MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2007 SOLUTION KEY

Round 1

A) $\bar{z} = a - bi$ Thus, $z + \bar{z} = 7a - 5bi = 7 + 3i \rightarrow 7a = 7$ and $-5b = 3 \rightarrow (a, b) = \left(1, -\frac{3}{5}\right)$

$\rightarrow \mathbf{1 + \left(-\frac{3}{5}\right)i}$ or $\mathbf{1 + (-0.6)i}$

B) Let $z = a + bi$. Then $(a + bi)^2 = 25(3 + 4i) \rightarrow a^2 - b^2 = 3$ and $2ab = 4 \rightarrow (a, b) = (2,1)$ or $(-2, -1)$
$\rightarrow z = 5(2 + i)$ or $5(-2 - i) \rightarrow$ **10 + 5*i*, -10 – 5*i***

C) $|-3 + 4i| = \sqrt{(-3)^2 + 4^2} = 5, |12 + 16i| = \sqrt{12^2 + 16^2} = 20, |7 - 24i| = \sqrt{7^2 + (-24)^2} = 25$
$5x^2 - 20x - 25 = 5(x^2 - 4x - 5) = 5(x - 5)(x + 1) = 0 \rightarrow x =$ **5, -1**