MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 - FEBRUARY 2015 SOLUTION KEY

Team Round - continued

D) 104 singles, 18 doubles, 2 triples and 6 homeruns in 400 at bats

My batting average was $BA = \dfrac{104+18+2+6}{400} = \dfrac{130}{400} = \dfrac{13}{40} = 0.325$

My slugging percentage was $SLG = \dfrac{104+2(18)+3(2)+4(6)}{400} = \dfrac{170}{400} = \dfrac{17}{40} = 0.425$

Thus, my on base percentage was $\dfrac{15}{40}$ or 0.375. Let x = *HBP* and y = *SF*. Then:

$BB + HBP = 11x$ and we have $\dfrac{130+11x}{400+11x+y} = 0.375 = \dfrac{3}{8} \Leftrightarrow 1040 + 88x = 1200 + 33x + 3y$

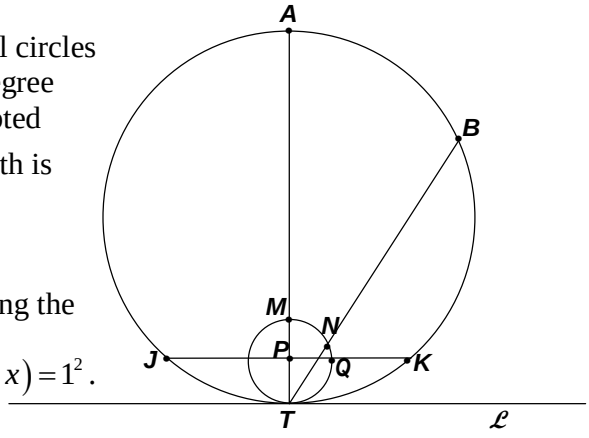
$\Leftrightarrow \Leftrightarrow 55x - 3y = 160$ (a linear function w/slope $\dfrac{55}{3}$) $\Leftrightarrow y = \dfrac{5(11x-32)}{3}$

Since 3 is not a factor of 5, it must be a factor of $11x - 32$. $x = 4 \Rightarrow y = \dfrac{5(12)}{3} = 20$

Increasing x by 3 and y by 55, we get additional pairs: $(7,75)$, $(10,130)$,...

However, since I had fewer than 100 sac flies, only **4** and **7** are acceptable x-values.

E) Let R and r denote the radii of the large and small circles respectively. As an inscribed angle $\angle ATB$, its degree measure is half the degree measure of its intercepted arc. Therefore, minor arc $\overset{\frown}{AB}$ is 72°, i.e. its length is $\dfrac{1}{5}$ of the circumference of the circle and $C = 5\left(\dfrac{4\pi}{5}\right) = 4\pi \Rightarrow R = 2$. Let $PT = x$. Applying the product-chord theorem in the larger circle, $x(4-x) = 1^2$.



$x^2 - 4x + 1 = 0 \Rightarrow x = \dfrac{4 \pm 2\sqrt{3}}{2} \Rightarrow PT = 2 - \sqrt{3}$

(the other root is extraneous)

$PT = 3 \cdot PM \Rightarrow TM = \dfrac{4}{3}\left(2-\sqrt{3}\right) \Rightarrow r = \dfrac{2}{3}\left(2-\sqrt{3}\right)$

Therefore, the required area is

$4\pi - \pi\left(\dfrac{2}{3}\right)^2\left(2-\sqrt{3}\right)^2 = \pi\left(4 - \dfrac{4}{9}\left(7-4\sqrt{3}\right)\right) = \pi\left(4 - \dfrac{28}{9} + \dfrac{16}{9}\sqrt{3}\right)$

$= \pi\left(\dfrac{8}{9} + \dfrac{16}{9}\sqrt{3}\right) = \dfrac{8}{9}\left(1+2\sqrt{3}\right)\pi \Rightarrow (A,B,C) = \underline{(\mathbf{8,9,2})}$.