MASSACHUSETTS MATHEMATICS LEAGUE
FEBRUARY 2004
ROUND 4: WORD PROBLEMS

ANSWERS

A) 15/2

B) 400

C) ±155

A) What is the larger of the two numbers whose sum is ten, if the sum of their reciprocals is 8/15.

$x, 10-x \qquad \frac{1}{x}+\frac{1}{10-x}=\frac{8}{15}$

$15(10-x)+15x = 8(10x-x^2)$

$150 = 80x - 8x^2, \quad 8x^2 - 80x + 150 = 0$

$4x^2 - 40x + 75 = 0 \quad (2x-5)(2x-15) = 0 \quad x = \frac{5}{2}, \frac{15}{2}$ ANS 15/2

B) An elevator went from the bottom of a tower to the top at a speed of 4 meters/second. It remained at the top for ninety seconds, and then returned to the bottom at a speed of 5 m/sec. If the total trip took 4.5 minutes, how high is the tower?

x = ht of elevator $\quad \frac{x}{4}+90+\frac{x}{5}=270, \quad \frac{x}{4}+\frac{x}{5}=180$

$5x + 4x = 180 \cdot 20$

$\frac{9x}{9} = \frac{180 \cdot 20}{9}, \quad x = 20 \cdot 20 = 400$

C) The sum of the squares of three positive odd integers is 967 more than the sum of the squares of the two even integers between them. Calculate the sum of the five consecutive integers. $x, x+2, x+4$

$x^2 + (x+2)^2 + (x+4)^2 = 967 + (x+1)^2 + (x+3)^2$

$x^2 + 4x + 4 + 8x + 16 = 967 + 2x + 1 + 6x + 9$

$x^2 + 12x + 20 = 8x + 977, \quad x^2 + 4x - 957 = 0$

$(x+33)(x-29) = 0, \quad x = 29, \quad x+1 = 30, \quad x+2 = 31, x+3 = 32, x+4 = 33$

OR $x = -33 \rightarrow -155$

ANS ±155