MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 – NOVEMBER 2012 SOLUTION KEY

Round 1

A) FOILing, $a+bi=(2-i)(a-i)=2a-2i-ai-1=(2a-1)+(-2-a)i$

Equating the real and imaginary coefficients, $\begin{cases} a=2a-1 \\ b=-2-a \end{cases} \Rightarrow$ (a, b) = **<u>(1, –3)</u>**

B) The only nonzero integers for which $\sqrt{a^2+b^2}=5$ are ±3 and ±4, four choices for *a* and four choices for *b*.
Thus, sixteen ordered pairs are possible, but since *a* > *b*, of these
only (4, 3), (4, –3), (3, –4) and (–3, –4) are acceptable.
Thus, there are only **<u>4</u>** ordered pairs.

C) Given: $\begin{cases} {z_1}^2+{z_2}^2=-41-6i \\ (2-i)z_1z_2=-15-20i \end{cases}$

Dividing $(2-i)$ and multiplying by 2, the second equation gives us

$$2z_1z_2=2\left(\frac{-15-20i}{2-i}\right)\cdot\frac{2+i}{2+i}=2\left(\frac{-5(3+4i)(2+i)}{5}\right)=2(-6-3i-8i+4)=-4-22i$$

Adding to the first equation, ${z_1}^2+{z_2}^2+2z_1z_2=(-41-6i)+(-4-22i)$

$\Leftrightarrow (z_1+z_2)^2=-45-28i$

Since the sum of two complex numbers is a complex number, let $z_1+z_2=a+bi$, for real numbers *a* and *b*.

Then: $a^2-b^2=-45$
$2ab=-28 \Rightarrow ab=-14$

Clearly, *a* and *b* have opposite signs.
The ordered pairs (2, –7) and (–2, 7) satisfy both equations and z_1+z_2 = **<u>2 – 7*i*</u>**, **<u>–2 + 7*i*</u>** .