MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2012 SOLUTION KEY

Round 4

A) $8^x = \sqrt[3]{\dfrac{2}{4^x}} \Leftrightarrow 2^{3x} = \left(\dfrac{2^1}{2^{2x}}\right)^{\frac{1}{3}} = \left(2^{1-2x}\right)^{\frac{1}{3}} = 2^{\frac{1-2x}{3}} \Leftrightarrow 3x = \dfrac{1-2x}{3} \Rightarrow 9x = 1-2x \Rightarrow x = \underline{\mathbf{\dfrac{1}{11}}}$.

Alternate solution:

Cubing both sides, $8^{3x} = \dfrac{2}{4^x} \Rightarrow 2^{9x} = \dfrac{2^1}{2^{2x}} = 2^{1-2x} \Rightarrow 9x = 1-2x \Rightarrow x = \underline{\mathbf{\dfrac{1}{11}}}$.

B) Since the x-intercept of $y = -3 + 4\log_{16} x$ is determined by letting $y = 0$, we have

$\log_{16} a = \dfrac{3}{4} \Leftrightarrow a = 16^{\frac{3}{4}} = 2^3 = 8$.

$$\left(\frac{\log_2 a^8}{\log_4 a^2}\right)^{\frac{1}{3}} = \left(\frac{\log_2 8^8}{\log_4 8^2}\right)^{\frac{1}{3}} = \left(\frac{\log_2\left(2^3\right)^8}{\log_4\left(4^3\right)}\right)^{\frac{1}{3}} = \left(\frac{24}{3}\right)^{\frac{1}{3}} = \underline{\mathbf{2}}$$

C) $y_2 = 81y_1 \Rightarrow y_2 = 81\left(2\left(4^x\right)\right) = 81\left(2^{2x+1}\right)$. Also, $y_2 = \dfrac{8^{x+2}}{4} = \dfrac{2^{3x+6}}{2^2} = 2^{3x+4}$

Thus, $81\left(2^{2x+1}\right) = 2^{3x+4}$

$\Rightarrow 81 = \dfrac{2^{3x+4}}{2^{2x+1}} = 2^{x+3} \Rightarrow x+3 = \log_2 81 = 2\log_2 9 = 4\log_2 3$

b as small as possible $\Rightarrow x = 4\log_2 3 - 3 \Rightarrow$ **<u>(4, 3, –3)</u>**.