MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2011 SOLUTION KEY

Team Round – continued

D) Using the quadratic formula to solve for y in terms of x:
Re-arranging the terms of $x^2 - xy - 6y^2 + x + 7y - 2 = 0$, we have
$\rightarrow 6y^2 + y(x-7) + \left(2 - x - x^2\right) = 0$
(This is a quadratic equation of the form $Ay^2 + By + C = 0$.)

Thus, $y = \dfrac{(7-x) \pm \sqrt{(x-7)^2 - 4\cdot 6\cdot\left(2-x-x^2\right)}}{12} = \dfrac{(7-x) \pm \sqrt{25x^2 + 10x + 1}}{12} = \dfrac{(7-x) \pm \sqrt{(5x+1)^2}}{12}$

$= \dfrac{(7-x) \pm (5x+1)}{12} = \dfrac{(8+4x)}{12}, \dfrac{6-6x}{12} = \underline{\boldsymbol{\dfrac{x+2}{3}, \dfrac{1-x}{2}}}$

Alternate Solution (Method of Indeterminant Coefficients)
Suppose $x^2 - xy - 6y^2 + x + 7y - 2 = \left(x + Ay + B\right)\left(x + Cy + D\right)$ for some constants A, B, C and D.
Multiplying out the trinomials,
$\left(x + Ay + B\right)\left(x + Cy + D\right) = x^2 + (A+C)xy + ACy^2 + (B+D)x + (AB+CD)y + BD$.
Equating the coefficients,

$$\begin{cases} (1)\quad A + C = -1 \\ (2)\quad AC = -6 \\ (3)\quad B + D = 1 \\ (4)\quad AD + BC = 7 \\ (5)\quad BD = -2 \end{cases}$$

(1), (2) $\rightarrow$ $(A, C) = (2, -3)$ or $(-3, 2)$, (3), (5) $\rightarrow$ $(B, D) = (2, -1)$ or $(-1, 2)$

Testing these possible ordered pairs in (4), the only combinations that works are
$(A, B, C, D) = (-3, 2, 2, -1)$ or $(2, -1, -3, 2)$.
Thus, $\left(x + Ay + B\right)\left(x + Cy + D\right) = (x - 3y + 2)(x + 2y - 1) = 0$
The second possibility just reverses the two factors.
Setting each factor equal to zero, we have $y = \underline{\boldsymbol{\dfrac{x+2}{3}, \dfrac{1-x}{2}}}$

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