MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2008 SOLUTION KEY

Round 6

A) The Thursdays in January fall on: 1, 8, 15, 22, 29, 36 (oops!) = Feb 5th
→ the Thursdays in February fall on: 5, 12, 19, 26 and 33 (oops!) = Mar 5th
→ the Thursdays in March fall on: 5, 12, 19, 26 and 33 (oops!) = April 2nd
→ the Thursdays in April fall on: 2, 9, 16, 23, 30, 37 (oops!) = May 7th

Alternate solution uses **John Conway's "Doom's Day" Formula.**
The following 11 dates fall on the **same** day of the week (DOW) as the last day of February.

3/7 (7 days later)
4/4 6/6 8/8 10/10 12/12 (even / day matches month)
7/11 and 11/7 5/9 and 9/5 (odd / month-day reversals)
A helpful mnemonic: "John is dyslexic and works from 9 to 5 at the '711'."

This takes care of every month except January
1/31 is 28 (or 29) days before the last day in February, depending on leap year status
Thus, **1/31 is the same DOW in a non-leap year and a day earlier in a leap year**.

Notice: The 11 listed dates all fall on Friday in 2008 (since 2/29 is a Friday) but 1/31 falls on a Thursday since 2008 is a leap year.

Proving this is not very difficult. Write up your arguments and give them to your coach!

The problem at hand: 1/1 = Thurs → 1/29 = Thurs → 1/31 = Sat.
The fact that 2025 is not a leap year means that "Doom's Day" in 2025 is on a Saturday.
3/1 = Sat → 1st Thurs = 5th, 4/4 = Sat → 4/2 = 1st Thurs, 5/9 = Sat → 5/7 = 1st Thurs
→ Month = **MAY**

B) 29 (prime) → sum = 1 [29 is a deficient number (as are all primes)]
28 → factors of 1, 2, 4, 7 and 14 → sum = 28 [28 is called a perfect number.]
27 → 1, 3 and 9 → sum = 13
26 → 1, 2, 13 → sum = 16
25 → 1 and 5 → sum = 6 [25, 26 and 27 are all deficient numbers]
24 → 1, 2, 3, 4, 6, 8 and 12 → sum = 36 Bingo! - **24** is the abundant number we want!

C) We could directly substitute the 7 ordered pairs in the definition of the operation ♦.
But a much better approach would be to determine what ordered pairs satisfied the equation
$a \;\blacklozenge\; b = b \;\blacklozenge\; a$ or $(a+1)(2-b)=(b+1)(2-a)$.
Multiplying out, $2a - ab + 2 - b = 2b - ab + 2 - a$ → $2a - b = 2b - a$ → $a = b$
Thus, we require ordered pairs (x, x^2) for which $x = x^2$.
$x = x^2$ → $x^2 - x = x(x-1) = 0$ → $x = 0, 1$ → **2** solutions [(0,0), (1,1)]