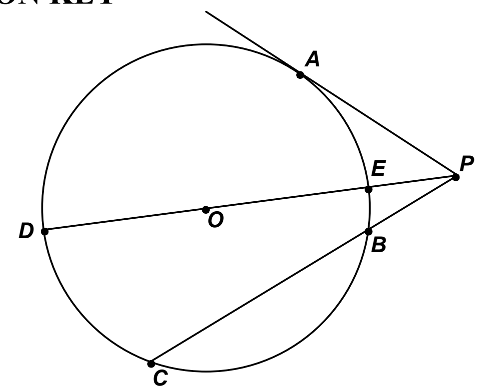
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2010 SOLUTION KEY



Round 5 - continued

C) Let $PB = x$ and $BC = 3x$. According to the tangent-secant relation, $PA^2 = PB(PC)$, i.e. outer(outer + inner)
$x(4x) = 14^2 \rightarrow x = 7 \rightarrow PE = 2\sqrt{7}$.
Let $OE = OD = r$. Then $14^2 = 2\sqrt{7}\left(2\sqrt{7}+2r\right)$.
$\rightarrow 196 = 28 + 4\sqrt{7}r \rightarrow r = \dfrac{168}{4\sqrt{7}} = \dfrac{42}{\sqrt{7}} = 6\sqrt{7}$.
Thus, the area of circle O is $\pi\left(6\sqrt{7}\right)^2$ = $\underline{\mathbf{252\pi}}$.

Round 6

A) The desired outcome could be either R_1B_2 or B_1R_2.
OR → ADD
Thus, the probability is $\dfrac{4}{9}\cdot\dfrac{4}{9} + \dfrac{5}{9}\cdot\dfrac{5}{9} = \dfrac{16+25}{81}$ = $\underline{\mathbf{\dfrac{41}{81}}}$.

B) $P = 1 - $ P(none of them solve the problem) = 1– P(not A)·P(not B)·P(not C) = $1-\dfrac{4}{5}\cdot\dfrac{3}{4}\cdot(1-x)$
$= \dfrac{2+3x}{5} = \dfrac{8+12x}{20}$.
Q = P(A not$B$ not$C$ or not$A$ B not$C$ or not$A$ not$B$ C) =
$\dfrac{1}{5}\cdot\dfrac{3}{4}\cdot(1-x)+\dfrac{4}{5}\cdot\dfrac{1}{4}\cdot(1-x)+\dfrac{4}{5}\cdot\dfrac{3}{4}\cdot x = \dfrac{7}{20}(1-x)+\dfrac{12x}{20} = \dfrac{7+5x}{20}$.
Thus, $P : Q = 8 : 5 \rightarrow \dfrac{8+12x}{7+5x} = \dfrac{8}{5} \rightarrow 40 + 60x = 56 + 40x \rightarrow 20x = 16 \rightarrow x = \underline{\mathbf{\dfrac{4}{5}}}$.

C) The 5th term is $\dbinom{7}{4}\left(\dfrac{1}{2}x^2\right)^3\left(Ax^{-1}\right)^4 = \dfrac{7\cdot6\cdot5}{1\cdot2\cdot3}\cdot\dfrac{1}{8}\cdot A^4\cdot x^6\cdot x^{-4} = \dfrac{35A^4}{8}x^2 = \dfrac{70}{81}x^T$
$\rightarrow T = 2$ and $A^4 = \dfrac{16}{81} \rightarrow (A, T) = \underline{\left(\pm\mathbf{\dfrac{2}{3}, 2}\right)}$.