MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2011 SOLUTION KEY

Round 2

A) 13 x 43 = 559 is a multiple of both 13 and 43.
Add 559 to 312 to produce another multiple of 13, namely 871.
Subtract 559 from 688 to produce another multiple of 43, namely 129.
The sum is unaffected by this transformation, so (a, b) = **(129, 871)**.

Algebraic solution (much more of a pain! – but it does limit the number of possible answers)

$13x + 43y = 43 \rightarrow 13x = 1000 - 43y \rightarrow x = \frac{1000-43y}{13} = 76 - 3y + \boxed{\frac{4(3-y)}{13}}$

The boxed fractional expression must evaluate to an integer. Clearly, this is true for $y = 3$.
Multiples of 13 are 13 apart, so $y = 3, 16, 29, \ldots$
$y = 3 \rightarrow x = 67 \rightarrow 13x = 871,\ 43y = 129$
$y = 16 \rightarrow x = 24 \rightarrow 13x = 312,\ 43y = 688$
$y = 29 \rightarrow x = -19$ (rejected, since both x and y must be positive)
Thus, the solution found by the arithmetic approach is unique.

B) $N = 33650ab97$ must be divisible by 9 and 11.
$\div 9 \rightarrow (3 + 3 + 6 + 5 + 0 + a + b + 9 + 7) = 33 + a + b$ is a multiple of 9
$\rightarrow a + b = 3$ or 12 (since $0 \leq a + b \leq 18$)
$\div 11 \rightarrow (b + 7 + 0 + 6 + 3) - (a + 9 + 5 + 3) = b - a - 1$ is a multiple of 11
Case 1: $a + b = 3 \rightarrow b = 3 - a$
Substituting, $b - a - 1 = 2 - 2a = 2(1 - a)$
$a = 1 \rightarrow b = 2$ OK, $a = 2 \rightarrow -2$ fails, $a = 3 \rightarrow -4$ fails, … $a = 9 \rightarrow -16$ fails (all values are even)

Case 2: $a + b = 12 \rightarrow b - a - 1 = 11 - 2a$
Only $a = 0$ produces a multiple of 11, but $a = 0 \rightarrow b = 12$ which is not an allowable digit

Therefore, the solution **(1, 2)** is unique.

C) Factoring 129600 as a product of primes, we get $2^6 \cdot 3^4 \cdot 5^2$.
In base 10, an integer ending in zero is divisible by 10 ($= 2\cdot5$).
Looking at the factorization ($2^6 \cdot 3^4 \cdot 5^2$), we see by inspection that the product contains 10^2, i.e., it must end in exactly two zeros.
In this case, we are limited by the number of factors of 5.
Ignore the 3s. Using all the factors of 5 and two of the factors of 2, we get two factors of 10.

In base 12, an integer ending in zero is divisible by 12 ($= 2^2\cdot3$).
We do not need to convert 129600 to base 12. Simply examine the factorization above and determine the maximum number of 12s that can be produced.
We are limited by the number of factors of 2. Ignore the 5s and consider only the 2s and 3s.
Since $2^6\cdot3^3 = (2^2\cdot3)^3 = 12^3$, 129600 will end in **3** zeros.

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