MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2008 SOLUTION KEY

Round 3

A) The required line is a line halfway between the given two parallel lines.

The given equations are equivalent to $\begin{cases} 4x-6y-10=0 \\ 4x-6y+1=0 \end{cases}$

Clearly, $4x-6y+\frac{-10+1}{2}=0$ is the required equation.

Multiplying by 2 to clear fractions: $8x - 12y - 9 = 0$ → **(8, –12, –9)**

B) Completing the square we have: $\begin{cases} (x-2)^2+(y+5)^2=25 \\ (x+6)^2+(y-10)^2=81 \end{cases}$

The shortest distance between the circles lies on the segment connecting the centers of the circles. The distance between the centers (2, –5) and (–6, 10) is 17.
Thus, the required distance is 17 – (5 + 9) = **3**.

C) $2x + 3y = 4$ → linear with slope = –2/3 and passes through (2, 0) Applying the slopes, we have additional solutions of: (5, –2), (8, –4) (–1, 2), (–4, 4) and (–7, 6)
For any other solutions, either the x- or y - coordinate (or both) has (have) more than one digit.
Adding, we have $A = (2 + 5 + 8 – 1 – 4 – 7) = 3$ and $B = 0 – 2 – 4 + 2 + 4 + 6 = 6$ → **(3, 6)**