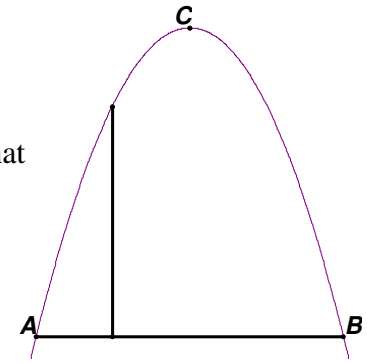
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2009 SOLUTION KEY

Round 1

A) Substituting, $A(7, a)$: $49 - a^2 = 24 \rightarrow a = \pm 5$ $B(b, 1)$: $b^2 - 1 = 25 \rightarrow b = \pm 5$
The longest distance is between $A(7, -5)$ and $B(-5, 1)$ $AB = \sqrt{(7 - {}^{-}5)^2 + ({}^{-}5 - 1)^2} = \sqrt{180} = \underline{\mathbf{6\sqrt{5}}}$

B) Let $A(0, 0)$, $B(12, 0)$ and $C(6, 8)$. Then the equation of the arch is $y = x(12 - x) \cdot c$, for some fudge-factor c to adjust the height.
Substituting, $8 = 6(12 - 6) \cdot c$ and $c = \frac{2}{9}$. The perpendicular segment that is a quarter of the way across the span connects $(3, 0)$ and $(3, h)$.
Thus, substituting, $h = \frac{2}{9} \cdot 3 \cdot (12 - 3) = \underline{\mathbf{6}}$



C) Unless the center of the circle is <u>above</u> $y = -2$, there would be no x-intercepts. Additionally, the center must be 5 units left or right of $x = 3$, i.e. at $(-2, 3)$ or $(8, 3)$.
When $y = 0$,
$(x+2)^2 + (y-3)^2 = 25 \rightarrow (x + 2)^2 = 16 \rightarrow x = -6, 2$
$(x-8)^2 + (y-3)^2 = 25 \rightarrow (x - 8)^2 = 16 \rightarrow x = 4, 12$
Thus, the required sum is $-6 + 2 + 4 + 12 = \underline{\mathbf{12}}$