MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2009 SOLUTION KEY

Round 2

A) $2\cdot\left[2,3,5,7,11,13,17,19,23,\cancel{29}\right] \rightarrow 9$

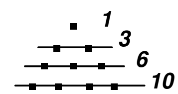
$3\cdot\left[3,5,7,11,13,\cancel{17}\right] \rightarrow 5$

$5\cdot\left[5,7,\cancel{11}\right] \rightarrow 2$

$7\cdot\left[7,\cancel{11}\right] \rightarrow 1$

$11\cdot\left[\cancel{11}\right] \rightarrow 0$

Thus, the number of semi-primes < 50 is **<u>17</u>**.



B) Note the numbers along the right edge, 1, 3, 6, 10, … . These are <u>triangular</u> numbers.

They numbers are generated by the formula $\dfrac{n(n+1)}{2}$. $n = 1 \rightarrow 1$, $n = 2 \rightarrow 3$, $n = 3 \rightarrow 6$ etc.

Thus, the last number in the 10$^{\text{th}}$ row is $\dfrac{10(11)}{2} = 55$ and we must sum

$46, 47, 48, \ldots, 53, 54, 55 = 5(101) =$ **<u>505</u>**

101
101
101

B) <u>Alternate solution #1</u>

Note the numbers along the left edge. 1, 2, 4, 7, … = 1, 1 + 1, 1 + (1 + 2), 1 + (1 + 2 + 3), …

These numbers are generated by $1+\dfrac{(n-1)n}{2} = \dfrac{n^2-n+2}{2}$ Thus, the first number in the 10$^{\text{th}}$ row is $\dfrac{10^2-10+2}{2} = 46$ and the solution follows as above.

<u>Alternate solution #2</u>

Note the number(s) in the "center" of each row. If there is a single number only, double it.
2(1), 2 + 3, 2(5), 8 + 9, … = 2, 5, 10, 17, … = 1 + 1, 1 + 4, 1 + 9, 1 + 16, …

These numbers are generated by $1 + n^2$. Thus, the "center" number is $\dfrac{1+n^2}{2}$ and, hence, the sum of the 10 numbers in the 10$^{\text{th}}$ row is $10\left(\dfrac{10^2+1}{2}\right) = 5(101) =$ **<u>505</u>**.