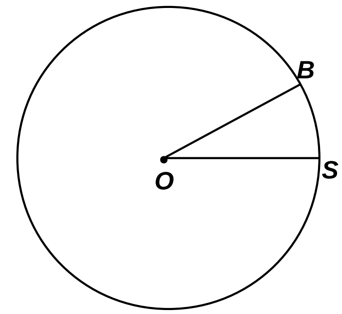
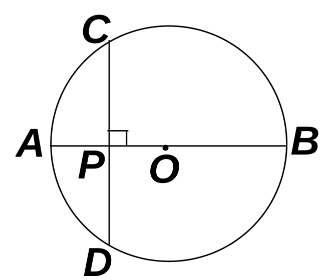
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2016 SOLUTION KEY

Round 5



A) $\frac{27}{360}=\frac{22.5}{A}\Rightarrow\frac{3}{40}=\frac{22.5}{A}\Rightarrow 3A=900\Rightarrow A=\underline{\mathbf{300}}$.

B) Since the perpendicular bisector of any chord passes through the center of the circle, $\overline{AB}$ must be a diameter.
Let $PC=PD=n$ and $PA:PB=m:4m$. Then: $AB=5m$
By the product-chord theorem, $n^2=4m^2\Rightarrow n=2m$.
$m=1\Rightarrow n=2$, but radius $OC=2.5$
$m=2\Rightarrow n=4, AB=10, OC=5 \Rightarrow A(\text{circle}\,O)=\underline{\mathbf{25\pi}}$.



C) $5n+12n+13n=180\Rightarrow n=6$
Thus, the measures of the angles of ΔABC are 30°, 72°, and 78°.

1) If $(m\angle A, m\angle B)=(30°,72°), \left(\overset{\frown}{AC},\overset{\frown}{BC}\right)=(60°,144°)$

$\Rightarrow m\angle APM=\frac{30°+72°}{2}=51°$ or $m\angle MPN=180°-51°=129°$

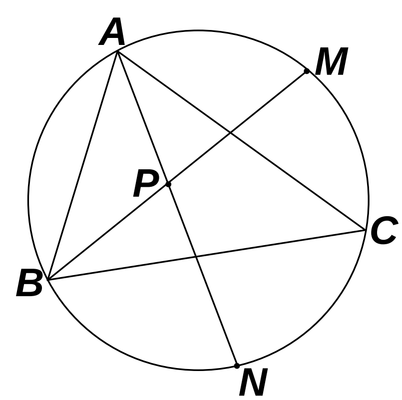
2) If $(m\angle A, m\angle B)=(30°,78°), \left(\overset{\frown}{AC},\overset{\frown}{BC}\right)=(60°,156°)$

$\Rightarrow m\angle APM=\frac{30°+78°}{2}=54°$ or $m\angle MPN=180°-54°=126°$

3) If $(m\angle A, m\angle B)=(72°,78°), \left(\overset{\frown}{AC},\overset{\frown}{BC}\right)=(144°,156°)$

$\Rightarrow m\angle APM=\frac{72°+78°}{2}=75°$ or $m\angle MPN=180°-75°=105°$

Thus, the largest obtuse angle measures $\underline{\mathbf{129}}$°.



An astute observer would have noted that to maximize $m\angle P$, $m\angle A$ and $m\angle B$ had to be minimized and , therefore case 1) above must give the maximum value of *P*.