MASSACHUSETTS MATHEMATICS LEAGUE CONTEST 4 - JANUARY 2008 SOLUTION KEY

Round 2

- A) $4x^2 + 8ax + 25 = (2x \pm 5)^2 = 4x^2 \pm 20x + 25 \implies 8a = \pm 20 \implies a = \pm \frac{5}{2}$
- B) -15 factors as (1)(-15), (-1)(15), (3)(-5), (-3)(5), $\Rightarrow a = \pm 14$ or ± 2 The corresponding factorizations are: $14(x^2 + 7)$, $-14(x^2 - 7)$, $2(x^2 + 49)$ and $-2(x^2 - 49)$ and only the latter has two distinct linear factors over the integers. Thus, a = -2

C)
$$\frac{2x^2 + x - 1}{x^2 - x - 2} = 1 - 2x \Rightarrow \frac{(2x - 1)(x + 1)}{(x - 2)(x + 1)} = 1 - 2x$$

Clearly, x = -1 is not a solution. Canceling, $\frac{(2x-1)}{(x-2)} = 1 - 2x \implies 2x - 1 = (x-2)(1-2x)$

$$2x - 1 = x - 2x^{2} - 2 + 4x \Rightarrow 2x^{2} - 3x + 1 = (x - 1)(2x - 1) = 0 \Rightarrow x = 1, \frac{1}{2}$$