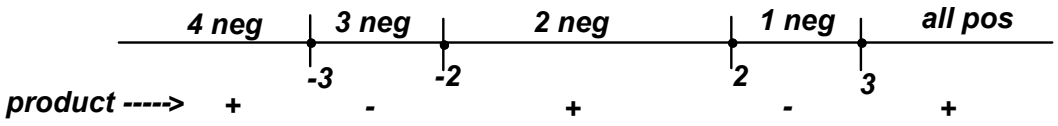
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2009 SOLUTION KEY

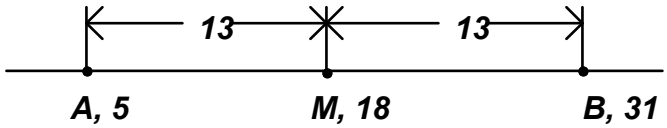
Round 5

A) $\left(x^2-4\right)\left(x^2-9\right)=(x+2)(x-2)(x+3)(x-3)$

Test each of the 5 sections on the number line determined by the critical values, ±2, ±3
The number of negative factors determines the sign of the product.



B) $16 \le 3k+2 \le 96$ → $14 \le 3k \le 94$ → $15 < 3k \le 93$ → $5 \le k \le 31$



This set of values is shown in the diagram at the right.
Since distance between two points on the number line is the absolute value of the difference of the coordinates of the points involved, we note the midpoint of the interval has coordinate 18 and the distance to each endpoint is 13.
Therefore, the equivalent absolute value representation is $|k-18| \le 13$ → $(a, b) =$ **(18, 13)**

C) If the units digit of A were 5 or any even digit, then A would not be prime.
The only digits that can be used to form A and B are 1, 3, 7 and 9.
Thus, there are 4 possible ordered pairs (A, B): (13, 31), (17, 71), (37, 73) and (79, 97)
The values of $|A-B|$ are: 18, 54, 36 and 18 → $C = 18$ and $D = 54$
The interval $72 < x < 216$ contains $216 - 72 - 1 =$ **143** integers

Round 6

A) $2^{-1}-\sqrt{\frac{25}{9}-\frac{64}{25}}+(3\cdot5)^{-1}=\frac{1}{2}-\sqrt{\frac{25^2-9(64)}{9(25)}}+\frac{1}{15}=\frac{1}{2}-\sqrt{\frac{625-576}{9(25)}}+\frac{1}{15}=\frac{1}{2}-\frac{7}{15}+\frac{1}{15}=\frac{1}{2}-\frac{2}{5}=$ **$\frac{1}{10}$**

B) $\begin{cases} a=\frac{2}{3}b \\ b=\frac{4}{5}c \end{cases}$ → $a=\frac{2}{3}\cdot\frac{4}{5}\cdot c=\frac{8}{15}c$

Substituting and multiplying through by 15, $8c + 12c + 15c = 35c = 15(70)$ → $c =$ **30**

C) Recall: $a*b=\begin{cases} a+ab, & \text{when } b \text{ is a proper fraction} \\ b-ab, & \text{when } b \text{ is an improper faction} \end{cases}$

Since $\left(6*\frac{2}{3}\right)=(6+4)=10$ and $\left(\frac{3}{4}*\frac{3}{2}\right)=\left(\frac{3}{2}-\frac{3}{4}\cdot\frac{3}{2}\right)=\frac{3}{2}-\frac{9}{8}=\frac{3}{8}$, we have

$\left(6*\frac{2}{3}\right)*\left(\frac{3}{4}*\frac{3}{2}\right)=\left(10*\frac{3}{8}\right)=10+10\left(\frac{3}{8}\right)=10+\frac{15}{4}=$ **$\frac{55}{4}$** $\left($**$13\frac{3}{4}$** or **13.75**$\right)$