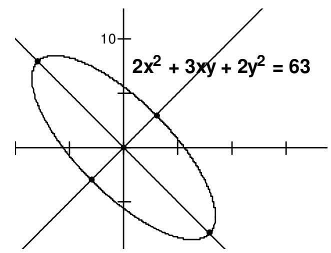
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2009 SOLUTION KEY

Team Round

A) The center of the ellipse is at (0, 0).
The graph is symmetric with respect to:
$y = x$, since interchanging x and y does not affect the equation.
$y = -x$, since replacing x and $-y$ and y by $-x$ does not affect the equation.
So the minor axis lies along one of these lines.
Replacing y by x, we have: $2x^2 + 3x^2 + 2x^2 = 63 \rightarrow x^2 = 9 \rightarrow x = \pm 3$
Replacing y by $-x$, we have: $2x^2 - 3x^2 + 2x^2 = 63$
$\rightarrow x^2 = 63 \rightarrow x = \pm 3\sqrt{7}$



Since $3\sqrt{7} > 3$, the endpoints of the minor axis are (–3, 3) and (3, 3) and its length is $\underline{\mathbf{6\sqrt{2}}}$.

<u>Notes for future contests:</u>

The general 2nd degree equation is $Ax^2 + Bxy + Cx^2 + Dx + Ey + F = 0$.
This equation normally graphs as a circle, a parabola, an ellipse or a hyperbola.
Bxy is a rotation term: If $B = 0$, all axes of symmetry are parallel to either the x – or y – axis.
If $B \neq 0$, then $B^2 - 4AC$ is called a discriminant.

$$B^2 - 4AC = \begin{cases} < 0 & \textit{ellipse} \\ > 0 & \textit{hyperbola} \\ = 0 & \textit{parabola} \end{cases} \quad (\underline{\text{graph}})$$

Possible *degenerate* cases

		$B^2 - 4AC$
no graph:	$(x-3)^2 + (y+2)^2 = -1 \Leftrightarrow x^2 + y^2 - 6x + 4y + 14 = 0$	-4
a single point:	$(x-3)^2 + (y+2)^2 = 0 \Leftrightarrow x^2 + y^2 - 6x + 4y + 13 = 0$	-4
a single line:	$(x-y+1)^2 = 0 \Leftrightarrow x^2 - 2xy + y^2 + 2x - 2y + 1 = 0$	0
a pair of parallel lines:	$(x-y+1)(x-y-1) = 0 \Leftrightarrow x^2 - 2xy + y^2 - 1 = 0$	0
a pair of intersecting lines:	$(x-y+1)(x+y+1) = 0 \Leftrightarrow x^2 - y^2 + 2x + 1 = 0$	$+4$