MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 – MARCH 2014 SOLUTION KEY

Round 1

A) $\begin{vmatrix} A-2 & 3 \\ 11 & 2A+1 \end{vmatrix} = (A-2)(2A+1)-3\cdot 11 = 2A^2-3A-35 = (2A+7)(A-5)=0 \Rightarrow A = \cancel{5},\ \underline{\mathbf{-\frac{7}{2}}}$

B) Let $A=\dfrac{1}{x}$ and $B=\dfrac{1}{y}$.

The given system is equivalent to $\begin{cases} \dfrac{1}{2}A+7B=1 \\ 2A+4B=1 \end{cases} \Leftrightarrow \begin{cases} 2A+28B=4 \\ 2A+4B=1 \ (***) \end{cases}$

Subtracting, $24B=3 \Rightarrow B=\dfrac{1}{8} \Rightarrow y=8$.

Substituting in (***), $2A+\dfrac{1}{2}=1 \Rightarrow A=\dfrac{1}{4} \Rightarrow y=4$

The required ordered pair is **<u>(4, 8)</u>**.

C) $\begin{cases} a^2+8b^2-12bc=36 \\ 4ab-9c^2=36 \end{cases}$ Subtracting, we have $a^2-4ab+8b^2-12bc+9c^2=0$.

Partitioning and sharing $8b^2$, $a^2-4ab\underline{+4b^2+4b^2}-12bc+9c^2=(a-2b)^2+(2b-3c)^2=0$
The sum of two non-negative quantities can only be zero if each binomial is zero!
Thus, $a=2b=3c$.

Substituting $a=3c,\ b=\dfrac{3}{2}c$ in the second equation, we have

$4(3c)\left(\dfrac{3}{2}c\right)-9c^2=36 \Leftrightarrow 9c^2=36 \Leftrightarrow c=\pm 2$

Therefore, (a, b, c) = **<u>(6, 3, 2)</u>**, **<u>(–6, –3, –2)</u>**.