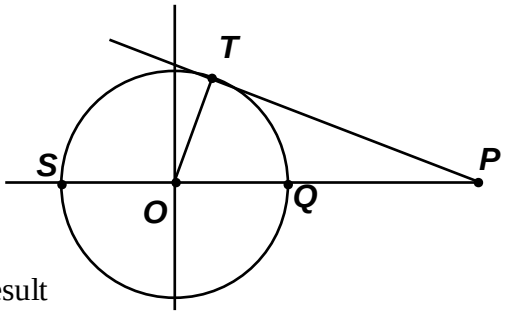
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2016 SOLUTION KEY

Round 5

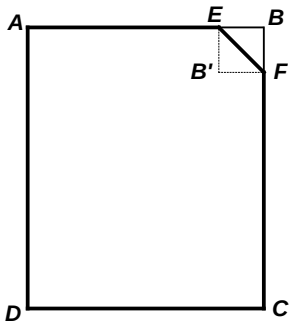


A) Since a radius is perpendicular to any tangent line at the point of tangency, ΔTOP is a right triangle.

Let $OT = OQ = r$. Then: $r^2 + 35^2 = (r+25)^2 \Rightarrow 35^2 = 50r + 25^2$

Taking advantage of the difference of perfect squares,
$50r = 35^2 - 25^2 = (35+25)(35-25) = 60 \cdot 10 = 600 \Rightarrow r = \underline{\mathbf{12}}$.

Alternately, $PQ \cdot PS = PT^2 \Rightarrow 25(25+2r) = 35^2$ and the same result follows.



B) Let $BE = x$. $\mathcal{A}(AEFCD) = \left(5+\frac{1}{4}\right)\left(6+\frac{1}{2}\right) - \frac{1}{2}x^2 = 33$

$\Rightarrow \left(5+\frac{1}{4}\right)\left(6+\frac{1}{2}\right) = 30 + 2.5 + 1.5 + \frac{1}{8} = 34\frac{1}{8} \Rightarrow$

$\frac{1}{2}x^2 = \frac{9}{8} \Rightarrow x^2 = \frac{9}{4} \Rightarrow x = \underline{\mathbf{\frac{3}{2}}}$ or $\underline{\mathbf{1.5}}$.

C) Let $AP = x$. Then:

$DR = 8x$ and $MR = AP \Rightarrow 8x - 14 = x \Rightarrow x = 2$.

$\begin{cases} AB = 4 \\ CD = 32 \end{cases} \Leftrightarrow QS = 18$

Clearly, *PQRS* is a rhombus and

$\mathcal{A}(\Delta SAP) = 24 \Rightarrow h = 24 \Rightarrow PR = 48$.

Since the area of a rhombus is half the product of its diagonals, we have the area of *PQRS* is

$\frac{1}{2} \cdot 18 \cdot 48 = \underline{\mathbf{432}}$.