MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2010
ROUND 7 TEAM QUESTIONS
ANSWERS

A) (___________ ,___________) D) (_____ , _____ , _____ , _____)

B) __________________________ E) (___________ ,___________)

C) (___________ ,___________) F) __________________________

****** NO CALCULATORS ON THIS ROUND ******

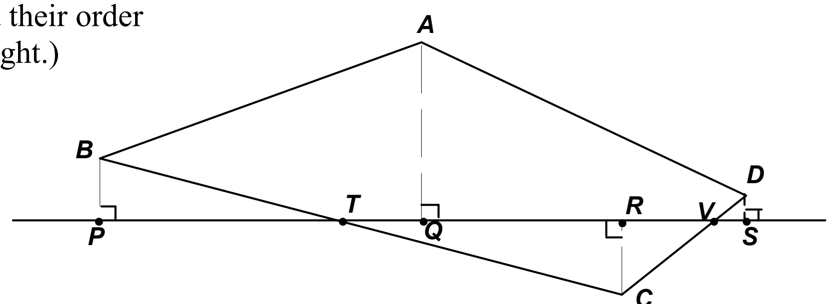
A) Let $z = a + bi$. Compute the ordered pair (a, b), if $\begin{cases} \dfrac{1}{z} = \overline{z} \\ a + b = 1.4 \\ a > b \end{cases}$.

B) If $\dfrac{1}{2x^3} - \dfrac{1}{x^2} - \dfrac{1}{2x} + 1 = 0$, find <u>all</u> possible values of $(x^2 + 1)^2$.

C) Given: $BP : CR : DS = 4 : 5 : 2$; and $\overline{BP}, \overline{AQ}, \overline{CR}$ and $\overline{DS} \perp \overleftrightarrow{TV}$.
Compute the unique ordered pair (a, b) for which the following statement is true:

$$Area(ABCD) = Area(ABPQ) + Area(ADSQ) - a \cdot Area(\Delta BPT) - b \cdot Area(\Delta DSV)$$

(P, T, Q, R, V and S are collinear and their order is as indicated in the diagram at the right.)



D) $x^{14k} - x^{8k} - x^{6k} + 1$ is factored completely over the integers, as a product of binomials and trinomials, where each lead coefficient is +1. The <u>sum</u> of these factors can be written in the form $Ax^{4k} + Bx^{2k} + Cx^k + D$. Determine the ordered quadruple (A, B, C, D).

E) Given: $\sin 54° = \dfrac{\sqrt{5} + 1}{4}$. In simplified form, $\sin 144° \sin 72° = \dfrac{\sqrt{A}}{B}$. Determine (A, B).

F) Given: $AB = BC$, $AD = AC$ and
$m\angle BAD$, $m\angle ADC$, $m\angle ADB$ form an increasing arithmetic progression,
where $(m\angle ADB - m\angle ADC)^2 = m\angle ADC + 60°$. Compute $m\angle BAD$.