MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2010 SOLUTION KEY

Round 4

A) B is the midpoint of $\overline{AC}$ → $\log n = \dfrac{\log 60 + \log 90}{2} = \dfrac{1}{2}\log 5400 = \log\sqrt{5400}$

Therefore, $n = \sqrt{5400} = \sqrt{100 \cdot 9 \cdot 6} = \underline{\mathbf{30\sqrt{6}}}$

B) $5^{2\log_5 x} - 12\left(4^{\log_2 \sqrt{x}}\right) - 27^{\log_3 4} = 0$ → $5^{\log_5 x^2} - 12\left(4^{\log_4 x}\right) - 3^{\log_3 (4^3)} = 0$

→ $x^2 - 12x - 64 = (x - 16)(x + 4) = 0$ → $x = \underline{\mathbf{16}}$ ($x = -4$ is extraneous.)

C) Let $a = 10^x$. Then $100^x - 3 \cdot 2^{x+1} \cdot 5^x + 5 = 0$ → $a^2 - 6a + 5 = (a - 5)(a - 1) = 0$

Thus, $10^x = 1$ → $x = \underline{\mathbf{0}}$ or $10^x = 5$ → $x = \underline{\mathbf{\log_{10} 5}}$ (or simply $\underline{\mathbf{\log 5}}$)

Round 5

A) y varies directly as x and z → $y = kxz$, for some constant k.

Substituting, $5 = k(3)(4)$ → $k = 5/12$.

Therefore, $y = \dfrac{5}{12} \cdot 36 \cdot 134 = 15(134) = \underline{\mathbf{2010}}$

B) The profit from the sale of the house was \$227000 – (\$180000 + \$12000) = \$35000

A total of \$5600 worth of repairs were done and Ben contributed $\dfrac{3600}{5600} = \dfrac{9}{14}^{\text{th}}$ of the money,

and Joe contributed $\dfrac{5}{14}^{\text{th}}$. $14k = 35000$ → $k = 12500$ → **Ben: \$22,500 Joe: \$12,500**

C) Let $A = 60 + x$ and $B = 20 + y$.

According to the first student, $\dfrac{10x + 6}{20 + y} = \dfrac{3}{2}$. According to the second student, $\dfrac{60 + x}{10y + 2} = \dfrac{3}{2}$.

Cross multiplying, $20x + 12 = 3y + 60$ → $20x - 3y = 48$ and

$120 + 2x = 30y + 6$ → $2x - 30y = -114$

$\begin{cases} -200x + 30y = -480 \\ 2x - 30y = -114 \end{cases}$ → $-198x = -594$ → $x = 3$, $y = 4$ → $(A, B) = \underline{\mathbf{(63, 24)}}$.