MASSACHUSETTS MATHEMATICS LEAGUE CONTEST 6 - MARCH 2008 SOLUTION KEY

Round 4

A) Subtracting 1 from both sides of the equation, $2 = \frac{2}{1} = \frac{2}{3 + \frac{1}{x}}$

Since the numerators are equal, we can equate the denominators.

$$1 = 3 + \frac{1}{x} \Rightarrow \frac{1}{x} = -2 \Rightarrow x = -\frac{1}{2}$$

B) Let 5x denote the number of additional questions. Then

$$\frac{24+2x}{75+5x} = 35\% = \frac{7}{20} \rightarrow 480 + 40x = 525 + 35x \rightarrow x = 9 \rightarrow \text{# questions} = 75 + 5(9) = \underline{120}$$

C)
$$y = \frac{2x-1}{5} \rightarrow 10 < x + \frac{2x-1}{5} < 20 \rightarrow 50 < 7x - 1 < 100 \rightarrow 7.285 < x < 14.428 < \dots x = 8, 9, \ldots 14$$

Substituting, x = 8 immediately gives an integer result.

Since the first equation represents a line with a slope of 2/5, incrementing x by 5 gives the next x for which y is also an integer. Thus, (8,3) and (13,5)