MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2009 SOLUTION KEY

Round 2

A) $A \rightarrow \dfrac{4 \cdot 6}{5}A = \dfrac{24}{5}A$ Thus, the actual increase is $\dfrac{24}{5}A - A = \dfrac{19}{5}A$ and $\dfrac{19}{5} = \dfrac{p}{100}$ (a p% increase)
$5p = 1900 \rightarrow p = \underline{\mathbf{380}}$

B) $2009 = 7(287) = 7^2 \cdot 41^1 \rightarrow$ 2009 has $(2 + 1)(1 + 1) = 6$ positive integer divisors, namely
$1 + 7 + 41 + 49 + 287 + 2009 = \underline{\mathbf{2394}}$

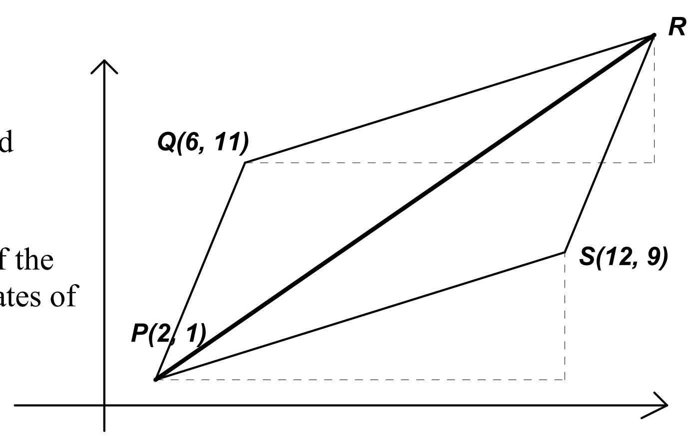
C) The number must be divisible by 3 and 11, so the sum of the digits must be divisible by 3, and $(9 + 4 + y) - (x + 3 + 5)$ must be divisible by 11. This means that $x + y$ could $= 0, 3, 6,$ etc. since $9 + 4 + 3 + 5 = 21$. Also $5 + y - x$ could $= 0, 11,$ etc. If $5 + y - x = 0$ and $y + x = 0$ or 3, we get negative values. So try $5 + y - x = 11$ and $y + x = 6$.
Solving, we get $y = 6$ and $x = 0 \rightarrow 904365 \rightarrow (x, y) = \underline{\mathbf{(0, 6)}}$

Round 3

A) Solving the linear equation for x (or y) and substituting in the quadratic equation would be messy. Instead, note that the center of the circle is $(2, -1)$ and the slope of the line is $\dfrac{4}{3}$.
Thus, the required points of intersection are:

$P(2 + 3, -1 + 4) = \underline{\mathbf{(5, 3)}}$ and $Q(2 - 3, -1 - 4) = \underline{\mathbf{(-1, -5)}}$

B) $AB^2 = (k - 5.9)^2 + 3.2^2 = (3.2\sqrt{5})^2 = 3.2^2(5) \rightarrow (k - 5.9)^2 = 3.2^2(5 - 1) = 3.2^2(2^2)$
$\rightarrow k - 5.9 = \pm 6.4 \rightarrow k = 5.9 \pm 6.4 \rightarrow \underline{\mathbf{-0.5, 12.3}}$



C) The slope of $\overline{PS}$ is 8/10.
R is then located by translating Q 10 units right and 8 units up. $R(16, 19)$
Thus, the slope of $\overleftrightarrow{PR}$ is $18/14 = 9/7$
and the equation is $9x - 7y + k = 0$ and the value of the constant is determined by substituting the coordinates of either P or R.

$P \rightarrow 18 - 7 + k = 0 \rightarrow k = -11$ or
$R \rightarrow 9(16) - 7(19) + k = 0 \rightarrow k = -144 + 123 = -11$
Thus, the required equation is $\underline{\mathbf{9x - 7y - 11 = 0}}$

or once the coordinates of R have been determined use the 2-point form of a straight line.