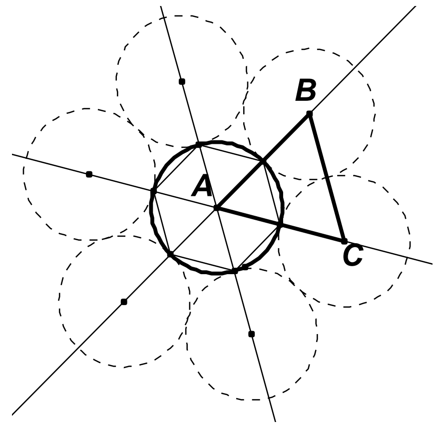
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2011 SOLUTION KEY

Round 5 (Exact equivalents in terms of π are acceptable.)

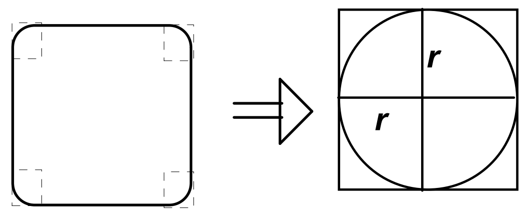
A) Clearly, $k = 6$.
Place a quarter tangent to circle of the innermost quarter at each vertex of a regular hexagon inscribed inside this circle. Every adjacent pair is tangent to each other as well as the innermost circle, since the lines connecting the centers (ex. ABC) form an equilateral triangle.



From the diagram, the radius of the covering coin is $\frac{3}{2}$.

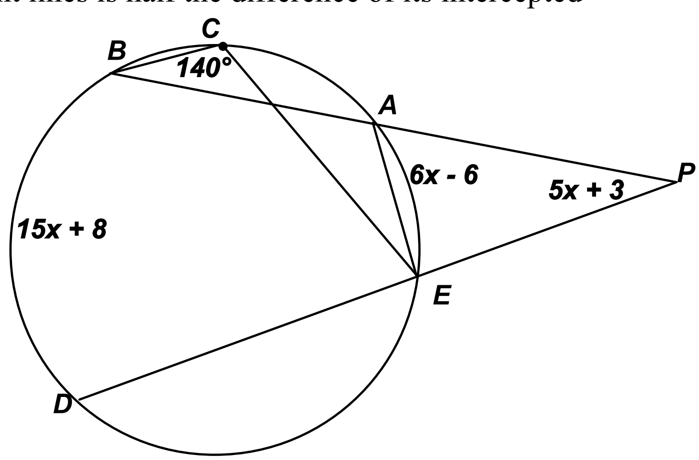
Therefore, the required area is $\underline{\boldsymbol{\frac{9\pi}{4}}}$.

B) Let x denote the side of the original square. Telescoping the four corners, the area lost equals the area of the region inside a square with edge $2r$ and outside a circle of radius r, i.e. $4r^2 - \pi r^2$



➔ $r^2(4-\pi) = 0.1x^2$ ➔ $\frac{x^2}{r^2} = \frac{(4-\pi)}{0.1}$ ➔ $\underline{\mathbf{10(4-\pi)}}$

C) The measure of an angle formed by two secant lines is half the difference of its intercepted arcs. Thus, $5x+3 = \frac{1}{2}(15x+8-6x+6)$



➔ $10x + 6 = 9x + 14$ ➔ $x = 8$
➔ $\overset{\frown}{BD} = 128°$, $\overset{\frown}{AE} = 42°$,
$\overset{\frown}{DE} = 280 - 128 = 152°$,
$\overset{\frown}{BCA} = 38°$
Finally, as an inscribed angle,
$m\angle AED = \frac{1}{2}(128+38) = \underline{\mathbf{83}}^\circ$

Alternate solution (Tuan Lee):
As above $x = 8$ ➔ $m\angle P = 43°$.
Since $\angle BCE$ and $\angle BAE$ are both inscribed angles intercepting the same arc $\overset{\frown}{BDE}$, each measures 140°. As an exterior angle of ΔAPE, $m\angle BAE = m\angle BPD + m\angle AEP$
$= m\angle BPD + 180 - m\angle AED$ and we have: $m\angle AED = 180 - 140 + 43 = \underline{\mathbf{83}}°$.