MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 – NOVEMBER 2013 SOLUTION KEY

Round 6

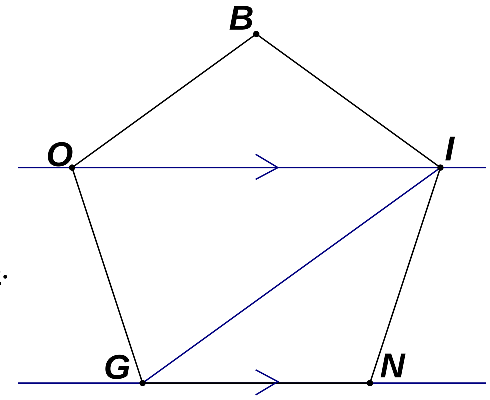
A) Ignoring the parallels, $m\angle ING = \dfrac{180\cdot 3}{5} = 108°$.

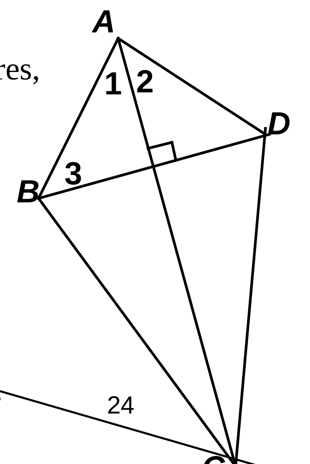
As base angles of isosceles triangles *BIO* and *ING*,

$m\angle BIO = m\angle NGI = \dfrac{180-108}{2} = 36°$

$\Rightarrow\ m\angle OGI = 108 - 36 = 72°$.

Therefore, $m\angle ING + m\angle BIO + m\angle OGI$ = 108 + 36 + 72 = **<u>216</u>**.





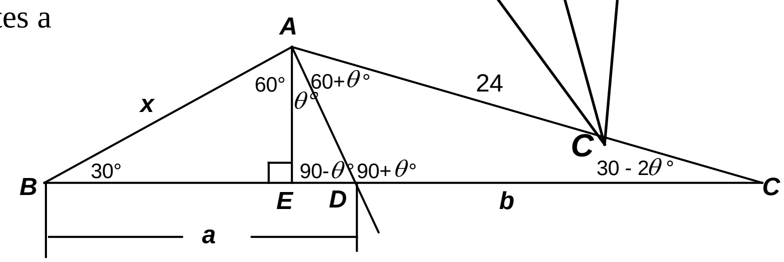
B) Since angles *BAC* (1) and *DAC* (2) have the same vertex and equal measures, they must be reflection angles across the main diagonal. To insure that the diagonals are perpendicular, angles *BAC* and *ABD* (3) must be complementary. Therefore,

$(2x+35)+(80-3x)=90 \Rightarrow 115-x=90 \Rightarrow x=\underline{\mathbf{25}}$.

C) Dropping a perpendicular from *A* to $\overline{BC}$ creates a 30-60-90 right triangle.

$AE = \dfrac{x}{2}, BE = \dfrac{x\sqrt{3}}{2}$ and $DE = \dfrac{x\sqrt{3}}{4}$

$\Rightarrow a = BD = \dfrac{x\sqrt{3}}{2} + \dfrac{x\sqrt{3}}{4} = \dfrac{3x\sqrt{3}}{4}$



Appling the angle bisector theorem,

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$$\frac{a}{x} = \frac{b}{24} \Leftrightarrow \frac{\frac{3x\sqrt{3}}{4}}{x} = \frac{b}{24} \Leftrightarrow b = \underline{\mathbf{18\sqrt{3}}}.$$

FYI: Without the fact that *BE* = 2*DE*, a unique value for $m\angle CAD$ could not have been determined. In ΔACD, we know two side lengths (*AC* and *DC*, but not the *<u>included</u>* angle). In fact, the only invariant in ΔCAD is *DC*. Changing *x* changes the length of $\overline{AD}$ and the measures of all three angles in ΔCAD, but the length of $\overline{DC}$ remains the same.

We know that $\theta < 30°$ (to insure that $\angle BAC$ is obtuse). In this problem, in ΔADE, there is a unique acute angle whose tangent has this value. Its exact designation is $Tan^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$ and its approximate value is 40.9°, so with this additional condition (*BE* = 2*DE*) , ΔCAD is <u>uniquely</u> determined.

An easy way to confirm this approximation would be to draw a right triangle with legs of length 2 and $\sqrt{3}$; then measure the smaller acute angle with a protractor!