Team Question A: Center C(4,-3)

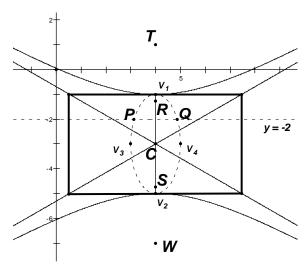
Since  $a = 2, b = 2\sqrt{3}$  and c = 4 for the hyperbola, and a = 2, b = 1 and  $c = \sqrt{3}$  for the ellipse.

$$R(4, -3 + \sqrt{3}), S(4, -3 - \sqrt{3})$$

$$T(4,1), W(4,-7)$$

$$V_3(3,-3), V_4(5,-3)$$

Since the slope of asymptotes are  $\pm \frac{2}{2\sqrt{3}} = \pm \frac{\sqrt{3}}{3}$ ,



the point-slope equations of the asymptotes are  $(y+3) = \pm \frac{\sqrt{3}}{3}(x-4)$ .

In the diagram below, graphically, it certainly appears as if the hyperbola approaches arbitrarily close to these lines. Analytically, why is this the case?

Staring with 
$$3(y+3)^2 - (x-4)^2 = 12 \Leftrightarrow \frac{(y+3)^2}{4} - \frac{(x-4)^2}{12} = 1$$
, solve for y.

$$(y+3)^2 = \frac{(x-4)^2}{3} + 4$$

As  $x \to \pm \infty$ , that is, as x becomes an arbitrarily large positive number (moving to the right on the graph) or an arbitrarily small negative number (moving to the left on the graph), the 4 becomes <u>negligible</u> and we have

$$(y+3)^2 \approx \frac{(x-4)^2}{3} \Leftrightarrow y+3 \approx \pm \frac{1}{\sqrt{3}}(x-4)$$

Thus, as  $x \to \pm \infty$ , the y-coordinate of a point on the straight lines (the asymptotes) becomes an increasingly better approximation of the y-coordinate of the corresponding point on corresponding branch of the hyperbola.

Q.E.D ('Quod erat demonstratum', Latin for "enough said", literally "that which was to be demonstrated")

