

**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2007 SOLUTION KEY**

Round 6

A) For $k = 2$ to 6, the expression $3k - 2$ produces the numbers 4, 7, 10, 13 and 16.
The sum of these 5 numbers is **50**.

B) Each of these terms is the sum of an infinite geometric sequence. Applying $\frac{a}{1-r}$,

$$t_1 = 1, t_2 = 1/2, t_3 = 1/3, t_4 = 1/4, \text{ etc} \\ 1 + 1/2 + 1/3 + 1/4 = 25/12 > 2 \rightarrow n = \underline{\mathbf{4}}$$

C) By definition, $x = \frac{2y}{1+y}$ and $y = \frac{4x}{x+2}$. Substituting for x in the second equation,

$$y = \frac{4\left(\frac{2y}{1+y}\right)}{\frac{2y}{1+y} + 2} = \frac{8y}{4y+2} = \frac{4y}{2y+1} \rightarrow 2y^2 + y = 4y \rightarrow 2y^2 - 3y = y(2y - 3) = 0 \rightarrow y = \frac{3}{2}$$

$$\text{and } x = \frac{3}{5/2} = \frac{6}{5}. \text{ Adding the required sum is } \frac{3}{2} + \frac{6}{5} = \frac{5+12}{10} = \underline{\mathbf{2.7}}$$