

**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 - FEBRUARY 2014 SOLUTION KEY**

Round 4

- A) Since Bonnie works three times as fast as Clyde, Clyde does $\frac{1}{4}$ of the job and Bonnie does $\frac{3}{4}$.

Since Bonnie take 6 minutes to do $\frac{3}{4}$ of the job, it will take her $\frac{4}{3}(6) = \underline{8}$ minutes to do the entire job alone.

- B) Let x denote my teenage son's age and shoe size back in the day.
Let y denote his current age. Then:

$$\begin{cases} y = x + 26 \\ x + 1 = \frac{3}{8}y \end{cases} \Rightarrow 8(x+1) = 3(x+26) \Rightarrow 5x = 78 - 8 \Rightarrow x = 14$$

Thus, his current shoe size is 15.

- C) Let A, B, C and D denote the unknown round scores.

Since there are 7 rounds, the median score is the 4th score when the scores are listed in increasing order: $A, 18, 18, 22, B, C, D$

The team score is 58 points plus the scores in the remaining 4 rounds.

A must be the team round score and must be less than 18, since the mode is 18 and it only occurred twice.

B, C and D must all be different (otherwise the mode would not be unique).

Also each must be larger than 22.

In the worst case scenario, $(A, B, C, D) = (0, 24, 26, 28) \Rightarrow$

total score = $58 + 3(26) = 136$

In the best case scenario, $(A, B, C, D) = (15, 26, 28, 30)$

\Rightarrow total score = $15 + 58 + 3(28) = 157$

Thus, $(M, m) = \underline{(157, 136)}$.