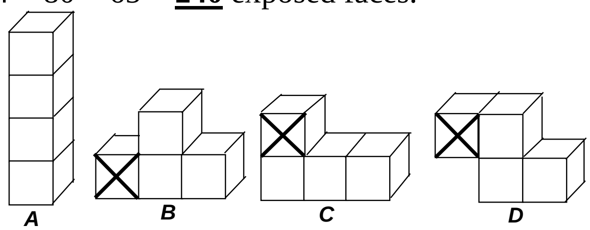
**MASSACHUSETTS MATHEMATICS LEAGUE**
**CONTEST 1 - OCTOBER 2014 SOLUTION KEY**

## Team Round

A) Four blocks have a total of 24 faces, but, after gluing, 6 exposed faces are lost. Thus, the maximum number of exposed faces is 18, if a stable position with exactly 2 edges in contact with the table top exists (which is the case for *B*, *C* and *D*).
*A* is stable resting on 1 or 4 faces, resulting in 17 or 14 exposed faces.
*B* is stable resting on 0, 1, 3 or 4 faces, resulting in 18, 17, 15 or 14 exposed faces.
*C* is stable resting on 0, 1, 2, 3 or 4 faces, resulting in 18, 17, 16, 15 or 14 exposed faces.
*D* is stable resting on 0, 1, 2 or 4 faces, resulting in 18, 17, 16, or 14 exposed faces.
Thus, we have 31 + 64 + 80 + 65 = **240** exposed faces.

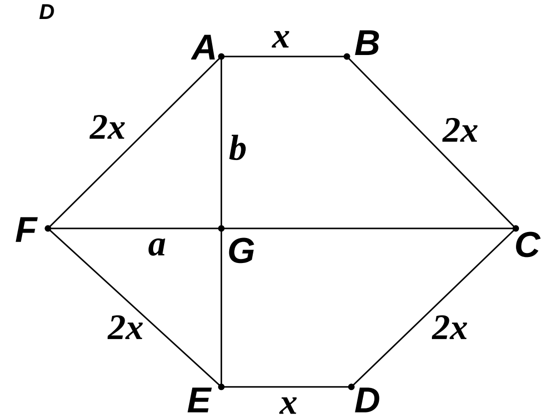



B) Let $AB = DE = x$, $BC = CD = EF = FA = 2x$
Let $G$ denote the intersection of $\overline{AE}$ and $\overline{FC}$
Let $FG = a$ and $AG = b$. Then:

$FC = AE + 1 \Rightarrow$ (1) $x + 2a = 2b + 1$

(2) $a^2 + b^2 = 4x^2$

(3) $area(\Delta AFE) = ab = 2.875 = \dfrac{23}{8}$




(1) $\Rightarrow \dfrac{x-1}{2} = b - a \Rightarrow (x-1)^2 = 4(a^2+b^2) - 8ab$ (***)

Substituting, using (2) and (3) in (***),

$(x-1)^2 = 4(4x^2) - 23 \Leftrightarrow 15x^2 + 2x - 24 = (5x-6)(3x+4) = 0$

Thus, $AB = x = \underline{\mathbf{\frac{6}{5}}}$ (or **1.2**).

C) Since $a:b = 4:7$, let $a = 4c$, $b = 7c$. Since $\begin{cases} x + ay = b^2 \\ x - by = a^2 \end{cases}$, after subtracting, we have

$(a+b)y = b^2 - a^2 = (b+a)(b-a) \Rightarrow y = b - a = 3c$ $\quad$ [ $a + b \neq 0$ ]

Substituting in the first equation,

$x + a(b-a) = b^2 \Rightarrow x = a^2 + b^2 - ab = 16c^2 + 49c^2 - 28c^2 = 37c^2$.

Thus, $\dfrac{x}{y} = \dfrac{37c^2}{3c} \Rightarrow x = \dfrac{37c}{3}y$ (Recall: $x$, $y$ and $c$ are all positive integers.)

To minimize both $x$ and $y$, we take $c = 1 \Rightarrow a = 4$, $b = 3$ and $x = \dfrac{37}{3}y \Rightarrow$

$(x,y) = (37,3) \Rightarrow x + y = \underline{\mathbf{40}}$. Check: $37 + 4\cdot 3 = 49 = 7^2$, $37 - 7\cdot 3 = 16 = 4^2$