MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2012 SOLUTION KEY

Team Round

C) $\begin{cases} 64A+16B+4C+D=204 \\ 27A+9B+3C+D=104 \\ 8A+4B+2C+D=46 \\ A+B+C+D=18 \end{cases}$ Number these equations (#1) through (#4) respectively.

(#1) – (#2) $\Rightarrow$ (#5) $37A + 7B + C = 100$ (#5) – (#6) $\Rightarrow$ (#8) $18A + 2B = 42$ (#8) – (#9) $\Rightarrow 6A = 12$
(#2) – (#3) $\Rightarrow$ (#6) $19A + 5B + C = 58$ (#6) – (#7) $\Rightarrow$ (#9) $12A + 2B = 30$
(#3) – (#4) $\Rightarrow$ (#7) $7A + 3B + C = 28$
Thus, $A = 2$. Substituting in (#9), $B = 3$. Substituting in (#7), $14 + 9 + C = 28 \Rightarrow C = 5$.
Substituting in (#4), $D = 8 \Rightarrow$ **(2, 3, 5, 8)**.

D) If he goes hitless, his average is $\frac{179}{448+x} < 0.393$

To lose over 7 points on his average, I guesstimate he made at least 7 plate appearances $\frac{179}{455} \approx 0.3934$. This average is too high, implying he made at least 8 plate appearances in the doubleheader and $x = 8$. Now we know $\frac{179+h}{456} \approx 0.406$. To maintain his average he needed at least 2 hits in 5 plate appearances; therefore, he must have gotten at least 4 hits to maintain a 0.400 average. To increase his average 6 points, I guesstimate he got $h = 6$ hits. $\frac{185}{456} = 0.4057$ and we have confirmation.

$(h, x) =$ **(6, 8)**.

E) The absolute value inside the radicals insures that the radicands will be positive, but since all real values of x will produce a nonnegative radicand, we must subdivide the domain into four regions and be on guard for extraneous answers. The critical values (where the radicands become zero) are –4, 1 and +4.

x +4----------0++
x - 1----------0++++++++++++++++++++++++++++++++++++
x - 4----------0+++++++++

- 4 **+ 1** **+ 4**

Squaring both sides, $|x+4|+|x-1|+2\sqrt{|x+4|}\sqrt{|x-1|} = |x-4|$

Case 1: $x \le -4$

$-x-4+1-x+2\sqrt{-x-4}\sqrt{1-x} = 4-x$

$\Rightarrow 2\sqrt{-x-4}\sqrt{1-x} = 4-x+2x+3 = x+7$

$\Rightarrow 4\left(x^2+3x-4\right) = x^2+14x+49$

$\Rightarrow 3x^2 - 2x - 65 = (3x + 13)(x - 5) = 0 \Rightarrow x =$ $\underline{-\frac{\mathbf{13}}{\mathbf{3}}}$ (5 is extraneous)