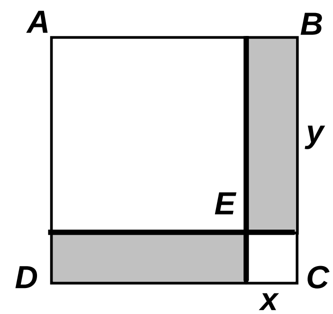
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2014 SOLUTION KEY

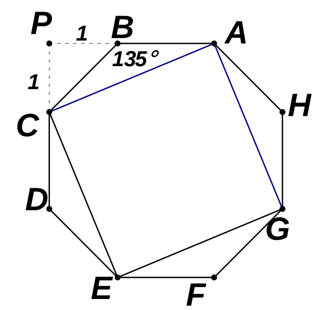
Round 3



A) $(x+y)^2 = 225 \Rightarrow AB = 15$

$CE = \sqrt{32} \Rightarrow x = 4, y = 11$

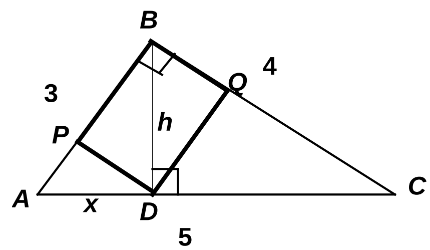
Therefore, the area of the shaded region is $2(4 \cdot 11) = \mathbf{\underline{88}}$.



B) Using Pythagorean Theorem on right ΔAPC,

$AC^2 = \left(1+\sqrt{2}\right)^2 + 1^2$

$\Rightarrow AC^2 = 1 + 2\sqrt{2} + 2 + 1 = \underline{\mathbf{4+2\sqrt{2}}}$



C) The area of ΔABC is $\frac{1}{2}\cdot 3\cdot 4 = \frac{1}{2}\cdot 5\cdot h \Rightarrow h = \frac{12}{5}$.

Proceed by the Pythagorean Theorem

$9 = x^2 + \left(\frac{12}{5}\right)^2 \Rightarrow x^2 = \frac{9\cdot 5^2 - 12^2}{5^2} = \frac{9(25-16)}{5^2} = \frac{9^2}{5^2}$

Thus, $AD = \frac{9}{5}$ and $CD = \frac{16}{5}$. Alternately,

$\Delta BAD \sim \Delta CAB \sim \Delta CBD \Rightarrow \frac{BA}{CB} = \frac{AD}{BD} \Rightarrow \frac{3}{4} = \frac{x}{h} \Rightarrow x = \left(\frac{3}{4}\right)\left(\frac{12}{5}\right) = \frac{9}{5}$ or, invoking the fact that the altitude to the hypotenuse is the mean proportional between the segments on the hypotenuse, $h^2 = x(5-x)$.

By similar arguments for triangles BAD and BCD, $DP = \frac{36}{25}$ and $DQ = \frac{48}{25} \Rightarrow$

$\frac{36}{25}\cdot\frac{48}{25} = \underline{\mathbf{\frac{1728}{625}}}$.