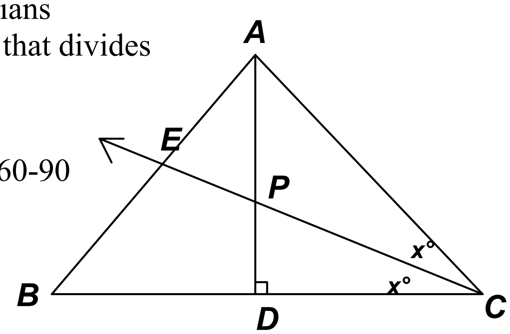
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2007 SOLUTION KEY

Round 6

A) Draw a line through C parallel to $\overrightarrow{BE}$. Since interior angles on the same side of a transversal crossing parallel lines are supplementary, $m\angle BCD = 24 + 88 =$ **<u>112°</u>**.

B) In an equilateral triangle, altitudes, angle bisectors and medians are one and the same! Medians always intersect at a point that divides the median into segments in a 2 : 1 ratio.
$AP = 12 \rightarrow PD = 6$. Thus, the altitude $\overline{AD}$ has length 18.
Since $\overline{AD}$ is also the side opposite the 60° angle in the 30-60-90 ΔBAD, $AB = \dfrac{18}{\sqrt{3}} \cdot 2 = \underline{\mathbf{12\sqrt{3}}}$



C) The measure of the interior angle of a 15-gon is $\dfrac{180(15-2)}{15} = 156°$.

The measure of the interior angle of a (15 + k)-gon is $\dfrac{180(15+k-2)}{15+k} = 180\left(1-\dfrac{2}{15+k}\right)$

Thus, $180\left(1-\dfrac{2}{15+k}\right)-156 = k+1 \rightarrow 24-\dfrac{360}{15+k} = k+1$

$360 + 24k - 360 = k^2 + 16k + 15 \rightarrow k^2 - 8k + 15 = (k-3)(k-5) = 0 \rightarrow k = \underline{\mathbf{3, 5}}$

Check:
$k = 3 \rightarrow$ 15-gon and 18-gon $\rightarrow$ interior angles: 156° and 160°, a 4° difference
$k = 5 \rightarrow$ 15-gon and 20-gon $\rightarrow$ interior angles: 156° and 162°, a 6° difference