MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2009
ROUND 7 TEAM QUESTIONS

******* CALCULATORS ARE PERMITTED IN THIS ROUND *******

ANSWERS

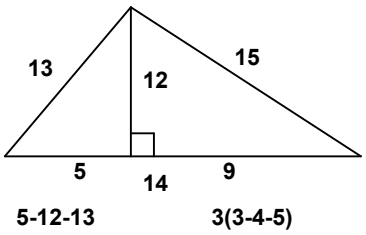
A) _____ : _____ D) _____ miles

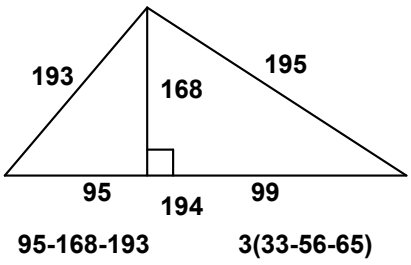
B) (_____ , _____ , _____) E) _____ units^2

C) (_____ , _____) F) (_____ , _____)

A) A rectangular block of ice cream has dimensions 7″, 8″ and 9″. It is completely covered with chocolate (like a Klondike Bar). Suppose it is then cut up into one inch cubes by making cuts parallel to the faces. Let A be the number of cubes with 2 or 3 faces covered with chocolate. Let B be the number of cubes with 0 or 1 face covered with chocolate. Compute the ratio of $A : B$.

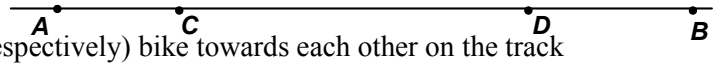
B) Several non-right triangles have all of these properties:
its sides have lengths that are consecutive integers
one of its altitudes has an integer length and
that altitude divides the triangle into two right triangles each with integer sides
Let S be a sequence of triplets (a, b, c) representing the lengths of sides of such triangles in increasing order of perimeter (where $a < b < c$).
The first term in this sequence is (13, 14, 15). The third term in this sequence is (193, 194, 195).





Determine the second term in this sequence.

C) Find the ordered pair (x, y) that satisfies $4x - 7y = 451$, x and y are positive integers and $x + y$ is the largest possible three-digit multiple of 3.



D) Alice and Barbara (starting at points A and B respectively) bike towards each other on the track above. Alice, biking 10 mph faster than Barbara, would meet Barbara at point D in 1 hour. If , however, Barbara increased her speed by 10 mph and Alice decreased her speed by k mph, they would meet at point C in 40 minutes. Alice's reduced speed is three-quarters of her original speed. Compute the distance (in miles) between C and D.

E) Compute the area of the region containing all points (x, y) that satisfy $\begin{cases} y \le 15 \\ y \ge (2x + 5)\left(\dfrac{|x|}{x}\right) \end{cases}$.

F) In each of the years from 1999 through 2008, 5 state quarters were issued at each of the mints in Philadelphia, Denver and San Francisco. Living on the east coast, I have found in circulation 100% of the quarters minted in Philadelphia and at least 75% of the quarters minted in Denver.
San Francisco only issues mint coins and I have found at most 12.5% of these quarters.
Let m and M denote the exact minimum and Maximum percentages of all the quarters minted that I have found. Compute the ordered pair (m, M).