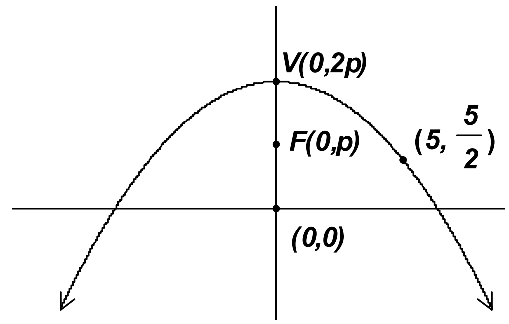
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2010 SOLUTION KEY

Team Round



A) $x^2 = -4p(y - 2p)$, where $p > 0$.

Substituting (5, 5/2), $25 = -4p(5/2) + 8p^2$

$\rightarrow 8p^2 - 10p - 25 = (4p + 5)(2p - 5) \rightarrow p = 5/2$

Thus, $x^2 = -10(y - 5) \rightarrow x^2 = 50$

$\rightarrow$ span = $\underline{\mathbf{10\sqrt{2}}}$

B) Since $C_1 = 2B$ and C_1 is prime, B must be 1. Likewise $C_3 = 3C \rightarrow C = 1$.

$$(2x+3y+A)(Bx+Cy+D) = (2x+3y+A)(x+y+D) = 2x^2+5xy+3y^2+(2D+A)x+(3D+A)y+AD$$

Since AD is prime, we must examine two cases:
1) $A = 1$ and D is prime
2) D = 1 and A is prime

The first case requires $(C_4, C_5) = (2D + 1, 3D + 1)$. $D = 1$ fails, but $D = 2 \rightarrow (5, 7)$
Any other prime values of D will be odd and this forces C_5 to be an even composite number.
The second case requires $(C_4, C_5) = (A + 2, A + 3)$. Both $A = 1$ and 2 fail. Likewise, any other prime values of A will be odd and this forces C_5 to be an even composite number.
Therefore, the only ordered triple is **<u>(5, 7, 2)</u>**

C) $\left(\cos^4 4x - \sin^4 4x\right)\left(1-2\sin^2 x\right) = \left(\cos^2 4x + \sin^2 4x\right)\left(\cos^2 4x - \sin^2 4x\right)\left(1-2\sin^2 x\right)$

Since the first factor is always equal to 1, it can be ignored and the original equation simplifies to $(\cos 8x)(\cos 2x) = 0$

$8x = \dfrac{\pi}{2} + n\pi \rightarrow x = \dfrac{(2n+1)\pi}{16} \rightarrow \dfrac{\pi}{16}, \dfrac{3\pi}{16}, \dfrac{5\pi}{16}, \dfrac{7\pi}{16}, \dfrac{9\pi}{16}, \dfrac{11\pi}{16}, \dfrac{13\pi}{16}, \dfrac{15\pi}{16}$

$2x = \dfrac{\pi}{2} + n\pi \rightarrow x = \dfrac{(2n+1)\pi}{4} \rightarrow \dfrac{\pi}{4}, \dfrac{3\pi}{4}$

Thus, $p = \dfrac{\pi}{16}$ and $q = \dfrac{3\pi}{4} \rightarrow \dfrac{q}{p} = \dfrac{3\pi}{4}\cdot\dfrac{16}{\pi} = \underline{\mathbf{12}}$