MASSACHUSETTS MATHEMATICS LEAGUE
MARCH 2004
ROUND 7: TEAM QUESTIONS - SOLUTIONS
ANSWERS

A) $x = 1, 2, -4$ D) 40 mph

B) $28/299$ E) $4\sqrt{3}$

C) $x = 2, 4, -3$ F) $2\sqrt{2} + \sqrt{3}$

A) $\begin{vmatrix} x & 2 & 1 \\ 1 & x & 1 \\ -x & 3 & x \end{vmatrix} \begin{matrix} x & 2 \\ 1 & x \\ -x & 3 \end{matrix} = \begin{vmatrix} 2x+1 & 1 \\ x+7 & 2 \end{vmatrix} = 4x + 2 - x - 7 = 3x - 5$

$x^3 - 2x + 3 - 2x - 3x + x^2 = x^3 + x^2 - 7x + 3$

$x^3 + x^2 - 7x + 3 = 3x - 5, \quad x^3 + x^2 - 10x + 8 = 0$

	1	1	−10	8
1	1	2	−8	0
2	1	4	0	
−4	1	0		

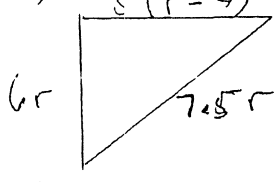
B) $\dfrac{{}_{12}C_3 \; {}_8C_2 \; {}_6C_0}{{}_{26}C_5} = \dfrac{12 \cdot 11 \cdot 10}{3 \cdot 2 \cdot 1} \cdot \dfrac{8 \cdot 7}{2 \cdot 1} \cdot \dfrac{5\,4\,3\,2\,1}{26 \cdot 25 \cdot 24 \cdot 23 \cdot 22} = \dfrac{4 \cdot 7}{13 \cdot 23} = \dfrac{28}{299}$

C) $3 - i$ is a root, so 3rd root is -6, Factors of $P(x)$ are

$(x+6)(x^2 - 6x + 10) = x^3 + 26x + 60 = 3x^2 - 16x + 36$

	1	−3	−10	24
2	1	−1	−12	0
4	1	3	0	
−3	1	0		

D)

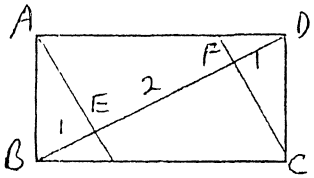


Times 2, $144r^2 + 100(r^2 - 8r + 16) = 225r^2$

$19r^2 - 800r + 1600 = 0$

$(19r - 40)(r - 40) = 0 \quad r = 40$ only

E)



$\dfrac{BE}{AB} = \dfrac{AB}{BD}, \quad \dfrac{1}{AB} = \dfrac{AB}{4}, \quad AB = 2$

$\dfrac{AE}{AD} = \dfrac{AD}{BD}, \quad \dfrac{3}{AD} = \dfrac{AD}{4}, \quad AD^2 = 12, \; AD = 2\sqrt{3}$

Area $= 4\sqrt{3}$

F) $\sqrt{3}\sqrt{3 + 2\sqrt{2}} + \sqrt{2}\sqrt{7 - 4\sqrt{3}} = \sqrt{3}(1 + \sqrt{2}) + \sqrt{2}(2 - \sqrt{3}) =$

$\sqrt{3} + \sqrt{6} + 2\sqrt{2} - \sqrt{6} = 2\sqrt{2} + \sqrt{3}$