

**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2012 SOLUTION KEY**

Team Round

$$C) \begin{cases} 64A + 16B + 4C + D = 204 \\ 27A + 9B + 3C + D = 104 \\ 8A + 4B + 2C + D = 46 \\ A + B + C + D = 18 \end{cases} \quad \text{Number these equations (\#1) through (\#4) respectively.}$$

$$(\#1) - (\#2) \Rightarrow (\#5) \quad 37A + 7B + C = 100 \quad (\#5) - (\#6) \Rightarrow (\#8) \quad 18A + 2B = 42 \quad (\#8) - (\#9) \Rightarrow 6A = 12$$

$$(\#2) - (\#3) \Rightarrow (\#6) \quad 19A + 5B + C = 58 \quad (\#6) - (\#7) \Rightarrow (\#9) \quad 12A + 2B = 30$$

$$(\#3) - (\#4) \Rightarrow (\#7) \quad 7A + 3B + C = 28$$

$$\text{Thus, } A = 2. \text{ Substituting in (\#9), } B = 3. \text{ Substituting in (\#7), } 14 + 9 + C = 28 \Rightarrow C = 5.$$

$$\text{Substituting in (\#4), } D = 8 \Rightarrow \underline{(2, 3, 5, 8)}.$$

$$D) \text{ If he goes hitless, his average is } \frac{179}{448+x} < 0.393$$

To lose over 7 points on his average, I guesstimate he made at least 7 plate appearances

$$\frac{179}{455} \approx 0.3934. \text{ This average is too high, implying he made } \underline{\text{at least}} \text{ 8 plate appearances in the}$$

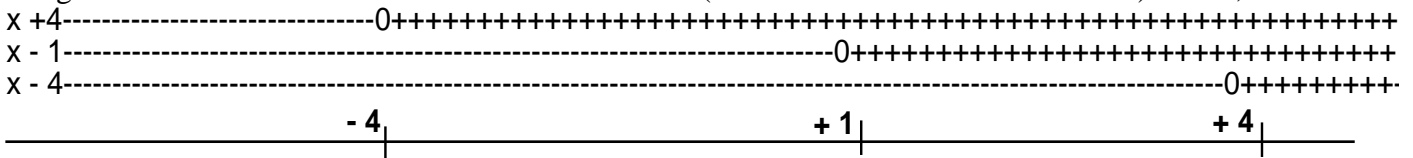
$$\text{doubleheader and } x = 8. \text{ Now we know } \frac{179+h}{456} \approx 0.406. \text{ To maintain his average he needed at least 2}$$

hits in 5 plate appearances; therefore, he must have gotten at least 4 hits to maintain a 0.400 average. To

$$\text{increase his average 6 points, I guesstimate he got } h = 6 \text{ hits. } \frac{185}{456} = 0.4057 \text{ and we have confirmation.}$$

$$(h, x) = \underline{(6, 8)}.$$

E) The absolute value inside the radicals insures that the radicands will be positive, but since all real values of x will produce a nonnegative radicand, we must subdivide the domain into four regions and be on guard for extraneous answers. The critical values (where the radicands become zero) are -4 , 1 and $+4$.



$$\text{Squaring both sides, } |x+4| + |x-1| + 2\sqrt{|x+4|}\sqrt{|x-1|} = |x-4|$$

Case 1: $x \leq -4$

$$-x-4+1-x+2\sqrt{-x-4}\sqrt{1-x} = 4-x$$

$$\Rightarrow 2\sqrt{-x-4}\sqrt{1-x} = 4-x+2x+3 = x+7$$

$$\Rightarrow 4(x^2+3x-4) = x^2+14x+49$$

$$\Rightarrow 3x^2-2x-65 = (3x+13)(x-5) = 0 \Rightarrow x = -\frac{13}{3} \text{ (5 is extraneous)}$$