MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2008 SOLUTION KEY

Round 5

A) A point on the circumference moves 24π inches per second.

Converting, $24\pi \frac{\cancel{\text{in}}}{\cancel{\text{sec}}} \cdot \frac{1\,\text{ft}}{12\,\cancel{\text{in}}} \cdot \frac{60\,\cancel{\text{sec}}}{1\,\text{min}} = \underline{\mathbf{120\pi}}$ ft/min

B) $(5x-3)^2 = (3x-1)(10x-12) \rightarrow 25x^2 - 30x + 9 = 30x^2 - 46x + 12$
$\rightarrow 5x^2 - 16x + 3 = (5x-1)(x-3) = 0 \rightarrow x = 3$ [1/5 is extraneous]
$PA = 12$, $PB = 8$, $BC = 10$, $AC = 9$
Thus, the perimeter of $\Delta APC = (12 + 18 + 9) = \underline{\mathbf{39}}$.

C) Let $x = PD$. Applying the product-chord theorem, $14x = 12(28)$
$\rightarrow x = 24$. Since E and F are midpoints, $AE = 20 \rightarrow PE = 8$ and
$CF = 19 \rightarrow PF = OE = 5$.
Thus, in right ΔPEO, $PO^2 = 8^2 + 5^2 \rightarrow PO = \sqrt{89}$ and
in right ΔAPD, $AD^2 = 12^2 + 24^2 \rightarrow AD = 12\sqrt{5}$
$\rightarrow AD - PO = \underline{\mathbf{12\sqrt{5} - \sqrt{89}}}$