Round Four:

A. If $x \neq 3$, we have $2x - 3 = 3x - 6 + x - 3$ so $6 = 2x$ thus $x = 3$. No solution.

B. $KE_q = CM_qV_q^2 = C(3M_d)(0.8V_d)^2 = 3(0.64)CM_dV_d^2 = 3.92(250) = 480$ J

C.

$$\frac{d+e}{ae-ec} = \frac{1}{a-c}\cdot\frac{d+e}{e} = \frac{1}{a-c}\cdot\left(1+\frac{d}{e}\right) = \frac{1}{a-c}\cdot\left(\frac{c}{c}+\frac{a+c}{c}\right) = \frac{a+2c}{(a-c)c}$$

Round Five:

A. As exterior angles, 1>4 and 4>2. If AD>BC then surely AD>DC so 2>3

B. Let x = half the smaller diagonal. Area is then $0.5(2x)(4x) = 4x^2$ while perimeter is $4\sqrt{5}\,x$ so $x^2 = p^2/80$ and area is $p^2/20$.

C. Let x = area ΔABC. ΔBDC has base DC = (4/5)AC and same ht as ΔABC so area is (4/5)x. ΔCED has base CE = (2/5)CB and same ht as ΔBDC so area of ΔCED is (2/5)(4/5)x Since x = 84 answer is (0.32)(84) = 26.88

Round Six:

A. Prob (notF and notS)= (210 – 20 – 28 +12)/210 = 174/210 = **29/35**

B. There are ${}_8C_3 = 56$ triangles ; there are ${}_4C_3 = 4$ triangles with only vowels for vertices so the answer is 52/56 = **13/14**

C. The x^3 term is ${}_{2005}C_{1004}\, x^{1004}(1/2x)^{1001}$; the x term is ${}_{2005}C_{1003}\, x^{1003}(1/2x)^{1002}$ so

$$\frac{2005!}{1004!(1001!)2^{1001}} \div \frac{2005!}{1003!(1002!)2^{1002}} = \frac{2(1002)}{1004} = \frac{501}{251}$$

Team Round:

A. $x - 6 = 2x^2 + 3 - 2y = 6 - y$ so $y = 12 - x$ and substituting in the first equality simplifies to $2x^2 + x - 15 = 0$ so $x = -3$ or 2.5

B. Expressing all as powers of 2 yields the following system: $x+y = 2(w+y) = 3(x - w - 2) = 4(x + y - w - 1)$ which solves to x = 14, y = -2, and w = 8.

C. $r^2 + s^2 + t^2 = (r+s+t)^2 - 2(rs + rt + st)$; Since $x^3 - ax^2 + bx - c$ has roots whose sum is a, product is c, and sum of pair-wise products is b, we have r+s+t= 39/3=13 and (rs+rt +st) = 120/3 yielding 169 – 2(40)=89

D. 150+60x = 0.8(350+45x) gives x = 65 / 12 while 0.8(150+60x) = 350+45x gives x = 230/3. Sum is 82 and 1/12 or 82 hrs 5 mins.

E. Stewart's Thm: $(AB)^2(DC) + (AC)^2(BD) = (AD)^2(BC) + (BD)(DC)(BC)$ so if BD=x we have $64x + 196x = 49(2x) + 2x^3$ so $2x^3 - 162x = 0 = 2x(x+9)(x-9)$ so x=9 and BC=18 OR drop perpendicular from A to BD at E let AE=x BE=y ED=z so subtract $x^2+z^2=49$ from $x^2+(2z+y)^2=196$ to get (1) $y^2+4yz+3z^2=147$; From $x^2+y^2=64$ subtract $x^2+z^2=49$ to get $y^2-z^2=15$ added to (1) gives $2y^2+4yz+2z^2=162$ so $y^2+2yz+z^2=81$ whence y+z=9 and BC=18.

F. $3^{1000} = 9^{500} = (10-1)^{500}$ which when expanded influences the last two digits only in the last two terms as all other terms contain at least two factors of 10 so by Bin. Thm. we have … $+ {}_{500}C_{499}\,(10)(-1)^{499} + {}_{500}C_{500}\,(-1)^{500} = -5000+1$ so last two digits are 01.