MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2013 SOLUTION KEY

Round 6

A) $\hat{7} = (2\cdot 7+1)! = 15!$

$3\#2 = \left(3^2+2^3\right)! = 17!$

$\frac{17!}{15!} = 17\cdot 16 = \underline{\mathbf{272}}$

B) Let's systematically list the possible sets of 3 digits.
Smallest digit 0: 019, 028, 037, 046, <u>*055*</u>
Smallest digit 1: <u>*118*</u>, 127, 136, 145
Smallest digit 2: <u>*226*</u>, 235, <u>*244*</u>
Smallest digit 3: <u>*334*</u>
If the 3 digits are distinct, there are 6 possible combinations.
If two of the digits are the same, there are 3 possible combinations.
8(6) + 5(3) = <u>**63**</u>

C) We must examine the integers from 7 through 27 inclusive.
All primes are deficient since the only proper divisor is 1.
Accordingly, 7, 11, 13, 17, 19 and 23 are deficient numbers.
The only abundant numbers are:

$12 = 2^2\cdot 3\ [1,2,3,4,6]$

$18 = 2\cdot 3^2\ [1,2,3,6,9]$

$20 = 2^2\cdot 5\ [1,2,4,5,10]$

$24 = 2^3\cdot 3\ [1,2,3,4,6,8,12]$

The proper divisors sum to 16, 21, 22 and 36 respectively.
Note that each of the abundant numbers has a repeated prime factor.
Since there are 21 numbers to be classified, the required ratio is <u>**17 : 4**</u>.