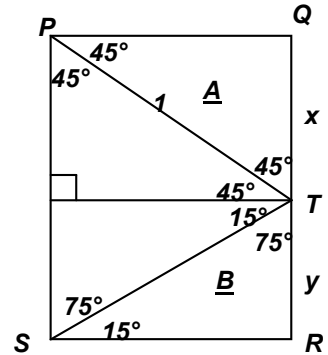**Team Round – continued**

E) From the diagram at the right, we see that $x = \sin 45$ and

$$\Delta PTS: \ \frac{\sin 75}{1} = \frac{\sin 45}{ST} \rightarrow ST = \frac{\sin 45}{\sin 75} \qquad \Delta TRS: \ \frac{y}{ST} = \sin 15$$

$$y = \frac{\sin 15 \cdot \sin 45}{\sin 75} = \frac{\sin 15 \cdot \sin 45}{\cos 15} = \tan 15 \sin 45$$

$$\frac{A}{B} = \frac{\frac{1}{2}PQ(x)}{\frac{1}{2}SR(y)} = \frac{x}{y} = \frac{\sin 45}{\tan 15 \sin 45} = \frac{1}{\tan 15} = \frac{1}{2-\sqrt{3}} = \mathbf{2+\sqrt{3}}$$

F) $4d + 5k = 540 \rightarrow d = (540 - 5k)/4 = (135 - k) - k/4$
k must be a multiple of 4.
$k = 4t \rightarrow m\angle E = 20t$
$d = m\angle A = m\angle B = m\angle C = m\angle D = (540 - 20t)/4 = 135 - 5t$
$m\angle EAD = (180 - 20t)/2 = 90 - 10t$
$m\angle DAB = (135 - 5t) - (90 - 10t) = 45 + 5t$
$$\frac{m\angle EAD}{m\angle DAB} = \frac{90 - 10t}{45 + 5t} = \frac{18 - 2t}{9 + t}$$
Trying values of t we get the following ordered pairs:

(1, 8/5) (2, 14/11) (3, 1) ….   This is a decreasing sequence and $t = 4 \rightarrow \underline{\dfrac{10}{13}}$
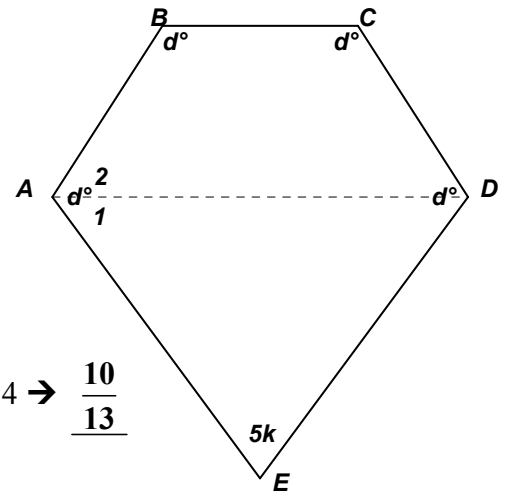
Alternate solution #1:

$5d + 5k = 540 \rightarrow d = \dfrac{540 - 5k}{4} = 135 - \dfrac{5}{4}k$.

Since d must be an integer, k must be divisible by 4.
Since $m\angle E = 5k < 180$, $k < 36$. Therefore $k = 4, 8, 12, 16, \dots , 32$.
The chart below indicates as k increases the required ratio decreases and the largest value less than 1 is highlighted.

k	d (m∠BAE)	m∠E	m∠1 (∠EAD)	m∠2 (∠DAB)	Ratio
4	130	20	80	50	8/5
8	125	40	70	55	14/11
12	120	60	60	60	1
16	115	80	50	65	**10/13**

Alternate solution #2:
**** $4d + 5k = 540 \rightarrow d$ must be a multiple of 5. For ADE to be a triangle, $0 < 5k < 180$ or $0 < k < 36$.
Substituting in ****, we have $90 < d < 135$. By trial and error (and the fact that d is a multiple of 5), we test 130, 125, 120, … . These results are contained in the table above, referencing the second column as the key field.