MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2006 SOLUTION KEY

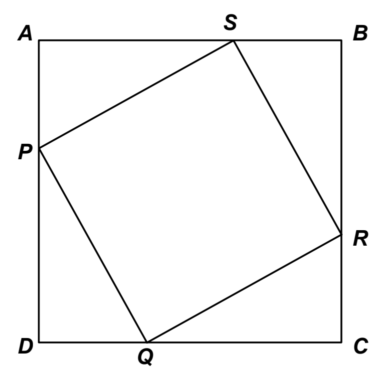
Team Round

A) $(3-4i)+\dfrac{3-4i}{25}=3.12-4.16i=\sqrt{z}+c$

To determine $\sqrt{z}$, $\sqrt{3+4i}=\sqrt{(a+bi)^2}=\sqrt{(a^2-b^2)+2abi}$ → $a^2-b^2=3$ and $ab=2$
→ $(a, b) = (2, 1)$ → $3.12 - 4.16i = 2 + i + c$ → $c = \underline{\mathbf{1.12 - 5.16}\boldsymbol{i}}$

B) Let (k, c) denote the rowing rate of the kayaker in still water and the current of the stream.
$(k - c) = (4/5)(k + c)$ → $5k - 5c = 4k + 4c$ → $k = 9c$ or $k : c = 9{:}1$
$R_{up}: R_{down} = (9x - x) + (9x + x) = 8x : 10x$ → $T_{up} : T_{down} = 10x : 8x$
$T_{up} + T_{down} = 18x = 3$ → $x = 1/6$ → $T_{up} = 5/3$ hour and $T_{down} = 4/3$ hour
Upstream rate: $k - c = 6/(5/3) = 18/5$ mph Downstream rate: $k + c = 6/(4/3) = 18/4$ mph
Solving simultaneously, $2k = 81/10$ → $k = 81/20 =$ **4.05** mph.

C) Let $SB = x$ → $BR = 12 - x$
With no loss of generality, assume $SB < SA$ as appears to be the case in the given diagram.
$x^2 + (12 - x)^2 = 128$ → $x^2 - 12x + 8 = 0$ → $SB = 6 - 2\sqrt{7}$
→ $SB^2 = 64 - 24\sqrt{7}$
$SD^2 = SA^2 + AD^2 = (6 + 2\sqrt{7})^2 + 12^2 = 64 + 24\sqrt{7} + 144 =$
$208 + 24\sqrt{7}$



It is true, in general, for any point S in the plane of a square $ABCD$, that

$$SA^2 + SC^2 = SB^2 + SD^2$$

Thus, $SA^2 + SB^2 + SC^2 + SD^2 = 2(SB^2 + SD^2) = 2(64 - 24\sqrt{7} + 208 + 24\sqrt{7}) = \underline{\mathbf{544}}$

D) $-x^{10} + x^4 + x - x^7 = x(-x^9 + x^3 + 1 - x^6) = x(x^3(1 - x^6) + (1 - x^6)) = x(1 + x^3)(1 - x^6)$
$= x(1 + x^3)^2(1 - x^3) = x[(1 + x)(1 - x + x^2)]^2(1 - x)(1 + x + x^2)$
$= \underline{\mathbf{x(1 + x)^2(1 - x)(1 - x + x^2)^2\ (1 + x + x^2)}}$ – or equivalent