MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2015 SOLUTION KEY

Round 3

A) $2(Ax+2)=3-(Bx-1)+x \Leftrightarrow 2Ax+4=4+(1-B)x$

This is true for all x if (and only if) $2A = 1 - B$ or $2A + B = 1$.
Thus, $2A + B + 3 =$ **4**.

B) Suppose the original attendance is N students. Then:

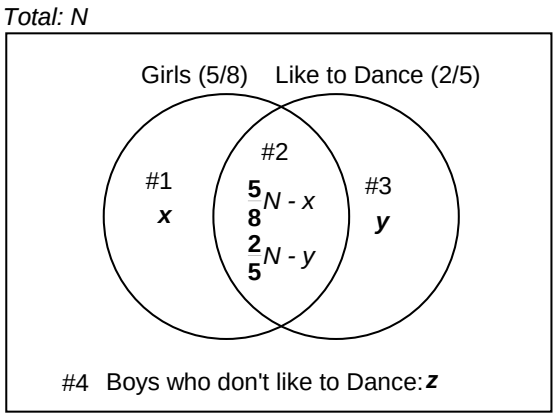
$\frac{5}{8}N+20=\frac{3}{4}(N+20) \Leftrightarrow 5N+160=6N+120 \Rightarrow N=40$

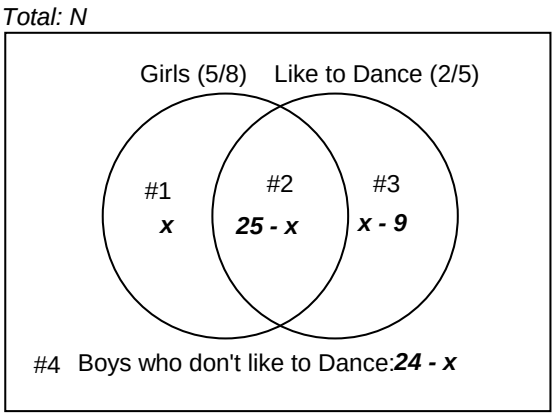
If $\frac{2}{5}$ of these students like to dance, then the total number of dancers is $\frac{2}{5}\cdot 40+20=36$

and $\frac{36}{60}=\frac{6}{10}=\frac{60}{100} \Rightarrow k=$ **60**.

Note: There would be multiple solutions to questions like:
x - How many girls from the original school did not like to dance?
y - How many boys from the original school liked to dance?
z - How many boys from the original school did not like to dance?

After determining that $N = 40$, the Venn diagram on the left simplifies to the one on the right.





We see that $9 \le x \le 24$ and 16 different ordered triples (x, y, z) satisfy the conditions of the original problem and all of them give us $k = 60$.

C) $\begin{cases} F+75d=4400 \\ F+400d=18050 \end{cases} \Rightarrow F=4400-75d=18050-400d \Rightarrow 325d=13650 \Rightarrow d=42$

$(F,d)=$ **(1250,42)**.