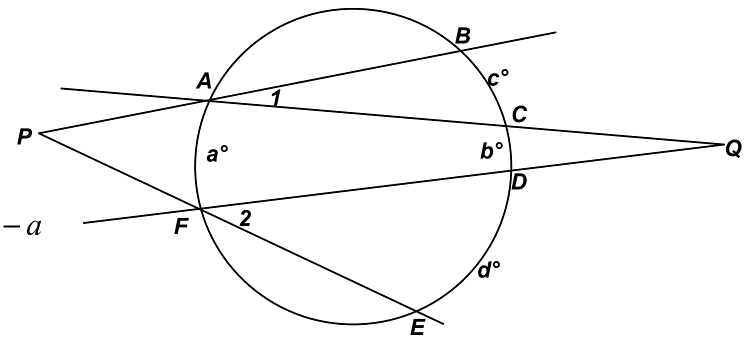
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 – MARCH 2011 SOLUTION KEY

Round 5 – continued



C) Let m$\angle P = 2x$, m$\angle Q = x$ and the degree measures of the minor arcs as <u>indicated in the diagram</u>

$$m\angle P = \frac{1}{2}(c+d+b-a) \rightarrow 4x = c+d+b-a$$

$$m\angle Q = x = \frac{1}{2}(a-b) \rightarrow 2x = a-b$$

Adding, $c+d=6x$.

$$m\angle 1 + m\angle 2 = \frac{1}{2}c + \frac{1}{2}d = 3x$$

Thus, without finding specific values for our variables and only using the first piece of given information, the required ratio $\dfrac{m\angle 1 + m\angle 2}{m\angle P}$ is $\dfrac{3x}{2x} = \dfrac{3}{2}$ ➔ <u>**3 : 2**</u>.

Round 6

A) MMMHH or any rearrangement of 3 Misses and 2 Hits

$$\binom{5}{3}(0.4)^2(0.6)^3 = \frac{10(4)^2(6)^3}{10^5} = \frac{2^4\cdot 2^3\cdot 3^3}{2^4\cdot 5^4} = \frac{2^3\cdot 3^3}{5^4} = \underline{\mathbf{\frac{216}{625}}}$$ (or <u>**0.3456**</u> or <u>**34.56%**</u>)

B) Since there are 12 terms in the expansion, the middle terms are the 6th and 7th terms.

The required ratio is $\dfrac{\binom{11}{5}(2x)^6(-3y)^5}{\binom{11}{6}(2x)^5(-3y)^6}$ or the reciprocal. Since the combinatorial terms are equal, we have $\dfrac{2x}{-3y} = \dfrac{2}{-3}\cdot\dfrac{3}{7} = \dfrac{2}{-7}$ and the required product is <u>**–14**</u>.

C) Given: P(T) = $\dfrac{1}{2}$, P(D) = $\dfrac{2}{3}$ and P(H) = k. Let ~ denote <u>not</u>.

P(at least 2) = P(exactly 2) + P(exactly 3)

= P(T)·P(D)·P(~H) + P(T)·P(~D)·P(H) + P(~T)·P(D)·P(H) + P(T)·P(D) ·P(H)

$$= \tfrac{1}{2}(\tfrac{2}{3})(1-k) + \tfrac{1}{2}(\tfrac{1}{3})(k) + \tfrac{1}{2}(\tfrac{2}{3})(k) + \tfrac{1}{2}(\tfrac{2}{3})(k) = \frac{3}{4}$$

$$= \frac{1-k}{3} + \frac{k}{6} + 2\left(\frac{k}{3}\right) = \frac{3}{4}$$ ➔ $4(1-k) + 2k + 8k = 9$ ➔ $6k = 5$ ➔ $k = \underline{\mathbf{\frac{5}{6}}}$

Created with nitroPDF professional
download the free trial online at nitropdf.com/professional