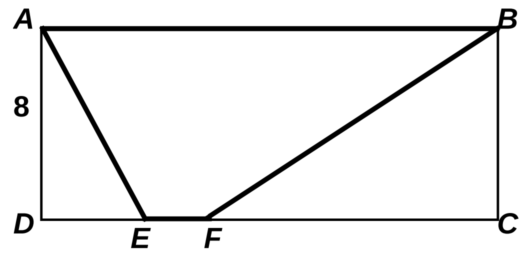
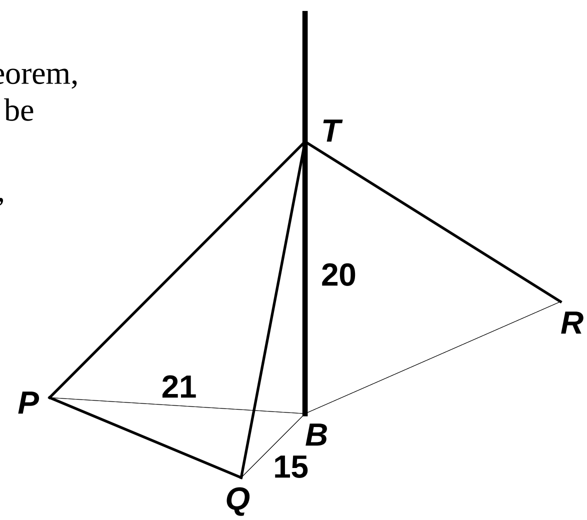
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2014 SOLUTION KEY

Round 2



A) $DE = 6, FC = 15, AB = 24 \Rightarrow EF = 3$

Thus, the area of $ABFE$ is $\frac{1}{2} \cdot 8 \cdot (3 + 24) = \underline{\mathbf{108}}$



B) Using special right triangles or the Pythagorean Theorem, $PT = 29$ and $QT = 25$. The triangle inequality must be satisfied in both ΔTPQ and ΔBPQ.

In ΔTPQ, $PQ < 29 + 25 = 54$; however, in ΔBPQ, $PQ < 21 + 15 = 36$.

Thus, the maximum integer value of PQ is 35 $\Rightarrow$ the maximum perimeter of ΔTPQ is **89**.

C) Let a denote the width of a stripe and b its length.
Let c and d denote the length of the diagonals on each flag.

Then: $\begin{cases} (7a)^2 + b^2 = c^2 & \text{(flag on left)} \\ a^2 + b^2 = d^2 & \text{(flag on right)} \end{cases}$ and $\begin{cases} 2c + 14d = 256 \\ c = 3d - 2 \end{cases}$

Substituting in the second pair of equations, $(3d - 2) + 7d = 128 \Rightarrow d = 13, c = 37$

$d = 13 \Rightarrow (5,12,13)$, $c = 37 \Rightarrow (35,12,37)$

and the dimensions of the flag are 12 x 35 producing an area of **420**.