

**MASSACHUSETTS MATHEMATICS LEAGUE  
CONTEST 6 – MARCH 2015  
ROUND 1 ALG 2: SIMULTANEOUS EQUATIONS AND DETERMINANTS**

**ANSWERS**

A) \_\_\_\_\_

B) \_\_\_\_\_

C) \_\_\_\_\_

A) Compute the largest integer value of  $k$  for which the determinant  $\begin{vmatrix} 3 & -5k \\ 4 & k+100 \end{vmatrix}$  is negative.

B) For what value(s) of  $c$  will the following system of equations have an infinite number of solutions?

$$14x + 3y - 7z = 8$$

$$-8x + 5y + 4z = c$$

$$-2x + 3y + z = 2$$

C) Given:  $A(10, -7)$ ,  $B(-6, 11)$ , and  $C(3, k)$

The area of  $\triangle ABC$  is given by the formula  $\frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$ , that is, half the absolute value

of the determinant of the  $3 \times 3$  matrix formed by the coordinates of the vertices, taken clockwise (or counterclockwise) with 1s filling the third column.

Determine all single-digit positive integral values of  $k$  for which the area of  $\triangle ABC$  is an integer perfect square.