MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2012 SOLUTION KEY

Round 1

A) The circle has radius 6 and area 36π.

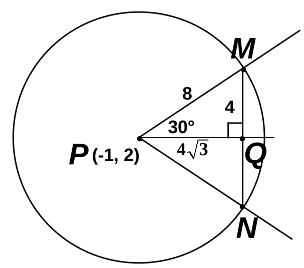
The ellipse $\left(\dfrac{x^2}{9}+\dfrac{y^2}{4}=1\right)$ has $a = 3$ and $b = 2 \Rightarrow$ area = 6π

Thus, the difference in areas is $\underline{\mathbf{30\pi}}$.

B) C_1 has its center at $(-1, 2)$ and a radius of 8.

Since $\overline{MN}$ is vertical, a horizontal line through P will be the perpendicular bisector of $\overline{MN}$ and the bisector of $\angle P$.

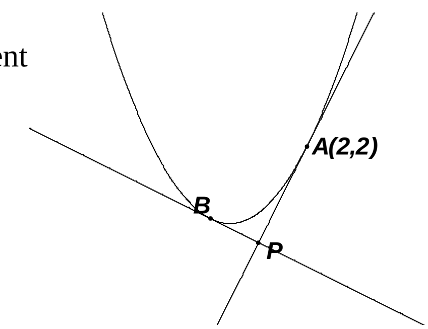
Thus, we have a $30 - 60 - 90$ right triangles and $MQ = 4$, $PQ = 4\sqrt{3}$ and the coordinates of M must be $\underline{\left(\mathbf{4\sqrt{3}-1,\ 6}\right)}$.



C) The tangent through $(2, 2)$ has the equation $2x - y = 2$.

If the coordinates of B are $\left(b, \dfrac{1}{2}b^2\right)$, then the equation of the tangent through B is $bx - y = \dfrac{b^2}{2}$. This line has slope b and is perpendicular to $2x - y = 2$, so $b = -1/2$.

The coordinates of B are then $\left(-\dfrac{1}{2}, \dfrac{1}{8}\right)$.



This second line then has equation $-\dfrac{1}{2}x - y = \dfrac{\left(-\frac{1}{2}\right)^2}{2}$ or $x + 2y = -\dfrac{1}{2} + \dfrac{1}{4} = -\dfrac{1}{4}$

and both equations simplify to $4x + 8y = -1$.

Solving $\begin{cases} 2x - y = 2 \\ 4x + 8y = -1 \end{cases}$, $x = \dfrac{3}{4}, y = -\dfrac{1}{2}$ and $\underline{P\left(\mathbf{\dfrac{3}{4}, -\dfrac{1}{2}}\right)}$.