MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2010
ROUND 7 TEAM QUESTIONS
ANSWERS

A) _____ D) (_____ , _____)

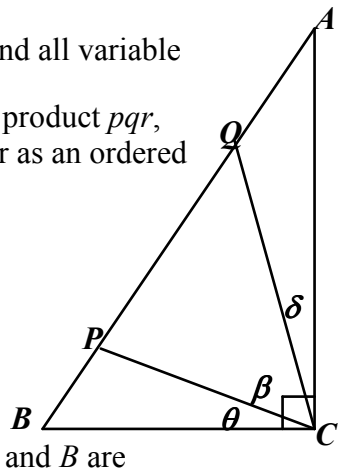
B) (_____ , _____ , _____) E) _____

C) _____ F) _____

******* NO CALCULATORS ARE PERMITTED IN THIS ROUND *******

A) Let functions f and g be defined by $f(x) = -k\,|\,x\,| + A$ and $g(x) = 4k\,|\,x\,| - 5B$.
If $k > 1$, $A = \begin{vmatrix} 1 & k \\ -k & 1 \end{vmatrix}$, $B = \begin{vmatrix} k & 1 \\ 1 & k \end{vmatrix}$ and the functions f and g have the same zeros, compute the area of the region bounded by f and g.

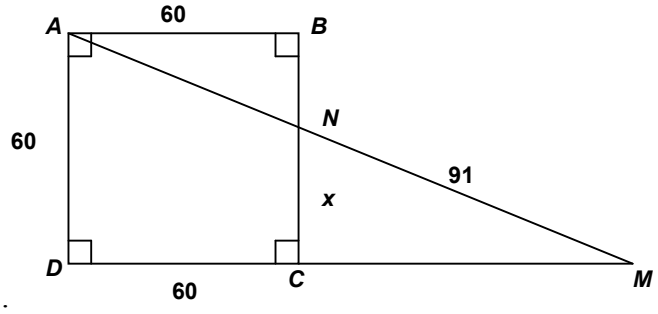
B) Suppose $\dfrac{\sqrt[4]{2\cdot 3^a}\cdot\sqrt[6]{3\cdot 2^b}}{\sqrt[3]{12}} = \sqrt[r]{2^p\cdot 3^q}$, each radical is in simplest form and all variable denote positive integers. Compute the number of distinct values of the product pqr, the maximum product and the minimum product. Express your answer as an ordered triple (count, max, min).



C) Points P and Q lie on the hypotenuse of right triangle ABC.
If $\sin\theta = \dfrac{3}{5}$ and $\sin\delta = \dfrac{7}{25}$, compute the value of $\sin\beta$.

D) Two bottles of equal volume contain rubbing alcohol and water. The ratios of alcohol to water in the two bottles are 3 : 1 and A : B, where A and B are relatively prime integers. The contents of the two bottles are mixed and the new ratio of alcohol to water is 27:13. Determine the ordered pair (A, B).

E) Given square $ABCD$ below with sides as indicated. Compute x, the length of $\overline{NC}$. Diagram is not necessarily drawn to scale.



F) Let N be the positive difference between the two largest coefficients in the expansion of $(3a+2b)^{11}$. Determine the prime factorization of N.
When writing the product, list the primes from smallest to largest.