MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 – MARCH 2014 SOLUTION KEY

Round 2

A) –1 cannot be an exponent, since this would result in $N = A^B + C^D$ being a non-integer.
So we consider $(-1)^3 + 2^4$, $(-1)^3 + 4^2$, $(-1)^4 + 2^3$, and $(-1)^4 + 3^2$
The winner (i.e. the minimum) is $(-1)^4 + 2^3 = \underline{\mathbf{9}}$.

B) Given $\dfrac{1+x}{x-\dfrac{5}{2x-3}}$, clearly $x=\dfrac{3}{2}$ causes division by 0. Considering the denominator of the complex fraction, $x-\dfrac{5}{2x-3}=\dfrac{2x^2-3x-5}{2x-3}=\dfrac{(2x-5)(x+1)}{2x-3}$ and this expression is zero for $x=-1, \dfrac{5}{2}$. Even though $(x+1)$ is a common factor and the expression simplifies to $\dfrac{2x-3}{2x-5}$, for $x=-1$, the original fraction becomes $\dfrac{0}{0}$ which is still undefined.

Thus the fraction is undefined for three values, namely, $\underline{\mathbf{-1, \frac{3}{2}, \frac{5}{2}}}$.

C) Given $\begin{cases} \dfrac{2^{x^2+Bx}}{4^{Ax}} = \dfrac{1}{32} \\ 3^{2A+B+4} = 5^{2(A+B)+3} = 7^{\frac{2A+5}{B}} \end{cases}$ The only way powers of 3, 5 and 7 can be equal for rational exponents would be if each exponent is zero. Therefore, $A=-\dfrac{5}{2}$ and substituting for the powers of 3 and 5, we have $B-1=0$ and $-5+2B-3=0 \Rightarrow B=1$

$$\frac{2^{x^2+Bx}}{4^{Ax}}=\frac{1}{32} \Leftrightarrow \frac{2^{x^2+Bx}}{2^{2Ax}}=2^{-5} \Leftrightarrow 2^{x^2+(B-2A)x}=2^{-5} \Rightarrow$$

$$x^2+(B-2A)x+5=0 \Leftrightarrow x^2+6x+5=(x+1)(x+5)=0 \Rightarrow x=\underline{\mathbf{-1,-5}}$$