MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2008 SOLUTION KEY

Round 3

A) Potential extraneous solutions: ($\cos x = 0$) $x \neq \pi/2 + n\pi$

$2(\cos x - \sin x) = 1 - \tan x = 1 - \dfrac{\sin x}{\cos x} \rightarrow 2\cos x(\cos x - \sin x) = \cos x - \sin x$

$(\cos x - \sin x)(2\cos x - 1) = 0$

$\rightarrow \cos x = \sin x \rightarrow x =$ **<u>$\pi/4, 5\pi/4$</u>**

$\rightarrow \cos x = 1/2 \rightarrow x =$ **<u>$\pi/3, 5\pi/3$</u>**

B) $\sin 140^\circ \cos 220^\circ = \dfrac{\cos x}{\sec 60^\circ} \rightarrow$

$\cos x = 2\sin 140^\circ \cos 220^\circ = \sin(A - B) + \sin(A + B) = 2\sin A \cos B$

$\rightarrow A = 140, B = 220$

Thus, $\cos x = \sin(-80) + \sin 360 = -\sin(80) = -\cos(10)$

Thus, x denotes a related value of 10° in quadrant 2 or 3 $\rightarrow x =$ **<u>170°, 190°</u>**

C) $\tan^2 x \cdot \sec^2 x - \tan^2 x - \sec^2 x + 1 = 0 \rightarrow \tan^2 x(\sec^2 x - 1) - (\sec^2 x - 1) = (\tan^2 x - 1)(\sec^2 x - 1) = 0$

$\rightarrow \tan x = \pm 1 \rightarrow x =$ 45°, 135°, 225°, 315° or $\sec x = \pm 1 \rightarrow x =$ 0°, 180°

$\rightarrow 900 - 6 =$ **<u>894</u>**