Team Round - continued

E) $\sin\left(\dfrac{5\pi}{3}\right)+\cos(240°)+\sin^2\left(\dfrac{\pi}{4}\right)-\tan^2(135°)-3\tan(180°)=-\dfrac{\sqrt{3}}{2}-\dfrac{1}{2}+\dfrac{1}{2}-1-0=-\dfrac{\left(\sqrt{3}+2\right)}{2}$

Since $\sqrt{1}<\sqrt{3}<\sqrt{4}$, we know $1<\sqrt{3}<2 \Leftrightarrow \dfrac{1}{2}<\dfrac{\sqrt{3}}{2}<1$

Adding 1, we have $\dfrac{3}{2}<\dfrac{\sqrt{3}+2}{2}<2$ and, therefore, $(a,\,b) = \textbf{(−2, −1)}$.

Some students remember that $\sqrt{3}\approx 1.732$ (the year of George Washington's birth) and,

therefore, $\dfrac{\left(\sqrt{3}+2\right)}{2}\approx\dfrac{3.732}{2}\approx 1.8^{+}$ and the same result follows.

F) $\overrightarrow{BP}$ is a <u>trisector</u> of $\angle B$, so that $m\angle PBC < m\angle ABP$.
$\overrightarrow{CQ}$ is a <u>bisector</u> of $\angle C$.
$m\angle BDC = k = 140°$
Case 1: A is the vertex angle
$\begin{cases}x+y=40\\3x=2y\end{cases} \Rightarrow (x,y)=(16,24)$ and $m\angle A=180-(2\cdot48)=\textbf{84°}$
Case 2: B is the vertex angle (No solution)
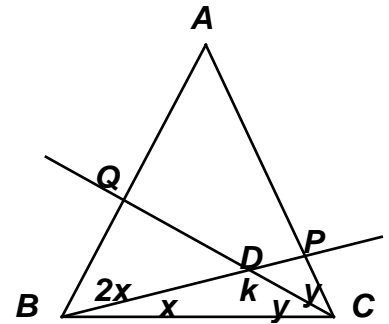$\begin{cases}x+y=40\\m\angle A=2y=180-(3x+2y)\end{cases}$

$\Rightarrow 3x+4y=180 \Rightarrow 3x+4(40-x)=180 \Rightarrow 160-x=180 \Rightarrow x=-20$

Case 3: C is the vertex angle
$\begin{cases}x+y=40\\m\angle A=3x=180-(3x+2y)\end{cases}$

$\Rightarrow 6x+2y=180 \Rightarrow 3x+(40-x)=90 \Rightarrow (x,y)=(25,15) \Rightarrow m\angle A=\textbf{75°}$

Additional Challenges:
Suppose that $m\angle BDC = k°$.
- Show that if $k=130$, $\triangle ABC$ is equilateral.
- Show that if A is the vertex angle, $105<k<180$.
- Show that if $k=125°$, there are 3 possible measures for $\angle A$, namely 30°, 48°, 52.5°.
- Is there a k-value which gives three different <u>integer</u> values for $m\angle A$?

Your analysis can start here:
$\begin{cases}x+y=180-k\\3x=2y\end{cases} \Rightarrow x=72-\dfrac{2k}{5},\ y=108-\dfrac{3k}{5}$. A as vertex angle gives $A=\dfrac{12}{5}k-252,\ B=C=216-\dfrac{6}{5}k$

Talk these questions over with your teammates.
Share your ideas with your coach and/or me (olson.re@gmail.com). Thanks.