MASSACHUSETTS MATHEMATICS LEAGUE CONTEST 1 - OCTOBER 2012 SOLUTION KEY

Round 6

A) A cubic foot is equivalent to a cube 1 foot or 12 inches on side. Therefore, $1 \text{ ft}^3 = 12^3 = 1728 \text{ in}^3$.

$$\frac{8 \cdot 9 \cdot 15}{1728} = \frac{2^3 \cdot 3^3 \cdot 5}{\left(2^2 \cdot 3\right)^3} = \frac{5}{2^3} = \frac{\mathbf{5}}{\mathbf{8}} \text{ ft}^3$$

or alternately converting each measure to a fractional number of feet, $\frac{2}{\cancel{\times}} \cdot \frac{\cancel{\times}}{4} \cdot \frac{5}{4} = \frac{10}{16} = \frac{5}{\cancel{8}}$ $(8") \quad (9") \quad (15")$

B)
$$\frac{\text{quarts}}{\text{persons served}} = \frac{7}{15} = \frac{x}{100} \Rightarrow \frac{7}{3} = \frac{x}{20} \Rightarrow x = \frac{140}{3} = 46^+$$

Thus, 47 quarts are required. Since the cost of 1 gallon is less than the cost of 4 quarts, we need to maximize the number of gallons purchased. We need 11 gallons and 3 quarts. 11(5.29) + 3(1.49) = \$62.66

C)
$$\frac{1}{2}\nabla\frac{8}{9} = \frac{2\cdot\frac{1}{2}\cdot\frac{8}{9}}{\frac{1}{2}+\frac{8}{9}} - \sqrt{\frac{1}{2}\cdot\frac{8}{9}} = \frac{\frac{8}{9}}{\frac{1}{2}+\frac{8}{9}} - \frac{2}{3} = \frac{8}{9}\cdot\frac{18}{25} - \frac{2}{3} = \frac{16}{25} - \frac{2}{3} = \frac{48-50}{75} = \frac{2}{75}$$

Also accept $\frac{-2}{75}$, $\frac{2}{-75}$, $-0.02\overline{6}$.