**MASSACHUSETTS MATHEMATICS LEAGUE**
**CONTEST 5 – FEBRUARY 2012 SOLUTION KEY**

## Round 1

A) $f\left(f(2)\right)+f\left(f(-2)\right)+f(0)=$ $f(10)+f(-10)+f(0)$ = 45 + (-34) + 10 = **<u>21</u>**.

B) $f\left(x\right)=(x+1)\left(x^2-6\right)\left(x^2+1\right)$

$$\frac{2f(-2)}{3f(3)}=\frac{2\left(-1\cdot-2\cdot5\right)}{3\left(4\cdot3\cdot10\right)}=\frac{20}{360}=\underline{\mathbf{\frac{1}{18}}}$$

C) Replace $f(x)$ with $y$, i.e. let $y=\dfrac{x+1}{x-1}$ and solve for $x$ in terms of $y$.

$\Rightarrow xy-y=x+1$. Solving for $x \Rightarrow y+1=xy-x=x(y-1) \Rightarrow x=\dfrac{y+1}{y-1}$

Thus, $f(2x)=\dfrac{2x+1}{2x-1}=\dfrac{2\dfrac{y+1}{y-1}+1}{2\dfrac{y+1}{y-1}-1}=\dfrac{2(y+1)+(y-1)}{2(y+1)-(y-1)}=\dfrac{3y+1}{y+3}=\underline{\mathbf{\dfrac{3f(x)+1}{f(x)+3}}}$

## Round 2

A) The notes are being played in repeating blocks of 8 in the sequence CDEFGFED.
$\dfrac{2011}{8}=251,\quad r=4$.
Thus, Engelbert completed 251 blocks and stops on the 4th note in the 252nd block
$\Rightarrow (F, N) =$ **<u>(R, F)</u>**.

B) The 3-digit number must be of the form $3x7$ or $4x7 \Rightarrow$ digit sum is $10 + x$ or $11 + x$.
For integers in the 300s, we must test only 317.
For integers in the 400s, we must test only 407
Since 407 = 11(37), we need only rigorously test 317.
We must test for divisibility only by primes smaller than $\sqrt{317}$. Since $19^2 = 361 > 317$, the divisors to be tested are: 2, 3, 5, 7, 11, 13 and 17.
The first three divisors are easily eliminated.
$7 \Rightarrow r=2$ $11 \Rightarrow r=9$ $13 \Rightarrow r=5$ $17 \Rightarrow r=11$
Thus, **<u>317</u>** is prime.

C) Each pointer cycles through the 13 numbered positions, before returned to #3.

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Pointer $A$: | 3 | 10 | 4 | 11 | 5 | 12 | 6 | 13 | 7 | 1 | 8 | 2 | 9 | 3 |
| Pointer $B$: | 3 | 11 | 6 | 1 | 9 | 4 | 12 | 7 | 2 | 10 | 5 | 13 | 8 | 3 |
| Pointer $C$: | 3 | 5 | 7 | 9 | 11 | 13 | 2 | 4 | 6 | 8 | 10 | 12 | 1 | 3 |
| Sums: | | 26 | 17 | 21 | 25 | $\boxed{29}$ | 20 | 24 | 15 | 19 | 23 | 27 | 18 | |

Thus, the maximum sum is 29 and it occurs first for $n = 5 \Rightarrow$ **<u>(5, 29)</u>**.