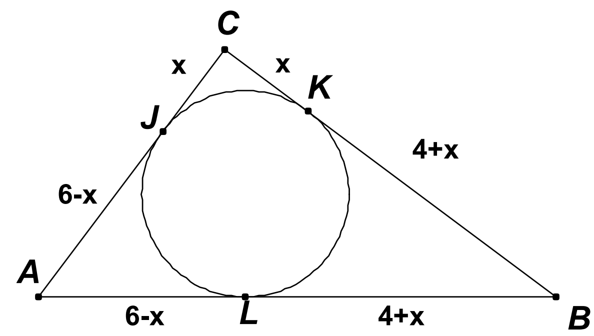
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 - FEBRUARY 2011 SOLUTION KEY

Team Round

E) Tangents from an external point to a circle are congruent.
Let $CJ = CK = x$.
$AC = 6 \rightarrow AJ = AL = 6 - x$ and
$AB = 10 \rightarrow BL = BK = 4 + x$.
$BC = x + (4 + x) = 8 \rightarrow x = 2$



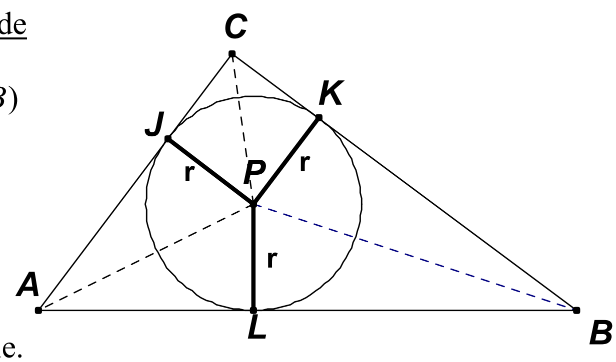
Since ΔABC is a right triangle, its area is $\frac{1}{2}\cdot 6\cdot 8 = 24$.

Notice that the radius of the inscribed circle is an <u>altitude</u> in triangles APC, BPC and APB.

$\text{area}(\Delta ABC) = \text{area}(\Delta APC) + \text{area}(\Delta BPC) + \text{area}(\Delta APB)$

$\rightarrow 24 = \frac{1}{2}\cdot 6\cdot r + \frac{1}{2}\cdot 8\cdot r + \frac{1}{2}\cdot 10\cdot r = 12r$

$\rightarrow 24 = r\frac{(6+8+10)}{2} = 12r \rightarrow r = 2$



Note: The line above illustrates an important relationship between <u>any</u> triangle and its inscribed circle.
Namely, the area of a triangle equals the product of its <u>s</u>emi-perimeter and the <u>r</u>adius of its inscribed circle. [$A(\Delta) = rs$] Semi-perimeter means half the perimeter.

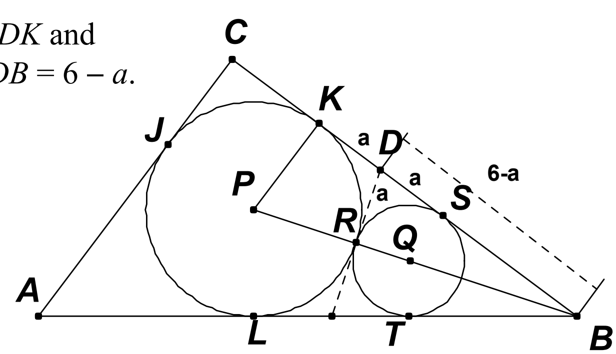
Applying the Pythagorean Theorem to ΔPKB, $PB = 2\sqrt{10}$.

Draw a line perpendicular to $\overline{PB}$ at R. Note that $DR = DK$ and $DR = DS$. They are all marked a in the diagram. Now $DB = 6 - a$.

In right ΔDRB, $a^2 + \left(2\sqrt{10} - 2\right)^2 = (6 - a)^2$

$\rightarrow 44 - 8\sqrt{10} = 36 - 12a \rightarrow 12a = 8\left(\sqrt{10} - 1\right)$

$\rightarrow a = \frac{2}{3}\left(\sqrt{10} - 1\right)$



Thus, $BS = BK - 2a = 6 - 2a = 6 - \frac{4}{3}\left(\sqrt{10} - 1\right) =$

$\underline{\mathbf{\frac{2}{3}\left(11 - 2\sqrt{10}\right)}}$ or (any exact equivalent)