

**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 - FEBRUARY 2010
ROUND 7 TEAM QUESTIONS
ANSWERS**

A) (_____ , _____) D) _____

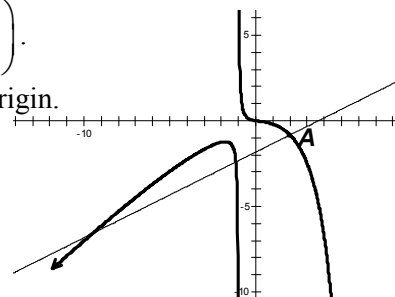
B) A B C E) (_____ , _____ , _____ , _____)

C) _____ F) _____

******* CALCULATORS ARE PERMITTED IN THIS ROUND *******

A) The graph of $f(x) = \frac{x^3 + x}{x^2 - 5x - 6}$ and $g(x) = \frac{3x - 11}{6}$ intersect at $A\left(2, -\frac{5}{6}\right)$.

Compute the coordinates (x, y) of the point of intersection furthest from the origin.



B) Which is larger $A = \sqrt[2008]{2008!}$, $B = \sqrt[2009]{2009!}$, $C = \sqrt[2010]{2010!}$

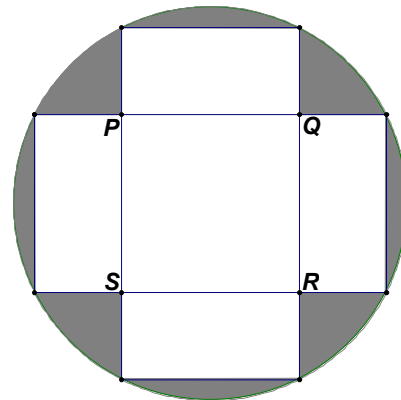
C) Compute $\cos\left(2\text{Arc}\cos\left(-\frac{3}{5}\right) + \text{Arc}\sin\left(-\frac{7}{25}\right)\right)$.

D) Sue is currently 24 years old. Eight years ago, the sum of the ages of her younger brother Al and her older sister Pam was 24. If all of these ages must be positive integers, compute the number of possible values for Al's current age.

E) Two $2 \times k$ rectangles are inscribed in a circle, where $k > 2$. Their intersection consists of 4 points P, Q, R and S which are vertices of a square. The maximum value of k for which the area of the shaded region is exactly half the area of the circle may be written in the form $\frac{A + B\sqrt{C - \pi(\pi + D)}}{\pi}$.

Determine the ordered quadruple (A, B, C, D) .

F) Assume the continued fraction
$$\frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \dots}}}$$
 converges to $\sqrt{2} - 1$.



Here are successive evaluations of this continued fraction: $\frac{1}{2}, \frac{1}{2 + \frac{1}{2}} = \frac{2}{5}, \frac{1}{2 + \frac{1}{2 + \frac{1}{2}}} = \frac{5}{12}, \dots$

That is, we have a sequence $(a_1, a_2, a_3, \dots) = \left(\frac{1}{2}, \frac{2}{5}, \frac{5}{12}, \dots\right)$. Adding 1, we have approximations of $\sqrt{2}$.

Thus, $(A_1, A_2, A_3) = \left(\frac{3}{2}, \frac{7}{5}, \frac{17}{12}\right)$ are the first three approximations of $\sqrt{2}$ generated by this continued fraction. Compute A_{10} .