MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 - FEBRUARY 2017 SOLUTION KEY

Round 1

A) $g(x)=\dfrac{8-4x}{3}+1=0 \Rightarrow 8-4x=-3 \Rightarrow x=\underline{\mathbf{\dfrac{11}{4}}}$.

B) Given: $y=f(x)=\dfrac{k}{x+2}$ To find the inverse function f^{-1}, we interchange x and y and then resolve for y in terms of x.

$x=\dfrac{k}{y+2} \Rightarrow xy+2x=k \Rightarrow y=f^{-1}(x)=\dfrac{k-2x}{x}$

Thus, $f(2)=f^{-1}(4)\cdot f(4) \Leftrightarrow \dfrac{k}{4}=\dfrac{k-8}{4}\cdot\dfrac{k}{6} \Leftrightarrow \dfrac{1}{4}=\dfrac{k-8}{24} \Rightarrow k=\underline{\mathbf{14}}$.

Solution #2:
Without bothering to explicitly find $y=f^{-1}(x)$, we note that any input to f^{-1} is output from f and solve $4=\dfrac{k}{x+2}$ for x. Cross multiplying, $k=4x+8 \Rightarrow x=\dfrac{k-8}{4}$ and the same result follows.

C) Since the given line with slope 9 passes through (2, 0), its equation must be $y=9x-18$.
At the points of intersection, $f(x)=x^3-6x^2-4x+24=9x-18 \Leftrightarrow x^3-6x^2-13x+42=0$
Since we know that $x=2$ is a root of this equation, we can use synthetic substitution to determine the other roots.

$$\begin{array}{lrrrr} & 1 & -6 & -13 & 42 \\ 2 & 1 & -4 & -21 & 0 \end{array}$$

$\Rightarrow x^3-6x^2-4x+42=(x-2)(x^2-4x-21)=(x-2)(x-7)(x+3)=0$

$\Rightarrow x=7,-3$

Substituting in $y=9x-18$, the coordinates of the points of intersection are $\underline{\mathbf{(-3,-45),\ (7,45)}}$.