MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2015 SOLUTION KEY

Team Round - continued

C) The lengths of each candle after x minutes are $16\left(1-\dfrac{x}{240}\right), 12\left(1-\dfrac{x}{360}\right)$

Since the tall (inexpensive) candle burns faster and the initial lengths are in a ratio of 4 : 3, we know that after S minutes, the inexpensive candle will be half as tall as the expensive candle.

$16\left(1-\dfrac{S}{240}\right)=\dfrac{1}{2}\cdot 12\left(1-\dfrac{S}{360}\right) \Leftrightarrow 16-\dfrac{S}{15}=6-\dfrac{S}{60} \Leftrightarrow \dfrac{S}{15}-\dfrac{S}{60}=10 \Leftrightarrow 3S=600 \Leftrightarrow S=200$

After T minutes, $16\left(1-\dfrac{T}{240}\right)+12\left(1-\dfrac{T}{360}\right)=10 \Leftrightarrow 16-\dfrac{T}{15}+12-\dfrac{T}{30}=10$

$\Leftrightarrow 28\cdot 30-3T=300 \Leftrightarrow \dfrac{T}{10}=28-10 \Leftrightarrow T=180$. Thus, $T-S=\underline{\mathbf{-20}}$.

D) Brute force:

$k =$	1	2	3	4	5	*6*
feet travelled in the kth second	2640	1320	660	330	165	82.5
Total feet travelled	2640	3960	4620	4950	5115	5197.5
Feet to go	2640	1320	660	330	165	82.5

$\dfrac{5197.5}{6}=866.25 \Rightarrow \underline{866}$. Note at an average speed of 866 ft/sec over 6 seconds, the projectile has travelled 5196 feet and is only 84 feet from the target, within the 88 foot requirement, so rounding <u>down</u> produced the correct nearest integer. Thus, $(k,S)=\underline{(\mathbf{6,866})}$

Solution #2:

After k seconds, the projectile has travelled $5280\left(\dfrac{1}{2}+\dfrac{1}{4}+\ldots+\dfrac{1}{2^k}\right)=5280\left(\dfrac{\frac{1}{2}\left(1-\left(\frac{1}{2}\right)^k\right)}{1-\frac{1}{2}}\right)$.

$=5280\left(\dfrac{2^k-1}{2^k}\right)$. The remaining distance to the target is $5280\left(1-\dfrac{2^k-1}{2^k}\right)=5280\left(\dfrac{1}{2^k}\right)$

$\Leftrightarrow 2^5\cdot 3\cdot 5\cdot 11\cdot 2^{-k} \Leftrightarrow 3\cdot 5\cdot 11\cdot 2^{5-k}<88 \Leftrightarrow 3\cdot 5\cdot 2^{2-k}<1 \Leftrightarrow 15<\dfrac{1}{2^{2-k}}=2^{k-2}=\dfrac{2^k}{4}$

$\Leftrightarrow 2^k>60 \Rightarrow k=6$ (ft/sec)

$$S=\frac{5280\left(\frac{2^k-1}{2^k}\right)}{k} \Rightarrow \frac{5280\left(\frac{63}{64}\right)}{6}=\frac{5197.5}{6}=866.25 \Rightarrow \underline{\mathbf{866}}$$