MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2016 SOLUTION KEY

Round 2

A) $2x = 3 + \dfrac{15}{x+2} \Rightarrow (2x-3)(x+2) = 15 \Leftrightarrow 2x^2 + x - 6 - 15 = 0$, provided $x \neq -2$.

$\Leftrightarrow 2x^2 + x - 21 = (x-3)(2x+7) = 0$

$\Rightarrow x = \underline{\mathbf{3, -\frac{7}{2}}}$.

B) $A^2 - B^2 = 72 \Rightarrow (A+B)(A-B) = 72 \Rightarrow \begin{cases} A+B \\ A-B \end{cases} = \begin{cases} 72 & \mathbf{36} & 24 & \mathbf{18} & \mathbf{12} & 9 \\ 1 & \mathbf{2} & 3 & \mathbf{4} & \mathbf{6} & 8 \end{cases}$

For <u>positive</u> <u>integers</u> *A* and *B*, it is always true that $A+B > A-B$, so we stop at $(A+B, A-B) = (9,8)$. If $A+B$ and $A-B$ were to have opposite parity, then neither *A* nor *B* would be integers. Thus, we consider only the bolded pairs.

Adding the equations, we have $2A = \begin{cases} 38 \\ 22 \\ 18 \end{cases} \Rightarrow A = 19, 11, 9 \Rightarrow (A,B) = \underline{\mathbf{(19,17), (11,7), (9,3)}}$

C) $(x^2+y)(x^2-y) + x^2(y^2-1)$ is a sum of two terms and, since there is no common factor between these two terms (other than 1), we have no option other than first multiplying out these terms $(x^2+y)(x^2-y) + x^2(y^2-1) = x^4 - y^2 + x^2y^2 - x^2$.

Grouping the first and third, and second and fourth terms, we have

$(x^4 + x^2y^2) - (y^2 + x^2) = x^2(x^2+y^2) - 1(y^2+x^2) = (x^2+y^2)(x^2-1) = \underline{\mathbf{(x^2+y^2)(x-1)(x+1)}}$

or $\underline{\mathbf{(x^2+y^2)({}^{-}1+x)(1+x)}}$. Each has exactly one minus sign.

Do <u>not</u> accept

$(x^2+y^2)(1-x)(-1-x)$, $(x^2+y^2)(1+{}^{-}x)({}^{-}1-x)$ or $(x^2+y^2)(1+{}^{-}x)({}^{-}1+{}^{-}x)$,

since each of these is considered to have three “minus” signs.