MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2009 SOLUTION KEY

Round 6

A) P(3 sen <u>and</u> 2 jr) = $\dfrac{({}_4C_2)({}_7C_3)}{{}_{11}C_5} = \dfrac{\dfrac{4!}{2!2!}\cdot\dfrac{7!}{3!4!}}{\dfrac{11!}{6!5!}} = 6\cdot\cancel{7}\cdot5\cdot\dfrac{\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot1}{11\cdot\cancel{10}^2\cdot\cancel{9}^3\cdot\cancel{8}\cdot\cancel{7}} = \boldsymbol{\underline{\dfrac{5}{11}}}$

B) The second term in the expansion is $\binom{5}{1}\left(\dfrac{x^3}{4}\right)^4\left(8x^{-n}\right)^1$. Expanding, this is

$5(4)^{-4}8^1\left(x^3\right)^4\left(x^{-n}\right) = 5\cdot2^{-8}\cdot2^3\cdot x^{12-n} = \dfrac{5}{32}x^0 = \dfrac{5}{32}$ provided $n = 12$ Thus, $(n, c) = \underline{\left(\mathbf{12}, \dfrac{\mathbf{5}}{\mathbf{32}}\right)}$

C) P(*Jones* winner) = 4/10 = 2/5
Exactly 6 *Jones* winners can occur in only one way (*JJJJJJ*).
Exactly 5 *Jones* winners could occur in 6 possible ways, each represented by a sequence of 5 *J*'s and 1 *S* (e.g. *JJJJJS*), since the *S* could occur in anyone of 6 positions in the sequence .
Exactly 4 *Jones* winners requires permuting the sequence *JJJJSS*. This can be done in
$\binom{6}{2} = \binom{6}{4} = \dfrac{6!}{2!\,4!} = \dfrac{6\cdot5}{2} = 15$ ways.
Thus, P($\geq$ 4 wins) = $\left(\dfrac{2}{5}\right)^6 + 6\left(\dfrac{2}{5}\right)^5\left(\dfrac{3}{5}\right) + 15\left(\dfrac{2}{5}\right)^4\left(\dfrac{3}{5}\right)^2 = \dfrac{64+576+2160}{5^6} = \dfrac{2800}{5^6} = \boldsymbol{\underline{\dfrac{112}{625}}}$