**MASSACHUSETTS MATHEMATICS LEAGUE**
**MARCH 2006 BRIEF SOLUTIONS**

**Round One:**

A. $(2 + 0 + x^2) – (0 + x^2 + x) = 5$ means $2 – x = 5$, so $x = –3$.

B. First equation simplifies to $x – 1 = 2y + 1$; sub for $2y + 1$ in second to get $2(x – 1)^2 = 50$, so $x – 1 = \pm 5$. If $x = 6$, $y = 2$; if $x = – 4$, $y = – 3$.

C. First eqtn: $y^2 = [(1 + x\sqrt{2})(1 – x\sqrt{2})]^2$ so $y = \pm(1 – 2x^2)$ so $3x + 1 = 1 – 2x^2$ meaning $x = 0$ or $x = –3/2$; or $3x + 1 = 2x^2 – 1$ meaning $x = 2$ or $x = –1/2$
The sum of the four numbers is 0.

**Round Two:**

A. $\sqrt{2^9} + \sqrt{2^8} = 2^{4.5} + 2^4 = 16 + 2^3 2^{1.5} = 16 + 8\sqrt{8}$ so $(a, b) = (16, 8)$.

B. Replace $2\sqrt{2}$ .with $\sqrt{8}$. Note $\dfrac{1}{\sqrt{x+1} + \sqrt{x}}\left(\dfrac{\sqrt{x+1} - \sqrt{x}}{\sqrt{x+1} - \sqrt{x}}\right) = \sqrt{x+1} - \sqrt{x}$ so

$$2\left(\sqrt{8} - \sqrt{7} + \sqrt{7} - \sqrt{6} + \sqrt{6} - \sqrt{5} + \ldots + \sqrt{3} - \sqrt{2}\right) = 2(\sqrt{8} - \sqrt{2}) = 2(2\sqrt{2} - \sqrt{2})$$

C. 2^(-6$x$) / 2^(15) = 2^(3($x$ + 4)$x$) so $–6x – 15 = 3x^2 + 12x$ etc.

**Round Three:**

A. $(k – 3)(-1)^3 + (2k – 5)(-1)^2 + (k – 7)(-1) + (k – 10) = 0$ simplifies to $k – 5 = 0$

B. $k(x – 1)(3x + 4)(2x – 3) = k(6x^3 – 7x^2 – 11x + 12) \rightarrow 12k = -12 \rightarrow k = -1$
$\rightarrow f(x) = -6x^3 + 7x^2 + 11x – 12 \rightarrow f(-1) = 6 + 7 – 11 – 12 = -10$.

C. $P(x) = Q_1(x)(x – 2)(x + 3) + ax + b$ [2nd degree divisor can leave a 1st degree remainder.]
$P(x) = Q_2(x)(x – 2) – 3 \rightarrow P(2) = -3 = 2a + b$
$P(x) = Q_3(x)(x + 3) + 2 \rightarrow P(-3) = 17 = -3a + b \rightarrow a = -4, b = 5$

**Round Four:**

A. $30y = 24{,}072 \rightarrow y$-intercept = (0, 802.4) $20x = 24{,}072 \rightarrow x$-intercept = (1203.6, 0)

B. $5s + 6p = 147$ and $40(0.60s) + 40(0.5p) = 578$ or $24s + 20p = 578$; system solves to $s = 12$, $p = 14.5$.

C. If $x$ – a is negative, $|x – a| = a – x = a + 2$ means $x = – 2$ violating $x > 0$. Thus, $x – a$ is nonnegative so $|x – a| = x – a = a + 2$, so $x + a = a + 2 + (2a) = 3a + 2$ maximized when $a = 668$, so $x = 2006$.

**Round Five:**

A. If $A$ is a base angle $180 = A + A + 7A$. $A = C = 20$; if $B$ is a base angle $180 = 7A + 7A +$ A. $A = 12$, $C = B = 7(12) = 84$.

B. $JK = 750$ (Pythagoras, or 3-4-5 scaled by 150) $\Delta MKP \sim \Delta PKJ$ so $MK/600 = 600/750$ and $MK = 480$. $\Delta MNK \sim \Delta JPK$, so $MN/450 = 480/750$ and $MN = 288$.