

**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2010 SOLUTION KEY**

Team Round - continued

F) Note: $a_n = \frac{1}{2 + a_{n-1}}$ So, rather than thinking of a_4 as $\frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2}}}}$,

$$\text{we look at } a_4 = \frac{1}{2 + a_3} = \frac{1}{2 + \frac{5}{12}} = \frac{12}{29}$$

Lining up the a -sequence evidence $\begin{array}{cc|cc} 1 & 2 & \overline{5} & \underline{12} \\ 2 & 5 & \underline{12} & 29 \end{array} \begin{array}{cc} X & Z \\ Y & T \end{array}$, we notice that $Y = Z$ and $T = X + 2Y$.

Therefore, the a -sequence continues $\frac{29}{2(29)+12} = \frac{29}{70}, \frac{70}{169}, \frac{169}{408}, \frac{408}{985}, \frac{985}{2378}, \frac{2378}{5741}$.

$$a_{10} = \frac{2378}{5741} \rightarrow A_{10} = 1 + \frac{2378}{5741} = \frac{\mathbf{8119}}{\mathbf{5741}}$$