MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2014 SOLUTION KEY

Round 2

A) The teen's <u>current</u> age, between 13 and 19 inclusive, can't be even because 2 years earlier or later he would not be"in his prime". We need test only <u>odd</u> cases.
$13 \Rightarrow 11$ (prime), 15 (not prime) satisfies the stated conditions.
$15 \Rightarrow 13$ (prime), 17 (prime) fails
$17 \Rightarrow 15$ (not prime), 19 (prime) fails
$19 \Rightarrow 17$ (prime), 21 (not prime)
Therefore, my current age is either 13 or 19 and I am no longer a teen in either 7 years or in 1 year. (M, m) = <u>**(7, 1)**</u>.

B) The least common multiple of 6 and 21 is 42.
We require that $42n > 2014 \Rightarrow n > 47^{+} \Rightarrow n_{\min} = 48$
$48 \cdot 42 = \underline{\mathbf{2016}}$. Adding 42, we get additional possibilities: <u>**2058**</u> and <u>**2100**</u>.

Or, alternately, dividing 2014 by 42 result in a quotient of 47 and a remainder of 40
By adding 2 to the dividend (numerator), we insure divisibility by 42 and the same result follows.

C) The rightmost digit of an odd prime with 2 or more digits is 1, 3, 7, or 9.
Squaring these, the rightmost digit must be 1 or 9.
Therefore, we have eliminated two of the 5 given numbers.
3027 and 9025 are not the squares of a prime. The other 4 numbers <u>must</u> be.
$4489 = \left(6\boxed{x}\right)^2$, where x must be 3 or 7$\Rightarrow$ 67
$5329 = \left(7\boxed{x}\right)^2$, where x must be 3 or 7$\Rightarrow$ 73
$7921 = \left(8\boxed{x}\right)^2$, where x must be 1 or 9$\Rightarrow$ 89
Grouping them as follows decreases the probability of botching the arithmetic,
(67 +73) + 89 = 140 + 89 = <u>**229**</u>.