MASSACHUSETTS MATHEMATICS LEAGUE
FEBRUARY 2004
ROUND 3: TRIG. IDENTITIES OR INVERSES

ANSWERS

A) $\csc\theta$

B) $30^\circ, 60^\circ, 210^\circ, 240^\circ$

C) $(2+2\sqrt{30})/15$

A) Simplify $\dfrac{(\cot\theta - \cos\theta)(1 + \sin\theta)}{\cos^3\theta}$ to the form T(θ) where T is one of the six trig functions.

$$\frac{\left(\frac{\cos\theta}{\sin\theta} - \cos\theta\right)(1+\sin\theta)}{\cos^3\theta} = \frac{(\cos\theta - \sin\theta\cos\theta)(1+\sin\theta)}{\sin\theta\cos^3\theta}$$

$$= \frac{\cos\theta(1-\sin^2\theta)}{\sin\theta\cos^3\theta} = \frac{1}{\sin\theta} = \csc\theta$$

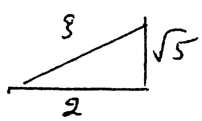
B) For $0^0 \le \theta < 360^0$, solve $\dfrac{2\tan\theta}{1+\tan^2\theta} = \dfrac{\sqrt{3}}{2}$.

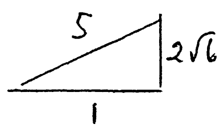
$$\frac{\frac{2\sin\theta}{\cos\theta}}{\sec^2\theta} = \frac{2\sin\theta}{\cos\theta}\cdot\frac{\cos^2\theta}{1} = 2\sin\theta\cos\theta = \sin 2\theta = \frac{\sqrt{3}}{2}$$

$2\theta = 60^\circ;\ 120^\circ,\ 420^\circ,\ 480^\circ$

$\theta = 30^\circ,\ 60^\circ;\ 210^\circ,\ 240^\circ$

C) Using principle values, express $\cos(\sec^{-1}\dfrac{3}{2} - \cos^{-1}\dfrac{1}{5})$ in simple radical form.





$\cos A\cos B + \sin A\sin B =$

$$\frac{2}{3}\cdot\frac{1}{5} + \frac{\sqrt{5}}{3}\cdot\frac{2\sqrt{6}}{5} = \frac{2+2\sqrt{30}}{15}$$