

**MASSACHUSETTS MATHEMATICS LEAGUE**  
**MARCH 2006**  
**ROUND 7: TEAM QUESTIONS**

**\*\*\*\*\* NO CALCULATORS ON THIS ROUND \*\*\*\*\***

**ANSWERS**

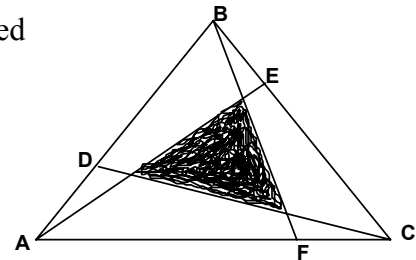
A) \_\_\_\_\_ D) \_\_\_\_\_  
 B) \_\_\_\_\_ E) \_\_\_\_\_  
 C) ( \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_ ) F) \_\_\_\_\_

- A) Let  $A$  be the product of all values for the constant  $k$  for which the system has no solutions for  $(x, y)$ . Let  $B$  be the product of all values for the constant  $k$  for which the system has infinitely many solutions for  $(x, y)$ . Find  $A + B$ .

$$\begin{aligned} 4x + k^2y &= -4 - 2k \\ (k^2 - 5)x - y &= 2 \end{aligned}$$

- B) Let  $A$  be a positive two-digit integer with the property that if the digits are reversed to form the smaller integer  $B$ , then  $A^2 - B^2$  is a perfect square. Find the sum of all values of  $A$  with this property.
- C) The zeros of  $y = f(x) = ax^3 + bx^2 + cx + 7$  are one more than the reciprocals of the zeros of  $y = g(x) = x^3 + x^2 - 5x + 2$ . Determine  $(a, b, c)$ .
- D)  $ABCD$  is a parallelogram. Three of the vertices are  $(1, 7)$ ,  $(-3, 1)$  and  $(9, 4)$ . The fourth vertex has several possible locations. If  $P$  is the one furthest from the line  $y = x$ , exactly how far is  $P$  from the origin?

- E)  $\triangle ABC$  is equilateral with  $AB = 26$ . Points  $D, E$  and  $F$  are placed so that  $AD = \frac{1}{4}(AB)$ ,  $BE = \frac{1}{4}(BC)$  and  $CF = \frac{1}{4}(CA)$  as shown. Find the exact area of the shaded region.



- F) Assume  $n$  is a positive integer. Find the sum of all different values of  $n$  for which the expansion of  $(4x^n + \frac{x^{-3}}{2})^{10}$  will contain an  $x$ -free term, i.e. a constant term.