MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2007 SOLUTION KEY

Round 1

A) The radicand must be positive to insure that the denominator is real and nonzero.
$4x - 3x^2 = x(4 - 3x) > 0 \rightarrow x > 0$ and $x < 4/3 \rightarrow$ **$\underline{0 < x < 4/3}$**

B) $f(1) = 1 - f(0) = 1$, $\quad f(2) = 2 - f(1) = 2 - 1 = 1$
$f(3) = 3 - f(2) = 3 - 1 = 2$, $\quad f(4) = 4 - f(3) = 4 - 2 = 2$
$f(5) = 5 - f(4) = 5 - 2 = 3$, $\quad f(6) = 6 - f(4) = 6 - 3 = 3$
In general, for even n, $f(n) = n/2$; for odd n, $f(n) = (n + 1)/2 \rightarrow f(2007) =$ **$\underline{1004}$**

C) $A(mx + b)^2 \equiv m(Ax^2) + b \rightarrow Am^2x^2 + 2Ambx + Ab^2 \equiv Amx^2 + b$
Note: $\equiv$ denotes this is an identity, not just an equation. It is true for all values of x.

Equating coefficients and recalling that $A \neq 0$, $m \neq 0$,
$Am^2 = Am \rightarrow Am(1 - m) = 0 \rightarrow m = 1$
$2Amb = 0 \rightarrow b = 0$
$Ab^2 = b \rightarrow A$ can be any nonzero digit.
Choosing $A = 9$, maximizes the 3-digit number: **$\underline{910}$**

Round 2

A) Substituting $A = 1,2,3,\ldots$produces the sequence 8, 15, 22, ….
The first multiple of 13 in this sequence occurs when $A = 11$, $B = 78/13 = 6$
Thus, $(A, B) =$ **$\underline{(11, 6)}$**.

B) The expressions $7n + 2$ and $11n + 4$ generate the sequences
2, 9, 16, 23, 30, 37, … and 4, 15, 26, 37, …
Clearly, the two-digit integer they have in common is 37.
The next common integer can be found by adding 77, the least common multiple of 7 and 11.
To find the largest three-digit integer A that they have in common, solve the inequality
$A = 37 + 77k < 1000$ over the integers. $k < 963/77 = 12^+ \rightarrow k = 12 \rightarrow A = 37 + 924 =$ **$\underline{961}$**

C) **111** = 3(37) – ok
222 = 2(3)(37) – ok
333 = 3(111) = 3^2(37) – fails because of the repeated prime.
444 = 4(111) = 2^2(3)(37) fails
555 = 3(5)(37) - ok
666 = (2)3^2(37) – fails
777 = 3(7)(37) – ok
888 = 2^3(3)(37) – fails
999 = 3^3(37) – fails
Thus, the required sum is (1 + 2 + 5 + 7)(111) = (15)(111) = **$\underline{1665}$**.