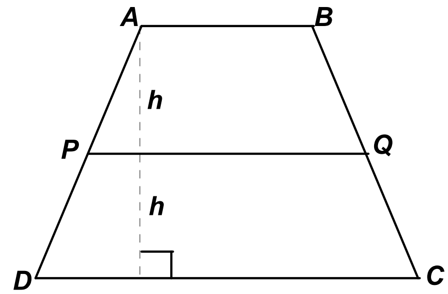
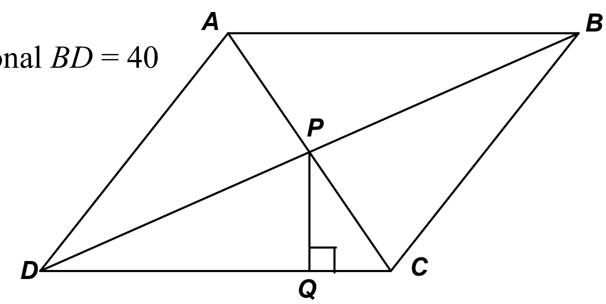
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 – NOVEMBER 2010 SOLUTION KEY

Round 3

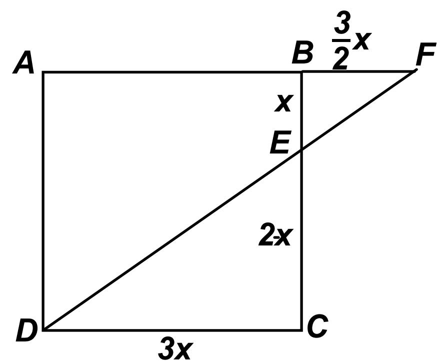


A) As a median, $PQ = \dfrac{12+20}{2} = 16$ and the altitudes of trapezoids *ABQP* and *PQCD* are equal in length. Thus, the required ratio is

$$\frac{\frac{1}{2}h(12+16)}{\frac{1}{2}h(16+20)} = \frac{28}{36} = \underline{\mathbf{\frac{7}{9}}}$$



B) $PQ = 12$, $QC = 9$ ➔ $PC = 15$ ➔ diagonal $AC = 30$
Perimeter = 100 ➔ $DC = 25$, $DQ = 16$, $DP = 20$ ➔ diagonal $BD = 40$
Thus the area of the rhombus = $\frac{1}{2}d_1 d_2 = \frac{1}{2}\cdot 30\cdot 40 = \underline{\mathbf{600}}$



C) $\Delta BEF \sim \Delta CED$, $\dfrac{BE}{CE} = \dfrac{1}{2}$ and $CD = 3x$ ➔ $BF = \dfrac{3}{2}x$

Therefore, $\frac{1}{2}x\cdot\frac{3}{2}x = 24$ ➔ $x^2 = 32$

➔ area(*ABCD*) = $3x\cdot 3x = 9x^2 = 9(32) = \underline{\mathbf{288}}$

<u>Alternate solution (MaryBeth McGinn / Tuan Le):</u>

$$\frac{area(\Delta BEF)}{area(\Delta DEC)} = \frac{BE^2}{EC^2} = \frac{1}{4}$$ ➔ area(ΔDEC) = 96

$$\frac{area(\Delta BEF)}{area(\Delta ADF)} = \frac{BE^2}{AD^2} = \frac{1}{9} \Rightarrow \frac{area(\Delta BEF)}{area(BADE)} = \frac{1}{8}$$ ➔ area(*BADE*) = 192 ➔ area(*ABCD*) = $\underline{\mathbf{288}}$.

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