

MASSACHUSETTS MATHEMATICS LEAGUE

DECEMBER 2003

ROUND 4: LOGS & EXPONENTIALS

NON-CALCULATOR

ANSWERS

A) 27

B) $3/2, -2$

C) -1

A) Solve for x : $\log_9 x = \log_{16} 320 - \log_{16} 5$

$$\log_9 x = \log_{16} \frac{320}{5} = \log_{16} 64 = \frac{3}{2}$$

$$x = 9^{3/2} = 27$$

B) Solve for x : $\left(\frac{1}{4}\right)^{x-x^2} = 8^{2-x}$

$$2^{-2(x-x^2)} = 2^{3(2-x)}$$

$$-2x + 2x^2 = 6 - 3x$$

$$2x^2 + x - 6 = 0$$

$$(2x-3)(x+2) = 0$$

$$x = 3/2, -2$$

C) Solve for x : $\log_2(-2x-1) - \log_{\sqrt{2}} 2 + \log_2(-x+3) = 0$

$$\log_2(-2x-1)(-x+3) = \log_{\sqrt{2}} 2 = 2$$

$$2x^2 - 6x + x - 3 = 2^2 = 4$$

$$2x^2 - 5x - 7 = 0 \quad (2x-7)(x+1) = 0$$

$$x = \frac{7}{2}, -1, \text{ only } -1 \text{ checks.}$$