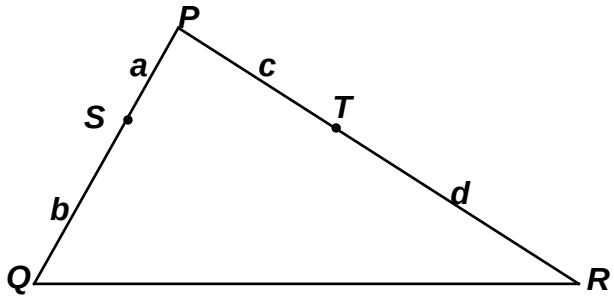
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2017 SOLUTION KEY

Team Round – continued



E)
Given: $PQ = 6$, $QR = 10$, $PS : SQ = a : b$, $PT : TR = c : d$

$$\frac{a}{b}=\frac{c}{d},\ \frac{c}{a+b}=\frac{a}{c+d}$$

Let $x = a + b$ and $y = c + d$.

$$\frac{a}{b}=\frac{c}{d}\Rightarrow\frac{a}{x-a}=\frac{c}{y-c}\Rightarrow ay-ac=xc-ac\Rightarrow ay=xc\Rightarrow c=\frac{ay}{x}$$

$$\frac{c}{a+b}=\frac{a}{c+d}\Rightarrow\frac{c}{x}=\frac{a}{y}=\Rightarrow ax=cy\Rightarrow ax=\left(\frac{ay}{x}\right)y=\frac{ay^2}{x}\Rightarrow x^2=y^2\Rightarrow x=y\Rightarrow PQ=PR$$

Thus, both proportions can be true if and only if ΔPQR is isosceles with base $\overline{QR}$; hence, the only possible perimeter is (6 + 6 + 10) = **22**.