MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2014
ROUND 7 TEAM QUESTIONS

ANSWERS

A) _____ D) _____

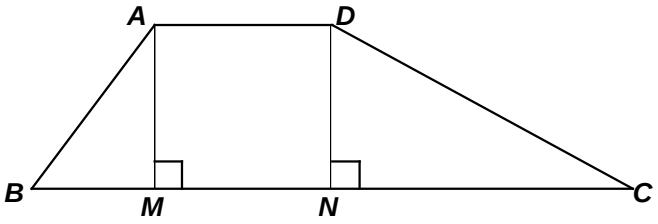
B) SUN MON TUE WED THU FRI SAT E) _____

C) _____ F) _____

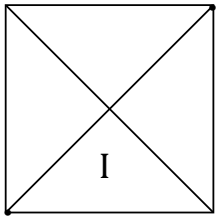
A) Let $N = \dfrac{1}{(1-i)^k}$ for integer values of k.
If $10 < k < 100$, determine for how many values of k, N is real.

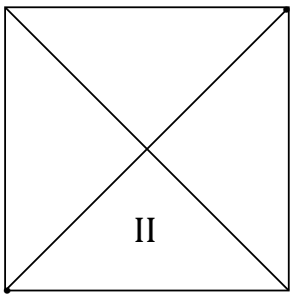
B) Misao Okawa, one of the oldest living persons in the world, celebrates his birthday in March. In 2014, his birthday fell on a Wednesday. On what day of the week did his birthday fall in 1898, the year he was born?
Recall that there are 365 days in a year, except in leap years. The extra day (2/29) is added only in non-century years divisible by 4 and in century years divisible by 400.

C) Given: Trapezoid $ABCD$ with $\overline{AD} \parallel \overline{BC}$ and $(AB, BC, CD, AM) = (30, 87, 51, 24)$
An isosceles trapezoid $PQRS$ has the same perimeter as $ABCD$, sides of integer length and an altitude equal in length to the altitude of $ABCD$. Compute all possible areas of trapezoid $PQRS$.



D) In each of the squares below, consider the lattice points within the triangular regions marked I and II. The lower left vertex in each square is the origin. The upper right vertices are (n, n) and $(n + 1, n + 1)$ respectively, where n is a positive integer. Compute all value of n for which the number of lattice points in region II is 5 more than the number of lattice points in region I.





E) Compute all possible values of $\sin\left(\dfrac{n\pi}{3} + \dfrac{m\pi}{6}\right)$, if m and n are both positive multiples of 3.

F) ΔABC is scalene and acute.
Its interior angles measure $x°$, $y°$ and $(3x - 2y)°$, where x and y are integers.
If $x + y < 120$, compute the number of possible values of x.