

Changes to original questions:

2C) The original problem was stated:

Usually radicals with different indices cannot be combined.

Given: A, B are integers and $0 < A, B < 4$

Compute the ordered triple (N, C, X) , where N, C and X are positive integers and C is as small as possible, for which

$$\sqrt[12]{16(27)(128)(1024)} + 3\sqrt[4]{2^A 3^B} = N(\sqrt[C]{X})$$

The appeal argued that $0 < A, B < 4$ was ambiguous and could be interpreted as $0 < A$ and $B < 4$.

The intent was that the values of both A and B were strictly between 0 and 4 and I thought single “and” connector made it clear that there were two conditions, not three. However, the appeal was not without merit and I decided to grant the appeal.

Here is the solution submitted by Amelia Paine (Winchester HS):

$$\sqrt[12]{16(27)(128)(1024)} + 3\sqrt[4]{2^A 3^B} = N(\sqrt[C]{X})$$

$$\sqrt[12]{2^{21} \cdot 3^3} + 3\sqrt[4]{2^A 3^B} =$$

$$2^{\frac{21}{12}} \cdot 3^{\frac{3}{12}} + 3 \cdot 2^{\frac{A}{4}} \cdot 3^{\frac{B}{4}} =$$

$$2^{\frac{7}{4}} \cdot 3^{\frac{1}{4}} + 2^{\frac{A}{4}} \cdot 3^{\frac{B+4}{4}}$$

Let $A = 7, B = -3$. Then:

$$2^{\frac{7}{4}} \cdot 3^{\frac{1}{4}} + 2^{\frac{7}{4}} \cdot 3^{\frac{1}{4}} = 2 \left(2^{\frac{7}{4}} \cdot 3^{\frac{1}{4}} \right) = 2^{\frac{11}{4}} \cdot 3^{\frac{1}{4}} = \sqrt[4]{2^{11} \cdot 3^1} = 4\sqrt[4]{2^3 \cdot 3} = 4\sqrt[4]{24}$$

Therefore, $(N, C, X) = \underline{(4, 4, 24)}$.