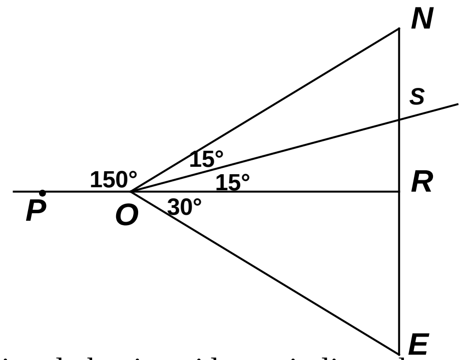
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2016 SOLUTION KEY

Round 5

A) $A = \begin{cases} (1)\ \ 30° + n(360°) \\ (2)\ \ 150° + n(360°) \end{cases}$

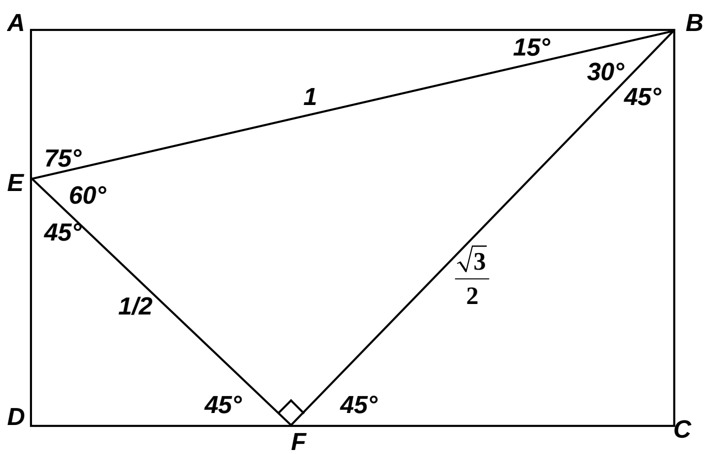
For (1), the minimum value of n is 3, producing 1110
For (2), the minimum value of n is 2, producing **870**.



B) $\dfrac{\tan 60° - \sin 270°}{\sin 210° + \cos 330° - \tan(-225°)} = \dfrac{\sqrt{3}-(-1)}{-\frac{1}{2}+\frac{\sqrt{3}}{2}-(-1)} = \dfrac{\sqrt{3}+1}{\frac{\sqrt{3}+1}{2}} = \underline{\mathbf{2}}$

C) Clearly, $m\angle POS = 165$.
The tangent of an obtuse angle is negative.
Since $\tan\theta = -\tan(180-\theta)$, $\tan 165° = -\tan 15°$.

Solution #1: Using only special angles (30°, 45° and 60°)
Consider rectangle $ABCD$ with an embedded 30 – 60 – 90 right triangle having sides as indicated.



1) $FC = BC = AD = \dfrac{\sqrt{3}}{2}\cdot\dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{3}}{2\sqrt{2}} = \dfrac{\sqrt{6}}{4}$ 2) $DE = DF = \dfrac{1}{2}\cdot\dfrac{1}{\sqrt{2}} = \dfrac{1}{2\sqrt{2}} = \dfrac{\sqrt{2}}{4}$

3) $AB = DF + FC = \dfrac{\sqrt{2}}{4}+\dfrac{\sqrt{6}}{4} = \dfrac{\sqrt{6}+\sqrt{2}}{4}$ 4) $AE = AD - DE = \dfrac{\sqrt{6}-\sqrt{2}}{4}$

5) $\tan 15° = \dfrac{AE}{AB} = \dfrac{\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}} = \dfrac{\left(\sqrt{6}-\sqrt{2}\right)^2}{6-2} = \dfrac{8-2\sqrt{12}}{4} = 2-\sqrt{3} \Rightarrow \tan 165° = \underline{\mathbf{\sqrt{3}-2}}$

Solution #2: (BORING expansion formulas!)

$\tan(A-B) = \dfrac{\tan A - \tan B}{1+\tan A\tan B} \Rightarrow \tan 15° = \dfrac{\tan 60° - \tan 45°}{1+\tan 60°\tan 45°} = \dfrac{\sqrt{3}-1}{1+\sqrt{3}} = \text{etc.}$