Team Round - continued

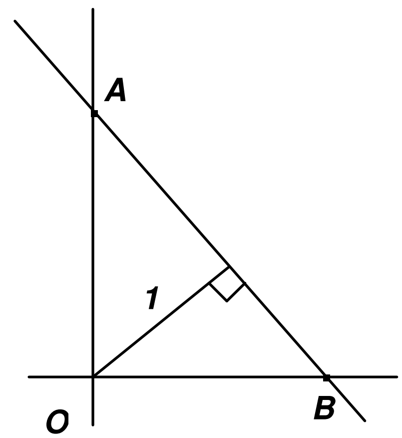
C) The distance form the origin to this line is measured along the perpendicular and this distance is given by:

$$\frac{|0(a+2)+0(a)+b|}{\sqrt{(a+2)^2+a^2}} = 1 \rightarrow b^2 = 2a^2 + 4a + 4 \rightarrow 2a^2 + 4a + (4 - b^2) = 0$$

Solving for a in terms of b using the quadratic formula, the discriminant must be nonnegative to guarantee a real solution. Thus, $16 - 8(4 - b^2) \geq 0 \rightarrow 8b^2 \geq 16 \rightarrow b^2 \geq 2$

Since $b > 0$, $b_{\min} = \sqrt{2}$. Substituting, $2a^2 + 4a + 2 = 2(a^2 + 2a + 1) = 2(a + 1)^2 = 0 \rightarrow a = -1$

$\rightarrow (a, b) = \underline{\left(\mathbf{-1, \sqrt{2}}\right)}$



Alternate solution: Note that $A\left(0, -\frac{b}{a}\right)$ and $B\left(\frac{-b}{a+2}, 0\right)$

The area of ΔAOB is $\frac{1}{2} \cdot \left|\frac{b}{a}\right| \cdot \left|\frac{b}{a+2}\right|$ using $\overline{OB}$ as the base or

$\frac{1}{2} \cdot 1 \cdot \sqrt{\left(\frac{b}{a}\right)^2 + \left(\frac{b}{a+2}\right)^2}$ using $\overline{AB}$ as the base. Equating, we have

$$\frac{b^2}{|a(a+2)|} = |b| \cdot \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{a+2}\right)^2} = |b| \cdot \sqrt{\frac{(a+2)^2 + a^2}{a^2(a+2)^2}} = \frac{|b|}{|a(a+2)|} \cdot \sqrt{2a^2 + 4a + 4} =$$

Cancelling, $b = \sqrt{2(a+1)^2 + 2} = \sqrt{2(a+1)^2 + 2}$

To minimize b is to minimize the radical expression, which is equivalent to minimizing the quadratic expression in the radicand. Clearly, $a =$ **<u>–1</u>** does this and the minimum value of b is $\underline{\mathbf{\sqrt{2}}}$.

D) $89.5 = 65 + (90 - 65)e^{-0.5k} \rightarrow 24.5 = 25e^{-0.5k} \rightarrow k = -2\ln\left(\frac{24.5}{25}\right) \approx 0.040405$

$98.6 = 65 + 25e^{-0.040405t} \rightarrow t = \frac{\ln\left(\frac{33.6}{25}\right)}{-0.040405} \approx -7.317 \rightarrow$ 7 hours 19^{+} minutes $\rightarrow$ **<u>11:30 pm</u>**