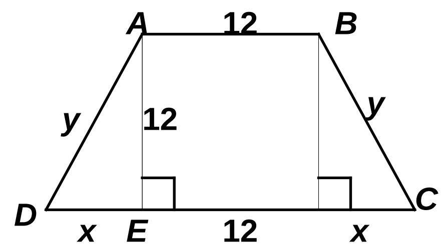
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 – NOVEMBER 2012 SOLUTION KEY

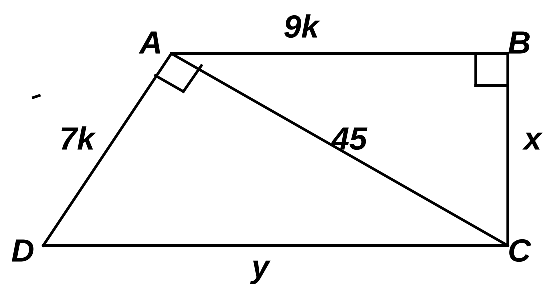
Round 3

A) Let $BE = x$. Then: $\dfrac{16-2x}{2x}=\dfrac{8-x}{x}=\dfrac{7}{1}\Rightarrow x=1\Rightarrow BE : CE =$ **<u>1 : 3</u>**.



B) $2(x + y + 12) = 60 \Rightarrow x + y = 18$
From knowledge of common Pythagorean triples,
$(x, y) = (5, 13)$ or noting that
$x^2+12^2=y^2\Rightarrow x^2+144=(18-x)^2\Rightarrow 144=324-36x\Rightarrow x=5$

The area of $ABCD$ is $\frac{1}{2}(12)(12+22)=6(34)=\mathbf{\underline{204}}$.



C) $\dfrac{\frac{1}{2}x(9k)}{\frac{1}{2}7k(45)}=\dfrac{27}{35}\Rightarrow \dfrac{x}{35}=\dfrac{27}{35}\Rightarrow x=27$
In ΔABC, $(27, 9k, 45) = 9(3, k, 5) \Rightarrow$
$k = 4 \Rightarrow AD = 28$
Thus, the area of $ABCD$ =
$\frac{1}{2}\cdot 36\cdot 27+\frac{1}{2}\cdot 28\cdot 45=18(27)+14(45)=18(27+35)=\mathbf{\underline{1116}}$

Note that finding the value of y was <u>not</u> necessary.
By Pythagorean Theorem, we could have computed the value of DC. It is actually 53.
Note also that $ABCD$ is <u>not</u> a trapezoid. If it were a trapezoid, the length of the altitude from A to $\overline{DC}$ (call it $\overline{AE}$) would be 27.
ΔADC has sides 28, 45 and 53.
Thus, as a right triangle, the area of ΔADC is $\frac{1}{2}\cdot 28\cdot 45$ or $\frac{1}{2}\cdot AE\cdot 53$, implying
$AE=\sqrt{495}=3\sqrt{55}\neq 27$.
Thus, $ABCD$ is definitely <u>not</u> a trapezoid!