MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2013 SOLUTION KEY

Round 4

A) $6\left(\frac{1}{x}\right)=x+5 \Leftrightarrow x^2+5x-6=(x+6)(x-1)=0 \Rightarrow x=$ $\underline{\mathbf{-6,\ 1}}$

B) Note: $x \neq 1, -2$
If a solution produces either of these values, they are extraneous.
Let $w=\frac{x+2}{x-1}$. Then $\frac{x+2}{x-1}-3=18\left(\frac{x-1}{x+2}\right) \Rightarrow w-3=18\left(\frac{1}{w}\right)$
$\Rightarrow w^2-3w-18=(w-6)(w+3)=0 \Rightarrow w=6, -3$.
$\frac{x+2}{x-1}=6 \Rightarrow 6x-6=x+2 \Rightarrow 5x=8 \Rightarrow x=$ $\underline{\mathbf{\frac{8}{5}}}$.
$\frac{x+2}{x-1}=-3 \Rightarrow -3x+3=x+2 \Rightarrow 4x=1 \Rightarrow x=$ $\underline{\mathbf{\frac{1}{4}}}$.

C) Let $N=\underline{X}\,\underline{Y}=10X+Y$. Then:
$N^2=(10X+Y)^2=100X^2+20XY+Y^2$
Subtracting the square of the sum of the digits $\left(X^2+2XY+Y^2\right)$, we have
$99X^2+18XY=9X(11X+2Y)=2655 \Rightarrow X(11X+2Y)=295$
Either X is divisible by 5 or $11X+2Y$ is!
Knowing that both X and Y are single-digit integers,
we try $X=5$, $55+2Y=59 \Rightarrow Y=2$
Could our unique two-digit number be 52?
To minimize number crunching, we take advantage of the *difference of perfect squares*.
Checking, $52^2-7^2=(52+7)(52-7)=(60-1)(45)=2700-45=2655$ Bingo!