MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2008 SOLUTION KEY

Round 6

A) $9 + 4d = 25 \rightarrow d = 4 \rightarrow t_{14} = t_{12} + 2(4) = \underline{\mathbf{57}}$

B) The first sequence is an <u>arithmetic</u> sequence with a common difference of 7, i.e. $t_n = 7n - 9$.
The second sequence is a <u>geometric</u> sequence with a common ratio of –2, i.e. $t_n = (3/8)(-2)^n$

The 10^{th} term in the geometric sequence is $(3/8)(-2)^{10} = 384$.
$7n - 9 > 384 \rightarrow n > 393/7 = 56+ \rightarrow n = 57 \rightarrow 7(57) - 9 = \underline{\mathbf{390}}$

C) Using the recursive part of the definition, $A_{N+2} = 2A_{N+1} + 3A_N$

$N = 3 \rightarrow A_5 = 2A_4 + 3A_3$

$N = 2 \rightarrow A_4 = 2A_3 + 3A_2$

Substituting for A_2 and A_5, $\begin{cases} 17 = 2A_4 + 3A_3 \\ A_4 = 2A_3 + 12 \end{cases} \rightarrow (A_3, A_4) = (-1, 10)$

$A_3 = 2A_2 + 3A_1 \rightarrow -1 = 8 + 3A_1 \rightarrow A_1 = -3$

$A_6 = 2A_5 + 3A_4 = 2(17) + 3(10) = 64$

Thus, $A_1 + A_6 = \underline{\mathbf{61}}$