MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2015 SOLUTION KEY

Round 2

A) Doing the addition is base 2, we have
$$\begin{array}{r} 1011010 \\ +\underline{1111111} \\ 11011001 \end{array}.$$

Converting this result to base 10, we have $2^7+2^6+2^4+2^3+2^0=128+64+16+8+1=217$.

Converting to base 5, we have
$$217-\mathbf{1}\left(5^3\right)=217-125=92$$
$$92-\mathbf{3}\left(5^2\right)=92-75=17$$
$$17-\mathbf{3}\left(5^1\right)=17-15=\mathbf{2}$$

Thus, $a_5=\underline{\mathbf{1332}}$

The same conversion can be obtained by repeatedly dividing by 5 and keeping track of the quotients and remainders until the quotient becomes 0. Reading the remainders from the bottom up gives us the above answer.

$$\begin{array}{rcl} 5\,|\,217 & & \\ 43 & \mathbf{2} & \\ & & \Uparrow \\ 8 & \mathbf{3} & \\ & & \Uparrow \\ 1 & \mathbf{3} & \\ & & \Uparrow \\ 0 & \mathbf{1} & \end{array}$$

B) Numbers divisible by both 6 and 15 are multiples of 30.
Let $n-1$, n and $n+1$ denote the three consecutive integers.
Then $3n=30k \Rightarrow n=10k$
$k=1,2,3 \Rightarrow n=10,20,30 \Rightarrow (9,10,11),(19,20,21),(29,30,31)$
Therefore, the required sum is $9+19+29=\underline{\mathbf{57}}$.

C) $A=125(45)^x=3^{2x}\cdot 5^{x+3} \Rightarrow n(A)=(2x+1)(x+4)$
$B=18(24)^x=2^{3x+1}\cdot 3^{x+2} \Rightarrow n(B)=(3x+2)(x+3)$
Thus, $\dfrac{(2x+1)(x+4)}{(3x+2)(x+3)}=\dfrac{3}{4} \Rightarrow 8x^2+36x+16=9x^2+33x+18 \Rightarrow x^2-3x+2=0$
$\Rightarrow (x-1)(x-2)=0 \Rightarrow x=\underline{\mathbf{1,2}}$