

**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2012 SOLUTION KEY**

Team Round

A) 3 faces: corner cubes (at the 8 vertices) $\Rightarrow 8$

2 faces: edge cubes – excluding the corners (12 edges) $\Rightarrow 4$ ea. @ $(n-2)$, $(n-3)$ and $(n-4)$
 $\Rightarrow 12n - 36$ or $12(n-3)$

1 face: interior cubes on each face (6 faces, i.e. 3 pairs of opposite faces)

$n \times (n-1)$ faces: $2[(n-2)(n-3)]$

$n \times (n-2)$ faces: $2[(n-2)(n-4)]$

$(n-1) \times (n-2)$ faces: $2[(n-3)(n-4)]$

$\Rightarrow 6n^2 - 36n + 52$ or $6(n-3)^2 - 2$

0 faces: interior cubes only $\Rightarrow (n-2)(n-3)(n-4)$ or $n^3 - 9n^2 + 26n - 24$

We can proceed by direct evaluation or algebraically (table is included below)

We require that $6n^2 - 36n + 60 = (n^3 - 9n^2 + 26n - 24) + 12(n-3) = n^3 - 9n^2 + 38n - 60$

$\Leftrightarrow n^3 - 15n^2 + 74n - 120 = 0 \Leftrightarrow (n-4)(n-5)(n-6) = 0 \Rightarrow n = \underline{\mathbf{4, 5, 6}}$.

The following table confirms our results:

<i>n</i>	(3)	[2]	(1)	[0]	
4	8	12	4	0	= 12
5	8	24	22	6	= 30
6	8	36	52	24	= 60
7	8	48	94	60	102 < 108
8	8	60	148	120	156 < 180
9	8	72	214	210	222 < 288
10	8	84	292	336	gap continues to widen