MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2013 SOLUTION KEY

Team Round

C) The slope of $\overline{AB}$ is $\dfrac{9x-3y}{8y-2x-2}=\dfrac{9x-3x-3}{8x+8-2x-2}=\dfrac{6x-3}{6x+6}=\dfrac{2x-1}{2x+2}$.

The slope of $\overline{AC}$ is $\dfrac{10x+10y-3-3y}{17y+9x-2x-1}=\dfrac{10x+7y-3}{7x+17y-1}=\dfrac{17x+4}{24x+16}$

Since A, B and C are collinear, the slopes of $\overline{AC}$ and $\overline{AB}$ must be equal.
Equating and cross multiplying,

$\dfrac{2x-1}{2x+2}=\dfrac{17x+4}{24x+16}\Rightarrow 48x^2+8x-16=34x^2+42x+8\Rightarrow 14x^2-34x-24=0$

$\Rightarrow\ 7x^2-17x-12=(7x+4)(x-3)=0\Rightarrow x=3$

$y=x+1\ \Rightarrow\ y=4$

$\Rightarrow A(2x+1,3y)\ \Rightarrow A$**<u>(7, 12)</u>**

(B(31,27) and C(95,67) are clearly further from the origin.)

D) $\dfrac{3}{(x+1)(x+2)^2}=\dfrac{A}{x+1}+\dfrac{B}{x+2}+\dfrac{C}{(x+2)^2}$

Multiplying through by $(x+1)(x+2)^2$, we have an equation which is true for **<u>all</u>** values of x, namely, $3=A(x+2)^2+B(x+1)(x+2)+C(x+1)$.

If $x = -2$, then two terms on the right hand side disappear and we have $3 = C(-1) \Rightarrow C = -3$.
If $x = -1$, we have $3 = A(1)^2 \Rightarrow A = 3$
Picking an arbitrary value of x, we can solve for B.

$x = 0 \Rightarrow\ 3=3(2)^2+B(1)(2)+(-3)(1)\Leftrightarrow 3=12+2B-3\Leftrightarrow B=-3$

Thus, $A^3+B+C=27-3-3=\underline{\mathbf{21}}$.