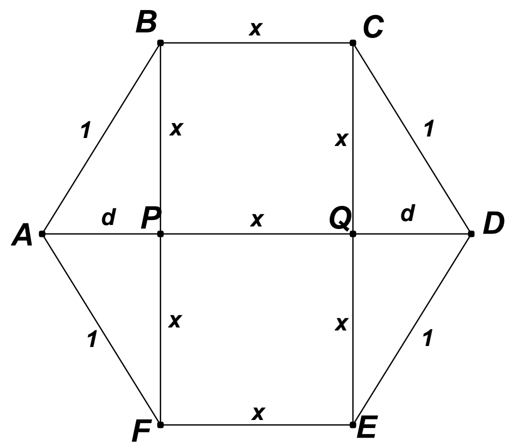
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2011 SOLUTION KEY

Team Round - continued

E) The measurements are indicated in the diagram at the right:



$d=\sqrt{1-x^2}$ ➔ $AD=\ x+2\sqrt{1-x^2}\ =2.2=\dfrac{11}{5}$

$10\sqrt{1-x^2}=11-5x$ ➔ $100(1-x^2) = 121 - 110x + 25x^2$

➔ $125x^2 - 110x + 21 = (25x - 7)(5x - 3) = 0$ ➔ $x=\dfrac{7}{25},\dfrac{3}{5}$

➔ $CE=$ $\underline{\boldsymbol{\dfrac{14}{25},\dfrac{6}{5}}}$ (or **<u>0.56</u>**, **<u>1.2</u>**)

Double check that both answers are OK.

Alternate solution (Norm Swanson)

$\left(2\sqrt{1-x^2}\right)^2=(2.2-x)^2$ ➔ $5x^2-4.4x+0.84=0$

Suppose the roots are multiplied by 10. Since the linear coefficient determines the sum of the roots and the constant term determines the product of the roots, this changes the linear coefficient by 10 and the constant term by 100.

➔ $5x^2-4.4(10)x+0.84(100)=5x^2-44x+84=(5x-14)(x-6)=0$

Thus, the roots of the original quadratic are $\dfrac{14}{50},\dfrac{6}{10}$ ➔ $\dfrac{7}{25},\dfrac{3}{5}$ ➔ $CE=$ $\underline{\boldsymbol{\dfrac{14}{25},\dfrac{6}{5}}}$.

F) Consider a specific case first, let $n = 2$.

$(A+B)^2+(C+D)^3+(E-F)^4=$

$\left(A^2+2AB+B^2\right)+\left(C^3+3C^2D+3CD^2+D^3\right)+\left(E^4-4E^3F+6E^2F^2-4EF^3+F^4\right)$ ****

Examining the coefficients only, (1 + 2 + 1) + (1 + 3 + 3 + 1) + (1 – 4 + 6 – 4 + 1) = 2 + 8 + 0 = 10

We notice:

- The coefficients are terms in Pascal's triangle.
- The first two sums are powers of 2.
- In the third sum, the signs of the coefficients alternate and the resulting sum is 0.

Were these coincidences?

If we want only the coefficients in **** above, we could let $A = B = C = D = E = F = 1$.

This forces all the literal parts to evaluate to 1 and we are left with the coefficients.

Therefore, the sum of the coefficients in S divided by 16 is

$$\frac{(1+1)^n+(1+1)^{n+1}+(1-1)^{n+2}}{16}=\frac{2^n+2^{n+1}+0}{16}=\left(\frac{2^n}{2^4}\right)(1+2)=3\left(2^{n-4}\right)$$

➔ $(k, x) =$ **<u>(3, n – 4)</u>**

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