MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2017 SOLUTION KEY

Team Round – continued

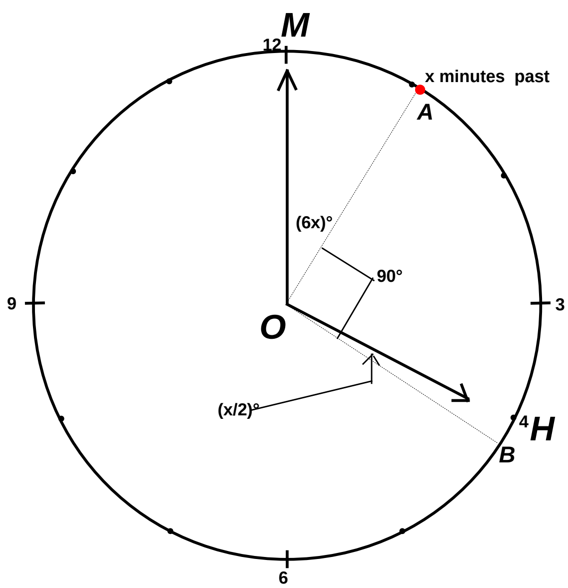
F) Enumerating times over a continuous domain of 1 hour when the angle is obtuse requires we determine when the hands are perpendicular. At 4PM the hands form an angle of 120° and, as time passes, the angle measure first gets smaller and then it gets larger again. Note the minute makes a complete revolution every hour, while the hour hand takes 12 hours to make a complete revolution. Thus, the minute hand moves 12 times as fast as the hour hand. In one minute, the minute hand turns $\frac{1}{60}$ of a revolution, i.e. turns through $\frac{1}{60}(360) = 6°$ and the hour hand turns through $\frac{1}{12}(6°) = 0.5°$.

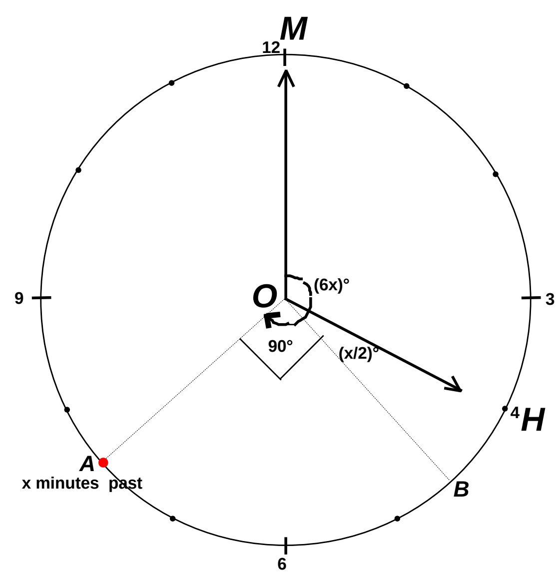
Case 1: (shortly after 4:05 PM) Case 2: (shortly after 4:35 PM)

A is the point to which the minute hand points at *x* minutes past the hour.

Not forgetting that the hour hand has also moved, we have the following diagrams:

[$m\angle MOA$ refers to an angle measured *clockwise* from *M* to *A*. On the left, it is an acute angle; on the right, it is a *reflexive* angle, i.e. an angle whose measure is greater than 180° (but less than 360°).]





From the diagram on the left, we have

$$m\angle MOA + m\angle AOB = m\angle MOH + m\angle HOB \Leftrightarrow 6x + 90 = 120 + \frac{x}{2} \Rightarrow x = \frac{60}{11}.$$

From the diagram on the right, we have

$$m\angle MOA - m\angle AOB = m\angle MOH + m\angle HOB \Leftrightarrow 6x - 90 = 150 + \frac{x}{2} \Rightarrow x = \frac{480}{11}$$

Thus, the probability is $\dfrac{\frac{60}{11} + \left(60 - \frac{420}{11}\right)}{60} = \frac{1}{11} + \left(1 - \frac{7}{11}\right) = \underline{\mathbf{\frac{5}{11}}}$.