MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 - FEBRUARY 2011 SOLUTION KEY

Team Round

D) Let x be the rate of runner #2. We have: $\frac{B}{x}=\frac{A}{R}$ → $x=\frac{B}{A}\cdot R$.

Since the two runners pass each other in 45 seconds when they run in opposite direction, they have completed 1 lap, i.e. covered a distance of 1320 feet in 45 seconds. Thus,

$$R\cdot 45+\frac{B}{A}\cdot R\cdot 45=\frac{1}{4}\cdot 5280=1320 \rightarrow R\left(1+\frac{B}{A}\right)=R\left(\frac{A+B}{A}\right)=\frac{1320}{45}=\frac{88}{3}$$

→ $R=\frac{88A}{3(A+B)}$ → A must be a multiple of 3

The factors of 88 are: 1, 2, 4, 8, 11, 22, 44 and 88.

Under the given restrictions,

- $A > B$,
- the sum $A + B$ can't be 1 or 2 and
- the difference $A - B$ must be 1 or 2

$A+B$	$(A,B)=$
4:	(3, 1) → R = 22 ft/sec
8:	~~(5,3)~~ - 5 is not a multiple of 3
11:	(6, 5) → R = 16 ft/sec
22:	~~(12,10)~~ - not relatively prime
44:	(23, 21) – 23 is not a multiple of 3
88:	(45, 33) → R = 15 ft/sec

Thus, R = **15, 16 or 22**.