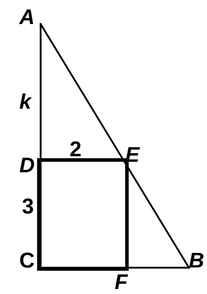
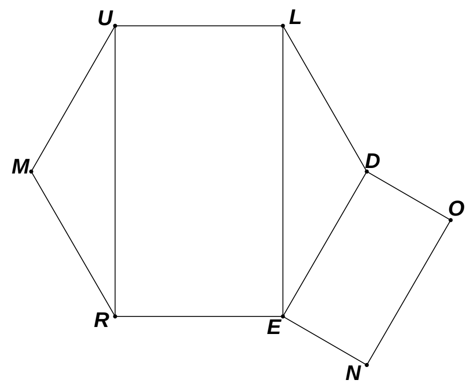
**MASSACHUSETTS MATHEMATICS LEAGUE**
**CONTEST 4 - JANUARY 2012 SOLUTION KEY**




## Round 5

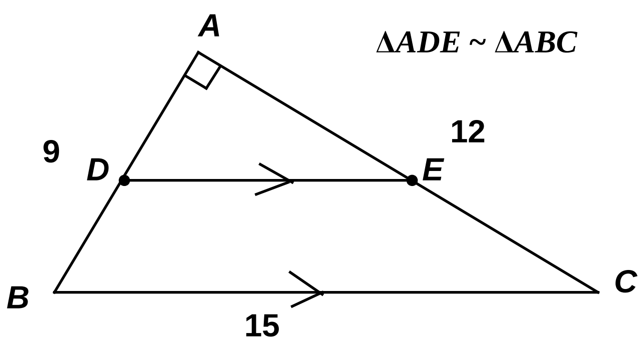
A) $\Delta ADE \sim \Delta EFB \Rightarrow \dfrac{AD}{DE} = \dfrac{EF}{FB} \Rightarrow \dfrac{k}{2} = \dfrac{3}{FB} \Rightarrow FB = \underline{\boldsymbol{\dfrac{6}{k}}}$



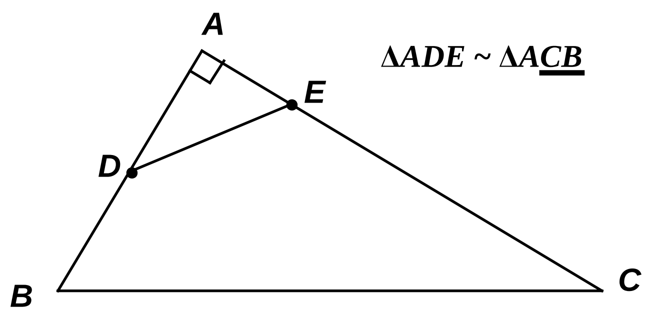

B) Drop a perpendicular from $M$ to $\overline{UR}$, intersecting $\overline{UR}$ in point $Q$, forming two 30 – 60 – 90 right triangles.
$MR = 2 \Rightarrow QU = \sqrt{3}$, $UR = 2\sqrt{3}$
Rectangles whose lengths and widths are in the same ratio are similar.

Thus, $\dfrac{\text{short}}{\text{long}} = \dfrac{UL}{UR} = \dfrac{DO}{DE} \Rightarrow \dfrac{2}{2\sqrt{3}} = \dfrac{DO}{2} \Rightarrow DO = \dfrac{2}{\sqrt{3}} = \underline{\boldsymbol{\dfrac{2\sqrt{3}}{3}}}$

**Δ*ADE* ~ Δ*ABC***




**Δ*ADE* ~ Δ*A<u>CB</u>***




C) Right angle at $A$ and $(AB, BC) = (9, 15) \Rightarrow AC = 12$.
If $\overline{DE} \parallel \overline{BC}$ ($\Delta ADE \sim \Delta ABC$), then $E$ is also a midpoint and $AE = 6 = \dfrac{6}{1} \Rightarrow a + b = \underline{\mathbf{7}}$.

If $\Delta ADE \sim \Delta ACB$ (<u>Note</u>: $D$ now corresponds to $C$, and $E$ to $B$), then $\dfrac{AD}{AC} = \dfrac{AE}{AB} \Rightarrow$

$\dfrac{4.5}{12} = \dfrac{AE}{9} \Rightarrow \dfrac{9}{24} = \dfrac{3}{8} = \dfrac{AE}{9} \Rightarrow AE = \dfrac{27}{8}$

$\Rightarrow a + b = \underline{\mathbf{35}}$.