MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2011 SOLUTION KEY

Round 4

A) $2^{3x+1} = 4^x = 2^{2x} \Leftrightarrow 3x+1 = 2x \Leftrightarrow x = -1$. Thus, (a, b) = $\underline{\left(\mathbf{-1, \frac{1}{4}}\right)}$.

B) $x = 0 \Rightarrow y = 3 - 5 = -2$

$y = 0 \Rightarrow \log_2\left(8(2x-1)^2\right) = 5 \Rightarrow 8(2x-1)^2 = 2^5 = 32 \Rightarrow 2x - 1 = \pm 2 \Rightarrow x = \frac{3}{2}, -\frac{1}{2}$

$(0, -2)$ to $\left(\frac{3}{2}, 0\right) \Rightarrow \underline{\mathbf{2.5}}$ $\quad$ $(0, -2)$ to $\left(-\frac{1}{2}, 0\right) \Rightarrow \underline{\mathbf{\frac{1}{2}\sqrt{17}}}$

C) Using the formula $\log a + \log b = \log ab$ for a, $b > 0$, we get

$$A = \log 2 + \log\frac{3}{2} + \log\frac{4}{3} + \ldots + \log\frac{2000}{1999} = \log\left(\frac{2\cdot 3\cdot 4\cdot \ldots \cdot 2000}{2\cdot 3\cdot 4\cdot \ldots \cdot 1999}\right) = \log 2000 = 3 + \log 2$$

$$1og_{1024}10 = k \Leftrightarrow \frac{1}{\log\left(2^{10}\right)} = k \Leftrightarrow \frac{1}{10\log 2} = k \Leftrightarrow \log 2 = \frac{1}{10k}$$

Thus, $A = 3 + \frac{1}{10k} = \underline{\mathbf{\frac{30k+1}{10k}}}$.