MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2010 SOLUTION KEY

Team Round – continued

B) The hypotenuse has length 10. Let h denote the length of the altitude to the hypotenuse.

Then: Area $= \frac{1}{2}\cdot 6\cdot 8 = \frac{1}{2}\cdot 10\cdot h \rightarrow h = 4.8$

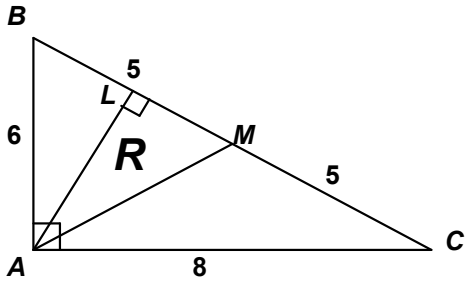
The length of the median is half the hypotenuse $\rightarrow$ $m = 5$

We avoid using the Pythagorean Theorem by looking for a special right triangle.

$(a, b, c) = (?, 4.8, 5) = \frac{1}{10}(?, 48, 50) = \frac{1}{5}(?, 24, 25)$

Special rt. Triangle (7, 24, 25) $\rightarrow$ $a = 7/5$ and the required ratio is

$$\frac{\frac{1}{2}(1.4)(4.8)}{\frac{1}{2}(6)(8)} = \frac{1.4}{10} = \underline{\mathbf{\frac{7}{50}}}$$



Alternate Solution (Tuan Le):

$\cos B = \frac{AB}{BC} = \frac{6}{10} = \frac{3}{5} = \frac{BL}{AB} = \frac{BL}{6} \rightarrow BL = \frac{18}{5}$

$\rightarrow LM = 5 - \frac{18}{5} = \frac{7}{5}$

Therefore, $\frac{R}{\text{area}(\Delta ABC)} = \frac{\frac{1}{2}(AL)(LM)}{\frac{1}{2}(AL)(BC)} = \frac{LM}{BC} = \frac{7/5}{10} = \underline{\mathbf{\frac{7}{50}}}$

C) Let r and c denote the row rate and the current respectively (in mi/hr). Then:

$\begin{cases} \text{downstream}: (r+c)1.5 = 12 \\ \text{upstream}: (r-c)4 = 12 \end{cases}$. Subtracting, $2.5r - 5.5c = 0 \rightarrow r = \frac{11c}{5}$ Substituting,

$\left(\frac{11c}{5} - c\right) = 3 \rightarrow \frac{6c}{5} = 3 \rightarrow c = 2.5, r = 5.5$

To travel 1 mile downstream, $8T = 1 \rightarrow T = 1/8$ hour = **7** minutes **30** seconds $\rightarrow$ **(7, 30)**

D) $\frac{22 - \frac{n}{2}}{7 - \frac{n}{2}} = c \rightarrow \frac{44 - n}{14 - n} = c$

The quotient is positive for $n < 14$ and $n > 44$.

For integer values of $n \geq 45$, the quotients are $\frac{1}{31}, \frac{2}{32}, \frac{3}{33}, \ldots$

Clearly none of these are integers. Thus, we restrict our attention to $1 \leq n \leq 13$.

$n = 1, 2, 3, 4$ produce $\frac{43}{13}, \frac{42}{12}, \frac{41}{11}$ and $\frac{40}{10} = 4$

$n = 13$ produces $\frac{31}{1} = 31 \rightarrow (L, S) =$ **(13, 4)**.