MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2014 SOLUTION KEY

Round 5

A) $\left|5-|x+1|\right|=2 \Leftrightarrow 5-|x+1|=\pm 2 \Leftrightarrow |x+1|=3,7$

Therefore, $x+1=\pm 3$ or $x+1=\pm 7$ and we have our 4 solutions: **2, ¯4, 6, ¯8** (in any order)

B) The equation simplifies to $2|x-1|+4|x-1|=6|x-1|=|5x+11|$

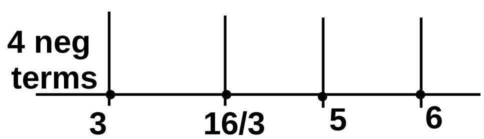
Over the stated domain $-2<x<2$, the original equation simplifies to $6|x-1|=5x+11$.

$$\Rightarrow\ 5x+11=\begin{cases}6(x-1) \text{ if } x\ge 1\\ 6(1-x) \text{ if } x<1\end{cases} \Rightarrow x=\begin{cases}17\\ -\dfrac{5}{11}\end{cases}$$

The only value in the stated domain is $\mathbf{-\frac{5}{11}}$.

C) $$\frac{6(x-5)+x-3-3(x-3)(x-5)}{(x-3)(x-5)}\ge 0 \Rightarrow \frac{7x-33-3x^2+24x-45}{(x-3)(x-5)}\ge 0$$

$$\Rightarrow \frac{3x^2-31x+78}{(x-3)(x-5)}\le 0 \Rightarrow \frac{(3x-13)(x-6)}{(x-3)(x-5)}\le 0$$



The critical points 3, 13/3, 5 and 6 divide the number line into 5 regions.
At the extreme left ($x<3$) all four terms are negative and, moving to the right, each time a boundary is crossed, one less term is negative. Thus, the regions with a negative quotient are $3<x<13/3$ and $5<x<6$ and equality occurs at $x=13/3$ and 6 $\Rightarrow$ $\mathbf{3<x\le\frac{13}{3},\ 5<x\le 6}$