

**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 – MARCH 2015 SOLUTION KEY**

Round 3

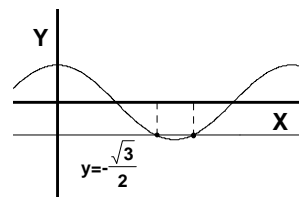
$$\text{A) } 5\theta = \begin{cases} 60^\circ + n \cdot 360^\circ \\ 120^\circ + n \cdot 360^\circ \end{cases} \Leftrightarrow \theta = \begin{cases} 12^\circ + n \cdot 72^\circ \\ 24^\circ + n \cdot 72^\circ \end{cases} \Rightarrow \begin{cases} 12, 84, 156 \\ 24, 96, 168 \end{cases} \Rightarrow 96 + 156 + 168 = \underline{420}$$

B) The first factor is 0 for $x = \frac{2\pi}{3}$ and $\frac{5\pi}{3}$, so these values are excluded.

Only the middle factor can be negative! $\cos x < -\frac{\sqrt{3}}{2}$

$\frac{\sqrt{3}}{2} \Rightarrow \frac{\pi}{6}$ family, Q2, 3 \Rightarrow open interval

$$\underline{\underline{\frac{5\pi}{6} < x < \frac{7\pi}{6} \text{ or } \left(\frac{5\pi}{6}, \frac{7\pi}{6}\right)}}$$



Note: The excluded values for the first factor are outside this interval.

C) Simplify the given expression before attempting to substitute.

$$\begin{aligned} & 1 + \underbrace{(\sin^2 x + \cos^2 x)} + \underline{\tan^2 x} + \underline{\cot^2 x} + \sec^2 x + \csc^2 x = \underline{\sec^2 x} + \underline{\csc^2 x} + \sec^2 x + \csc^2 x \\ & = 2(\sec^2 + \csc^2 x) \\ & = 2\left(\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}\right) = 2\left(\frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}\right) = \frac{2}{\frac{1}{4}(2\sin x \cos x)^2} = \frac{8}{\sin^2 2x} \end{aligned}$$

$$\text{Now we can substitute, } \frac{8}{\left(\sin \frac{5\pi}{4}\right)^2} = \frac{8}{\left(-\frac{\sqrt{2}}{2}\right)^2} = \underline{16}.$$