MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 – MARCH 2007 SOLUTION KEY

Team Round

A) $9a + 2 = 2b + 9 \rightarrow b = (9a - 7)/2 \rightarrow (1, 1)$ which is rejected. Slope $m = 9/2$ and
$|b - 87| < 14 \rightarrow -14 < b - 87 < +14 \rightarrow 73 < b < 101$
$\rightarrow$ value of b must be between 74 and 100 inclusive $\rightarrow$ min ordered pair to be tested is
$(1 + 9(2), 1 + 9(9)) = (19, 82)$ ok
Next $(21, 91)$ rejected (GCF = 7), $(23, 100)$ ok
$\begin{vmatrix} 19 & 82 \\ 23 & 100 \end{vmatrix} = 19(100) - 23(82) = 1900 - 1886 = \underline{\mathbf{14}}$

B) $34^2 + 33 = 1189$ and $35^2 + 34 = 1259 \rightarrow x = 34$

Testing $y = 24$. Is $5 - \sqrt{24} < .1$? $5 - \sqrt{24} < .1 \rightarrow 4.9 < \sqrt{24} \rightarrow 24.01 < 24$
Oops, 24 fails, but just barely. Thus, $y = 25$.
$(x, y) = (34, 25) \rightarrow$ required product $= 34(59) \rightarrow$ prime factors $= 2, 17, 59 \rightarrow \underline{\mathbf{78}}$

C) Since the coefficients are real, complex roots must occur in conjugate pairs. Thus, $(-1 - i)$ is also a root. Using the sum and product of the roots relation to the coefficients, $x^2 + 2x + 2$ must be a factor of $p(x)$. Since the cubic term of $p(x)$ is missing (i.e. $0x^3$) and the constant term is -6, the other factor of $p(x)$ must be $x^2 - 2x - 3$.
Multiplying, $p(x) = x^4 - 5x^2 - 10x - 6 \rightarrow (A, B) = \underline{\mathbf{(-5, -10)}}$

D) Solving for A in terms of $B \rightarrow A = 2 + \dfrac{160}{3B+2}$
$160 = 2^5 5^1 \rightarrow 160$ has 12 factors: 1, 2, 4, 5, 8, 10, 16, 20, 32, 40, 80 and 160.
Equating $3B + 2$ to each of these values produces an integer value of $B = \cancel{1}, 2, 6, 10$ and 26
which correspond to $A = \cancel{34}, 22, 10, 7$ and $4 \rightarrow$ the ordered pairs **<u>(22, 2), (10, 6), (7, 10), (4, 26)</u>**