

**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2011
ROUND 7 TEAM QUESTIONS**

ANSWERS

A) _____ D) _____

B) _____ E) _____

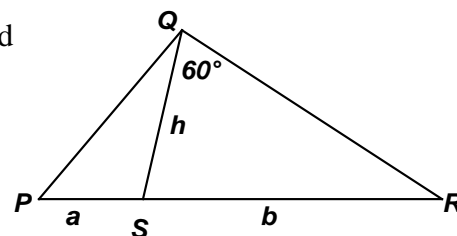
C) _____ F) _____

A) $\triangle PQR$ is a right triangle with right angle at Q .

$PS = a$, $RS = b$ and \overline{QS} divides $\angle Q$ into 30° and 60° as indicated in the diagram at the right.

For positive integers L , M and N , $h^2 = \frac{La^2b^2}{Ma^2 + Nb^2}$.

Compute the ordered triple (L, M, N) for which the sum $L + M + N$ is a minimum.



B) Consider two groups of Pythagorean triples:

Group A: row 1: 3, 4, 5 row 2: 5, 12, 13 row 3: 7, 24, 25 row 4: 9, 40, 41

Group B: row 1: 8, 15, 17 row 2: 12, 35, 37 row 3: 16, 63, 65 row 4: 20, 99, 101

Compute the quotient of the third term in the 13th row of group A to the second term in the 36th row in group B.

C) Compute the coordinates of all points $P(x, y)$, where x and y are positive integers and P is twice as far from $A(5, 2)$ as it is from $B(11, 8)$

D) Compute all possible real values of x for which $3^{\left(\log_3 x^4 - \frac{1}{\log_x 3}\right)} + 2^{2\log_2 x} + 7^{5\log_7 x} = 3x^4$.

E) In the spring, the ratio of girls to boys was 7 : 11 in the junior class at a local high school. Over the summer two boys joined this class and three girls in this class moved out of the district, making the new ratio of girls to boys 5 : 8. All these students advanced to the next grade. If there are no additional changes, in the fall, how many seniors will be attending this local high school?

F) P and Q are regular polygons. Q has five times as many sides as P . The ratio of the measure of an interior angle of Q to the measure of an interior angle of P is 7 to 5. Regular polygon R has more sides than polygon Q and interior angles whose measures are an integral number of degrees. Compute the minimum number of degrees in one of the interior angles of R .