MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 – NOVEMBER 2014 SOLUTION KEY

Round 5

A) The value of the fraction is undefined only when the denominator is zero, or when one of the trig functions is undefined. We do not consider values of *x* for which the numerator is zero, namely 45°, since for this value the denominator is not 0.

$2\sin x - 1 = 0 \Rightarrow \sin x = \frac{1}{2} \Rightarrow x = \underline{\mathbf{30°, 150°}}$

$\cos x = 0 \Rightarrow x = \underline{\mathbf{90°}}$ (Also, $\tan x$ is undefined for $x = 90°$.)

$\tan^2 x - 3 = 0 \Rightarrow \tan x = \pm\sqrt{3} \Rightarrow x = \underline{\mathbf{60°, 120°}}$

B) Since sin(–*x*) = –sin(*x*) and sin(–2*x*) = –sin(2*x*), it follows that
$f(-x) = \sin(-x)\sin(-2x) = \sin(x)\sin(2x) = f(x)$. Thus, since
$\frac{602\pi}{3} = 200\frac{2}{3}\pi$, with a period of 2π, we can disregard 200π.

$$f\left(\frac{-602\pi}{3}\right) = f\left(\frac{602\pi}{3}\right) = f\left(\frac{2\pi}{3}\right) = \sin\left(\frac{2\pi}{3}\right)\sin\left(\frac{4\pi}{3}\right) = \frac{\sqrt{3}}{2}\cdot\frac{-\sqrt{3}}{2} = -\frac{3}{4}$$

$$f\left(\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right)\cdot\sin\left(\frac{\pi}{2}\right) = \frac{\sqrt{2}}{2}\cdot 1 = \frac{\sqrt{2}}{2}$$

With these observations, the given expression evaluates to $2\left(-\frac{3}{4}\right) - \frac{1}{2} = \underline{\mathbf{-2}}$.

(Ask your coach/teammates about even and odd functions.)

C) $BF + FG + GC = BC \Rightarrow 2x\sqrt{3} + x = 11\sqrt{3} \Rightarrow x = \frac{11\sqrt{3}}{2\sqrt{3}+1}\cdot\frac{2\sqrt{3}-1}{2\sqrt{3}-1} = \frac{66 - 11\sqrt{3}}{11} = 6 - \sqrt{3}$

Therefore, the area of *EFGH* is

$\left(6 - \sqrt{3}\right)^2 = 36 - 12\sqrt{3} + 3 = 39 - 12\sqrt{3}$

$\Rightarrow (M, N) = \underline{(\mathbf{39, 12})}$.