MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2012
ROUND 7 TEAM QUESTIONS
ANSWERS

A) (______ , ______ , ______) D) ____________________________

B) ____________________________ E) _______________________ units

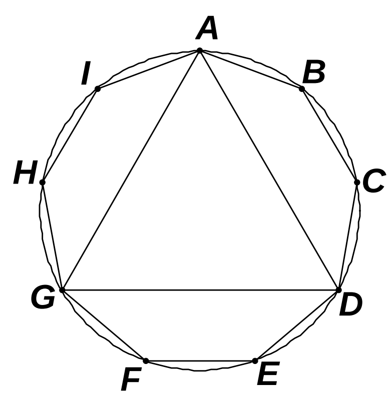
C) (___________ , ___________) F) _______________________ units

A) Let $Z = \cfrac{1}{i-\cfrac{1}{i-\cfrac{1}{i-\cfrac{1}{i-\ \ldots\ \cfrac{1}{i-\cfrac{1}{i}}}}}}$, where the complex fraction contains k instances of i and $k > 2$.

For example, for $k = 2$, $Z = \dfrac{1}{\boldsymbol{i}-\boxed{\dfrac{1}{\boldsymbol{i}}}}$.

For some minimum value of k this expression simplifies to $-\frac{A}{B}i$, where A and B are positive integers and A is a perfect square ($A \neq 1$). Determine the ordered triple (k, A, B).

B) Dick, Joe and Norm are practicing for a big math contest. They are very competitive and equally talented and on a set of 100 practice questions, each was able to correctly answer 60 questions and no question stumped all three mathletes.
A question is defined to be *hard* if exactly one mathlete got it right.
A question is defined to be *easy* if all three mathletes got it right.
Some questions are neither easy nor hard.
There were k more *hard* questions than *easy* questions. Compute k.



C) In a regular nonagon *ABCDEFGHI*, ΔADG has area $36\sqrt{3}$.
The area of the nonagon is $k\sin\theta°$. Find the ordered pair (k, θ°), where both k and θ are positive integers and θ is acute.

D) For how many integer values of x between 0 and 100 inclusive, does the quotient $\dfrac{8x+4}{\dfrac{2}{x+1}+\dfrac{14}{x-3}}$ produce an integer value?

E) In polar coordinates, the equation $r = \cos\theta + \sqrt{3}\sin\theta$ defines a circle which passes through the origin. $\theta = 30°$ and $\theta = 60°$ defines lines through the origin which make angles of 30° and 60° respectively, measured counterclockwise from the positive x-axis. Let B and C be the points in the first quadrant where these lines intersect the circle. Compute the distance between B and C.

F) Scalene triangle *ABC* has sides of integer length.
$\overline{AD}$ is the altitude to side $\overline{BC}$.
If $AB = 12$ and $m\angle BAD = 30°$, compute all possible perimeters of ΔABC.