Round Five:

A. Power of pt M = 9(4)=36=MK(MN) so MK=MN=6. Power of pt L= LK(LN) = 25(13) = LP^2.

B. Equil. triangle of side 6 has area $9\sqrt{3}$ plus three 300° sectors = 3(5/6)9π

C. IB is alt to hyp of rt triangle EIF so IF is geom.. mean of FB & FE = $8\sqrt{6}$. HF:IF = DF:CF = 2:3 so HF = (2/3) $8\sqrt{6}$.

Round Six:

A. Constant difference is 4/2=2; $a_3 = a_0 + 3(2)$ so $a_0 = -2006$ and therefore $a_{2005} = -2006 + 2005(2)$.

B. $81d = g_0 r^3$ while $9d = g_0 r$; dividing gives $r = \pm 3$. If r = 3, $g_n = 2 \cdot 3^n$ and since $a_9 = 6$, $a_n = \frac{2}{3}n$ so $a_2 + g_2 = \frac{58}{3}$. If r = –3, $a_n = \frac{-2}{3}n$ and $a_2 + g_2 = \frac{50}{3}$.

C. Will accumulates \$20x12x15=\$3,600. Shauna has the geometric sum 100*1.08 + $100*(1.08)^2 + 100*(1.08)^{15} = 100*1.08*(1.08^{15} - 1)/0.08 \approx$ \$2,932.43.

Team Round:

A. $g^{-1}(3) = 20$. If $f^{-1}(3) = 20$ then $f(20) = \frac{20a+7}{20-1} = 3$ so $a = 2.5$.

$f^{-1}(x) = \frac{x+7}{x-2.5} = g^{-1}(x) = 7x - 1$ so $(7x-1)(x-2.5) = (x+7)$ with solutions of x = 3 and x = –3/14.

B. $T=k^2$ while S=j(j+1) so $S–T = j^2 + j - k^2$. Maximize j at 99 gives $S–T=9900–k^2$ which is a multiple of 25 if k^2 is so k is divisible by 5 thus get max if k=95. The only pairs (j,k) left whose sum exceeds 99+95 are (98,97) and (97,98) both of which fail by inspection. Thus the answer is **194**

C. Take tangent of both sides and apply tangent of a sum theorem to get $\frac{A + 4/A}{1 - A(4/A)} = \frac{-5}{3}$ so $A + \frac{4}{A} = 5$ thus A= 4 or 1 making the left side of the original eqtn $\frac{\pi}{4} + Tan^{-1}(4)$ which is between $\frac{\pi}{2}$ and $\frac{3\pi}{4}$. Since $Tan^{-1}(\frac{-5}{3})$ is in the fourth quadrant, we need n=1.

D. Let x= time at 60mph . Total dist = 60x + 45(2x) +2(60x)=270x. Total time=x+2x+(120x/36)=(19/3)x . Avg speed is 270x / (19/3)x = 810/19.