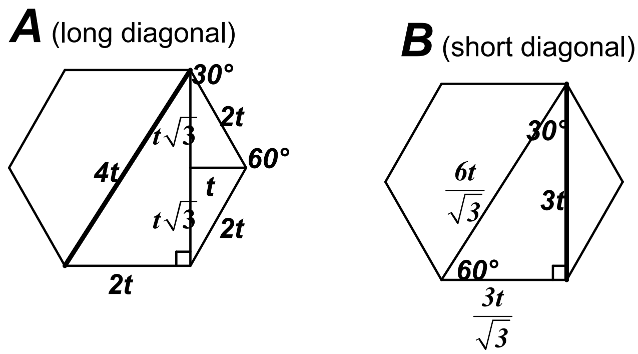
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2010 SOLUTION KEY

Round 5 - continued

B)



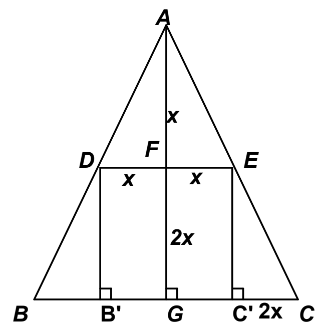
Study the diagrams at the right:

$A_{\text{short}} : B_{\text{long}} = 2\sqrt{3}t : \dfrac{6t}{\sqrt{3}}$ → $\underline{\mathbf{1:1}}$

C) $\dfrac{FG}{AG} = \dfrac{2}{3}$ → $\dfrac{AF}{AG} = \dfrac{1}{3}$ → $\dfrac{area(\Delta ADE)}{area(\Delta ABC)} = \dfrac{1}{9}$ → $\dfrac{area(\Delta AFD)}{area(\Delta ABC)} = \dfrac{1}{18}$

$\dfrac{area(DECB)}{area(\Delta ABC)} = \dfrac{8}{9}$

$\dfrac{area(DEC'B')}{area(\Delta ABC)} = \dfrac{4x^2}{\frac{1}{2}\cdot 6x\cdot 4x} = \dfrac{4}{9}$ → $\dfrac{1}{18} : \dfrac{4}{9} : \dfrac{8}{9}$ → $\underline{\mathbf{1:8:16}}$

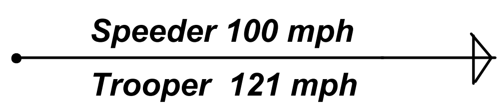


Round 6

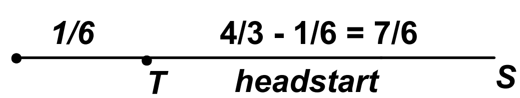
A) $\left(\sqrt{8} - \dfrac{1}{\sqrt{2} - \frac{1}{\sqrt{2}}}\right)^2 = \left(2\sqrt{2} - \dfrac{1}{\frac{2-1}{\sqrt{2}}}\right)^2 = \left(\sqrt{2}\right)^2 = \underline{2}$

B) $\left(\dfrac{1}{a+b} - \dfrac{1}{a-b}\right)\left(a^{-1} - b^{-1}\right) = \left(\dfrac{(a-b)-(a+b)}{(a+b)(a-b)}\right)\left(\dfrac{1}{a} - \dfrac{1}{b}\right) = \left(\dfrac{-2b}{(a+b)(a-b)}\right)\left(\dfrac{b-a}{ab}\right)$

$= \left(\dfrac{+2b}{(a+b)(b-a)}\right)\left(\dfrac{b-a}{ab}\right) = \underline{\dfrac{\mathbf{2}}{\mathbf{a(a+b)}}}$



C) $\dfrac{48}{60} = \dfrac{4}{5}$ mi/min $= \dfrac{4}{300}$ mi/hour · 100 mi/hr → $\dfrac{4}{3}$ mi.



Let t denote the fraction of an hour required to catch the speeder (after the trooper reached his cruising speed.)

$121t = 100t + \left(\dfrac{4}{3} - \dfrac{1}{6}\right)$ → $21t = \dfrac{7}{6}$ → $\dfrac{1}{18}$ hour · 60 = $\dfrac{10}{3}$ minute

= 3 minutes 20 sec.

Therefore, it took exactly **<u>4 minutes 8 seconds</u>** to overtake Mario.