Algorithm for Extracting Square Root sans Calculator

An example: *Determine the best two-decimal place approximation of* $\sqrt{8.15}$.

Group digits to the left and to the right of the decimal point into blocks of two.
Since we want accuracy to two decimal places, we write 8.15 as 08.15 00 00
The third decimal place will tell us if we need to round up.

The first digit is the largest N for which $N^2 \leq$ leftmost twosome. $N^2 \leq 08 \Rightarrow N = 2$
Square N, subtract, and bring down the next twosome. Call this value X. $X = 415$
Double the current approximation (2) and write this value (4) in the space at the left
Let d denote the next digit in the approximation.
We want $(4\,d)\cdot d$ to be less than or equal to X, i.e. forty-something times something $\leq$ 415
$(48)\cdot 8 = 384 < 415$, but $(49)9 = 441 > 415$, so the next digit is 8.

This is summarized in the following template:

$$\begin{array}{lr} & 2.d \\ & \sqrt{08.150000} \\ & \underline{4} \\ 4\underline{d} & 415 \\ & \underline{384} \\ & \boxed{d=8} \end{array}$$

Continue repeating these steps until the required number of decimal places have been determined

- Double the current approximation
- Determine the next digit [largest d for which $(\ldots\, d)d \leq X$]
- Multiply / Subtract / Bring down the next twosome

The devil is in the details which are shown in the diagrams below:

$$\begin{array}{ll} & 2.\ 8\ \ d \\ & \sqrt{08.150000} \\ & \underline{4} \\ 48 & 415 \\ & \underline{384} \\ 56\underline{d} & 3100 \\ \multicolumn{2}{l}{(565\cdot 5 = 2825 < 3100)} \\ \multicolumn{2}{l}{(566\cdot 6 = 4396 > 3100)} \\ & \boxed{d=5} \end{array}$$

$$\begin{array}{ll} & 2.\ 8\ \ 5\ \ d \\ & \sqrt{08.150000} \\ & \underline{4} \\ 48 & 415 \\ & \underline{384} \\ 565 & 3100 \\ & \underline{2825} \\ 570\underline{d} & 27500 \\ \multicolumn{2}{l}{(5704\cdot 4 = 22816)} \\ \multicolumn{2}{l}{(5705\cdot 5 > 27500)} \\ & \boxed{d=4} \end{array}$$

In practice, the calculations to determine d are not shown and all the computations are combined into a single template.
Thus, rounded to two decimal places, $\sqrt{8.15} = 2.85$.