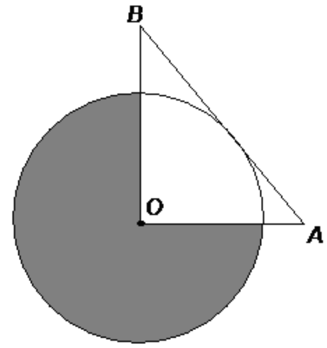
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 - FEBRUARY 2014 SOLUTION KEY

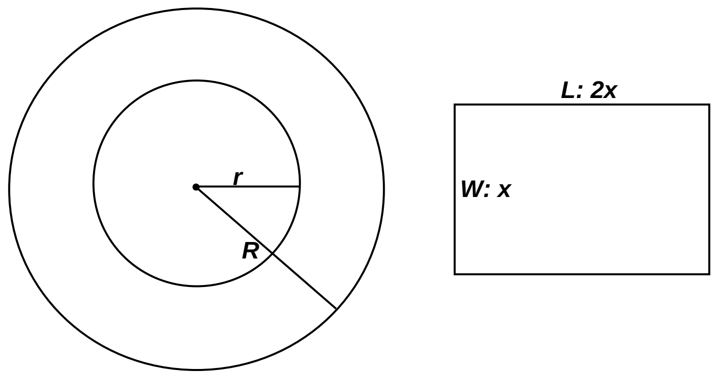
Round 5



A) The radius of circle O is the altitude to the hypotenuse.

$\frac{1}{2}\cdot 3\cdot 4=\frac{1}{2}\cdot 5\cdot h \Rightarrow h=\frac{12}{5}$

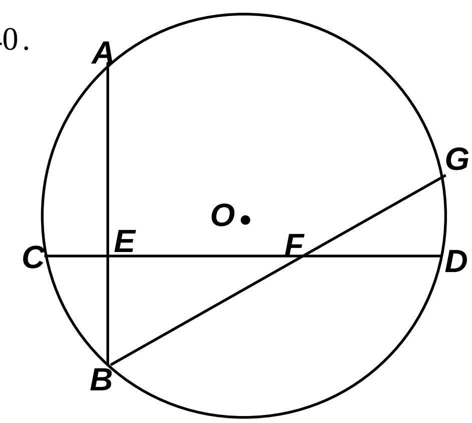
The shaded region is $\frac{3}{4}$ of the circle. $\frac{3}{4}\cdot\pi\cdot\left(\frac{12}{5}\right)^2=\underline{\boldsymbol{\frac{108\pi}{25}}}$.



B) Let x denote the width of the rectangle,
r the radius of the smaller circle and
R the radius of the larger circle. We are given that $D_{small}=2r=\frac{2}{3}R$ and $\mathrm{A}(\text{ring})=A(\text{rect})$.

$\Rightarrow R=3r$ and $8\pi r^2=2x^2 \Rightarrow x=2\sqrt{\pi}r$

Thus, $\frac{L}{D_{large}}=\frac{2x}{6r}=\frac{x}{3r}=\frac{2\sqrt{\pi}r}{3r}=\underline{\boldsymbol{\frac{2\sqrt{\pi}}{3}}}$.



C) According to the product chord theorem, $AE\cdot BE=2\cdot 20=40$.
Since AE and BE are integers and $AE>BE$, the possible ordered pairs (AE,BE) are: $(8,5),(10,4),(20,2),(40,1)$

The shortest possible length of $\overline{AB}$ is $8+5=13$ and $(BE,BF)=(5,13)\Rightarrow EF=12$.

Using F as a division point and re-applying the Product-Chord theorem, $CF\cdot FD=BF\cdot FG \Rightarrow$

$(2+12)(22-14)=13\cdot FG \Rightarrow FG=\frac{14\cdot 8}{13}=\underline{\boldsymbol{\frac{112}{13}}}$.