

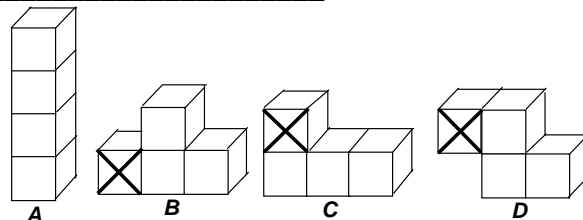
**MASSACHUSETTS MATHEMATICS LEAGUE**  
**CONTEST 1 - OCTOBER 2014**  
**ROUND 7 TEAM QUESTIONS**  
**ANSWERS**

A) \_\_\_\_\_ D) ( \_\_\_\_\_ , \_\_\_\_\_ )

B) \_\_\_\_\_ E) \_\_\_\_\_

C) \_\_\_\_\_ F) \_\_\_\_\_

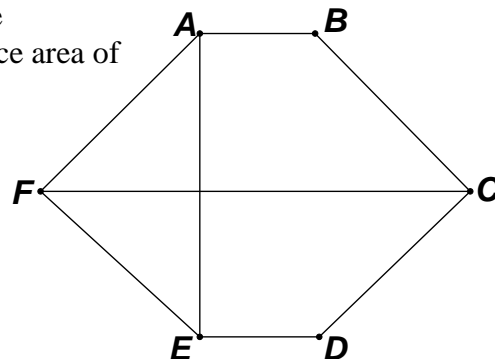
- A) Four unit blocks ( $1 \times 1 \times 1$ ) are glued together face-to-face in a single layer to form different geometric solids. This may be done in 4 different ways. Multiple copies of each solid are available. Copies of each solid are to be placed in all possible stable positions with a different number of surfaces in contact with the table top.



Assume that any solid with 1, 2, 3 or 4 faces, or exactly 2 edges in contact with the table, is in a stable position. In some copies of  $B$ ,  $C$  and  $D$ , the cubes marked with an  $X$  are weighted. Only glued faces and faces that lie in the plane of the table top are considered to be not exposed. The sum of the total exposed surface area of all these solids is  $K$ . Compute  $K$ .

- B) In hexagon  $ABCDEF$ ,  $AB = DE$  and  $BC = CD = EF = FA = 2 \cdot AB$

$A$  is the reflection of  $E$  and  $B$  is the reflection of  $D$  across  $\overleftrightarrow{FC}$   
 If  $FC = AE + 1$  and the area of  $\triangle AFE$  is 2.875, compute  $AB$ .



- C) Given:  $\begin{cases} x + ay = b^2 \\ x - by = a^2 \end{cases}$  for positive integers  $x, y, a$  and  $b$ .

If  $a : b = 4 : 7$ , compute the minimum sum of  $x + y$ .

- D) Let  $x$  be the smallest integer for which  $y = \frac{7}{x+4} - \frac{3}{x-3} > 0$ . Determine the ordered pair  $(x, y)$ .

- E) For  $n = 1$ ,  $\frac{n^3 - 32}{n^2 + 30}$  is an integer. Determine the largest even positive integer for which  $\frac{n^3 - 32}{n^2 + 30}$  is also an integer.

- F) A 10 gallon container  $A$  is full of water and containers  $B$  and  $C$ , with 7 gallon and 4 gallon capacities respectively, are empty. Water can only be transferred from one container to another if one of the following conditions is satisfied:

- the container into which water is poured is completely filled
- the container from which the water is poured is completely emptied

Let the initial state be  $(A, B, C) = (10, 0, 0)$ . After the first transfer, we would have  $(3, 7, 0)$  or

$(6, 0, 4)$ . After a total of  $k$  transfers, I can get  $(5, 5, 0)$ . Compute the minimum possible value of  $k$ .