MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2008 SOLUTION KEY

Round 6

A) Pick one blue from the 5 available and two red from the 4 available without regard for order. The total number of [possible drawings is 3 from the 9 available also without regard for order. AND → MULTIPLY. Thus, this is simply a ratio of combinations.

$$p = \frac{\binom{5}{1}\cdot\binom{4}{2}}{\binom{9}{3}} = \frac{5\cdot 6}{\frac{9\cdot 8\cdot 7}{3\cdot 2\cdot 1}} = \frac{5\cdot 6}{3\cdot 4\cdot 5} = \underline{\mathbf{\frac{5}{14}}}$$

B) $(3+2i)^5 = 3^5 + 5\cdot 3^4(2i) + 10\cdot 3^3(2i)^2 + 10\cdot 3^2(2i)^3 + 5\cdot 3(2i)^4 + (2i)^5$

$= 243 + 405(2i) + 270(4i^2) + 90(8i^3) + 15(16i^4) + 32i^5$

$= 243 + 810i - 1080 - 720i + 240 + 32i$

→ $b = 122$ → $\sqrt{b-1} = \underline{\mathbf{11}}$

C) The number must be 96, 75, 72 or 81.
The probability of one of these numbers being generated is

$(1/2)(1/4) + 1/3(3/5) + (1/3)(1/10) + (1/6)(3/8) = 1/8 + 1/5 + 1/30 + 1/16$

$= (30 + 48 + 8 + 15)/240 = \underline{\mathbf{101/240}}$