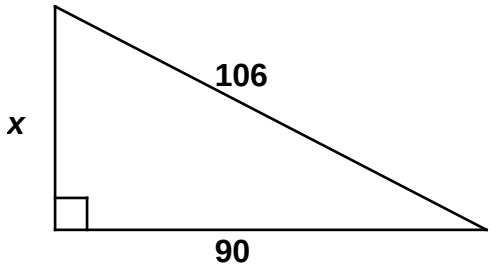
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2013 SOLUTION KEY

Round 2

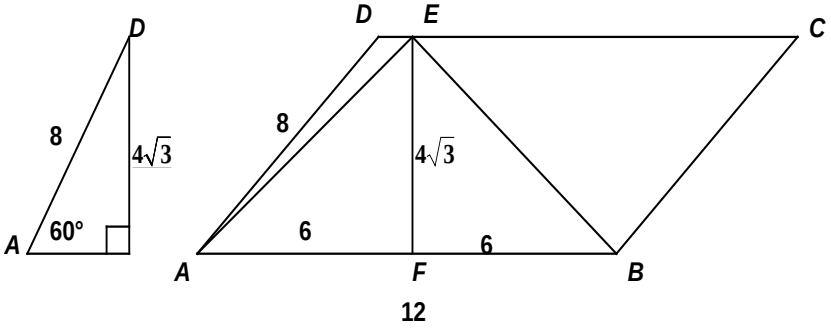


A) Using the Pythagorean Theorem,
$x^2 = 106^2 - 90^2$
Resisting the temptation to play arithmetic,
we factor the right hand side of the equation.
$x^2 = (106 + 90)(106 - 90) = 196(16) = 14^2 4^2$.
Thus, $x = 14(4) =$ **$\underline{56}$**.

B) $AF = FB = 6$, $EF = 4\sqrt{3}$

$$AE^2 = 6^2 + \left(4\sqrt{3}\right)^2 = 36 + 48 = 84$$

$$\Rightarrow AE = \underline{\mathbf{2\sqrt{21}}}$$



C) $AD + DB = AB \Rightarrow 4x + 5y - 5 = 7x + 2y + 1$
$\Rightarrow 3x - 3y + 6 = 0 \Rightarrow x = y - 2 \Rightarrow CD = 4(y - 1)$
$CD^2 + DB^2 = BC^2 \Rightarrow 16(y - 1)^2 + (5y - 3)^2 = 4(3y - 1)^2$
$16y^2 - 32y + 16 + 25y^2 - 30y + 9 = 36y^2 - 24y + 4$
$5y^2 - 38y + 21 = (5y - 3)(y - 7) = 0$
$\Rightarrow y = 3/5$ (extraneous) $y = 7 \Rightarrow$ **$\underline{(5, 7)}$**