

**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2009 SOLUTION KEY**

Round 6

A) Clearly, the gaps between successive terms in every row and in every column are increasing by 2.

In the 2nd row, the numbers are: 4, 10, 18, 28, 40, 54, 70, 88

In the 3rd row, the numbers are: 8, 16, 26, 38, 52, 68, 86, 106

In the 1st column, the numbers are: 2, 4, 8, 14, 22, 32, 44, 58

In the 2nd column, the numbers are: 6, 10, 16, 24, 34, 46, 60, 76

In the 8th row, the numbers are: 58, 76, 96 Thus, the difference is 10.

B) $\frac{13x+6}{5x} = \frac{5x}{x+2} \rightarrow 3x^2 - 8x + 3 = (3x+1)(x-3) = 0 \rightarrow x = 3, -1/3$

$x = -1/3, r = -1 \rightarrow$ terms alternate positive, then negative; hence the 4th term is $-\frac{5}{3}$

$x = 3 \rightarrow r = 3 \rightarrow 5, 15, 45, \underline{135}$

C) The recursive rule states that the next term is twice the last known term minus three times the term before that. Thus, two terms must be known before the rule may be applied.

$4, 5 \rightarrow a_3 = 2(5) - 3(4) = 10 - 12 = -2$

$\rightarrow a_4 = 2(-2) - 3(5) = -4 - 15 = -19$

$\rightarrow a_5 = 2(-19) - 3(-2) = -38 + 6 = -32$

$\rightarrow a_6 = 2(-32) - 3(-19) = -64 + 57 = -7$

$\rightarrow a_7 = 2(-7) - 3(-32) = -14 + 96 = +82 \rightarrow (k, a_k) = \underline{(7, 82)}$