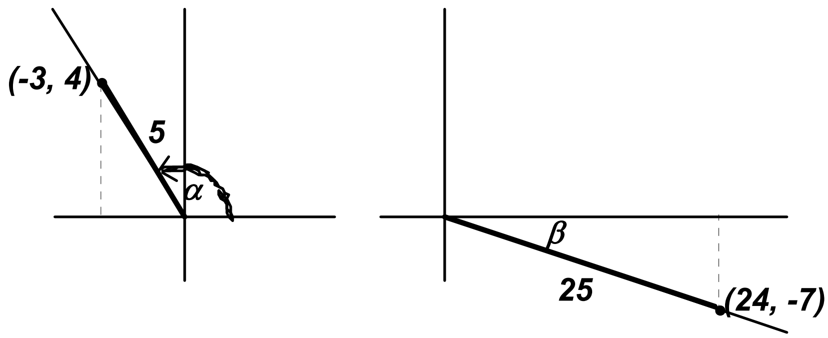
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2010 SOLUTION KEY

Team Round - continued

B) Raising A and B to the 2008*2009 power gives us $2008!^{2009}$ and $2009!^{2008}$ respectively.
Dividing by $2008!^{2008}$, we have 2008! and 2009^{2008}.
Observe that 2008! = 2008·2007· … ·2·1 (2008 factors), but $2009^{2008} = \underbrace{2009 \cdot 2009 \cdot \ldots \cdot 2009}_{\text{2008 factors}}$
Thus, $B = \sqrt[2009]{2009!}$ is larger. Similarly $C > B$ and we have **C** is the largest.

C) Let $\alpha = Arc\cos\left(-\frac{3}{5}\right)$ and $\beta = Arc\sin\left(-\frac{7}{25}\right)$. As indicated in the diagram below,
$90 < \alpha < 180$ (quadrant 2) and $-90 < \beta < 0$ (quadrant 4).



$$\cos\left(2Arc\cos\left(-\frac{3}{5}\right) + Arc\sin\left(-\frac{7}{25}\right)\right) = \cos(2\alpha+\beta) = \cos 2\alpha \cos\beta - \sin 2\alpha \sin\beta$$

$$= \left(1-2\sin^2\beta\right)\cos\beta - \left(2\sin\alpha\cos\beta\right)\sin\beta = \left(1-2\left(\frac{4}{5}\right)^2\right)\cdot\frac{24}{25} - 2\cdot\frac{4}{5}\cdot-\frac{3}{5}\cdot-\frac{7}{25}$$

$$= \left(1-\frac{32}{25}\right)\cdot\frac{24}{25} - \frac{24\cdot 7}{25^2} = -\frac{2\cdot 24\cdot 7}{25^2} = \underline{\mathbf{-\frac{336}{625}}} \text{ (or } \underline{\mathbf{-0.5376}}\text{)}$$