

Addenda:

1B – (Thanks to Sung Ahn and Sam Solomon - Canton)

An interesting alternative approach:

By solving the 3rd equation for z and substituting for z in the 1st equation, we see that y must be $\frac{11}{12}$.

$$14x + 3y - 7z = 8$$

$$14x - 7z = k_1$$

$-8x + 5y + 4z = c$ becomes $-8x + 4z = k_2$, where each k is just a constant, namely,

$$-2x + 3y + z = 2$$

$$-2x + z = k_3$$

$$k_1 = 8 - 3y, k_2 = c - 5y, \text{ and } k_3 = 2 - 3y$$

Divide the equations through by 7, -4 and -1 respectively.

$$2x - z = k_1 / 7$$

We have $2x - z = -k_2 / 4$

$$2x - z = -k_3$$

If the constants on the right hand side are all different, we have 3 parallel lines, which would produce no solutions.

If exactly two of the constants were equal, we would have 2 parallel lines (again no solution). The only way the system could produce an infinite number of solutions is if all three constants are the same.

We require that $\frac{8-3y}{7} = \frac{5y-c}{4} = 3y-2$.

For $y = \frac{11}{12}$, the first and last expressions evaluate to $\frac{3}{4}$.

$$\frac{5y-c}{4} = \frac{3}{4} \Rightarrow c = 5y - 3 = 5\left(\frac{11}{12}\right) - 3 = \frac{55-36}{12} = \frac{19}{12}.$$

Team D has been rewritten to request the parameters of the required conditions as an ordered triple.

The listed answer to the original question was incorrect and the requested conditions were much more involved than anticipated.

The original question was:

For nonzero real constants a and b , the linear equations $y = ax + b$ and $\frac{x}{a} - \frac{y}{b} = 1$ intersect along $y = x$. Compute all ordered pairs (a, b) for which this is not possible.