MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2008 SOLUTION KEY

Round 4

A) $49^{\log_7 3-4\log_7 2} = 49^{\log_7 3-\log_7 16} = 49^{\log_7\frac{3}{16}} = 7^{2\log_7\frac{3}{16}} = 7^{\log_7\frac{9}{256}} = \underline{\mathbf{\frac{9}{256}}}$

B) $2^x = \dfrac{1}{2^x} - \dfrac{3}{2}$ Let $N = 2^x$. Then: $N = \dfrac{1}{N} - \dfrac{3}{2}$ → $2N^2 + 3N - 2 = (2N-1)(N+2) = 0$

→ $N = \dfrac{1}{2}, -2$ Since a power function never produces a negative value, the latter value is extraneous. Thus, $2^x = \dfrac{1}{2}$ → $x = \underline{\mathbf{-1}}$

C) Since $A^{\log_A B} = B$, the equation simplifies to $3x^5 - 2x^4 + 8x^2 = 12x^3$
→ $x^2(3x^3 - 2x^2 - 12x + 8) = 0$
→ $x^2[x^2(3x - 2) - 4(3x - 2)] = x^2(3x - 2)(x^2 - 4) = x^2(3x - 2)(x + 2)(x - 2) = 0$
→ $x = 0, 2/3, \pm 2$
Since x is the argument of the log function, $x > 0$. Thus, 0 and -2 are rejected → **<u>2/3, 2</u>** only