

Team Question A: Center $C(4, -3)$

Since $a = 2, b = 2\sqrt{3}$ and $c = 4$ for the hyperbola, and
 $a = 2, b = 1$ and $c = \sqrt{3}$ for the ellipse.

$$R(4, -3 + \sqrt{3}), S(4, -3 - \sqrt{3})$$

$$T(4, 1), W(4, -7)$$

$$V_3(3, -3), V_4(5, -3)$$

Since the slope of asymptotes are $\pm \frac{2}{2\sqrt{3}} = \pm \frac{\sqrt{3}}{3}$,

the point-slope equations of the asymptotes are $(y + 3) = \pm \frac{\sqrt{3}}{3}(x - 4)$.

In the diagram below, graphically, it certainly appears as if the hyperbola approaches arbitrarily close to these lines. Analytically, why is this the case?

Starting with $3(y + 3)^2 - (x - 4)^2 = 12 \Leftrightarrow \frac{(y + 3)^2}{4} - \frac{(x - 4)^2}{12} = 1$, solve for y .

$$(y + 3)^2 = \frac{(x - 4)^2}{3} + 4$$

As $x \rightarrow \pm\infty$, that is, as x becomes an arbitrarily large positive number (moving to the right on the graph) or an arbitrarily small negative number (moving to the left on the graph), the 4 becomes negligible and we have

$$(y + 3)^2 \approx \frac{(x - 4)^2}{3} \Leftrightarrow y + 3 \approx \pm \frac{1}{\sqrt{3}}(x - 4)$$

Thus, as $x \rightarrow \pm\infty$, the y -coordinate of a point on the straight lines (the asymptotes) becomes an increasingly better approximation of the y -coordinate of the corresponding point on corresponding branch of the hyperbola.

Q.E.D ('Quod erat demonstratum', Latin for "enough said", literally "that which was to be demonstrated")

