MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 - FEBRUARY 2017 SOLUTION KEY

Team Round

A) Given: $y = \dfrac{1-2x}{x-6}$

Since the denominator cannot be zero, the equation of the vertical asymptote is $x = 6$.

Writing $\dfrac{1-2x}{x-6}$ as $\dfrac{\frac{1}{x}-2}{1-\frac{6}{x}}$ we note that as $x \to \pm\infty$ (i.e. gets arbitrarily large), $\dfrac{1}{x}$ and $\dfrac{6}{x}$ both approach zero and the quotient approaches –2.

Thus, the equation of the horizontal asymptote is $y = -2$.

Note that:

A reflection of any point through the origin changes the sign of both coordinates of the point.

A reflection of any point across $y = x$ interchanges (swaps) the coordinates of the point.

$P_1(6,-2)$ undergoes the following transformations:

$\Rightarrow A(6,2) \Rightarrow B(-6,2) \Rightarrow C(-6,2k-2) \Rightarrow D(2h+6,2k-2)$

$\Rightarrow E(-2h-6,-2k+2)$

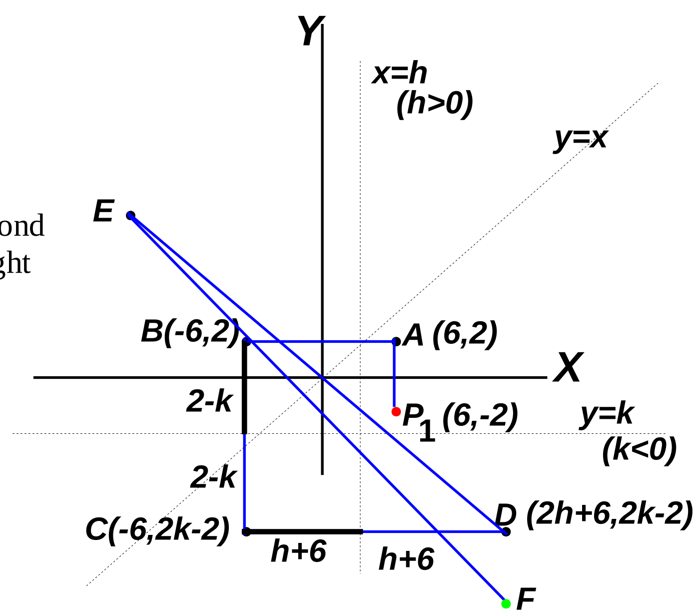
$\Rightarrow F(-2k+2,-2h-6) = (10,-12)$

Therefore, $(h,k) = \underline{(\mathbf{3,-4})}$.

Take care with the order! Equating the first coordinates gives us k, while equating the second coordinates gives us h. The diagram at the right illustrates the sequence of transformations.

Follow the blue line

$P_1(red) \gg A \gg B \gg C \gg D \gg E \gg F(green)$



Check:

$$(6,-2) \underset{x-axis}{\overset{across}{\Rightarrow}} (6,2) \underset{y-axis}{\overset{across}{\Rightarrow}} (-6,2) \underset{\substack{y=-4 \\ \Delta y=-6}}{\overset{across}{\Rightarrow}} (-6,-10) \underset{\substack{x=3 \\ \Delta x=+9}}{\overset{across}{\Rightarrow}} (12,-10) \underset{origin}{\overset{thru}{\Rightarrow}} (-12,10) \underset{y=x}{\overset{across}{\Rightarrow}} (10,-12)$$