MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 – MARCH 2013 SOLUTION KEY

Round 1

A) There will only be an infinite number of solutions if
these equations which look different are actually equivalent equations for the <u>same</u> line.
Multiplying the second equation by 4, we have $2x - y = 4k \Rightarrow y = 2x - 4k$.

Equating the expressions for y, $4k = 3 \Rightarrow k = \underline{\mathbf{\frac{3}{4}}}$.

B) Substituting, $\begin{cases} 1 = 2(-2) + A + B \\ 1 - A = \frac{1}{2}(-2 - B) \end{cases} \Leftrightarrow \begin{cases} A + B = 5 \\ 1 - A = \frac{-(2+B)}{2} \end{cases} \Leftrightarrow A - 1 = \frac{(7 - A)}{2} \Leftrightarrow A = 3, B = 2 \Rightarrow$ **<u>(3, 2)</u>**.

C) $\begin{vmatrix} 3k-5 & 3 \\ 4 & k-2 \end{vmatrix} = (3k-5)(k-2) - 12 = 3k^2 - 11k - 2$

Evaluating the 3 x 3 determinant using the weaving method:
Append copies of the entries in the left and middle columns to the original matrix.
Sum the three diagonal down-products. Call it S_1
Sum the three diagonal up-products. Call it S_2.
Subtract $(S_1 - S_2)$.

$$\begin{vmatrix} 1 & k-1 & -2 \\ 1 & 2 & -1 \\ 7-2k & -3 & 0 \end{vmatrix} \Rightarrow \begin{matrix} 1 & k-1 & -2 & 1 & k-1 \\ 1 & 2 & -1 & 1 & 2 \\ 7-2k & -3 & 0 & 7-2k & -3 \end{matrix}$$

$$\Leftrightarrow \left(1 \cdot 2 \cdot 0 + (k-1) \cdot -1 \cdot (7-2k) + (-2 \cdot 1 \cdot -3)\right) - \left((7-2k) \cdot 2 \cdot -2 + (-3 \cdot -1 \cdot 1 + 0 \cdot 1 \cdot (k-1)\right)$$

$$\Leftrightarrow \left(\left(2k^2 - 9k + 7\right) + 6\right) - \left(-28 + 8k + 3\right) = 2k^2 - 17k + 38$$

Equating and re-arranging terms, $k^2 + 6k - 40 = (k-4)(k+10) = 0 \Rightarrow k = \underline{\mathbf{4, -10}}$.