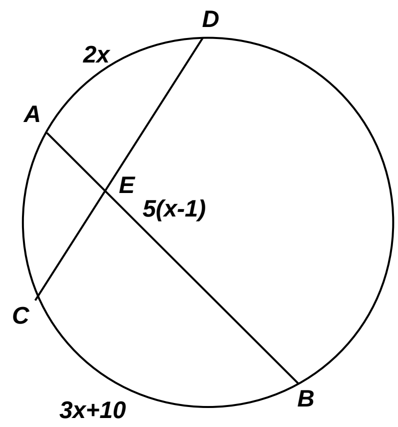
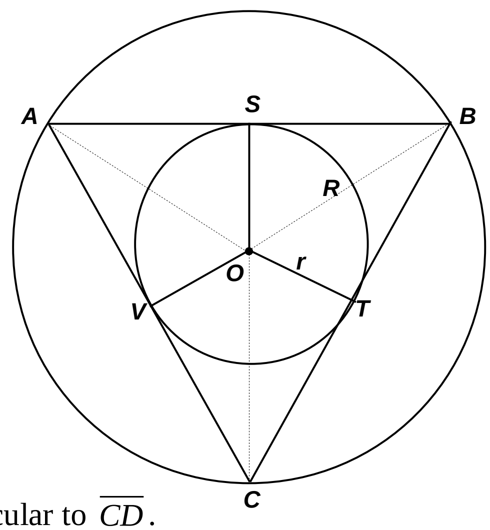
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 - FEBRUARY 2017 SOLUTION KEY

Round 5



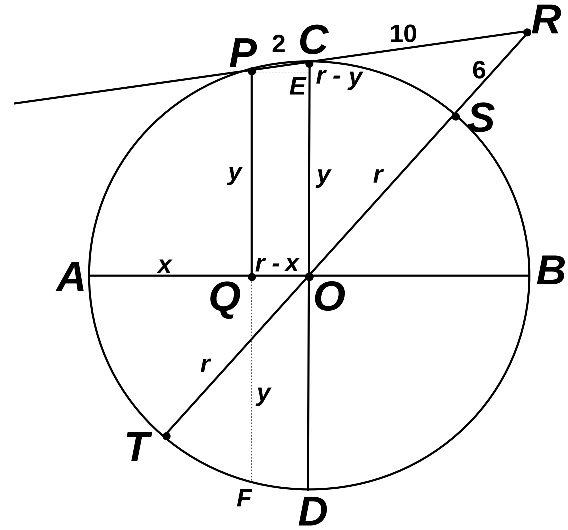
A) Since the measure of the vertical angles formed by two intersecting chords is the average of the intercepted arcs, we have

$$\frac{(2x)+(3x+10)}{2}=180-5(x-1)$$
$$\Rightarrow 5x+10=360-10x+10$$
$$\Rightarrow 15x=360 \Rightarrow x=\underline{\mathbf{24}}$$



B) In an equilateral triangle, the medians and altitudes are one and the same segments. Since the medians intersect at a point that divides each median into a 2 : 1 ratio, we see that $r+R=3r$ must equal an altitude of the equilateral triangle, namely, $6\sqrt{3}$. Therefore, $r=2\sqrt{3}$ and $R=4\sqrt{3}$, producing circles with areas of 12π and 48π, and a ring with area $\underline{\mathbf{36\pi}}$.

C) Let $AQ = x$, $PQ = y$ and $TO = OS = r$. Draw $\overline{PE}$ perpendicular to $\overline{CD}$.
Applying the secant-secant rule (outer segment times outer plus inner, i.e. $RC \cdot RP = RS \cdot RT$), we have $10 \cdot 12 = 6(6+2r) \Rightarrow r = 7$.



Applying the product-chord theorem,

$$AQ \cdot QB = PQ \cdot QF \Rightarrow y^2 = x(14-x) = \boxed{14x-x^2}$$

Applying the Pythagorean Theorem in ΔPEC, we have

$$(7-x)^2+(7-y)^2=2^2$$

Expanding and substituting,

$$\left(49-14x+x^2\right)+\left(49-14y+y^2\right)=4$$
$$94-\left(14x-x^2\right)-14y+y^2=0$$
$$94-14y=0 \Rightarrow y=\underline{\mathbf{\frac{47}{7}}}$$