MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2015
ROUND 6 ALG 2: PROBABILITY AND THE BINOMIAL THEOREM

ANSWERS

A) _____

B) _____

C) (_____ , _____)

A) Five fair coins are tossed. Compute the probability that the result will be 3 heads and two tails or one head and 4 tails.

B) The expansion of $\left(4x^2+k\right)^3$ contains the term $432x^2$ and the constant term is negative. Compute the value of *k*.

C) The formula $\frac{n!}{k!\cdot(n-k)!}$ works nicely for evaluating coefficients in the binomial expansion of $\left(A+B\right)^n$, when *n* is an integer. If *n* is a fraction, factorials are undefined, but we can use the following equivalent $\frac{n(n-1)(n-2)\cdot\ldots\cdot(n-k+1)}{1\cdot2\cdot3\cdot\ldots\cdot k}$, where both the numerator and denominator contain *k* factors. This formula can be used to expand $\sqrt{a+x}=\left(a+x\right)^{\frac{1}{2}}$.
This is particularly useful when *x* is very much less than *a* so that the first few terms provide a good approximation of the value of the expression.
The first three terms in the expansion of $\sqrt{1+x}$ are $1+\frac{1}{2}x-\frac{1}{8}x^2$.
The next two terms are $\underline{Ax^3-Bx^4}$.
Compute the ordered pair (*A*, *B*), each entry being a ratio of relatively prime integers.