MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2007 SOLUTION KEY

Round 4

A) $$\frac{2\frac{7}{16} - 3\frac{3}{4}}{5\frac{5}{8} + 7\frac{1}{2}} = \frac{\left(\frac{39}{16}-\frac{15}{4}\right)16}{\left(\frac{45}{8}+\frac{15}{2}\right)16} = \frac{39-60}{90+120} = \frac{-21}{210} = \mathbf{-\underline{\frac{1}{10}}} \text{ or } -\underline{\mathbf{0.1}}$$

B) Multiplying by the LCD = $(x + 1)(x - 1)$, $(x - 3)(x - 1) - 2x + 8 - 3x - 3 = 0$
→ $x^2 - 4x + 3 - 5x + 5 = 0$ → $x^2 - 9x + 8 = (x - 1)(x - 8) = 0$ → $x = \underline{\mathbf{8}}$ ($x = 1$ is extraneous)

C) $$\frac{(n+1)!(2n-2)!}{(n-1)!(2n)!} = \frac{(n+1)n\cancel{(n-1)!}\cancel{(2n-2)!}}{\cancel{(n-1)!}(2n)(2n-1)\cancel{(2n-2)!}} = \frac{(n+1)n}{(2n)(2n-1)} = \frac{2}{7}$$ → $7n^2 + 7n = 8n^2 - 4n$
→ $n^2 - 11n = n(n - 11) = 0$ → $n = \underline{\mathbf{11}}$

Here's the original question C (nixed by the proofreaders):
For extremely large positive values of n, the following fraction approaches a fixed value L. Compute L.

$$\frac{(n+1)!(2n-2)!}{(n-1)!(2n)!}$$

You might want to try your hand at solving this question before peeking at the solution below.

$$\frac{(n+1)!(2n-2)!}{(n-1)!(2n)!} = \frac{(n+1)n\cancel{(n-1)!}\cancel{(2n-2)!}}{\cancel{(n-1)!}(2n)(2n-1)\cancel{(2n-2)!}} = \frac{n^2+n}{4n^2-2n}$$

Dividing numerator and denominator by n^2, we have $\dfrac{1+\frac{1}{n}}{4+\frac{2}{n}}$

As n takes on extremely large positive values (i.e. approaches infinity), the fractional terms in the numerator and denominator each approach zero and the overall fraction approaches **<u>1/4</u>** or **<u>0.25</u>**