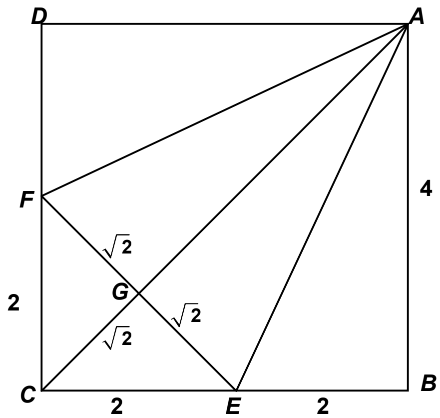
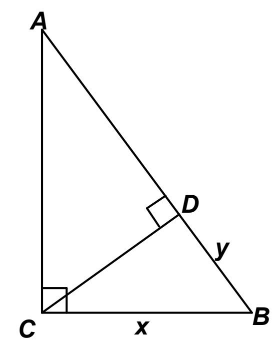
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2009 SOLUTION KEY

Round 2



A) Solution #1 (arithmetic only)
Square – 3 right triangles!
The area of ΔABE is 16 – (4 + 4 + 2) = 16
Thus, the required ratio of 6 : 16 = **<u>3 : 8</u>**

Solution #2 (algebraic)
In isosceles triangle CFE, $CG = GF = GE = \sqrt{2}$
Since the diagonal $AC = 4\sqrt{2}$, $AG = 3\sqrt{2}$
Area of $\Delta AEF = \frac{1}{2}\left(2\sqrt{2}\right)\left(3\sqrt{2}\right) = 6$ ➔ 6 : 16 = **<u>3 : 8</u>**



B) $AC = 10$. Using the Pythagorean Theorem in ΔABC and ΔBCD,

$$\begin{cases} (1)\ x^2 + 100 = (y+8)^2 \\ (2)\ x^2 - y^2 = 36 \end{cases}$$

Expanding (1), we have $x^2 - y^2 = 16y + 64 - 100$
Substituting for $x^2 - y^2$, $16y = 72$ ➔ $y = 9/2$
Then $x^2 = 36 + \frac{81}{4} = \frac{225}{4}$
➔ $x = 15/2$ and the required ordered pair is $\underline{\left(\mathbf{\frac{15}{2}, \frac{9}{2}}\right)}$

Alternative method: Note $\Delta ADC \sim \Delta CDB$ ➔ $\frac{AD}{CD} = \frac{DC}{DB} = \frac{AC}{CB}$ ➔ $\frac{8}{6} = \frac{6}{y} = \frac{10}{x}$

➔ $(x, y) = \underline{\left(\mathbf{\frac{15}{2}, \frac{9}{2}}\right)}$

C) Let x and y denote the lengths of the original right triangle.

Then $\begin{cases} (1)\ x^2 + y^2 = 65^2 \\ (2)\ (x+4)^2 + (y-8)^2 = 65^2 \end{cases}$

Expanding and subtracting (2) – (1), $8x + 16 - 16y + 64 = 0$ ➔ $x = 2y - 10$
Substituting in (1), $5y^2 - 40y = 65^2 - 100$ ➔ $y^2 + 8y = 13(65) - 20$
➔ $y^2 - 8y - 825 = (y - 33)(y + 25) = 0$ ➔ $y = 33$, $x = 56$ ➔ Per = 89 + 65 = **<u>154</u>**