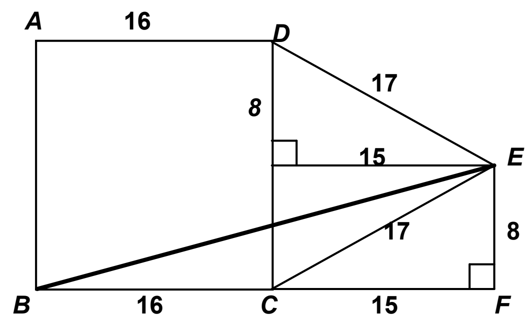
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2010 SOLUTION KEY

Round 2

A) $\overline{AB}$ is not the hypotenuse, but either of the other two sides could be.
Thus, either $AC^2 = 81 + 169$ or $169 - 81$ ➔ 250 or 88
$AC = \underline{\mathbf{5\sqrt{10}}}$, $\underline{\mathbf{2\sqrt{22}}}$

B) Method #1:
Noting the 8 – 15 – 17 special right triangle and using right triangle ΔBEF, we have
$BE^2 = 31^2 + 8^2 = 961 + 64 = 1025 = 25(41)$
➔ $BE = \underline{\mathbf{5\sqrt{41}}}$



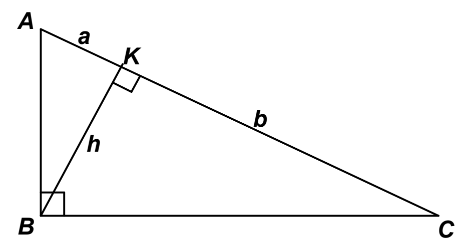
Method #2:
$\cos(\angle ECF) = 15/17$ ➔ $\cos(\angle ECB) = -15/17$
By the law of cosines, $BE^2 = 16^2 + 17^2 - 2(16)(17)(-15/17) = 256 + 289 + 480 = 1025 = 25(41)$.
➔ $BE = \underline{\mathbf{5\sqrt{41}}}$

C) $AC^2 = \left(2\sqrt{6}\right)^2 + \left(5\sqrt{3}\right)^2 = 24 + 75 = 99$ ➔ $AC = 3\sqrt{11}$.

The area of $\Delta ABC = \frac{1}{2}\left(2\sqrt{6}\right)\left(5\sqrt{3}\right) = \frac{1}{2}\left(3\sqrt{11}\right)h$ ➔ $10\sqrt{18} = 3h\sqrt{11}$ ➔ $10\sqrt{2} = h\sqrt{11}$

➔ $h = \frac{10\sqrt{2}}{\sqrt{11}} = \frac{10\sqrt{22}}{11}$ ➔ $x + y + z = \underline{\mathbf{43}}$.

Alternate Solution (Tuan Le)



$AC^2 = \left(2\sqrt{6}\right)^2 + \left(5\sqrt{3}\right)^2 = 24 + 75 = 99$ ➔ $AC = a + b = 3\sqrt{11}$.

$AB^2 = 24 = h^2 + a^2$

$BC^2 = 75 = h^2 + b^2$ (***)

Subtracting, $51 = b^2 - a^2 = (b + a)(b - a) = 3\sqrt{11}(b - a)$

➔ $b - a = \frac{17}{\sqrt{11}}$.

Solving simultaneously for b, $b = \frac{25}{\sqrt{11}}$.

Substituting in (***), $h^2 = 75 - \left(\frac{25}{\sqrt{11}}\right)^2 = 75 - \frac{25^2}{11} = \frac{825 - 625}{11} = \frac{2\left(10^2\right)11}{11^2}$ ➔ $h = \frac{10\sqrt{22}}{11}$

➔ $x + y + z = \underline{\mathbf{43}}$.