

MASSACHUSETTS MATHEMATICS LEAGUE
NOVEMBER 2003
ROUND 7: TEAM QUESTIONS
SOLUTIONS

ANSWERS

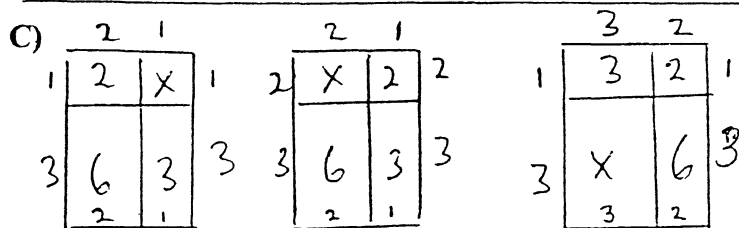
- A) $8\sqrt{2}$ D) $30 \times 40 \times 20$
B) 7 E) $500 + 500\sqrt{3}$
C) $1, 4, 9$ F) 540°

A) $9i^{14} = -9$, $(1+i)^3 = -2+2i$, $(\sqrt{2}-i)^2 = 1-2\sqrt{2}i$.

$$\frac{-9(-2+2i)}{1-2\sqrt{2}i} \cdot \frac{1+2\sqrt{2}i}{1+2\sqrt{2}i} = \frac{-9}{9} \frac{(-2-4\sqrt{2}+2i-4\sqrt{2}i)}{2+4\sqrt{2}-2i+4\sqrt{2}i} = \frac{-9}{2+4\sqrt{2}-2i+4\sqrt{2}i}$$

$$a+b = 8\sqrt{2}$$

- B) 2 (5) 8 19 (23) 27 53 (89) 125
 7 (17) 27 17 (71) 125
 11 (19) 27 41 (83) 125 Ans: 7



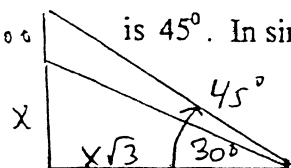
D) The top and base of a fish tank are rectangles whose length is 10 inches more than the width. If the sum of the height and width of the tank is 50 inches, and the combined areas of the top and base is 400 square inches less than the total area of the four sides, what are the dimensions of the tank?

$w = x$, $l = x+10$, $h = 50-x$ $2x(x+10) = 2(x+10)(50-x) + 2x(50-x) - 400$

Simplifies to $3x^2 - 80x - 300 = 0$ $(3x+10)(x-30) = 0$
 $w = x = 30$, $l = 40$, $h = 20$

E) The pilot of an airplane calculates the angle of depression of an airport to be 30° . The angle of depression to the airport from a second airplane 1000 feet directly above the first

is 45° . In simple radical form, what is the altitude of the first airplane?



$x\sqrt{3} = x + 1000$, $x = \frac{1000}{\sqrt{3}-1} = 500(\sqrt{3}+1) = 500 + 500\sqrt{3}$

F) The sides of a heptagon are extended to form a seven pointed star. What is the sum of the angles formed at the points of the star?

use reg. hept. Star $\angle = 180^\circ - 2 \cdot 51 \frac{3}{7} = 77 \frac{1}{7}$. Sum = $7 \cdot 77 \frac{1}{7} = 540^\circ$