

**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2009 SOLUTION KEY**

Round 4

A) Let x denote the correction factor. Then: $5x = 6$ or $x = 6/5$

Since $1/5$ of 10 should be 2, applying the correction factor, we have $2x = \frac{12}{5}$

B) $\frac{19}{57+k} = \frac{1}{5} \rightarrow 95 = 57 + k \rightarrow k = 38$ Thus, the new solution will contain $(19 + 38)$ gallons of acid and the solution is $\frac{19+38}{57+38} = \frac{57}{95} = \frac{3}{5}$ acid, i.e. **60%**

C) $1 - \frac{1}{1 - \frac{3}{2 + \frac{1}{x}}} = 2x \rightarrow x \neq 0$ (to avoid division by zero)

$$1 - \frac{1}{1 - \frac{3}{2 + \frac{1}{x}}} = 1 - \frac{1}{1 - \frac{3x}{2x+1}} = 1 - \frac{1}{\frac{2x+1-3x}{2x+1}} = 1 - \frac{2x+1}{1-x} = 2x \rightarrow \frac{1-2x}{1} = \frac{2x+1}{1-x}$$

Cross multiplying, $2x+1 = 1-3x+2x^2 \rightarrow 2x^2 - 5x = x(2x-5) = 0 \rightarrow x = \frac{5}{2}$

Alternate solution:

Replace $1/x$ with x (and vice-versa).

Then $1 - \frac{1}{1 - \frac{3}{2+x}} = \frac{2}{x} \rightarrow 1 - \frac{x+2}{x-1} = \frac{2}{x} \rightarrow \frac{-3}{x-1} = \frac{2}{x} \rightarrow -3x = 2x - 2 \rightarrow x = 2/5$

But x was replaced by $1/x$ (and vice versa)! Letting the real x stand up, $x = 5/2$

The only possible extraneous roots would have resulted from attempts to divide by zero and this would have occurred only if x came out to be 0, -2 or +1.

Therefore, our answer will check and the actual substitution below is unnecessary!

$$1 - \frac{1}{1 - \frac{3}{2 + \frac{2}{5}}} = 1 - \frac{1}{1 - \frac{3}{\frac{12}{5}}} = 1 - \frac{1}{1 - \frac{5}{4}} = 1 - (-4) = \boxed{5}$$

$$2(2.5) = \boxed{5}$$