MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - NOVEMBER 2011 SOLUTION KEY

Team Round - continued

F) Let $x = m\measuredangle A$.
P has (10 + *n*) sides, i.e. at least 11 sides and at most 19 sides.
Thus, $180((10 + n) - 2) = 10(x + k) + nx = x(10 + n) + 10k$
$\Rightarrow (180 - x)(10 + n) = 360 + 10k \Rightarrow x = 180 - \dfrac{360 + 10k}{10 + n}$
As *n* increases from 1 to 9, the denominator of the fraction increases from 11 to 19.
If $n = 1$, then the numerator must be the smallest possible multiple of 11.
This occurs for $k = 8$ and we have $x = A = 180 - 440/11 = 140$.
$k = 19$ also insures divisibility by 11, but $x = 180 - 550/11 = 130$ and it's not as large as possible.

$n = 2$: $\dfrac{360 + 10k}{12} = 30 + \dfrac{5k}{6} \Rightarrow k = 6, x = 180 - 35$, $(A, B) = (145,151)$
$n = 3$: $\dfrac{360 + 10k}{13} = \dfrac{351}{13} + \dfrac{10k + 9}{13} \Rightarrow k = 3, x = 180 - 30$, $(A, B) = (150,153)$
$n = 4$: $\dfrac{360 + 10k}{14} = \dfrac{2 \cdot 5(36 + k)}{2 \cdot 7} \Rightarrow k = 6, x = 180 - 30$, $(A, B) = (150,156)$
$n = 5$: $\dfrac{360 + 10k}{15} = 24 + \dfrac{2k}{3} \Rightarrow k = 3, x = 180 - 26$, $(A, B) = (154,157)$
$n = 6$: $\dfrac{360 + 10k}{16} = \dfrac{2 \cdot 5(36 + k)}{2 \cdot 8} \Rightarrow k = 4, x = 180 - 25$, $(A, B) = (155,159)$
$n = 7$: $\dfrac{360 + 10k}{17} = \dfrac{357}{17} + \dfrac{10k + 3}{17} \Rightarrow k = 15, x = 180 - 30$, $(A, B) = (150,165)$
$n = 8$: $\dfrac{360 + 10k}{18} = 20 + \dfrac{5k}{9} \Rightarrow k = 9, x = 180 - 25$, $(A, B) = (155,164)$
$n = 9$: $\dfrac{360 + 10k}{19} \Rightarrow k = 2, x = 180 - 20$, $(A, B) = (160,162)$
Thus, $(m, M) =$ **<u>(2, 15)</u>**.

Checks that $10(m\angle B) + n(m\angle A) = 180(n - 2)$
(not necessary, but included just to be on the safe side!)
$n = 2$: $1510 + 2(145) = (12 - 2)180 = 1800$ $n = 3$: $1530 + 3(150) = (13 - 2)180 = 1980$
$n = 4$: $1560 + 4(150) = (14 - 2)180 = 2160$ $n = 5$: $1570 + 5(154) = (15 - 2)180 = 2340$
$n = 6$: $1590 + 6(155) = (16 - 2)180 = 2520$ $n = 7$: $1650 + 7(150) = (17 - 2)180 = 2700$
$n = 8$: $1640 + 8(155) = (18 - 2)180 = 2880$ $n = 9$: $1620 + 9(160) = (19 - 2)180 = 3060$