

**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2012 SOLUTION KEY**

Round 1

A) Let x denote the length of an edge. Then: $6x^2 = \frac{1}{3}x^3 \Rightarrow x = \underline{\mathbf{18}}$ (since $x > 0$).

B) Let the dimensions be l , w and h . Then:

$$\begin{cases} lw = 54 \\ lh = 36 \\ wh = 24 \end{cases} \Rightarrow \frac{l}{w} = \frac{36}{24} = \frac{3}{2} \Rightarrow \frac{3}{2}w^2 = 54$$

$$\Rightarrow w^2 = 36 \Rightarrow (l, w, h) = (9, 6, 4)$$

$$\Rightarrow V = 9 \cdot 6 \cdot 4 = \underline{\mathbf{216}}$$

C) Let s denote the length of the side of the cube. Then the radius of the inscribed sphere is $\frac{s}{2}$.

$$\text{The required ratio is } \frac{\frac{4}{3}\pi\left(\frac{s}{2}\right)^3}{s^3 - \frac{4}{3}\pi\left(\frac{s}{2}\right)^3} = \frac{\frac{\pi s^3}{6}}{s^3 - \frac{\pi s^3}{6}} = \frac{\pi}{\underline{\mathbf{6-\pi}}}$$

$$\text{Since } \pi > 3, \frac{\pi}{6-\pi} \approx \frac{3^+}{3^-} > 1 \Rightarrow P > Q \Rightarrow \left(\frac{\pi}{\underline{\mathbf{6-\pi}}}, P \right).$$