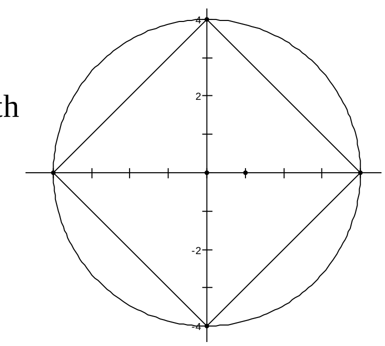
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2016 SOLUTION KEY

Round 1

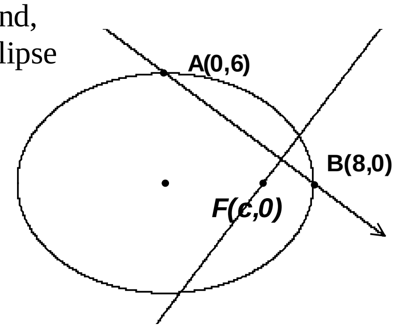
A) The x-intercepts of the parabola $y=(x+3)(x-h)$ at $A(-3,0)$ and $B(h,0)$ are images of each other across the axis of symmetry $x=8$. Thus, $(8,0)$ is the midpoint of $\overline{AB}$

$\Rightarrow \dfrac{-3+h}{2}=8 \Rightarrow h=2\cdot 8+3=\underline{\mathbf{19}}$.



B) The given equations determine a circle of radius 4 and a square with diagonals of length 8. Thus, the area of the enclosed region is

$16\pi-\dfrac{1}{2}\cdot 8\cdot 8=16(\pi-2) \Rightarrow (A,B)=\underline{\mathbf{(16,2)}}$.



C) The line 3x + 4y = 24 passes through points A(0, 6) and B(8, 0) and, since $a>b$, the ellipse is horizontal. Thus, the equation of the ellipse is $\dfrac{x^2}{64}+\dfrac{y^2}{36}=1$. The focus is at ($c$, 0) and since, for an ellipse, $a^2=b^2+c^2$, we have $c=\sqrt{64-36}=\sqrt{28}=2\sqrt{7}$.
The equation of the perpendicular line must be of the form 4x – 3y = n, for some constant n. Since the focus is at $\left(2\sqrt{7},0\right)$,

we have 4x – 3y = $8\sqrt{7}$. Therefore, k = $\underline{-\dfrac{\mathbf{8\sqrt{7}}}{\mathbf{3}}}$.