MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2016 SOLUTION KEY

Team Round

A) The summation $\sum_{n=1}^{n=32}(1+i)^n$ consists of 32 terms.

Since powers of i cycle in blocks of 4, consider any series of 4 consecutive terms.
Consider the simplest 4-block $(1+i)^0+(1+i)^1+(1+i)^2+(1+i)^3$.
This simplifies to $1+(1+i)+2i+2i(1+i)=2+3i+2i-2=5i$. Thus, expressions of the form $(1+i)^k+(1+i)^{k+1}+(1+i)^{k+2}+(1+i)^{k+3}$ simplify to $(1+i)^k\cdot 5i$, for <u>any</u> integer *k*.
The index for the given summation starts at 1.
$k=1\Rightarrow(1+i)^1\cdot(5i)=5i+5i^2=-5(1-i)$
For the given summation, k = 1, 5, 9, … , 29 generate 32 terms, 8 blocks of 4 terms each.
With a little effort, verify that the 4-block sums for k = 5, 9 and 13 are
$20(1-i)$, $-80(1-i)$ and $320(1-i)$. The coefficients of the common binomial term form a geometric sequence with a common multiplier of –4. We must sum 8 terms from this sequence.

The required sum is $\dfrac{a(1-r^n)}{1-r}(1-i)=\dfrac{-5\left(1-(-4)^8\right)}{1-(-4)}(1-i)$.

$$\Rightarrow k=\frac{5(4^8-1)}{5}=4^8-1=2^{16}-1=2^{10}\cdot 2^6-1=1024\cdot 64-1=\underline{\mathbf{65535}}.$$

B) The 2-digit primes are: 11, 13, 17, 19, 23, 29, 31
37, 41, 43, 47, 53, 59, 61
67, 71, 73, 79, 83, 89, 97

There are 21 2-digit primes.
The following chart summarizes the possible digit-sums and their corresponding frequencies

2	4	5	7	8	10	11	13	14	16	17
1	2	2	2	3	3	3	1	1	2	1
11	13 31	23 41	43 61	17 53 71	19 37 73	29 47 83	67	59	79 97	89

The fact that the 11 frequencies add up to 21 is a double check.
Thus, $(K,N)=\underline{\mathbf{(11,3)}}$, $S=\underline{\mathbf{\{8,10,11\}}}$. (The elements in the set *S* may be listed in any order.)