MASSACHUSETTS MATHEMATICS LEAGUE
MARCH 2004
ROUND 2: EXPONENTS & RADCALS

ANSWERS

A) $x = -1$

B) $2\sqrt{2}$

C) $x = -2$ or 3

A) Find the exact value of x: $\sqrt{4+\frac{1}{4}+\frac{4}{9}} = 2+\frac{1}{2}+\frac{x}{3}$

$\sqrt{\frac{144+9+16}{36}} = \sqrt{\frac{169}{36}} = \frac{13}{6} = \frac{5}{2}+\frac{x}{3}, \quad \frac{13-15}{6} = \frac{x}{3}$

$\frac{x}{3} = -\frac{2}{6} = -\frac{1}{3}, \quad x = -1$

B) Convert to simplified radical form $\frac{\sqrt{6}-\sqrt{2}}{\sqrt{3}+1}+\frac{2\sqrt{3}}{\sqrt{2}}$

$\frac{(\sqrt{6}-\sqrt{2})(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} + \sqrt{2}\sqrt{3} = \frac{3\sqrt{2}-2\sqrt{6}+\sqrt{2}+2\sqrt{6}}{2} =$

$\frac{4\sqrt{2}}{2} = 2\sqrt{2}$

C) Solve for x $8^{\frac{x+2}{x}} = 16^{\frac{x+2}{4}}$

I. $2^{\frac{3(x+2)}{x}} = 2^{\frac{4(x+2)}{4}} \qquad 3x+6 = x(x+2), \quad 3x+6 = x^2+2x,$

$x^2-x-6=0, \quad (x+2)(x-3)=0, \quad x=-2,3.$

II $x=-2$ since $8^0 = 16^0 = 1$, Then $8^{\frac{1}{x}} = 16^{\frac{1}{4}} = 2, \quad x=3$

III. $8^{\frac{x+2}{x}} = 8^{\frac{4}{3}}, \quad \frac{x+2}{4} = 8^{\frac{x+2}{3}} \qquad x = -2$ or 3