MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2010 SOLUTION KEY

Round 4

A) $2(8^2)+3(8)+6=1x^2+3x+4$ ➔ $158 = x^2 + 3x + 4$ ➔ $x^2 + 3x - 154 = (x - 11)(x + 14) = 0$
➔ $x =$ **<u>11</u>**, $\cancel{-14}$.

B) Mom and Alice are now $x + 18$ and x years old respectively. Clearly, 18 years ago, Mom was as old as Alice is now and Alice was only $x - 18$ years old.
$x = 2(x - 18)$ ➔ $x = 36$. Thus, in n years the required ratio is satisfied, namely $\dfrac{36+n}{54+n}=\dfrac{7}{10}$
➔ $360 + 10n = 378 + 7n$ ➔ $n =$ **<u>6</u>**.

C) $4A(48A+1)=5(16-201A)$ ➔ $192A^2+4A=80-1005A$ ➔ $192A^2+1009A-80=0$
No doubt the coefficients are intimidating, BUT…
Since the roots are <u>rational</u>, this quadratic must factor.
How do we avoid tedious trial and error?
The key observations are:

$192=3\cdot 2^6$, $80=5\cdot 2^4$ (each with exactly one odd factor) and
the coefficient of the middle term of the trinomial is **odd**.

Since the <u>OI</u> in FOIL produces the middle term, we must have an odd outer product and an even inner product (or vice versa). Thus, the factorization must be $(3A + 16)(64A - 5) = 0$
➔ $A = \underline{\boldsymbol{-\frac{16}{3}, \frac{5}{64}}}$.

Round 5

A) Two coplanar lines determine at most one point of intersection.
A third line will determine at most two more, if it crosses both of the existing lines.
Likewise, each subsequent line adds a maximum number of new intersection points if it crosses each of the existing lines.
Thus, for 10 lines $N_{\max} = 1 + 2 + 3 + 4 + \ldots + 9 = 9(10)/2 =$ **<u>45</u>**.

B) Let p = perimeter.

Area of triangle = ½(base)(height)
$= \frac{1}{2}\left(\frac{p}{3}\right)\left(\frac{p}{3}\right)\frac{\sqrt{3}}{2}$
$= \dfrac{p^2\sqrt{3}}{36}$

Area of hexagon = 6(1/2)(base)(height)
$= 6\left(\frac{1}{2}\right)\left(\frac{p}{6}\right)\left(\frac{p}{6}\right)\frac{\sqrt{3}}{2}$
$= \dfrac{p^2\sqrt{3}}{24}$

Ratio of triangle to hexagon is **<u>2 : 3</u>**