MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2007 SOLUTION KEY

Team Round

A) $f(x) = 3x - 1$ and $g(x) = 5x + 2 \rightarrow f^{-1}(x) = (x + 1)/3$ and $g^{-1}(x) = (x - 2)/5$

$$g^{-1}(f(x)) = g^{-1}(3x - 1) = \frac{3x-1-2}{5} = \frac{3x-3}{5}$$

$$f^{-1}(g(x)) = f^{-1}(5x + 2) = \frac{(5x+2)+1}{3} = \frac{5x+3}{3}$$

Equating and cross multiplying $\rightarrow 9x - 9 = 25x + 15 \rightarrow 16x = -24 \rightarrow x =$ **<u>-3/2</u>**.

B) $S = 25(26)/2 = 325$

$\left[\frac{25}{5}\right] + \left[\frac{25}{25}\right] = 6 \rightarrow$ the rightmost 6 digits of 25! are zeros.

Note: $[x]$ denotes the greatest integer in x, i.e. the largest integer $\leq x$
(5, 10, 15 and 20 each contain 1 factor of 5 and 25 contains 2 factors of 5)
Thus, the rightmost 7 digits of N are $x000325$, where we need only determine x.
Since there are no more factors of 5 in 25! and an excess of 2s, we know that x must be nonzero and even. Which is it? 2, 4, 6 or 8?

The prime factorization of 25! is $2^{x_1}3^{x_2}5^{x_3}7^{x_4}11^{x_5}13^{x_6}17^{x_7}19^{x_8}23^{x_9}$
We have already determined that $x_3 = 6$ and the following are easily verified:
$x_6 \ldots x_9 = 1$, $x_5 = 2$, $x_4 = 3$, $x_2 = 10$ and $x_1 = 16$

$x_5 = \left[\frac{25}{11}\right] + \left[\frac{25}{121}\right] = 2$ $\qquad x_4 = \left[\frac{25}{7}\right] + \left[\frac{25}{49}\right] = 3$

$x_2 = \left[\frac{25}{3}\right] + \left[\frac{25}{9}\right] + \left[\frac{25}{27}\right] = 8 + 2 = 10$

$x_1 = \left[\frac{25}{2}\right] + \left[\frac{25}{4}\right] + \left[\frac{25}{8}\right] + \left[\frac{25}{16}\right] + \left[\frac{25}{32}\right] = 12 + 6 + 3 + 1 = 22$

Thus, the prime factorization of 25! $= 2^{22}3^{10}5^{6}7^{3}11^{2}13^{1}17^{1}19^{1}23^{1}$.
Pulling out the factors of 10, 25! $= 10^{6}(2^{16}3^{10}7^{3}11^{2}13^{1}17^{1}19^{1}23^{1})$
The rightmost digit of the product in parentheses is the digit x we need.

$2^{16} = (2^4)^4 = (\_6)^4$ ends in 6
$3^{10} = (3^4)^2 3^2 = (\_1)^2 3^2$ ends in 9
7^3 ends in 3
It's left to you to verify that $11^{2}13^{1}17^{1}19^{1}23^{1}$ ends in 7
The product (_6)(_9)(_3)(_7) ends in 4.
Thus, the last 7 digits are 4000325 and the sum is **<u>14</u>**.