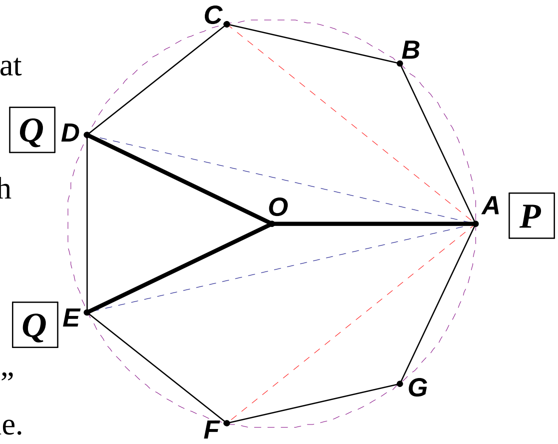
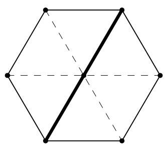
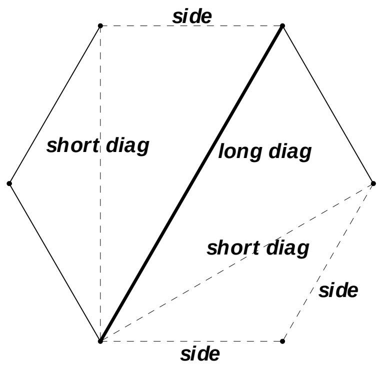
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2014 SOLUTION KEY

Round 6



A) A quick sketch of heptagon *ABCDEFG* will confirm that diagonals $\overline{AC}$ and $\overline{AF}$ have the same length, as do the longer diagonals $\overline{AD}$ and $\overline{AE}$.
There are 2 pairs of congruent diagonals starting at each vertex and ending at some other vertex.
That's total of $\frac{4\cdot 7}{2}=14$ diagonals, 7 short and 7 long.
A short diagonal "skips" one vertex;
a long one "skips" two. As a long diagonal $\overline{PQ}$ "skips" two vertices. Think of the heptagon inscribed in a circle.
Each side determines a central angle at O of $\frac{360}{7}^{\circ}$.
$\angle POQ$ consists of 3 of these central angles or $\frac{1080}{7}^{\circ}=154\frac{2}{7}^{\circ}$.
Thus, $a=\underline{\mathbf{154}}$.





B) From the diagram at the right, a triangle is made from either two sides of the hexagon and a short diagonal, or a side of the hexagon, a short diagonal and a long diagonal.
The difference is $\left|(s+d_s+d_l)-(2s+d_s)\right|=\left|d_l-s\right|$, but $\underline{d_l=2s}$!!
The difference is simply $s=\underline{\mathbf{\frac{4}{5}}}$, so we have nothing to do. Amen.

C) For all the pieces, the measures of the interior angles are either 45°, 90° or 135°.
Since the sides of the isosceles right triangle are always in a $1:1:\sqrt{2}$ ratio, we have

$$AC=4\sqrt{2}\ ,CD=1\Rightarrow CE=\frac{\sqrt{2}}{2}\quad EG=DR=8\sqrt{2}$$

$$PR=8,\ PQ=4\sqrt{2}\Rightarrow QR=8-4\sqrt{2}\Rightarrow FG=TR=\frac{QR}{\sqrt{2}}=4\sqrt{2}-4\quad GB=QS-FG=4\sqrt{2}+4$$

$$AB=AC+CE+EG+GB=$$

$$4\sqrt{2}+\frac{\sqrt{2}}{2}+8\sqrt{2}+\left(4\sqrt{2}+4\right)=16.5\sqrt{2}+4=\frac{33\sqrt{2}+8}{2}$$

$\Rightarrow (a,b,c)=\underline{(\mathbf{33,8,2})}$.