

**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2009
ROUND 7 TEAM QUESTIONS**

******* CALCULATORS ARE PERMITTED IN THIS ROUND *******

ANSWERS

- A) _____ D) _____
B) _____ E) _____
C) _____ F) _____

A) There are two possible triangles with sides $x - 1$, $x + 3$ and $2x - 3$ and an angle with measure of 120° . Compute the smaller of the two possible perimeters.

B) Consider the following list of Pythagorean triples:

Row 1:	9	40	41
Row 2:	11	60	61
Row 3:	13	84	85

The first numbers in each row form an increasing arithmetic progression.
Compute the sum of the squares of the numbers in the 11th row.

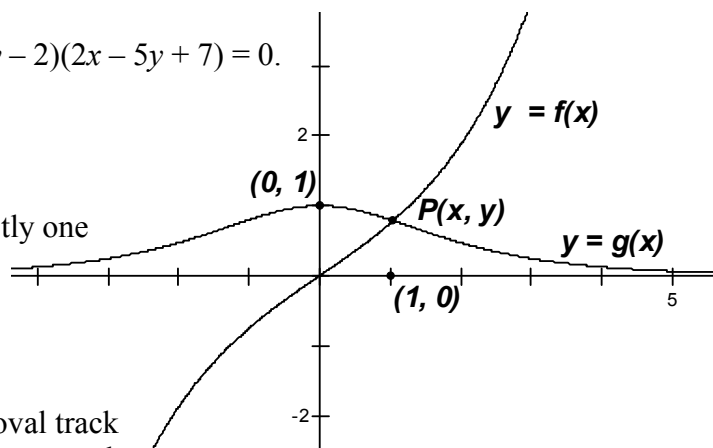
C) How many points are determined by the intersection of the graphs of

$$(x - y + 2)(3x + y - 4) = 0 \text{ and } (x + y - 2)(2x - 5y + 7) = 0.$$

D) Let $f(x) = \frac{2^x - 2^{-x}}{2}$ and $g(x) = \frac{2}{2^x + 2^{-x}}$

The graphs of $y = f(x)$ and $y = g(x)$ indicate exactly one point of intersection at $P(x, y)$.

The exact x -coordinate of point P can be expressed in the form $\log_2 N$. Compute N .



E) Abbott and Costello are running laps around an oval track in opposite directions. Their starting points are staggered by half a lap. Assuming both runners move at a constant speed and Abbott completes 5 laps in the time it takes Costello to complete 4, how many laps has Abbott completed when the runners have passed each other 100 times?

F) $\triangle ABC$ has sides a , b and c with integer lengths and $a \leq b$.

The median m to side c also has integer length.

Determine all possible values of m if the perimeter of $\triangle ABC$ is 24 and $c = 8$ or 10.