Round 1 - continued

C) – continued

Solution #2 shows how we can compute the required product **without specifically knowing** A, B, C and D. Expanding the product, $(1+A)(1+B)(1+C)(1+D)=$
$1+(A+B+C+D)+(AB+AC+AD+BC+BD+CD)+(ABC+ABD+ACD+BCD)+ABCD$

Normalizing a polynomial function (making its lead coefficient 1) does not change its zeros.
Normalizing $f(x)$ whose zeros are A, B, C and D, we have $F(x) = (x - A)(x - B)(x - C)(x - D)$
Expanding, we have
$x^4-(A+B+C+D)x^3+(AB+AC+AD+BC+BD+CD)x^2-(ABC+ABD+ACD+BCD)x+ABCD$

Lo and behold the coefficients match the expressions we need to evaluate!
After normalizing, each of these sums can be determined by inspection.

$$\underline{\mathbf{3}}x^4 \ \underline{-\mathbf{8}}x^3 \ \underline{-\mathbf{11}}x^2+\underline{28}x\ \underline{-\mathbf{12}}=0$$

$(A+B+C+D)$ = 8/3 (the opposite of the cubic coefficient divided by the lead coefficient)
$(AB+AC+AD+BC+BD+CD)$ = –11/3 (the quadratic coefficient divided by …)
$(ABC+ABD+ACD+BCD)$ = –28/3 (the opposite of the linear coefficient divided by …)
$ABCD = -12/3 = -4$ (the constant term divided by …)

Thus, the required product is $1+\left(\dfrac{8-11-28-12}{3}\right)=1-\dfrac{43}{3}=\underline{-\dfrac{\mathbf{40}}{\mathbf{3}}}$.

[Using this relationship between the coefficients and the zeros, you can verify that for the unfactorable polynomial, the computation is simply $(3+5+7+11)/2=\underline{\mathbf{13}}$.

Alternative Solution #3 A REAL GEM! (Norm Swanson): By synthetic substitution,

$$\begin{array}{r|rrrrr} -1 & 3 & -8 & -11 & 28 & -12 \\ \hline & 3 & -11 & 0 & 28 & \boxed{-40} \end{array}$$

. Divide by 3 and we have our answer. Why does this work?

Since A is a zero, $3A^4-8A^3-11A^2+28A-12=0$ and similarly for B, C and D.
Consider $g(x) = 3(x-1)^4-8(x-1)^3-11(x-1)^2+28(x-1)-12$.
Since $g(1 + A) = 3A^4-8A^3-11A^2+28A-12=0$, $1 + A$ is a zero of g and so is $1 + B$, etc.

In the expansion of $g(x)$, we only need to know the constant term which is determined by letting $x = 0$ or evaluating the original polynomial expression for $x = -1$.

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