MASSACHUSETTS MATHEMATICS LEAGUE
OCTOBER 2005 BRIEF SOLUTIONS

Round One:

A. Base has side 2.5, area 6.25 Vol = 9(6.25)/3 = 18.75

B. Hypotenuse KA = $\sqrt{98}$ while DQ and AQ are each $\sqrt{8^2 + 4^2} = \sqrt{80}$ Since $\overline{KD} \perp \overline{DQ}$ hypotenuse KQ = $\sqrt{80 + 49} = \sqrt{129}$.

C. #1 ht = 30, radius = 40, vol = 16000π. #2 ht = 40, radius = 30, vol = 12000π. Other 2 have radius = 24 (40x30/50) and hts of 18 and 32, vol=9600π.

Round Two:

A. Throws along hypotenuse of rt triangle (from shortstop to second to first). $\sqrt{90^2 + 30^2} = \sqrt{9000} = 94.8683... \approx 94.9$

B. $AE^2 = 12^2 + EC^2$; $EC^2 = EB^2 - 4^2$; $EB^2 = 16^2 - AE^2$ combined gives $AE^2 = 144 + 256 - AE^2 - 16$ so $AE^2 = 192$ (or AE is geom. mean of AC and AB)

C. Two diagonals create rt triangles with leg of 9 and hypotenuse of 41 or 15 so other leg is 40 or 12 so area is ½(52)(18) or ½(28)(18) difference is 216.

Round Three:

A. $16(x + 2) - 9(x - 1) = 144$; $7x + 33 = 144$; $7x = 103$; $x = 14\frac{5}{7}$

B. x = second leg avg. speed. Total distance was $60(2.5) + x(3.5) = 39(6)$. Solve x= 24

C. If x=0, $15 - 5B = 65$ so B = –10. if x=1, $3A + 10A + 65 = 117$, A = 4. Sum is –6

Round Four:

A. $(8\frac{1}{2} \times 11 \times 2\frac{1}{8}) / 500 = (17 \times 11 \times 17) / (2 \times 6 \times 500) = \frac{3179}{8000}$ which is irreducible.

B. 1/b = 4/9 – 5/18 =1/6 so b=6 , a =9.

C. ½ = n/8 – n/14 yields n = 28/3 = 9 and 1/3 hours after 1 p.m.