MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2014 SOLUTION KEY

E) continued

$k = 15 \Rightarrow a = -2 + \sqrt{676} = -2 + 26 = 24$ and we have this solution

The area is $\frac{1}{2} \cdot 45 \cdot 52 = 45 \cdot 26 = 1170$. Thus, $(P, A) = \underline{(\mathbf{156,1170})}$.

Additional triangles occur for

$k = 56 \Rightarrow a = 95$ (193-194-195 / altitude 168)

$k = 209 \Rightarrow a = 360$ (723-724-725 / altitude 627)

The next value of k "discovered" by the TI-84+ is $k = 362$, but then $3k^2 + 1$ would end in 3.

How many perfect squares do you know ending in 3?

Oops, the TI-84+ has exceeded its limits, again!

What is the next k-value?

F)

> It deos not mttaer in waht oredr the ltteers in a wrod are, the olny iprmoatnt tihng is taht the frist and lsat ltteer be in the rghit pclae. The rset can be a taotl mses and you can sitll raed it wouthit a porbelm. Tihs is bcuseae the bairn deos not raed ervey lteter by istlef, but the wrod as a wlohe. Taht is the phaonmneal pweor of the hmuan mnid!

The thirty-five words of 1, 2 or 3 letters can be ignored.

The arrangement of the letters in each of the fifteen 4-letter words can be done in 2 ways, since no interior letters are duplicated. $\Rightarrow 2^{15}$

5-letter words (12): order/thing/first/right/place/total/still/brain/every/whole/power/human

The arrangement of the letters in all these 5-letter words can be done in 6 ways, since no interior letters are duplicated. $\Rightarrow 6^{12} = 2^{12} \cdot 3^{12}$

There are four 6-letter words (two separate occurrences of the word 'letter')

itself $\left[4! = 2^3 3\right]$ matter $\left[\frac{4!}{2!} = 12 = 2^2 3\right]$ letter $\left[\left(\frac{4!}{2!2!}\right)^2 = (3!)^2 = 2^2 3^2\right]$

There are four 7 letter words

letters $\left[\frac{5!}{2!2!} = 2 \cdot 3 \cdot 5\right]$ without/problem/because $\left[(5!)^3 = \left(2^3 \cdot 3 \cdot 5\right)^3 = 2^9 3^3 5^3\right]$

There is one 9-letter word – important $\left[7! = 2^4 3^2 5^1 7^1\right]$

There is one 10-letter word – phenomenal $\left[\frac{8!}{2!2!} = 2 \cdot 7! = 2^5 3^2 5^1 7^1\right]$

$\Rightarrow 2^{15+12+3+2+2+1+9+4+5} \cdot 3^{12+1+1+2+1+3+2+2} \cdot 5^{1+3+1+1} \cdot 7^{1+1} = 2^{53} 3^{24} 5^6 7^2$

Thus, $(A, B, C, D) = \underline{(\mathbf{53, 24, 6, 2})}$

I always thought this T-shirt was 1 in a million, but it's actually more like

1 in 1 950 000 000 000 000 000 000 000 000 000 000 000 (or 1 : 1.95 x 10^{33}) .