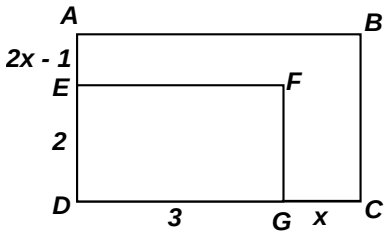
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2014 SOLUTION KEY

Round 5



A) $ABCD \sim EFGD \Rightarrow \dfrac{2}{3}=\dfrac{2x+1}{x+3} \Rightarrow 2x+6=6x+3 \Rightarrow x=$ **$\dfrac{3}{4}$**

Check: $2\left(\dfrac{3}{4}\right)-1=\dfrac{1}{2}<2$

$ABCD \sim FGDE \Rightarrow \dfrac{2}{3}=\dfrac{x+3}{2x+1} \Rightarrow 4x+2=3x+9 \Rightarrow x=7$ (extraneous) Check: $2(7)-1=13 \not< 2$

B) $\dfrac{(x+1)^2}{(2-x)^2}=\dfrac{9}{4} \Rightarrow 4x^2+8x+4=9x^2-36x+36 \Rightarrow 5x^2-44x+32=0$

Factoring, $(5x-4)\cancel{(x-8)}=0$

$x=8$ is rejected since the side of the second square would be negative.

Thus, $x=$ **$\dfrac{4}{5}$** .

$\dfrac{(x+1)^2}{(2-x)^2}=\dfrac{4}{9} \Rightarrow 9x^2+18x+9=16-16x+4x^2 \Rightarrow 5x^2+34x-7=0$

Factoring, $(5x-1)\cancel{(x+7)}=0$

$x=-7$ is rejected since the side of the first square would be negative.

Thus, $x=$ **$\dfrac{1}{5}$** .

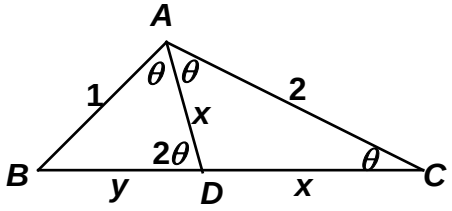
C) Let $DC = x$, $BD = y$ and $m\angle DAC = m\angle DCA = \theta$.

As an exterior angle of ΔADC , $m\angle BDA = 2\theta$

$m\angle DAC = m\angle DCA \Rightarrow DA = DC$

$\Delta BAC \sim \Delta BDA \Rightarrow m\angle BCA = m\angle BAD = \theta$

Thus, $\overrightarrow{AD}$ is an angle bisector and $\dfrac{y}{1}=\dfrac{x}{2} \Rightarrow x=2y$.



$\Delta BAC \sim \Delta BDA \Rightarrow \dfrac{BC}{BA}=\dfrac{AC}{DA} \Leftrightarrow \dfrac{x+y}{1}=\dfrac{2}{x}$

Cross multiplying, $x^2+xy=2 \Leftrightarrow 4y^2+2y^2=2 \Leftrightarrow y^2=\dfrac{1}{3} \Rightarrow y=\dfrac{\sqrt{3}}{3} \Rightarrow AD=x=$ **$\dfrac{2\sqrt{3}}{3}$** .