

**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2010 SOLUTION KEY**

Round 3

A) $(0.72)(8) + 0.57x = 0.65(x + 8) \rightarrow 72(8) + 57x = 65x + 65(8) \rightarrow 8x = (72 - 65)8 \rightarrow x = \underline{7}$

B) Solving the system of equations, $(A, B) = (67, 60)$.

L is a linear function (a line) given in intercept-intercept form.

The x -intercept is $(67, 0)$ and the y -intercept is $(0, 60)$.

Thus, the area of the triangular region is $\frac{1}{2} \cdot 67 \cdot 60 = 67(30) = \underline{2010}$.

C) $30\left(\frac{1}{2}y = \frac{2}{3}x + \frac{3}{5}\right) \rightarrow 15y = 20x + 18 \rightarrow 20x = 15y - 18 \rightarrow x = \frac{15y - 18}{20} \rightarrow (A, B, C) = (15, -18, 20)$

However, since $A < 0$, we must multiply through by -1 . $\rightarrow (A, B, C) = (-15, +18, -20)$

$$\frac{\frac{ABC}{AB+AC}}{\frac{ABC}{A(B+C)}} = \frac{\frac{ABC}{A(B+C)}}{\frac{BC}{B+C}} \text{ (since } A \neq 0) \rightarrow \frac{18(-20)}{18+(-20)} = 18(10) = \underline{180}$$

Round 4

A) The average is $\frac{\frac{1}{4} + \frac{6}{5} + \frac{3}{2}}{3} \cdot \frac{20}{20} = \frac{5 + 24 + 30}{60} = \underline{\frac{59}{60}}$.

B) $a \oplus b = \frac{a+2b}{2a-b} = 2 \rightarrow 4a - 2b = a + 2b \rightarrow 3a = 4b$

Substituting $a = b + 2$, we have $3(b + 2) = 4b \rightarrow b = 6 \rightarrow (a, b) = \underline{(8, 6)}$.

C) Let d denote the distance between home and office and r the average return rate in mph.

The overall average is the total distance traveled divided by the total time required.

The total distance traveled is $2d$ and the time required is the sum of the time going and the time

returning, i.e. $\frac{d}{40} + \frac{d}{r}$. Therefore, $\frac{2d}{\frac{d}{40} + \frac{d}{r}} = 55 \rightarrow \frac{2}{\frac{1}{40} + \frac{1}{r}} = 55 \rightarrow \frac{80r}{40+r} = \frac{55}{1} \rightarrow \frac{16r}{40+r} = \frac{11}{1}$.

Cross multiplying, $440 + 11r = 16r \rightarrow 5r = 440 \rightarrow r = \underline{88}$

You might wonder why a simple average $\frac{40+r}{2} = 55 \rightarrow r = 70$ is incorrect.

A simple average assumes that you have traveled at each of these speeds for the same time.

Clearly, this is not the case, since returning home over the same route at a faster speed will take less time. The overall average of 40mph and 70 mph would be closer to 40 than 70, i.e. less than 55, since you traveled at 40 mph for a longer time. The required average is a weighted

average and is given by the formula $\frac{2r_1r_2}{r_1+r_2}$. It's called a harmonic average. Check it out.

$$\frac{2 \cdot 40 \cdot 88}{40 + 88} = \frac{80 \cdot 88}{128} = \frac{\cancel{10}^5 \cdot \cancel{88}^{11}}{\cancel{16}} = 55$$

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