MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2006 SOLUTION KEY

Team Round - continued

D) Appealing to the graph of the common log function (i.e. base 10), $\log_{10} \frac{2a-1}{2-a} \le 0$

is equivalent to : $0 < \frac{2a-1}{2-a} \le 1$

$\frac{2a-1}{2-a} \le 1 \rightarrow \frac{2a-1}{2-a} - 1 \le 0 \rightarrow \frac{2a-1-2+a}{2-a} \le 0 \rightarrow \frac{3a-3}{2-a} \le 0 \rightarrow \frac{a-1}{2-a} \le 0 \rightarrow a \le 1$ or $a > 2$

$0 < \frac{2a-1}{2-a} \rightarrow$ ½ < a < 2 Taking the overlap, we have $\boxed{\frac{1}{2} < a \le 1}$.

E) Let a, b, c and d denote the rates of each of the 4 runners and T, the elapsed time when the leader (runner A) crosses the finish line.

D	C	B	A
(20-k)	15	18	20

Since *distance* = *rate* x *time*, $T = \frac{20}{a} = \frac{18}{b} = \frac{15}{c} = \frac{20-k}{d}$

When runner C reaches the finish line, runner D is 1 km behind and, therefore, $\frac{20}{c} = \frac{19}{d}$.

Thus, $\frac{d}{c} = \frac{20-k}{15} = \frac{19}{20} \rightarrow 400 - 20k = 19 \cdot 15 = 285 \rightarrow k = 115/20 =$ **23/4** = **5.75.**

F) $6 + x + y + z = 15 + (4 - x) + (8 - y) + z$
$\rightarrow 2x + 2y = 21$
But $\overline{PQ} \parallel \overline{AD} \parallel \overline{BC}$
$\rightarrow \frac{x}{4-x} = \frac{y}{8-y}$
$\rightarrow 8x - xy = 4y - xy$
$\rightarrow y = 2x$
Thus, $6x = 21 \rightarrow x = 7/2 \rightarrow PB : PA = 0.5 : 3.5 =$ **1 : 7**.