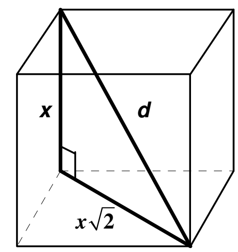
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2008 SOLUTION KEY

Round 1

A) Let x and d denote the edge and diagonal respectively of the cube.
Notice from the diagram at the right that $d = x\sqrt{3}$.



The TSA = $6x^2 = 1$. Substituting, $6\left(\dfrac{d}{\sqrt{3}}\right)^2 = 1 \rightarrow 2d^2 = 1 \rightarrow d = \underline{\boldsymbol{\dfrac{\sqrt{2}}{2}}}$.

B) Let r denote the <u>inner</u> radius of the ball.

$684\pi = \dfrac{4}{3}\pi(r+3)^3 - \dfrac{4}{3}\pi r^3$

Canceling the common factor of π and multiplying by $\dfrac{3}{4}$,

$3(171) = 513 = (r + 3)^3 - r^3 = 9r^2 + 27r + 27 \rightarrow 9r^2 + 27r - 486 = 0$
$\rightarrow 9(r^2 + 3r - 54) = 9(r + 9)(r - 6) = 0 \rightarrow r = 6 \rightarrow$ outside diameter = 2(6 + 3) = **<u>18</u>** cm

C) The region bounded by the plane and the three faces intersecting in the common vertex is a pyramid with a right triangular base.
V(cube) : V(Pyramid) = 8 : (1/3)(1/2)(1)
$\rightarrow$ the required ratio (8 – 1/6) : 1/6 = 47 : 1 $\rightarrow$ **<u>46</u>**