MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2012 SOLUTION KEY

Round 5

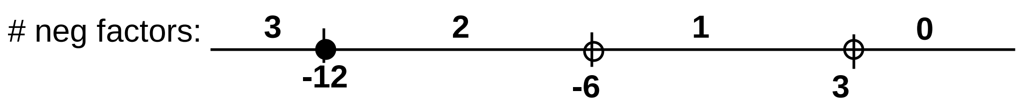
A) $1 \le 2x - 3 \le 7 \Leftrightarrow 4 \le 2x \le 10 \Leftrightarrow 2 \le x \le 5$

Since $y = 10 - 3x$, the minimum value of y occurs for the largest possible value of x.

For $x = 5$, $y = 10 - 3(5) =$ **<u>–5</u>**.

B) $\dfrac{2}{x+6} \ge \dfrac{5}{x-3} \Rightarrow \dfrac{2}{x+6} - \dfrac{5}{x-3} \ge 0 \Leftrightarrow \dfrac{2(x-3)-5(x+6)}{(x+6)(x-3)} \ge 0 \Leftrightarrow \dfrac{-3x-36}{(x+6)(x-3)} \ge 0 \Leftrightarrow \dfrac{(x+12)}{(x+6)(x-3)} \le 0$

The critical values are −12, −6 and +3.
Since the latter two values cause division by zero, these values must be excluded from any solution.
On the number line, all factors assume negative values to the left and positive values to the right.
Moving from left to right, as each critical value is passed, there is one less negative factor.
This is summarized in the diagram below:



Since the quotient must be negative or zero, we require an <u>odd</u> number of negative factors.
From the diagram, we see that the solution set is $x \le -12$ or $-6 < x < 3$.

C) $|x^2 - 3x - 1| \le 3 \Leftrightarrow -3 \le x^2 - 3x - 1 \le +3 \Leftrightarrow \begin{cases} x^2 - 3x + 2 \ge 0 \\ x^2 - 3x - 4 \le 0 \end{cases} \Leftrightarrow \begin{cases} (x-1)(x-2) \ge 0 \\ (x-4)(x+1) \le 0 \end{cases}$

We must take the intersection between $x \le 1$ or $x \ge 2$ (2 rays) and $-1 \le x \le 4$ (a segment)
The intersection is two segments: $-1 \le x \le 1$ or $2 \le x \le 4$

$|2x| \ge 5 \Leftrightarrow x \le -\dfrac{5}{2}$ or $x \ge +\dfrac{5}{2}$

Taking the intersection between these two constraints, we have $\underline{\boldsymbol{\dfrac{5}{2} \le x \le 4}}$.