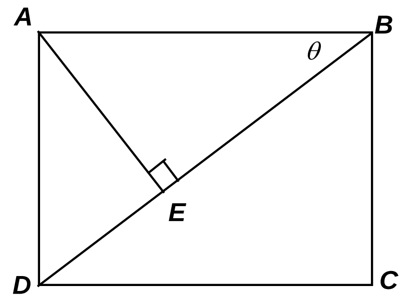
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 – NOVEMBER 2011 SOLUTION KEY

Round 5



A) In ΔDBA, $\cos\theta = \dfrac{AB}{BD}$

$AD = 5$ and $AE = 4 \Rightarrow DE = 3$

$\Delta ABE \cong \Delta DBA$ (by AA)

$\Rightarrow \dfrac{AB}{DB} = \dfrac{AE}{DA} = \underline{\boldsymbol{\dfrac{4}{5}}}$

[Using the dimensions of right ΔABE is a distraction, since computing $BE = \dfrac{16}{3}$ and $AB = \dfrac{20}{3}$ is unnecessary.]

B) $\left(\dfrac{\sqrt{2}}{2} - 1\right)^4 = \left(\dfrac{\sqrt{2}}{2}\right)^4 - 4\left(\dfrac{\sqrt{2}}{2}\right)^3 + 6\left(\dfrac{\sqrt{2}}{2}\right)^2 - 4\left(\dfrac{\sqrt{2}}{2}\right) + 1 = \dfrac{1}{4} - \sqrt{2} + 3 - 2\sqrt{2} + 1 = \dfrac{17}{4} - 3\sqrt{2}$

$= \dfrac{17 - 12\sqrt{2}}{4} \Rightarrow (A, B, C) =$ **<u>(17, 12, 4)</u>**

C) $\cos(270° + x) = \sin(-600°) \Leftrightarrow \sin x = \sin(120°) \Leftrightarrow x = \begin{cases} 120° + 360n \\ 60° + 360n \end{cases}$

$1500 \le 120 + 360n \le 1900 \Leftrightarrow \dfrac{138}{36} \le n \le \dfrac{178}{36} \Rightarrow n = 4\,(\text{only}) \Rightarrow 1560°$

$1500 \le 60 + 360n \le 1900 \Leftrightarrow \dfrac{144}{36} \le n \le \dfrac{184}{36} \Rightarrow n = 4, 5 \Rightarrow 1500°, 1860°$

Adding, **<u>4920°</u>**.

Alternate Solution:

$\cos(270° + x) = \sin(-600°) \Leftrightarrow$

$\cos(270° + x) = \sin(-600° + 720°) = \sin(120°) = \sin 60° = \cos(\pm 30°)$

$270 + x = \pm 30 + 360n \Rightarrow x = \begin{cases} -240 + 360n \to 120, 480, 840, 1200, \underline{1560}, 1920 \\ -300 + 360n \to 60, 420, 780, 1140, \underline{1500, 1860} \end{cases} \Rightarrow$ **<u>4920</u>**.