MASSACHUSETTS MATHEMATICS LEAGUE

DECEMBER 2003

ROUND 7: TEAM QUESTIONS

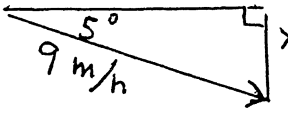
ANSWERS

A) 69.5 D) (2, 4), (0, −2)

B) 25 E) 30

C) 2 F) 9

A) A submarine submerging at an angle of 5 degrees is propelled at the rate of 9 mph. Calculate to the nearest tenth how many seconds it would take the submarine to reach a depth of 80 feet



$x = 9\sin 5° \frac{mi}{hr} = \frac{9\sin 5°(5280)}{3600} \frac{ft}{sec}$

$= 1.1505 \frac{ft}{sec}$, $t = 80/1.1505 = 69.5$ sec

B) Al and Bob each have 16 cards after playing four rounds of Flip-A-Card. In each round of this game, the loser gives the winner enough cards from his own to double the number of cards in the winner's hand. Al lost the first two rounds, but won the last two. How many cards did Al start with?

Al	Bob
16	16
8	24
4	28

Al	Bob
18	14
25	7

C) If $6^p = 3$, calculate the value of $(\log_p 36^p)(\log_3 p)$.

$\log_p 36^p = \log_p 6^{2p} = 2\log_p(6^p) = 2\log_p 3$ so $2\log_p 3 \cdot \log_3 p = 2 \cdot 1 = 2$

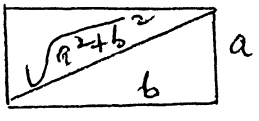
D) Determine both points on the line $y = 3x - 2$ which are $\sqrt{5}$ units from the line $2x + y = 3$

$2x + y = C$, $2x + y = 3$ $\quad d = \frac{|C-3|}{\sqrt{2^2+1^2}} = \sqrt{5}$, $|C-3| = 5$, $C = -2, 8$

$2x + y = 8$ (2, 4) $\qquad 2x + y = -2$ (0, −2)

$3x - y = 2$ $\qquad 3x - y = 2$

E) In a rectangle, the ratio of a diagonal to the perimeter is 5 to 14 while the area is 432 Calculate the length of diagonal.



$\frac{\sqrt{a^2+b^2}}{2a+2b} = \frac{5}{14}$, $14\sqrt{a^2+b^2} = 10a + 10b$

$24(a^2+b^2) = 50(432)$

$49a^2 + 49b^2 = 25a^2 + 50ab + 25b^2$, $a^2 + b^2 = 900$

F) The smallest interior angle of a certain polygon is 120 degrees, and successive angles increase by 5 degrees. That is, the angles are 120, 125, 130, etc. Calculate both possibilities for the number of sides of the polygon.

$120 + 125 + \cdots + (5n + 115) = 180(n-2)$, $\frac{n}{2}(5n + 235) = 180n - 360$

$n(n + 47) = 72n - 144$, $n^2 - 25n + 144 = 0$ $(n-9)(n-16) = 0$

16 is extraneous.