MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2015 SOLUTION KEY

Round 4

A) $182a-12a^2-2a^3=2a\left(91-6a-a^2\right)=\underline{\mathbf{2a(13+a)(7-a)}}$, $\underline{\mathbf{-2a(a+13)(a-7)}}$ or equivalent.

B) As the difference of perfect squares,
$x^4-(13x-30)^2 \Leftrightarrow \left(x^2+13x-30\right)\left(x^2-13x+30\right)=(x+15)(x-2)(x-3)(x-10)$
Setting equal to zero, we have $x=\underline{\mathbf{-15, 2, 3, 10}}$ (in any order).

C) Completing the squares in both the x- and y-expressions, we have the difference of perfect squares. Note the "fudge factors", namely $+4$ and $-16\left(\frac{1}{4}\right)$ sum to zero, so the original polynomial has not been changed!!

$$x^2+4x-16y^2+16y \Leftrightarrow \left(x^2+4x+4\right)-16\left(y^2-y+\frac{1}{4}\right)$$
$$\Leftrightarrow (x+2)^2-4^2\left(y-\frac{1}{2}\right)^2=(x+2)^2-(4y-2)^2$$
$$\Leftrightarrow (x+2+4y-2)(x+2-4y+2)=\underline{\mathbf{(x+4y)(x-4y+4)}}$$

Alternately, grouping the quadratic terms and the linear terms, we have

$$x^2+4x-16y^2+16y$$
$$\Leftrightarrow \left(x^2-16y^2\right)+4(x+4y)$$
$$\Leftrightarrow (x+4y)(x-4y)+4(x+4y)$$
$$\Leftrightarrow \underline{\mathbf{(x+4y)(x-4y+4)}}$$