Round 2 - continued

C) $13x + 11y = 316 \rightarrow y = \dfrac{316-13x}{11} = 28 - x + \dfrac{8-2x}{11} = 28 - x + 2\left(\dfrac{4-x}{11}\right)$

Thus, the smallest positive integer x to try is 4.
$x = 4 \rightarrow y = 24 + 2(0) = 24 \rightarrow$ partition $316 = 52 + 264 \rightarrow$ **(52, 264)**
$x = 15 \rightarrow y = 13 + 2(-1) = 11 \rightarrow$ partition $316 = 195 + 121 \rightarrow$ **(195, 121)**
Since the slope of the given linear equation is $-13/11$, we note that when x is increased by 11, y is decreased by 13. Therefore, x and y will no longer both be positive integers and the search stops.

Alternate solution (using congruence notation):
If you familiar with numerical congruence, read on!
If not, ask a teammate or teacher before continuing.

$11x + 13y = 316 \rightarrow 11x \equiv 316 \pmod{13}$ [$\equiv$ is the congruence operator]
Removing the largest multiple of 13, we have $11x \equiv 4 \pmod{13}$
The solution to this congruence is $x \equiv 11 \pmod{13}$, since $11(11) = 121 = 13(9) + 4$ and we see that 121 leaves a remainder of 4 when divided by 13. You may wish to verify that over the integer interval [0, 12], $x = 11$ is the only solution to $11x \equiv 4 \pmod{13}$.

$x = 11$ generates the partition $121 + 195 = 316 \rightarrow$ **(195, 121)**
$x = 11 + 13 = 24$ generates the partition $264 + 52 = 316 \rightarrow$ **(52, 264)**

In each ordered pair, the multiple of 13 must be listed first.