MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2016 SOLUTION KEY

Team Round

D) continued

Solution #2: Indeterminate Coefficients (or Systematic Guess and Check)

Key Concept: Parity: Even + Odd = Odd / Even x Odd = Even, etc.
Signs ($\pm$) are not so important, since interchanging positive and negative factors in a product maintains the negative result.
Matching the coefficients of

$$(Ax+By+C)(Dx+Ey+F)=ADx^2-(AE+BD)xy+BEy^2+(AF+DC)x+(BF+CE)y+CF$$

with the coefficients of $36x^2-3xy-60y^2+18x+38y-4$, we get an *exciting* system of 6 equations in the 6 unknown constants.

$$\begin{cases} (1)\ x^2 \quad AD=36 \\ (2)\ xy \quad AE+BD=-3 \\ (3)\ y^2 \quad BE=-60 \\ (4)\ x \quad AF+DC=18 \\ (5)\ y \quad BF+CE=38 \\ (6) \quad CF=-4 \end{cases}$$

There are lots of possibilities. To minimize the guesswork, we zero in on equation #2 (the only one with an *odd* sum) and #6 (*fewest* number of factors) and start "guessing".

If $C=1$ and $F=4$, then $\begin{cases} 4A+D=18 \\ 4B+E=38 \end{cases} \Rightarrow$ both D and E are even and this contradicts equation #2, since the sum $AE+BD$ is supposed to be odd.

Therefore, we definitely know that $C=2,\ F=-2$ (**or vice versa**).

Equations #3, 5 $\Rightarrow \begin{cases} BE=-60 \\ \boxed{B-E=19} \end{cases} \Rightarrow (B,E)=(-15,4)$

Equations #1, 2 $\Rightarrow \begin{cases} \boxed{-4A+15D=-3} \\ AD=36 \end{cases} \Rightarrow (A,D)=(12,3)$

Checking in equation #4, $12\cdot(-2)+3\cdot 2=-18$ Oops! It must have been $(C,F)=(-2,2)$

Voila! The factors are $(12x+15y-2)$ and $(3x-4y+2)$.

If $AB<0$, the required 6-tuple is $\underline{\mathbf{(3,-4,2,12,15,-2)}}$.

<u>Challenge:</u> If the question had asked for $AB+AE+AF+BD+BE+BF+CD+CE+CF$, it would have been MUCH easier. Why?