MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2007 SOLUTION KEY

Team Round

A) An acute triangle must have 3 acute angles.
Therefore, $\cos A$, $\cos B$ and $\cos C$ must each be positive.

The triangle inequality requires that $a + c > b \rightarrow 7 + c > 13 \rightarrow c \geq 7$.
Using the Law of Cosines,

$$\cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{218-c^2}{2(7)(13)} \geq 0 \rightarrow c \leq 14$$

$$\cos B = \frac{a^2+c^2-b^2}{2ac} = \frac{c^2-120}{2(7)(c)} \geq 0 \rightarrow c \geq 11$$

$$\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{120+c^2}{2(13)(c)} \geq 0 \text{ (true for all values of } c \text{ in consideration)}$$

Thus, for $11 \leq c \leq 14$, ΔABC is acute $\rightarrow$ **11, 12, 13, and 14**

B) $\begin{cases} 8a+b \\ 10a+b \\ 12a+b \end{cases}$ each represent a prime number.

The possible values of a and b are limited to the digits 0.. 7 (allowable digits in all 3 bases).
b can't be even (otherwise each expression would generate a nonprime)
$b \neq 5$ (otherwise $10a + b$ would not be prime)
$b \neq 3$ (otherwise $12a + b$ would not be prime)
Thus, we exam only those cases where $b = 1$ or 7.

		b = 1			b=7	
a	***8a+b***	***10a+b***	***12a+b***	***8a+b***	***10a+b***	***12a+b***
1	9	x	x	15	x	x
2	17	21	x	23	27	x
3	25	x	x	**31**	**37**	**43**
4	33	x	x	39	x	x
5	41	51	x	47	57	x
6	49	x	x	55	x	x
7	57	x	x	63	x	x

Thus, the only ordered pair producing 3 primes is **(3, 7)**.