MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2016 SOLUTION KEY

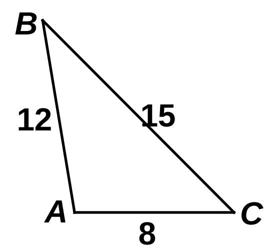
Round 1

A) Let the hypotenuse and long leg have lengths (x + 2) and x.

Then: $16^2 + x^2 = (x+2)^2 \Leftrightarrow 256 + x^2 = x^2 + 4x + 4 \Rightarrow 64 = x + 1 \Rightarrow x = 63$

Thus, the area is $\frac{1}{2} \cdot 16 \cdot 63 = 8 \cdot 63 = \underline{\mathbf{504}}$.

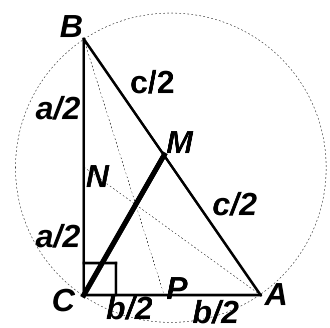
B) According to the Law of Sines,



$$\frac{\sin A}{15} = \frac{\sin B}{8} = \frac{\sin C}{12} = n \Rightarrow \begin{cases} \sin A = 15n \\ \sin B = 8n \\ \sin C = 12n \end{cases}$$

Therefore, $\frac{\sin B + \sin C}{\sin A} = \frac{8n + 12n}{15n} = \frac{20}{15} = \underline{\mathbf{\frac{4}{3}}}$.

C) In right triangles BCP and ACN,



$$\begin{cases} a^2 + \left(\frac{b}{2}\right)^2 = 8 \Rightarrow 4a^2 + b^2 = 32 \\ b^2 + \left(\frac{a}{2}\right)^2 = 27 \Rightarrow a^2 + 4b^2 = 108 \end{cases} \Rightarrow a^2 + b^2 = \frac{140}{5} = 28$$

But $a^2 + b^2 = c^2 = 28 \Rightarrow c = 2\sqrt{7} \Rightarrow CM = \underline{\sqrt{7}}$.

FYI:
The midpoint of the hypotenuse is the center of the circumscribed circle, i.e. the circle which passes through the 3 vertices of the right triangle ABC.
The medians in ANY triangle are concurrent, i.e. pass through a common point.
The point of concurrency divides each median into segments whose lengths are in a 2 : 1 ratio.