MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2015 SOLUTION KEY

Round 4

A) Since $\frac{Y}{4}$ is a completely reduced proper fraction for $Y > 0$, Y = 1 or 3.

Thus, $(21 - A)\cdot 4 = 52Y \Rightarrow 21 - A = 13Y = 13$ or $39 \Rightarrow A = 8, -18 \Rightarrow$ (A, Y) = **(8, 1)**, **(–18, 3)**.

B) $$\frac{A}{2x-3}+\frac{B}{x+1}=\frac{A(x+1)+B(2x-3)}{(2x-3)(x+1)}=\frac{(A+2B)x+(A-3B)}{2x^2-x-3}=\frac{20x+k}{2x^2-x-3}$$

This is an identity in x if and only if $A+2B=20$ and $k=A-3B$.

$A:B=4:3 \Rightarrow A=4C, B=3C$

Now $A+2B=10C=20 \Rightarrow C=2 \Rightarrow A=8, B=6$ and $k=A-3B=8-3(6)=\underline{\mathbf{-10}}$.

Alternate Solution:

Using $\frac{A}{B}=\frac{4}{3}$, after expressing the sum over a common denominator, we equate numerators,

$$A(x+1)+\left(\frac{3A}{4}\right)(2x-3)=20x+k\;.\;\begin{cases} x=-1 \Rightarrow -\frac{15A}{4}=-20+k \\ x=0 \Rightarrow -\frac{5A}{4}=k \end{cases} \Rightarrow -20+k=3k \Rightarrow k=\underline{\mathbf{-10}}$$

C) Dick travelled (126d1 – 12345) in 4 hours and 45 minutes.

$(600+10d+1)-345=256+10d$ miles

His average speed was $k=\frac{256+10d}{4.75}=\frac{4(256+10d)}{19}=\frac{8(128+5d)}{19}$ mph.

For $d=1$, we have $k=\frac{8(133)}{19}=8\cdot 7=56 \Rightarrow (k,d)=\underline{(\mathbf{56,1})}$.