MASSACHUSETTS MATHEMATICS LEAGUE CONTEST 5 - FEBRUARY 2017 SOLUTION KEY

Round 4

A)
$$0.05(50) + 0(x) = 0.03(50 + x)$$

 $\Leftrightarrow 250 = 150 + 3x \Leftrightarrow x = \frac{100}{3}$

B) The entire trip was 400 miles.

Rate x Time = Distance
$$\Rightarrow$$
 Rate = $\frac{\text{Distance}}{\text{Time}}$

$$\Rightarrow \frac{400}{\frac{200}{48} + \frac{200}{52}} = \frac{2}{\frac{1}{48} + \frac{1}{52}} = \frac{2}{\frac{52 + 48}{48 \cdot 52}} = \frac{2 \cdot 48 \cdot 52}{100} = 0.96(52) = 49.92$$

Thus, my average was **0.08** mph less than 50 mph.

C) Points A and B are travelling on the circumference of concentric circles.

1 revolution for point A is 6π inches; for B, it is $\frac{5}{8}(6\pi) = \frac{15}{4}\pi$ inches.

At 1024 RPM, in $\frac{1}{10,000}$ of a second, point *B* makes $\frac{1024/60}{10,000}$ of a revolution which is

$$\frac{256}{15 \cdot 10^4} \cdot \frac{15\pi}{4} = \frac{2^8}{2^4 \cdot 5^4} \cdot \frac{\pi}{2^2} = \frac{2^2}{5^4} \pi = \frac{4\pi}{625} \implies k = \frac{4}{625}.$$