Round 1

A) The volume of the cube is $8^3 = 512$.
A sphere of radius 4 could be inscribed in the cube of side 8 and, therefore, would have a smaller volume. Trying a radius of 5,
$$V_{sphere} = \frac{4}{3}\pi \cdot 5^3 = \frac{500\pi}{3} = \frac{1000\pi}{6} \approx \frac{1000(3.1416)}{6} = \frac{3141.6}{6} = 523^+ > 512. \text{ Thus, } r = \underline{\mathbf{5}}.$$
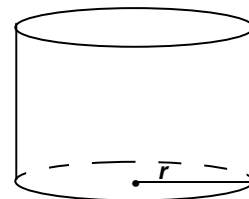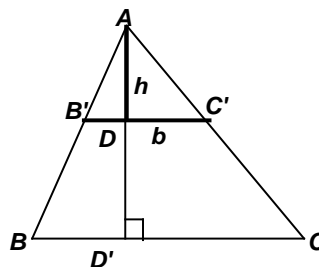
B) $\Delta AB'C' \sim \Delta ABC$ with a ratio of similitude of $\frac{1}{3}$.

Let $h = AD$ and $b = B'C'$.
$$r = \frac{7}{8}h \Rightarrow AD' = 3h = 3\left(\frac{8}{7}r\right) = \frac{24r}{7}$$
$$b = \frac{5}{4}r \Rightarrow BC = 3b = \frac{15r}{4} \quad \text{Thus,}$$

the area(circle) $= \pi r^2$ and area(ΔABC) $= \frac{1}{2}\left(\frac{24}{7}r\right)\left(\frac{15}{4}r\right) = \frac{45}{7}r^2 \Rightarrow \underline{\mathbf{\frac{7\pi}{45}}}$.

C) The volume of the tank is the volume of a cylinder plus the volume of a sphere.
$$V = \pi \cdot 18^2 \cdot 72 + \frac{4}{3}\cdot\pi\cdot 18^3 = 18^3 \cdot \pi \cdot \left(4 + \frac{4}{3}\right) \text{ inches}^3.$$

Converting to cubic feet, we have $\dfrac{18^3 \cdot \pi \cdot \left(4 + \dfrac{4}{3}\right)}{12^3} = \left(\dfrac{3}{2}\right)^3 \cdot \pi \cdot \dfrac{16}{3} = 18\pi$.

Converting to gallons, we have $18\pi \cdot \dfrac{15}{2} = \underline{\mathbf{135\pi}}$ gallons.