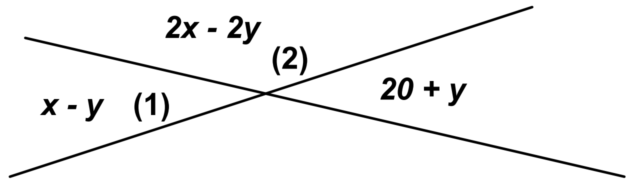
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2010 SOLUTION KEY

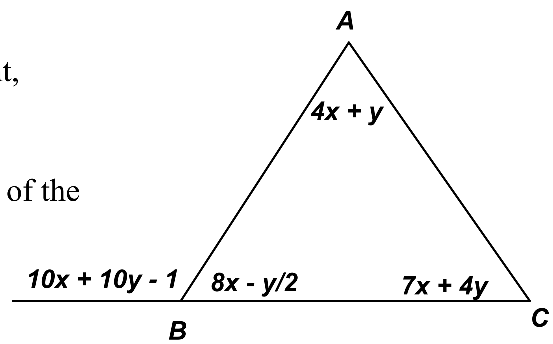
Round 6

A) $2x - 2y = 2(x - y)$
$\angle$s 1 and 2 are supplementary
$(x - y) + 2(x - y) = 3(x - y) = 180 \rightarrow x - y = 60$
Therefore, without having to solve for x and y,
the larger of the vertical angles is **<u>120</u>**°.

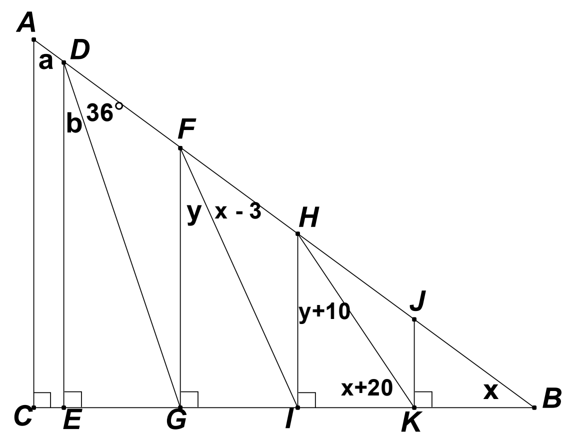


Alternate Solution:
Vertical angles $\rightarrow x - y = 20 + y \rightarrow x = 20 + 2y$
Linear Pair ($\angle$s 1 and 2) $\rightarrow$ $3x - 3y = 180 \rightarrow x - y = 60$
Substituting, $20 + 2y - y = 60 \rightarrow y = 40, x = 100 \rightarrow$ larger vertical angles **<u>120</u>**°.

B) Solve for (x, y) given $AB = AC$.
Since base angles of an isosceles triangle are congruent,
$7x + 4y = 8x - \frac{y}{2} \rightarrow x = \frac{9}{2}y$
Since the exterior angle of any triangle equals the sum of the measures of the two remote interior angles,
$10x + 10y - 1 = (4x + y) + (7x + 4y) \rightarrow x = 5y - 1$
Substituting, $5y - 1 = \frac{9}{2}y \rightarrow y = 2 \rightarrow (x, y) =$ **<u>(9, 2)</u>**.



C) In ΔHIK, $x + y = 60$.
Applying the fact that the measure of exterior angle equals the sum of the measures of the two remote interior angles to ΔHKB forces $m\angle KHB = 20$.
$\angle GFH$ and $\angle IHJ$, as corresponding angles of parallel lines, forces $x + (y - 3) = (y + 10) + 20$
$\rightarrow x = 33 \rightarrow \begin{cases} a = 57 \\ y = 33 \end{cases}$ and $m\angle DFG = 123$.
As alternate interior angles of parallels,
$m\angle DGF = m\angle EDG = b$.
Therefore, in ΔDFG, $b = 180 - (36 + 123) = 21$
$\rightarrow a + b =$ **<u>78</u>**.



Alternate Solution (Tuan Le)
As an exterior angle, $m\angle HKI = m\angle HKI + m\angle HKI$
$\rightarrow m\angle BHK = 20°$
In ΔHIB, $x + y + 30 = 90 \rightarrow x + y = 60$.
Since $\overline{HI} \parallel \overline{FG}$, $m\angle BFG = m\angle BHI \rightarrow$
$x + y - 3 = y + 30 \rightarrow x = 33, y = 27$ and $a = 57$.
$\overline{DE} \parallel \overline{AC} \rightarrow a = b + 36 \rightarrow b = 21$. Thus, $a + b = 51 + 27 =$ **<u>78</u>**.