Round One:

A. If $f^{-1}(z) = z$ then $f(z) = z$ too so $z = 4z + 6$ thus $z = -2$.

B. $f(2005)= 1 +f(1995) = \ldots = 200 + f(5) = 202$

C. $f^{-1}(x)=2x-3$; $g^{-1}(x) = 1/x + 3$; $f^{-1}(2)=1$; $g^{-1}(1)= -0.5$; h cancels h^{-1}; $f(-0.5)=1.25$; $g(1.25)=3.8$

Round Two:

A. $520=2^3 x5x13$ so $4x2x2=16$ factors. $5292=2^2 x3^3 x7^2$ so $3x4x3=36$ factors, **20** more.

B. An organized list gives 2x3x5, 2x3x7, ... 2x3x31 (9 values); then 2x5x7, ..., 2x5x19 (5 values); then 2x7x11, 2x7x13 (2 values); 3x5x7,..3x5x13(3 values); for **19** values with exactly 3 distinct prime factors. Note that without the distinct requirement we would have additional values such as 2x2x3x5

C. Divisibility by 9 requires 23+a+b be a multiple of 9 or 5+a+b=9 or 18 so a+b=4 or 13. Divisibility by 4 requires ab be a multiple of 4. Only possibilities are 04, 40, and 76 so products are 0 or 42

Round Three:

A. $(\tan^2(x) + 1) + \tan(x) =1 \Rightarrow \tan(x)=0$ (but then csc(x) undefined) OR $\tan(x)= -1$ so $\csc(x) = \pm\sqrt{2}$

B. $2x+1 = \dfrac{5}{27x+3}$ so $54x^2 + 33x - 2 =0$ so $x = -2/3$ or $1/18$. Since $2x + 1$ must be in the domain of the $\sin^{-1}$ function, only $-2/3$ is valid.

C. $\dfrac{5}{x+2} = \pm(2x+1) + 2n\pi$ but x must be rational so n=0. $\dfrac{5}{x+2} = -(2x+1)$ yields no real solutions. $\dfrac{5}{x+2} = 2x+1$ gives $5 =2x^2+5x +2$ so $x = \frac{1}{2}$ or -3 making $\dfrac{5}{x+2} = 2$ or -5 . Only the first is in the range of $\cos^{-1}$ so the only solution is $\frac{1}{2}$

Round Four:

A. The ratio of alcohol is $0.15 = \dfrac{0.75}{1+n}$ solving yields n = 4.

B. I am x now. "Then" my mother was 2x and my brother x-12 so 2x = x – 12 + 26 so x=14 any my brother is 22. My mother was 8 yrs older when I was born or 34.

C. Let r= paddling speed in still water, s = speed of current. 6(r-s) = s + 2(r+s) so s=(4/9)r and upstream rate is (5/9)r, downstream (13/9)r. Immediate return would have taken $\dfrac{Dist}{time} = \dfrac{6(r - s)}{r + s} = \dfrac{6(5/9)r}{(13/9)r} = \dfrac{30}{13}$ instead of the three hrs it took (1 resting)