Team Round – continued

F) $A=\sqrt{x+37}$ and $B=\sqrt{x-N}$ ➔ $\begin{cases} x+37=A^2 \\ x-N=B^2 \end{cases} \rightarrow 37+N=A^2-B^2=(A+B)(A-B)$

Thus, we must look at the factors of (37 + *N*). We know that for any pair of unequal factors of (37 + *N*), (*A* + *B*) will equal the larger factor (call it *L*) and (*A* – *B*), the smaller (call it *S*).

Specifically, $\begin{cases} A+B=L \\ A-B=S \end{cases}$ ➔ $A=\dfrac{L+S}{2}, B=\dfrac{L-S}{2}$. To insure that *A* and *B* are integers, *L* and *S* must be of the same parity, i.e. both even or both odd. We need to find the smallest value of *N* for which there are three such factor pairs (*L*, *S*).

By trial and error,
N = 1 ➔ 38 (none – no same parity factor pairs)
N = 2 ➔ 39 (only 39·1, 13·3) *N* = 3 ➔ 40 (only 20·2, 10·4)
N = 4 ➔ 41 (prime) *N* = 5 ➔ 42 (no same parity factor pairs) *N* = 6 ➔ 43 (prime)
N = 7 ➔ 44 (22·2 only) *N* = 8 ➔ 45 (45·1, 15·3, 9·5) – Bingo!

$$\begin{cases} A+B=45 \quad 15 \quad 9 \\ A-B=\ \ 1 \quad\ \ 3 \quad 5 \end{cases} \rightarrow (A,B)=\ (23,22),(9,6),(7,2) \rightarrow x=492,\ 44,\ 12$$

Thus, (*k*, *T*) = (8, 492 + 44 + 12) = **<u>(8, 548)</u>**.

We could have made the observation that we were looking for a number with 6 odd factors. The smallest such positive number would have been $3^2 \cdot 5^1 = 45$ which would have avoided and laborious plug and check.

Created with nitroPDF professional
download the free trial online at nitropdf.com/professional