MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2009 SOLUTION KEY

Team Round

C) Solving for x, $x = \dfrac{451+7y}{4} = 112 + y + \dfrac{3(1+y)}{4}$.

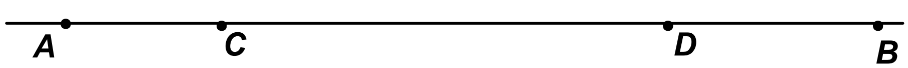
Thus, y must be picked so that $\dfrac{3(1+y)}{4}$ is an integer.

This occurs when $y = 3$ and correspondingly $x = 112 + 3 + 3 = 118$.
Since the given equation is a straight line with slope 4/7, an increase of 7 in the value of x corresponds to an increase of 4 in the value of y. Thus, a general solution is $(118 + 7t, 3 + 4t)$ and $x + y = 121 + 11t = 11(11 + t)$ The expression is a multiple of 3 for $t = 1, 4, 7, 10, \ldots$, i.e for t of the form $3k + 1$. $11(12 + 3k) < 1000 \Rightarrow k < 26.3 \Rightarrow k = 26 \Rightarrow t = 79 \Rightarrow (x, y) =$ **<u>(671, 319)</u>**

Alternate solution:
The largest possible value of $x + y$ is 999.
If we try 999, we notice that $4(999 - y) - 7y = 451$ or $4(999) - 451 = 11y$ requires divisibility by 11.

$4(999) - 451 = 3996 - 451 = 3545$ which fails [since $(5 + 5) - (3 + 4) = 3$]
$4(996) - 451 = 3533$ which fails [since $(5 + 3) - (3 + 3) = 2$]
$4(993) - 451 = 3521$ which fails [since $(5 + 1) - (3 + 2) = 1$]
$4(990) - 451 = 3509$ BINGO! [since $(5 + 9) - (3 + 0) = 11$, a multiple of 11]

Thus, $\begin{cases} 4x - 7y = 451 \\ x + y = 990 \end{cases} \Rightarrow (x, y) =$ **<u>(671, 319)</u>**



D) Let x denote Barbara's original rate (in mph).
Then $AB = AD + DB = (x + 10)(1) + x(1) = 2x + 10$
Also $AB = AC + CB = (x + 10 - k)(2/3) + (x + 10)(2/3)$

Equating these expressions for AB, $2x + 10 = \dfrac{2(2x + 20 - k)}{3} \Rightarrow 2x = 10 - 2k \Rightarrow x = 5 - k$

Alice's reduced rate $\Rightarrow \dfrac{x + 10 - k}{x + 10} = \dfrac{3}{4} \Rightarrow 4x + 40 - 4k = 3k + 30 \Rightarrow x = 4k - 10$

$5 - k = 4k - 10 \Rightarrow k = 3, x = 2, AB = 14, AD = 12, DB = 2, AC = (2 + 10 - 3)(2/3) = 6$
$\Rightarrow CD = 14 - 2 - 6 =$ **<u>6</u>**