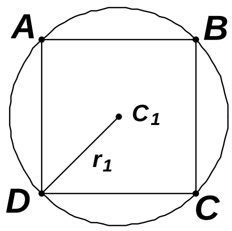
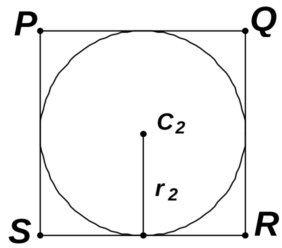
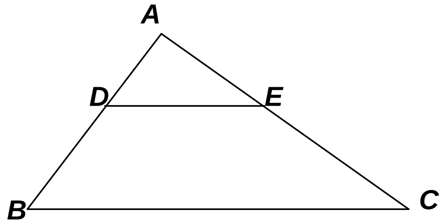
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2015 SOLUTION KEY

Round 5



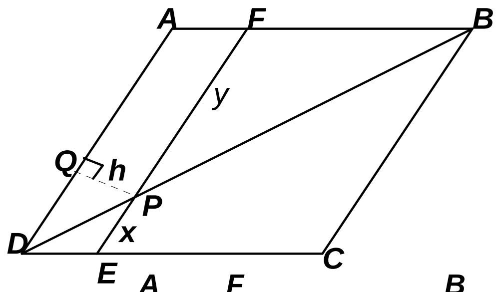


A) The diagonal of *ABCD* = $2r_1 = 4$ and its side must be $\frac{4}{\sqrt{2}} = 2\sqrt{2}$. In *PQRS*, $PS = 2r_2 = 4$. Thus, the perimeters are in the same ratio as the sides, namely, $\frac{2\sqrt{2}}{4} = \underline{\boldsymbol{\frac{\sqrt{2}}{2}}}$.



B) If the area of ΔADE : area of *DECB* is 4 : 21, then the area of ΔADE : area of ΔABC is 4 : (4 + 21) = 4 : 25 which implies the ratio of corresponding sides is 2 : 5.
Thus, $\frac{8}{BC} = \frac{2}{5} \Rightarrow BC = \underline{\mathbf{20}}$.

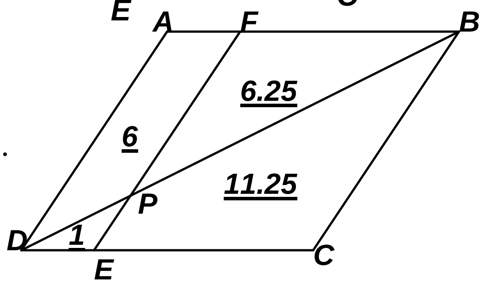
C) Let $h = PQ$ denote the distance between the parallels $\overline{AD}$ and $\overline{EF}$.



$$\frac{area(\Delta DPE)}{area(ADPF)} = \frac{\cancel{\frac{1}{2}}hx}{\cancel{\frac{1}{2}}h\left(y+(x+y)\right)} = \frac{1}{6}$$

$$\Rightarrow \frac{x}{x+2y} = \frac{1}{6} \Rightarrow 6x = x + 2y \Rightarrow \frac{x}{y} = \frac{2}{5}$$

$$\Delta DPE \sim \Delta BPF \Rightarrow \frac{area(\Delta DPE)}{area(\Delta BPF)} = \left(\frac{2}{5}\right)^2 = \frac{4}{25}$$

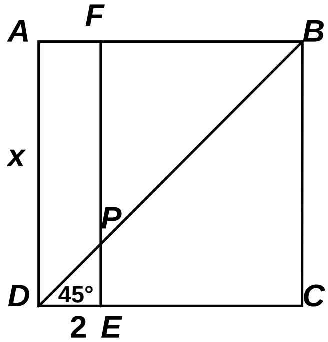


If the area of ΔDPE be 1, then $area(\Delta BPF) = \frac{25}{4} = 6.25$.

Since $\Delta BAD \cong \Delta DCB$, $area(CEPB) = 11.25$ and the required ratio is $\frac{6.25}{11.25} = \frac{25}{45} = \underline{\boldsymbol{\frac{5}{9}}}$.

Alternate Solution (Norm Swanson – Hamilton-Wenham - retired)
Assume *ABCD* is a square. (A square is a rhombus.) Let *DE* = *EP* = 2, and *AD* = *x* $\Rightarrow$ *PF* = *x* – 2 $\Rightarrow$ area(ΔDPE) = 2 $\Rightarrow$ area(*ADPF*) = 12

$$\frac{1}{2}(2)\left(x+(x-2)\right) = 12 \Rightarrow x = AD = 7 \Rightarrow PF = FB = 5$$



Thus, the required ratio is $\frac{12.5}{35-12.5} = \frac{25}{70-25} = \underline{\boldsymbol{\frac{5}{9}}}$.