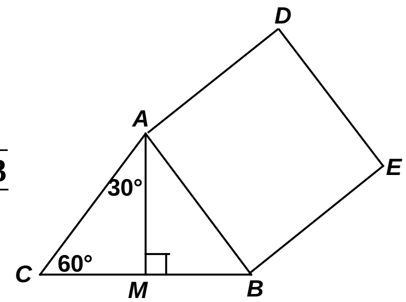
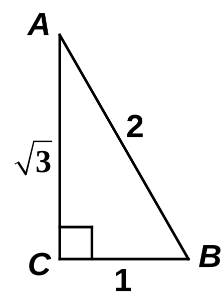
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 – NOVEMBER 2013 SOLUTION KEY

Round 5

A) $\overline{AM}$ is also an altitude and an angle bisector, forcing ΔAMB to be 30° – 60° – 90°.
Area($ABED$) = 256 $\Rightarrow AB = 16 \Rightarrow MB = 8 \Rightarrow AM =$ $\underline{\mathbf{8\sqrt{3}}}$



B) $AB = 2BC \Rightarrow \Delta ABC$ is 30°-60°- 90° or let AB = 2 and BC = 1, solve for AC ($=\sqrt{3}$) and use SOHCAHTOA to determine values for $\csc B$ and $\cot A$.
Since adding (or subtracting) multiples of 360° to (or from) the arguments of any trig function gives us a coterminal angle, the value of the trig function is unchanged.



$$\left(\csc B \cot A \sin(750°) \tan(-480°) \sin(570°)\right)^5 =$$

$$\left(\frac{2}{\sqrt{3}} \cdot \sqrt{3} \sin 30° \tan 240° \sin 210°\right)^5$$

$$\left(2 \cdot \frac{1}{2} \cdot \sqrt{3} \cdot \left(-\frac{1}{2}\right)\right)^5 = \left(-\frac{\sqrt{3}}{2}\right)^5 = \underline{-\frac{\mathbf{9}}{\mathbf{32}}\sqrt{\mathbf{3}}}.$$

C) Note that $\sin 10° = \cos 80°, \sin 30° = \cos 60°, \sin 50° = \cos 40°$

So $\dfrac{\sin 10° \sin 30° \sin 50° \sin 70°}{\cos 20° \cos 40° \cos 80° \cos 120°} = \cancel{\dfrac{\sin 10°}{\cos 80°}} \cdot \cancel{\dfrac{\sin 50°}{\cos 40°}} \cdot \cancel{\dfrac{\sin 70°}{\cos 20°}} \cdot \dfrac{\sin 30°}{\cos 120°} = \dfrac{\cos 60°}{-\cos 60°} = -1.$

$$\sin 30° = \frac{1}{2} \Rightarrow \sin 150° = \frac{1}{2},\ \sin 210° = -\frac{1}{2},\ \sin 330° = -\frac{1}{2}$$

Therefore, the original equation is equivalent to $\sin\theta < -\dfrac{1}{2}$. From the graph of the sine function over the specified interval, we have a solution set of $\underline{\mathbf{210° < \theta < 330°}}$.