MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 – MARCH 2012 SOLUTION KEY

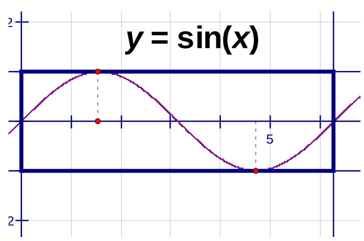
Round 3

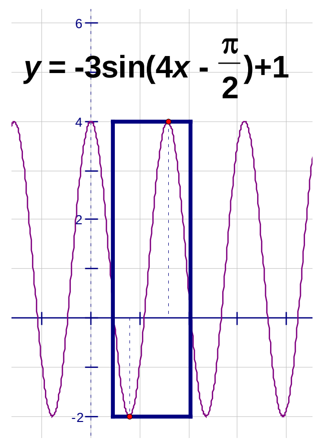
A) Using the Law of Sines, $\frac{\sin 40}{6}=\frac{\sin C}{9}\Rightarrow \sin C=\frac{3}{2}\sin 40°=1.5\cos 50° \Rightarrow (r,\theta)=\underline{\mathbf{(1.5,\ 50°)}}$.

B) The sine function normally assumes values between a minimum of –1 (at $\frac{3\pi}{2}$) and a maximum of 1 (at $\frac{\pi}{2}$). The factor of –3 "flips" the graph over the *x*-axis and increases the fluctuation to 3 units above and below the center line (normally the *x*-axis). The +1 shifts the entire graph vertically 1 unit. So the maximum value is +4 and we need only determine for what value(s) of *x* it occurs. The maximums occur when $4x-\frac{\pi}{2}=\frac{\mathbf{3\pi}}{\mathbf{2}}+2n\pi$ (Remember the graph is flipped.)

Thus, $x=\frac{2\pi+2n\pi}{4}=\frac{\pi(n+1)}{2}$. For *n* = 0, we have a maximum at $\underline{\left(\frac{\boldsymbol{\pi}}{\mathbf{2}},\mathbf{4}\right)}$.

In each case, the rectangle encloses one period of the graph. For $y=\sin(x)$ the period is 2π and the rectangle extends left to right from 0 to 2π, and top to bottom from –1 to +1. Divide the rectangle into quarters and we have located a maximum, a zero and a minimum. For $y=-3\sin\left(4x-\frac{\pi}{2}\right)+1$ the period is $\frac{\pi}{2}$ and the rectangle extends left to right from $\frac{\pi}{8}$ to $\frac{5\pi}{8}$, and top to bottom from –2 to +4. Likewise, dividing the rectangle into quarters we can locate the critical points. Here are the graphs of these two functions.





C) Converting to trig form, $-\sqrt{3}+i=2cis\left(\frac{5\pi}{6}\right)$ [$r^2=\left(\sqrt{3}\right)^2+1^2$, $\theta\in$QII and $\tan\theta=-\frac{1}{\sqrt{3}}$]

$=\left(-\sqrt{3}+i\right)^{400} = 2^{400}cis\left(\frac{1000\pi}{3}\right)=2^{400}cis\left(\frac{4\pi}{3}\right)=2^{400}\left[\cos\left(\frac{4\pi}{3}\right)+i\sin\left(\frac{4\pi}{3}\right)\right]=$

$2^{400}\left(-\frac{1}{2}\right)\left[1+i\sqrt{3}\right]=A+Bi$ Evaluating $\left(\frac{A}{B}\right)^4$, the factors of 2^{400} and $-\frac{1}{2}$ will cancel!

Thus, $\left(\frac{A}{B}\right)^4=\left(\frac{A}{A\sqrt{3}}\right)^4=\underline{\frac{\mathbf{1}}{\mathbf{9}}}$.