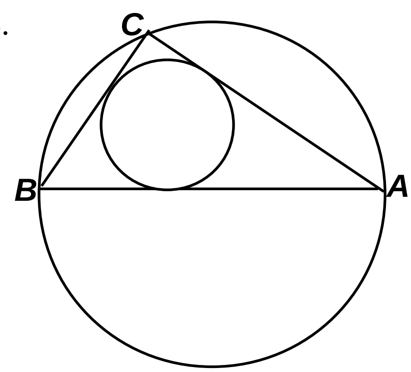
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2015 SOLUTION KEY

Team Round - continued



Here's the scoop on the challenge in the Team Round Question B.

We know that in any triangle $r_{ic} = \frac{A}{s}$. For right triangle *ABC*,

$$r_{ic} = \frac{\frac{1}{2}BC \cdot AC}{\frac{BC + AC + AB}{2}} \Rightarrow d_{ic} = \frac{2BC \cdot AC}{BC + AC + AB}$$

We wish to show that this is equivalent to $AC + BC - AB$

$\frac{2BC \cdot AC}{BC + AC + AB} = BC + AC - AB$ if and only if

$2BC \cdot AC = (BC + AC - AB)(BC + AC + AB)$ (since the denominator cannot be zero)

Multiplying out the right side, $(BC + AC)^2 - AB^2 = BC^2 + 2BC \cdot AC + AC^2 - AB^2$

Rearranging terms, we have

$(BC^2 + AC^2 - AB^2) + 2BC \cdot AC$

But, since *ABC* is a right triangle. $BC^2 + AC^2 = AB^2$ and the parenthesized expression is 0.
Therefore,

The diameter of a circle inscribed in any right triangle equals the sum of the lengths of the legs minus the length of the hypotenuse.