## MASSACHUSETTS MATHEMATICS LEAGUE CONTEST 2 – NOVEMBER 2008 SOLUTION KEY

## Round 6

- A)  $360/n = 4.5 \implies n = 80$  and the sum of the interior angles is determined by 180(n-2). 180(78) = 14040
- B)  $x = 49 \rightarrow y = 49$ As vertical angles,  $m \angle 3 + m \angle 4 = y$ . Since  $m \angle 3 = m \angle 4$ ,  $m \angle 4 = y/2$ .  $m \angle 1 + m \angle 2 = 180 - y \rightarrow m \angle 2 = 90 - y/2$ Draw a line through the vertex of the angle whose measure is  $z^{\circ}$  parallel to  $L_1$ . As alternate interior angles of  $\| s \|$ ,  $m \angle 2 = m \angle 5$  and  $m \angle 4 = m \angle 6$ . Thus, z = y/2 + (90 - y/2) = 90 and y + z = 139
- C) By the triangle angle bisector theorem,  $\frac{BD}{DC} = \frac{AB}{AC}$ .

Since 
$$\triangle DEF \sim \triangle DAC$$
,  $\frac{FE}{AC} = \frac{DF}{DC}$ .

Using 
$$BD = DF$$
, we have  $\frac{FE}{AC} = \frac{BD}{DC}$ 

Using transitivity, we have a third proportion  $\frac{FE}{AC} = \frac{AB}{AC}$ .

Since the denominators are equal, we have  $FE = AB = \underline{7}$ . No computations were necessary!

