MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2007 SOLUTION KEY

C) Let $C(-1, 1)$ be the reflection of point A across the y-axis.

$$\begin{cases} QA = QC \\ BC = BQ + QC \end{cases} \rightarrow BC = BQ + QA$$

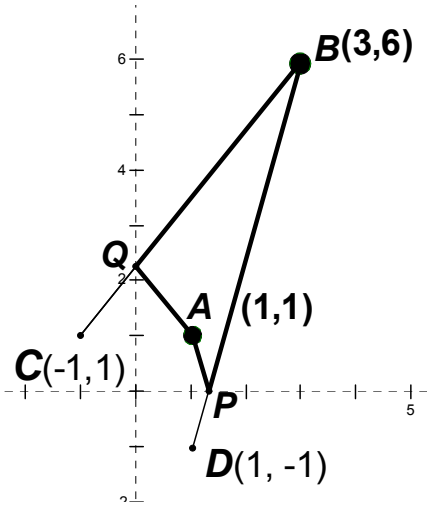
Thus, Q is the point on the y-axis that minimizes $AQ + QB$.

$\overleftrightarrow{BC}$: slope = 5/4 equation: $5x - 4y + 9 = 0 \rightarrow b = 9/4$

Similarly, let $D(1, -1)$ be the reflection of point A across the x-axis.

$\overleftrightarrow{BD}$: slope = 7/2 equation: $7x - 2y - 9 = 0 \rightarrow a = 9/7$

$$\frac{9}{4} + \frac{9}{7} = \frac{63+36}{28} = \underline{\mathbf{\frac{99}{28}}}$$



D) $\dfrac{\log A}{\log B} = \dfrac{\cancel{\log A}}{\cancel{\log B}} = \dfrac{A}{B} = \dfrac{2}{3} \rightarrow B = \dfrac{3A}{2}$ Then $\dfrac{\log A}{\log\left(\frac{3A}{2}\right)} = \dfrac{2}{3}$

$$\rightarrow 3\log A = 2\log\left(\frac{3A}{2}\right) = 2(\log 3 + \log A - \log 2)$$

$$\rightarrow \log A = 2(\log 3 - \log 2) = 2\log\left(\frac{3}{2}\right) = \log\left(\left(\frac{3}{2}\right)^2\right) = \log\left(\frac{9}{4}\right)$$

Since the log function is a one-to-one function, $A = \dfrac{9}{4}$ and $B = \dfrac{3 \cdot \frac{9}{4}}{2} = \dfrac{27}{8} \rightarrow \underline{\left(\mathbf{\frac{9}{4}}, \mathbf{\frac{27}{8}}\right)}$