MASSACHUSETTS MATHEMATICS LEAGUE CONTEST 3 - DECEMBER 2008 SOLUTION KEY

Round 6

A) Rhombus *PQRS* must also be a square and $PS = 2\sqrt{2}$ *JKLM* must also be a square.

If *J* is the midpoint of \overline{PS} , then $PJ = \sqrt{2} \rightarrow JK = \sqrt{2} \cdot \sqrt{2} = 2$ Thus, the perimeter of $JKLM = \underline{8}$

- B) A must be a square and B must be a pentagon, since these are the only regular polygons in which all diagonals could have the same length. Thus, C has 9 sides. n = 360/9 = 40 and $m = 180 40 = 140 \rightarrow m$: n = 140 : 40 = 7 : 2
- C) Since the diagonals of a rhombus are perpendicular, the fact that the diagonals have lengths in a 4 : 3 ratio implies that the sides of the rhombus must have lengths 5a. Thus,

$$\frac{Area(Rhom)}{Per(Rhom)} = \frac{\frac{1}{2}8a \cdot 6a}{20a} = \frac{9}{4} \Rightarrow \frac{6a}{5} = \frac{9}{4} \Rightarrow a = \frac{15}{8}$$

and the long diagonal has length 15.

