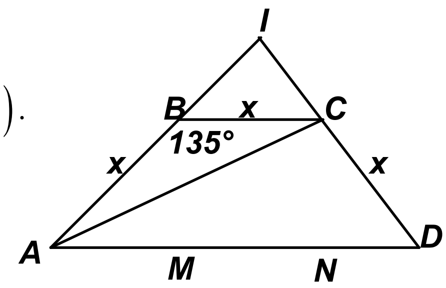
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2010 SOLUTION KEY

Round 1 – continued

C) Alternate solution #2 (Norm Swanson)



Using the law of cosines on ΔABC, $AC^2 = AD = x^2\left(2+\sqrt{2}\right)$.

Extend $\overline{AB}$ and $\overline{DC}$ to meet at point I.
ΔIBC and ΔIAD are 45-45-90 right triangles with $BI = \dfrac{x}{\sqrt{2}}$. By similar triangles, $\dfrac{AI}{BI} = \dfrac{AD}{BC}$ or

$$\frac{x+\dfrac{x}{\sqrt{2}}}{\dfrac{x}{\sqrt{2}}} = \frac{x^2\left(2+\sqrt{2}\right)}{x} \rightarrow \sqrt{2}+1 = x\left(2+\sqrt{2}\right) = x\sqrt{2}\left(\sqrt{2}+1\right)$$

$$\rightarrow 1 = x\sqrt{2} \rightarrow x = \underline{\boldsymbol{\frac{\sqrt{2}}{2}}}$$

Created with nitro PDF professional
download the free trial online at nitropdf.com/professional