MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2017 SOLUTION KEY

Round 3

A) $\sin(x+6°) = -\cos(36°) \Leftrightarrow \sin(x+6°) = -\sin(54°) = \sin(180+54) = \sin(234°)$

$x + 6° = 234° \Rightarrow x = \underline{\mathbf{228}}°$.

B) $4\sin^3 x - 4\sin^2 x - 3\sin x + 3 = 4\sin^2 x(\sin x - 1) - 3(\sin x - 1) = (\sin x - 1)(4\sin^2 x - 3 = 0)$

$\Rightarrow \sin x = 1, \pm\frac{\sqrt{3}}{2} \Rightarrow x = \underline{\boldsymbol{\frac{\pi}{2}, \frac{\pi}{3}, \frac{2\pi}{3}}}$. [$\sin x = -\frac{\sqrt{3}}{2}$ is impossible over the stated domain.]

C) $\left(4^{\sin^2 x}\right)\left(4^{\cos^2 x}\right)\left(4^{\tan^2 x}\right) = 4^{\sin^2 x + \cos^2 x + \tan^2 x} = 4^{1+\tan^2 x} = 4^{\sec^2 x} = 2^{2\sec^2 x} = \sqrt[3]{128} = \sqrt[3]{2^7} = 2^{\frac{7}{3}}$

$\Rightarrow \sec^2 x = \frac{7}{6} \Rightarrow \cos^2 x = \frac{6}{7}$.

Since $\cos 2x = 2\cos^2 x - 1$, we have $2\left(\frac{6}{7}\right) - 1 = \frac{12}{7} - 1 = \underline{\mathbf{\frac{5}{7}}}$.