MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2015
ROUND 7 TEAM QUESTIONS

ANSWERS

A) _____ D) _____

B) _____ E) _____

C) _____ F) _____

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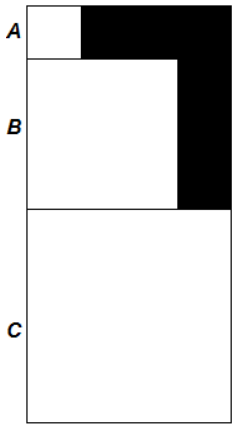
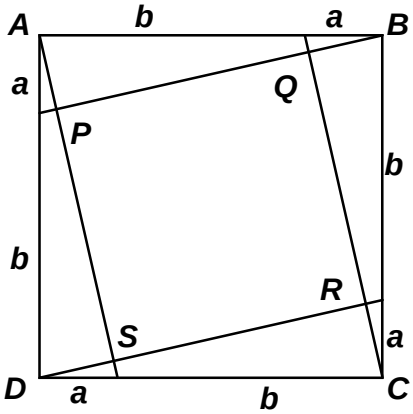
A) Consider the parabola P whose equation is $y = 2x^2 - 8x + C$.
The graph of P passes through the point $A(1, 1)$.
The line $\overleftrightarrow{AF}$, where F is the focus of P, intersects P in a second point Q.
Compute the x-coordinate of Q.



B) Compute <u>all</u> possible ordered pairs of real numbers (x, y) for which
$$y = \sqrt{x-1} \text{ and } x = \sqrt{72y+1}.$$

C) Specify all values (in radians) over $0 < x < 2\pi$ for which $\left|8\sin^2 x - 5\right| < 1$.

D) For positive integers A, B and C, $A^2 + B^2 = C^2$, where $B > A$ and $C = B + 2$. Three squares with <u>sides</u> A, B and C are stacked as in the diagram at the right.
The area of the shaded region is 910.
Compute $A + B + C$.



E) The ratio of the area of square $ABCD$ to the area of the square $PQRS$ can be expressed as $1 + \frac{2k}{b^2}$.
Determine an expression for k in terms of a and b.

F) 16, $5a - 4$ and a^2 are three <u>distinct</u> rational numbers.
For specific values of a, one of these rational numbers is the arithmetic mean of the other two. Compute the arithmetic mean of <u>all</u> the possible a-values.