Team Round - continued

D) Simplifying $3^{\left(\log_3 x^4 - \frac{1}{\log_x 3}\right)} + 2^{2\log_2 x} + 7^{5\log_7 x}$, the 2nd and 3rd terms are clearly, x^2 and x^5, but let's look carefully at the 1st term. Since x occurs as a base of the logarithm, $x \neq 1$.

$\log_3 x^4 - \dfrac{1}{\log_x 3} = \log_3 x^4 - \log_3 x = \log_3\left(\dfrac{x^4}{x}\right) = \log_3 x^3$ Thus, the 1st term is, in fact, x^3.

Thus, we have $x^3 + x^2 + x^5 = 3x^4 \Rightarrow x^2(x^3 - 3x^2 + x + 1) = 0$
Since x is the argument of the log function, $x > 0$ and the only roots come from the cubic factor.

By synthetic substitution, $1 \rfloor$
$$\begin{array}{rrrr} 1 & -3 & 1 & 1 \\ & 1 & -2 & -1 \\ \hline 1 & -2 & -1 & 0 \end{array}$$
$\Rightarrow (x-1)(x^2 - 2x - 1) = 0 \Rightarrow \cancel{x=1}$ or $x = \dfrac{2 \pm 2\sqrt{2}}{2}$

$\Rightarrow x = \underline{\mathbf{1+\sqrt{2}}}$ $\qquad (1-\sqrt{2} < 0$ and is, therefore, also extraneous.)

E) First condition: $\dfrac{G}{B} = \dfrac{7}{11} \Leftrightarrow G = \dfrac{7}{11}B$

Second condition : $\dfrac{G-3}{B+2} = \dfrac{5}{8}$

Cross multiplying, $8G - 24 = 5B + 10$

$\Leftrightarrow 8\left(\dfrac{7}{11}B\right) - 24 = 5B + 10 = \dfrac{55}{11}B + 10 \Leftrightarrow \dfrac{B}{11} = 34 \Leftrightarrow B = 374$

According to the first condition, $G = \dfrac{7}{11}(34 \cdot 11) = 238$

Thus, in the fall, the total number of students is $(374 + 2) + (238 - 3) = \underline{\mathbf{611}}$.

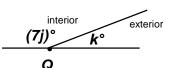F) Since Q has five times as many sides as P, the exterior angle of $P\left(\dfrac{360}{n}\right)$ is five times as large as the exterior angle of $Q\left(\dfrac{360}{5n}\right)$. The relationship between the interior and the exterior angles of these two polygons is summarized in the following diagram:

Thus, $5j + 5k = 180 = 7j + k \Rightarrow j = 2k$
Substituting, $7(2k) + k = 180 \Rightarrow k = 12$.
Since an exterior angle of Q measures 12°,
Q must have 30 sides. As the number of sides increases,
the measure of the interior angle increases.
This, if R is an N-gon, N must be the smallest factor of 360 larger than 30, i.e. 36°, producing an exterior angle of 10° and an interior angle of $\underline{\mathbf{170}}$°.