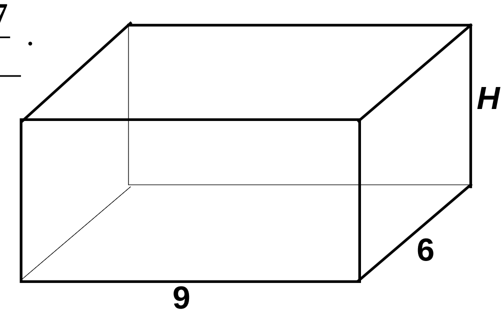
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2013 SOLUTION KEY

Round 1

A) $2\left(6H+9H\right)=1.5(2\cdot 9\cdot 6)\Rightarrow 30H=3\cdot 54\Rightarrow H=\frac{54}{10}=\underline{\mathbf{5.4}}$ or $\underline{\mathbf{\frac{27}{5}}}$.



B) Since $\text{SA}=2\pi rh+2\pi r^2$ and we are given $r = h$, we have
$484\pi=2\left(2\pi r^2\right)\Rightarrow r^2=121\Rightarrow r=11$.
Thus, $V=Bh=\left(\pi r^2\right)r=\pi r^3=\pi\left(11\right)^3=\underline{\mathbf{1331\pi}}$.

C) $\text{SA}_{\text{sphere}}=4\pi r^2=96\pi \Rightarrow r^2=24\Rightarrow r=2\sqrt{6}$
The pyramid with maximum volume will have its vertex P directly above the center of the base. Since the diagonals of the hexagon divide the hexagon into 6 equilateral triangles, the long diagonal of the hexagon is a diameter of the great circle (and of the sphere). The altitude from P to the base will also have length $2\sqrt{6}$. Recall that the area of an equilateral triangle is given by $\frac{s^2\sqrt{3}}{4}$. Thus, the volume of the pyramid is given by

$$\frac{1}{3}Bh=\frac{1}{3}\cdot 6\left(\frac{\left(2\sqrt{6}\right)^2\sqrt{3}}{4}\right)\cdot 2\sqrt{6}=24\sqrt{18}=\underline{\mathbf{72\sqrt{2}}}.$$