MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2016 SOLUTION KEY

Round 6

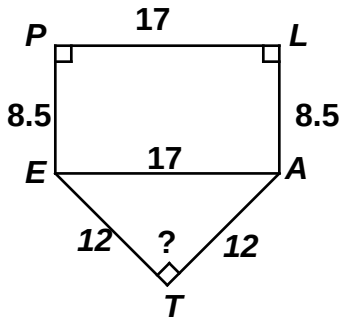
A) $\dfrac{360}{180(n-2)} = \dfrac{2}{n-2} = \dfrac{1}{8} \Rightarrow n-2 = 16 \Rightarrow n = 18$.

Diagonals in a polygon <u>from</u> any single vertex can be drawn <u>to</u> any <u>other</u> vertex, except the two adjacent vertices, eliminating 3 vertices. Thus, 18 - 3 = **<u>15</u>**.

B) Reflexive refers to an angle whose measure is greater than 180° and less than 360°.
The <u>exterior</u> angle at *T* is a reflexive angle.
The interior angle must be either acute or obtuse.

If $\angle ETA$ *were* a right, then angle $EA^2 = \left(12\sqrt{2}\right)^2 = 144 \cdot 2 = 288$.

However, $EA^2 = 17^2 = 289$. Since the square of the actual length is slightly longer, $\angle ETA$ must be **<u>obtuse</u>**.



C) Either by invoking the Pythagorean theorem,
$x^2 + 9(x+1)^2 = 625 \Rightarrow 10x^2 + 18x - 616 = 0 \Rightarrow 5x^2 + 9x - 308 = (5x+44)(x-7) = 0 \Rightarrow x = 7$,
or, recognizing special right triangles (7-24-25, 18-24-30), we have $CE = 18, AE = 24$
$\Rightarrow BD = 10 \Rightarrow DE = \sqrt{51} \Rightarrow DC^2 = 51 + 18^2 = 375 \Rightarrow DC = 5\sqrt{15}$
$d = 25 - \left(24 - \sqrt{51}\right) = 1 + \sqrt{51}$

Thus, the required ordered pair is $\underline{\left(\mathbf{5\sqrt{15},\ 1+\sqrt{51}}\right)}$.