

The original Team D) question:

For exactly two irrational values of the constant B , the equation $(2x-3)(Bx-1)=5$ has exactly one real root. **Approximate the larger of these two values to the nearest hundredth.**

A solution starts out the same:

$$(2x-3)(Bx-1)=5 \Leftrightarrow 2Bx^2-(2+3B)x-2=0$$

To insure exactly one root, we set the discriminant equal to zero.

$$b^2-4ac = (-(2+3B))^2 - 4(2B)(-2) = 0 \Rightarrow 9B^2 + 28B + 4 = 0$$

$$\Rightarrow B = \frac{-28 \pm \sqrt{28^2 - 4(36)}}{18} = \frac{-28 \pm \sqrt{4^2(7^2 - 9)}}{18} = \frac{-28 \pm 8\sqrt{10}}{18} = \frac{-14 \pm 4\sqrt{10}}{9}$$

The larger of the two values is $\frac{-14 + 4\sqrt{10}}{9}$.

We need to approximate $\sqrt{10}$.

Since $3.2^2 = 10.24$ (an overestimate by 0.24) and $3.1^2 = 9.61$ (an underestimate by 0.39), we know $\sqrt{10}$ lies between 3.1 and 3.2, closer to 3.2 than 3.1, i.e. $\boxed{3.15 < \sqrt{10} < 3.20}$.

Since $3.16^2 = 9.9856$ (slightly under our target value of 10), we have an outstanding approximation of $\sqrt{10}$ to two decimal places, an error of only 144 (actually 0.0144). For the cautious, since $3.17^2 = 10.0489$ (an error of 520), 3.16 is definitely the best two decimal place approximation.

Substituting, $\frac{-14 + 4(3.16)}{9} = \frac{-1.36}{9} = -0.15\bar{1} \approx \underline{\underline{-0.15}}$.

For comparison, the actual value is approximately -0.150099 to 6 decimal places.

For those who would like to know how to compute square roots directly, READ ON!

Be patient!

Study the two examples worked out in detail and the accompanying dialogue. Then:

Try the four suggested problems.

ENJOY!