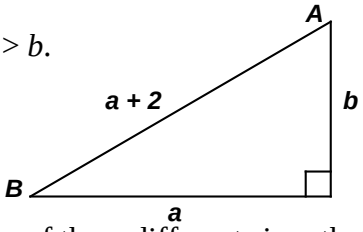
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2012
ROUND 7 TEAM QUESTIONS
ANSWERS

A) _____ D) (_____ , _____)

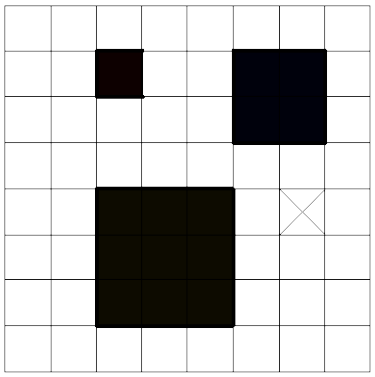
B) _____ E) _____

C) _____ F) _____

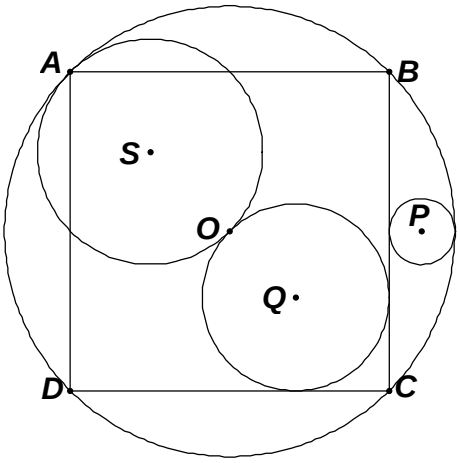
A) Given: a, b are relatively prime integers, $a > b$.
Compute the minimum perimeter of ΔABC
in which $\left(\cos \angle A\right) < \frac{1}{10}$.



B) In the 8 x 8 grid at the right there are squares of three different sizes that do not contain the "X".
Consider all possible squares on this grid from 1 x 1 through 8 x 8 inclusive. How many of these squares do not contain the "X" ?



C) Let R denote the radius of circle O which is circumscribed about square $ABCD$.
Let $\left(r_1, r_2, r_3\right)$ be the radii of the circles centered at P, Q and S respectively.
Circle Q and S are tangent at O.
Circle Q is also tangent to two sides of the square.
Circle P is externally tangent to square $ABCD$ and internally tangent to circle O.
If $r_1 + r_2 + r_3 = kR$, compute k as a simplified fraction.



D) Given: $f(x) = \begin{cases} 2 \cdot 4^x + A + 3 & \text{, if } x \geq 2 \\ A \log_4 x + B & \text{, if } 0 < x < 2 \end{cases}$ is a piecewise function and $A + B = 17$.
Compute the ordered pair of integers (A, B) for which this function is continuous at $x = 2$.

E) Given: $x = 2$, $y < 0$, $z > 0$ and $\frac{x+y}{z} = \frac{y+z}{x} = \frac{x+z}{y}$
Compute the largest possible value of y.

F) Compute the maximum number of sides in a regular polygon in which the number of diagonals is less than the degree measure of an interior angle.