Team Round - continued

D) $\frac{4}{5} < \frac{a}{2400-a} < \frac{5}{6}$ → $24(2400 - a) < 30a < 25(2400 - a)$

→ $54a > 24(2400)$ → $a > \frac{4}{9}(2400) \approx 1066.6+$ → min 1067

→ $55a < 25(2400)$ → $a < \frac{5}{11}(2400) \approx 1090.9+$ → max 1090

An even integer in the numerator would produce an even number in the denominator and the fraction would be reducible.
Likewise for numerators that are multiples of 5.
Thus, the following fractions must be examined for reducibility:

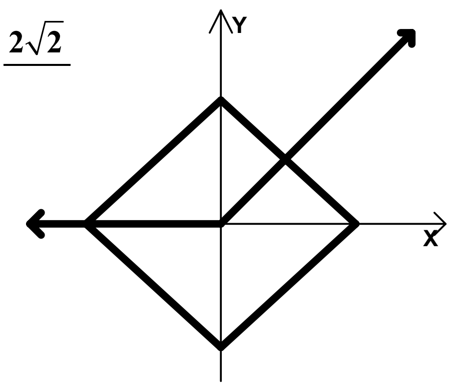
$\frac{1067}{1333}, \frac{1069}{1331}, \cancel{\frac{1071}{1329}}, \frac{1073}{1327}, \cancel{\frac{1077}{1323}}, \frac{1079}{1321}, \frac{1081}{1319}, \cancel{\frac{1083}{1317}}, \frac{1087}{1313}, \cancel{\frac{1089}{1311}}$

Each of the crossed out fractions is reducible, since both numerator and denominator are divisible by 3. The remaining **6** fractions are irreducible.
How do you verify this with a calculator? without a calculator?

E) For the first equation, if $x \leq 0$, $y = 0$ and for $x \geq 0$, $y = x$.
The second equation is a square with vertices on the axes at $(\pm A, 0)$ and $(0, \pm A)$
The points of intersection are at $(-A, 0)$ and $(A/2, A/2)$. Using the distance formula,

$d = \sqrt{\left(\frac{3A}{2}\right)^2 + \left(\frac{A}{2}\right)^2} = \frac{1}{2}\sqrt{10A^2} = 2\sqrt{5}$ → $10A^2 = 80$ → $A = \mathbf{2\sqrt{2}}$

($-\mathbf{2\sqrt{2}}$ is rejected, since A is the sum of two absolute values.)



F) By definition, $T_1 = T_0 + 2T_{-1}$ → $T_{-1} = \frac{T_1 - T_0}{2}$ Thus, we set out to find these two values.
Since we were given $T_n = T_{n-1} + 2T_{n-2}$, $T_2 = 1.5$ and $T_4 = 1.5T_0$
(1) $T_2 = T_1 + 2T_0 = 1.5$ → $T_1 = 1.5 - 2T_0$
(2) $T_4 = T_3 + 2T_2 = T_3 + 2(T_1 + 2T_0) = T_3 + 2(1.5 - 2T_0 + 2T_0) = T_3 + 3 = 1.5T_0$
(3) $T_3 = T_2 + 2T_1 = 1.5 + 2(1.5 - 2T_0) = 4.5 - 4T_0$
Thus, $1.5T_0 - 3 = 4.5 - 4T_0$ → $5.5T_0 = 7.5$ → $T_0 = 15/11$
Substituting in (1) above, $T_1 = 1.5 - 30/11 = -27/22$

$T_{-1} = \frac{-\frac{27}{22} - \frac{15}{11}}{2} = \frac{-27-30}{44} = \mathbf{-\frac{57}{44}}$