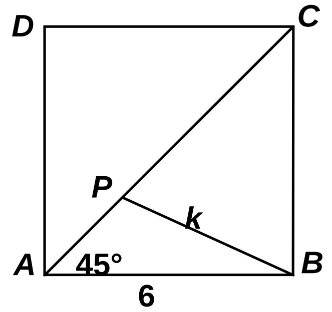
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2014 SOLUTION KEY

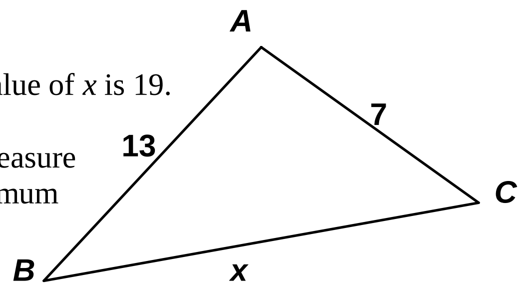
Round 1

A) Since $195 = 3 \cdot 5 \cdot 13$, the only integral right triangles with a hypotenuse of length 195 are triangles similar to a 3-4-5 or 5-12-13 right triangle, namely, triangles with sides of length $39(3,4,5) = (117,156,195) \Rightarrow$ **468** or $15(5,12,13) = (75,180,195) \Rightarrow$ **450**.

B) $AC = 6\sqrt{2}$. Since P cannot coincide with A, $k < 6$. When $\overline{BP} \perp \overline{AC}$, $k = 3\sqrt{2}$.
Thus, $3\sqrt{2} < BP < 6$ and the only integer value in this range is 5.
Using the Law of Sines on ΔAPB, $\dfrac{\sin 45°}{k} = \dfrac{\sin \angle APB}{6}$.
Substituting, $\sin \angle APB = \dfrac{3\sqrt{2}}{5} \Rightarrow \underline{\left(\mathbf{5, \dfrac{3\sqrt{2}}{5}}\right)}$.



C) Let x denote the length of $\overline{BC}$.
Clearly, to satisfy the triangle inequality, the maximum value of x is 19.
As the value of x increases, so does the measure of $\measuredangle A$.
However, since the value of the cosine decreases as the measure of the angle increases from 90° to 180°, we want the minimum possible value of x for which $\measuredangle A$ is obtuse!
Using the Law of Cosines, we have



$$x^2 = 7^2 + 13^2 - 2 \cdot 7 \cdot 13 \cdot \cos A \Rightarrow \cos A = \frac{218 - x^2}{2 \cdot 7 \cdot 13}.$$

If $\measuredangle A$ is obtuse, then $\cos A < 0$ and this occurs when $218 - x^2 < 0 \Leftrightarrow x^2 > 218 \Rightarrow x > 14$.

Thus, the minimum value of x is 15 and $\cos A = \dfrac{218 - 15^2}{2 \cdot 7 \cdot 13} = \dfrac{-7}{2 \cdot 7 \cdot 13} = \underline{-\dfrac{\mathbf{1}}{\mathbf{26}}}$.