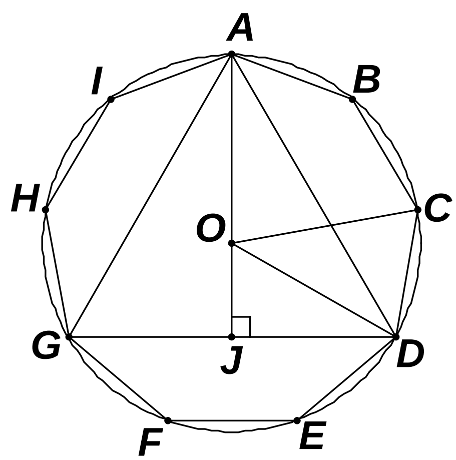
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2012 SOLUTION KEY

Team Round



C) ΔADG is equilateral and its area is $\dfrac{s^2\sqrt{3}}{4} = 36\sqrt{3} \Rightarrow s = 12$.

Since ΔDOJ is a 30-60-90 right triangle,

$JD = 6$, $OJ = 2\sqrt{3}$, $OD = 4\sqrt{3}$

$m\measuredangle COD = \dfrac{360°}{9} = 40°$

Using $A(\Delta) = \dfrac{1}{2}ab\sin\theta$, area(ΔCOD) =

$\dfrac{1}{2}\left(4\sqrt{3}\right)^2 \sin 40° = 24\sin 40°$

Therefore, the area of the nonagon is $9\left(24\sin 40°\right)$

Since both k and θ are positive integers and θ is acute, (k, θ°) = **<u>(216, 40)</u>**.

D) Of the 101 integers in the given range 3 must be excluded, since it causes division by zero. Simplifying the complex fraction, we have

$$\frac{8x+4}{\dfrac{2}{x+1}+\dfrac{14}{x-3}} = \frac{4(2x+1)}{\dfrac{2(x-3)+14(x+1)}{(x+1)(x-3)}} = \frac{4(2x+1)}{\dfrac{16x+8}{(x+1)(x-3)}} = 4(2x+1)\cdot\frac{(x+1)(x-3)}{8(2x+1)} = \frac{(x+1)(x-3)}{2}$$

Clearly, the numerator must be even and this happens if and only if x is odd.
Between 0 and 100 inclusive, there are 51 even integers and 50 odd integers.
Thus, the quotient is integral for **<u>49</u>** values of x.

E) $\theta = 0 \Rightarrow r = 1+\sqrt{3}\cdot 0 = 1$

$\theta = 90 \Rightarrow r = 0+\sqrt{3}\cdot 1 = \sqrt{3}$

Thus, in the Cartesian coordinate system, where points are located with x- and y-coordinates, $X(1,0)$, $Y\left(0,\sqrt{3}\right)$, ΔYOX is a 30-60-90 right triangle, and $XY = 2$.

The circle is circumscribed about ΔYOX and $\overline{XY}$ is a diameter and the midpoint of $\overline{XY}$ is the center of the circle.