MASSACHUSETTS MATHEMATICS LEAGUE
DECEMBER 2005
ROUND 7: TEAM QUESTIONS

ANSWERS

A)_____ **D)**_____

B)_____ **E)**_____

C)_____ **F)**_____

A) A right prism has a square base. Consider the set of all angles formed by a diagonal of a face and a diagonal of the prism. If one such angle has a tangent of 2, find the sum of the tangents of all the angles in the set.

B) Twin primes are primes that differ by 2. There are 12 pairs of twin primes between 1 and 180. Find the largest and smallest value of N if N – 1 and N + 1 are twin primes less than 180 and N has exactly eight factors.

C) The point P(a, b) is the first quadrant lattice point on $17x - 23y - 2 = 0$ closest to the origin. Find the area of the triangle with vertices P, (4,0) and (26, 1).

D) Solve for A in terms of B and C if A, B, and C > 1 and

$$\log_C AB - \frac{1}{\log_A C} = \log_C A + \log_{\sqrt{12}} 12$$

E) Al, Bob, and Carl each have different amounts of money. They share it as follows: Al gives Bob and Carl each 1/3 of his money. Bob then gives Al and Carl each 1/3 of his new total. Carl then gives Al and Bob each 1/3 of his new total. The result is that Al ends up with \$45 more than Bob and Bob ends with \$45 more than Carl. If after the first sharing Carl had \$78, how much money did Al end up with?

F) A, B, and C are consecutive vertices of a regular polygon with less than 25 sides. A rotation centered at B maps A onto C and C onto P. If A, B, and P are not collinear find the difference between the maximum and minimum possible values of $m\angle ABP$.