MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2007 SOLUTION KEY

Team Round - continued

C) $x^2 + 1 = y^2$

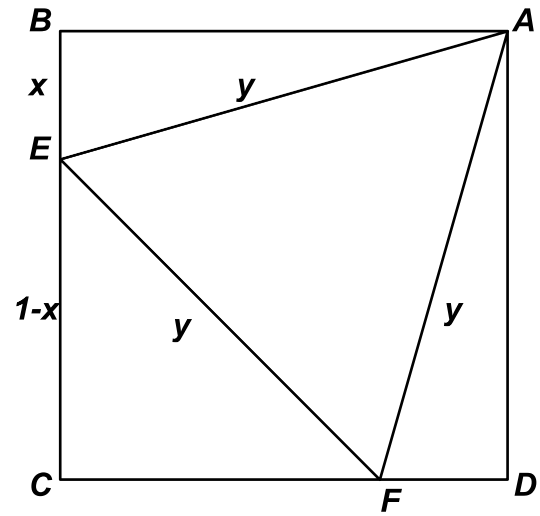
$(1 - x)^2 + (1 - x)^2 = y^2$

$\rightarrow 2(1 - x)^2 = x^2 + 1$

$\rightarrow x^2 - 4x + 1 = 0$

$\rightarrow x = \dfrac{4 \pm \sqrt{16-4}}{2} = 2 - \sqrt{3}$ ($2 + \sqrt{3}$ is rejected)

Thus, $x^2 = 7 - 4\sqrt{3} \rightarrow y^2 = 8 - 4\sqrt{3}$



Since the area of an equilateral triangles is given by $\dfrac{\text{side}^2\sqrt{3}}{4}$, we have $\dfrac{(8-4\sqrt{3})\sqrt{3}}{4} = \boxed{\mathbf{2\sqrt{3} - 3}}$

Alternate solution (finding y is not necessary):
Let K denote the area of equilateral ΔAEF.

$m\angle BAE = 15° \rightarrow x = \tan(15°) = 2 - \sqrt{3}$

$2\text{Area}(\Delta ABE) + \text{Area}(\Delta ECF) + K = 1$

$\rightarrow 2\left(\dfrac{1}{2} \cdot x \cdot 1\right) + \dfrac{1}{2}(1-x)^2 + K = 1 \rightarrow K = (1 - x) - \dfrac{1}{2}(1-x)^2 = \dfrac{1}{2}(1 - x^2)$

$= \dfrac{1}{2}\left(1 - \left(2 - \sqrt{3}\right)^2\right) = \dfrac{1}{2}\left(1 - 4 + 4\sqrt{3} - 3\right) = \underline{\mathbf{2\sqrt{3} - 3}}$