

**MASSACHUSETTS MATHEMATICS LEAGUE  
CONTEST 3 - DECEMBER 2016 SOLUTION KEY**

**Round 5**

A)  $\frac{50}{3 \cdot 5} = \frac{10}{3}$  lumens per in<sup>2</sup>.  $\frac{10}{3}(24 + 45) = 80 + 150 = \underline{\underline{230}}$  lumens.

B)  $\frac{A+B}{C+D} = \frac{\frac{7}{9}B+B}{\frac{5}{3}D+D} = \frac{\frac{16}{9}B}{\frac{8}{3}D} = \frac{2B}{3D} = \frac{2(4D)}{3D} = \underline{\underline{\frac{8}{3}}}$ .

C) Assume currently I have hit  $x$  3-pointers in  $y$  attempts. Then:

$$\begin{cases} \frac{x}{y} = 0.6 = \frac{3}{5} \\ \frac{x-8}{y} = \frac{1}{2} \end{cases} \Leftrightarrow \begin{cases} 5x = 3y \\ y = 2x - 16 \end{cases} \Leftrightarrow 5x = 3(2x - 16) \Rightarrow x = 48, y = 80$$

$$\frac{48+k}{80+k} \geq 0.70 = \frac{7}{10} \Leftrightarrow 480 + 10k \geq 560 + 7k \Rightarrow 3k \geq 80 \Rightarrow k_{\min} = \underline{\underline{27}}.$$

Check:  $k = 26 \Rightarrow \frac{74}{106} \approx 0.6981, k = 27 \Rightarrow \frac{75}{107} \approx 0.7009$