^2 - 36x + 9 = 32x^2 - 16x \Leftrightarrow$

$4x^2 - 20x + 9 = (2x-1)(2x-9) = 0 \Rightarrow x = \frac{1}{2}, \frac{9}{2}$.

Checking is a must when squaring both sides of an equation since extraneous answers may be introduced.

$x = \frac{1}{2} \Rightarrow \sqrt{0} - \sqrt{1} \neq 1$, rejected.

$x = \frac{9}{2} \Rightarrow \sqrt{16} - \sqrt{9} = 1$, check. Thus, $x = \underline{\mathbf{\frac{9}{2}}}$ only.

C) Let $F = \frac{(x-8)(x^2-8x+12) + (x-6)(x^2-10x+16)}{x - \sqrt{20x-96}} = \frac{N}{D}$. $N = 0 \Leftrightarrow$

$(x-8)(x-6)(x-2) + (x-6)(x-2)(x-8) = 2(x-2)(x-6)(x-8) = 0 \Rightarrow x = \cancel{2}, 6, 8$.

(2 is excluded because of the domain restriction.)

$D = 0 \Leftrightarrow x - \sqrt{20x-96} = 0 \Leftrightarrow x = \sqrt{20x-96} \Rightarrow x^2 = 20x - 96$

$\Rightarrow x^2 - 20x + 96 = (x-8)(x-12) = 0 \Rightarrow x = 8, 12$.

Therefore, the problematic x -values are:

$x = \underline{\mathbf{8}}$ (for which the fraction becomes $\frac{0}{0}$, an indeterminate value) and

$x = \underline{\mathbf{12}}$ (for which the fraction becomes $\frac{-52}{0}$ which is undefined). Thus, $S = \{\underline{\mathbf{6, 8, 12}}\}$.