MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2010 SOLUTION KEY

Round 2

A) $\dfrac{6x^2-5x-6}{10+15x} = \dfrac{(2x-3)\,\cancel{(3x+2)}}{5\,\cancel{(2+3x)}} = \underline{\dfrac{2x-3}{5}}$

B) Given: $\dfrac{12x^2+12x-45}{9-4x^2} = \dfrac{4x-A}{7}$ and $x=2$ The right hand side is $\dfrac{8-A}{7}$.

The left hand side is $\dfrac{3(2x-3)(2x+5)}{(3+2x)(3-2x)} = \dfrac{-3(2x+5)}{(3+2x)} = \dfrac{-27}{7}$ for $x=2$.

Equating, $A = \underline{\textbf{35}}$.

C) Verify that to avoid division by zero, we require that $x \neq 0,\ 2,\ 12$ or $\dfrac{6}{5}$.

$\dfrac{4}{5-\dfrac{3+x}{3}} = \dfrac{16}{4+\dfrac{8}{3-\dfrac{6}{x}}} \;\rightarrow\; \dfrac{4}{\dfrac{15-(3+x)}{3}} = \dfrac{16}{4+\dfrac{8}{\dfrac{3x-6}{x}}} \;\rightarrow\; \dfrac{4}{\dfrac{12-x}{3}} = \dfrac{16}{4+\dfrac{8x}{3x-6}} \;\rightarrow\; \dfrac{1}{\dfrac{12-x}{3}} = \dfrac{4}{4+\dfrac{8x}{3x-6}}$

Cross multiplying, $4\left(\dfrac{12-x}{3}\right) = 4 + \dfrac{8x}{3x-6}$.

$\rightarrow 16 - \dfrac{4x}{3} = 4 + \dfrac{8x}{3x-6}$ $\rightarrow 12 - \dfrac{4x}{3} = \dfrac{8x}{3x-6}$ $\rightarrow 3 - \dfrac{x}{3} = \dfrac{2x}{3(x-2)}$

Multiplying through by $3(x-2)$, $9(x-2) - x(x-2) = 2x$.

$9x - 18 - x^2 + 2x = 2x$ $\rightarrow x^2 - 9x + 18 = (x-3)(x-6) = 0$ $\rightarrow x = \underline{\textbf{3, 6}}$