MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2007 SOLUTION KEY

Round 1

A) Method 1 is straightforward (but tedious).

Pick a point on one of the lines → $P(3,0)$ is on the 2nd line $3x + 4y = 9$

Determine the slope of the perpendicular line through this point (negative reciprocal) → $\underline{+\frac{4}{3}}$

Determine the equation of this perpendicular line → $\underline{4x - 3y = 12}$

Determine point of intersection of this line and the first line → $Q\left(\frac{63}{25}, \frac{-16}{25}\right) = Q(2.52, -0.64)$

Using the distance formula, find the distance between P and Q

$\sqrt{(3-2.52)^2+(0+0.64)^2} = \sqrt{.48^2+.64^2} = \sqrt{.16^2(3^2+4^2)} = .16(5) =$ **0.8 or 4/5**

Method 2 uses the point to line distance formula .

The distance from point $P(x_1, y_1)$ to the line $Ax + By + C = 0$ is $\frac{|Ax_1+Bx_2+C|}{\sqrt{A^2+B^2}}$

Note that all terms in the equation must be on the same side.

Thus, using (3, 0) as the point and $3x + 4y - 5 = 0$ as the line we have

$$d = \frac{|3(3)+4(0)-5|}{\sqrt{3^2+4^2}} = \underline{\mathbf{4/5 \text{ or } 0.8}}$$

Method 3: Determine the hypotenuse of a triangle with legs given by A and B, the coefficients of x and y. [(3, 4) → 5] Subtract this value from C, the constant term. [9 – 5 = 4] Divide this value by the hypotenuse [4/5] That's all folks! The proof is left to you!

B) The coordinates (1, 0), (5, 0) and (0, 2) must satisfy the equation $y = ax^2 + bx + c$.
The last ordered pair implies $c = 2$.
(5, 0) → $25a + 5b + 2 = 0$
(1, 0) → $a + b + 2 = 0$ → $5a + 5b + 10 = 0$
Subtracting, we have $20a - 8 = 0$ → $a = 2/5$
Substituting, $2/5 + b + 2 = 0$ → $b = -12/5$

Alternately, x-intercepts of 1 and 5 → $y = k(x - 1)(x - 5)$

A y-intercept of 2 → constant terms $5k = 2$ → $k = 2/5$ Multiplying, $y = \frac{2}{5}x^2 - \frac{12}{5}x + 2$

Thus, $(a, b, c) = \underline{\left(\mathbf{\frac{2}{5}, \frac{-12}{5}, 2}\right)}$

C) $4(x-1)^2 + 2500(y+2)^2 = 10000$ → $\frac{(x-1)^2}{2500} + \frac{(y+2)^2}{4} = 1$ → $a = 50$, $b = 2$

Thus, the exact area is 100π and the overestimate is LW = 100(4) = 400
The percent error is $(400 - 100\pi)/100\pi = (4 - \pi)/\pi \approx 0.2732+$ → **27.3%**