MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2009 SOLUTION KEY

Round 6

A) Each of the expressions 8^2, 4^6, 16^3 and 64^2 is equivalent to 2^{12}. Thus, we have:
$\sqrt{4\cdot 2^{12}}\cdot\sqrt{x^2}=\sqrt{2^{14}}\cdot\sqrt{x^2}=2^7\cdot|x|=128|x|=8^3 \rightarrow 2^7|x|=2^9 \rightarrow |x|=4 \rightarrow x=$ **± 4**

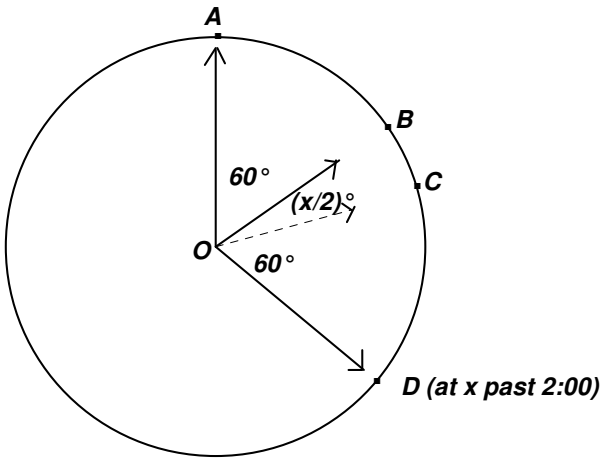
B) In 1 minute the minute hand travels through 6°.
In x minutes the minute hand travels through $(6x)$ °.
The hour hand travels at 1/12 the rate of the minute hand and, therefore travels through $(x/2)$°
.

From the diagram at the right, we see that:

$60+\frac{x}{2}+60=6x$

$\rightarrow 240=11x \rightarrow x=$ **$\frac{240}{11}$**

Note: A 60° is not formed again before 3:00. During the remainder of the hour, the angle between the hour and minute hand increases to 180° (when they point in diametrically opposite directions) and then decreases to 90° at 3:00.



C) $\overline{(x\circ y)}=\overline{2x-y}=(2x-y)^2$
$\bar{x}\circ\bar{y}=x^2\circ y^2=2x^2-y^2$
Expanding and equating we have: $4x^2-4xy+y^2=2x^2-y^2$
$\rightarrow 2x^2+2y^2-4xy=0 \rightarrow 2\left(x^2-2xy+y^2\right)=0$ or $2(x-y)^2=0$
$\rightarrow y=$ **x**