MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 – MARCH 2010 SOLUTION KEY

Round 2

A) $\sqrt{2^{\,4}\left(5^{-2}-13^{-2}\right)}=\sqrt{2^4\left(\frac{1}{5^2}-\frac{1}{13^2}\right)}=\sqrt{2^4\left(\frac{13^2-5^2}{5^2\cdot 13^2}\right)}=\sqrt{2^4\left(\frac{12^2}{5^2\cdot 13^2}\right)}=\frac{4\cdot 12}{5\cdot 13}=\frac{48}{65}$

→ $A + B =$ **113**.

B) $\left(\frac{2\sqrt{5}-\sqrt{10}}{\sqrt{10}-\sqrt{5}}\right)^2=\frac{20-4\sqrt{50}+10}{10-2\sqrt{50}+5}=\frac{30-4\sqrt{50}}{15-2\sqrt{50}}=\frac{30-20\sqrt{2}}{15-10\sqrt{2}}=\frac{10\left(3-2\sqrt{2}\right)}{5\left(3-2\sqrt{2}\right)}=$ **2**.

C) First and foremost - $\sqrt{\frac{9}{16}-\frac{9}{25}}\neq\frac{3}{4}-\frac{3}{5}$!!! Squaring both sides,

$\sqrt{\frac{9}{16}-\frac{9}{25}}=\frac{3}{4}-\frac{3}{x}$ → $\frac{9}{16}-\frac{9}{25}=\left(\frac{3}{4}-\frac{3}{x}\right)^2=\frac{9}{16}-2\left(\frac{3}{4}\right)\left(\frac{3}{x}\right)+\frac{9}{x^2}=\frac{9}{16}-\frac{9}{2x}+\frac{9}{x^2}$

→ $\frac{9(25-16)}{16(25)}=\frac{9}{16}-\frac{9}{2x}+\frac{9}{x^2}$ → $\frac{9}{16(25)}=\frac{1}{16}-\frac{1}{2x}+\frac{1}{x^2}$ →

$\frac{9}{16(25)}-\frac{25}{16(25)}=-\frac{1}{25}=-\frac{1}{2x}+\frac{1}{x^2}$.

Multiplying through by $-50x^2$, $2x^2 = 25x - 50$ → $2x^2 - 25x + 50 = (2x - 5)(x - 10) = 0$
→ $x = 10$.

$\sqrt{\frac{2^x\cdot 4^{3x+4}}{8^{2x+2}}}=\sqrt{\frac{2^x\cdot 2^{6x+8}}{2^{6+6x}}}=\sqrt{2^{x+2}}$ Since $x = 10$, we have $\sqrt{2^{12}}=2^6=$ **64**.