MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2015 SOLUTION KEY

Round 2

A) $3725_8 \;+\; 452_6 = 3(8)^3 + 7(8)^2 + 2(8)^1 + 5 = 3(512) + 7(64) + 16 + 5 = 1536 + 448 + 21 = 2005$

$452_6 = 4(6)^2 + 5(6)^1 + 2 = 4(36) + 30 + 2 = 144 + 32 = 176$

Thus, the sum is <u>**2181**</u>.

B) List the primes that are less than 60: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59
The possible ordered pairs for (A, B) are (53, 7), (47, 13), (43, 17), (41, 19), (37, 23) and (31, 29)
The possible differences are 46, 34, 26, 22, 14 and 2.
Therefore the average of the possible differences is 144/6 = <u>**24**</u>.

C) Since 999,999,999 + 1 = 1,000,000,000 = 10^9 and we are searching for the largest integer N for which $N^2 < 10^9$, we note that

$N = \sqrt{10^9} = 10^4 \cdot \sqrt{10} < 10^4(3.1622\cdots) \Rightarrow 31622^2 = 999{,}9\_\,\_{,}\_\,\_\,\_$

$31622^2 = (31600 + 22)^2 = (31600)^2 + 2(22)(31600) + 22^2$

Since we only need the 5 least significant digits, the first term contributes 60000, the second term contributes 90400 and the third term contributes 00484. Summing, the rightmost five digits are 50884 $\Rightarrow$ a digit sum of <u>**25**</u>.

Alternate Solution (submitted by the famous Roman mathematician Brutus Forcus]
Square 31622 by hand (arghhhhh!!!).

$$\begin{array}{r} 31622 \\ \underline{31622} \\ 63244 \\ 632440 \\ 18973200 \\ 31622000 \\ \underline{948660000} \\ 9999\boldsymbol{50884} \end{array}$$

Since $\sqrt{10} \approx 3.16228$, the $\sqrt{10}$ is "sandwiched" between 3.1622 and 3.1623, and the next best estimate is 8/10 of the distance between these two values. Since $\sqrt{N}$ increases as N increases, 31623^2 will be a larger value, but will it exceed 1,000,000,000 and overflow the display???

$31623^2 = (31622+1)^2 = \boxed{31622^2} + 2(31622) + 1$, an increase of 63245 over the value calculated above. This increase will push the total over 9 digits.

Thus, 31622^2 is the largest displayable perfect square and the required sum is <u>**25**</u>.