MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 – MARCH 2011 SOLUTION KEY

Round 3

A) $\pi \cdot \pi = \pi^2$ It's just that simple – no conversion necessary!
If the question had been multiply the inch equivalent of 3 feet by the foot equivalent of 3 inches, the product would be $36(1/4) = 9 = 3^2$.

For the disbelievers,
In the case of degree-radian conversion, 180° = π radians (halfway around a circle)

Therefore, $\pi° = \dfrac{\pi^2}{180}$ radians and $\dfrac{\pi^2}{180}\cdot 180 = \underline{\boldsymbol{\pi^2}}$

B) $2\sin(x)+3\cot(x)=0 \rightarrow \dfrac{2\sin^2(x)+3\cos(x)}{\sin(x)}=0 \rightarrow 2\left(1-\cos^2(x)\right)+3\cos(x)=0$

$\rightarrow 2\cos^2(x)-3\cos(x)-2=\left(2\cos x+1\right)\left(\cos x-2\right)=0 \rightarrow \cos x=-\dfrac{1}{2} \rightarrow x = \underline{\mathbf{120}}, \underline{\mathbf{240}}$.

C) Let $z^4=\left(rcis\theta\right)^4=r^4cis\left(4\theta\right)=-8+8\sqrt{3}i=Acis\alpha$, where $\alpha \in Q2$

Converting to trigonometric form, $-8+8\sqrt{3}i=16cis\left(\dfrac{2\pi}{3}\right)$, because for $x+yi$

$A^2=x^2+y^2$ and $\tan\alpha=\dfrac{y}{x} \rightarrow A^2 = 64 + 64\cdot 3 = 256 \rightarrow A = 16$ and $\tan\alpha = \sqrt{3} \rightarrow \theta=\dfrac{2\pi}{3}$

Thus, $r^4cis(4\theta)=16cis\left(\dfrac{2\pi}{3}\right) \rightarrow \begin{cases} r^4=16 \\ 4\theta=\dfrac{2\pi}{3}+2n\pi \end{cases} \rightarrow \begin{cases} r=2 \\ \theta=\dfrac{\pi\left(1+3n\right)}{6} \end{cases}$, for $n=0,1,2,3$

Thus, the roots are $2cis(\theta)$, where $\theta = \dfrac{\pi}{6},\dfrac{2\pi}{3},\dfrac{7\pi}{6}$ and $\dfrac{5\pi}{3}$.

Converting back to rectangular form, $2cis\left(\dfrac{\pi}{6}\right)=2\left(\cos\dfrac{\pi}{6}+i\sin\dfrac{\pi}{6}\right)=2\left(\dfrac{\sqrt{3}}{2}+\dfrac{1}{2}i\right)=\sqrt{3}+i$ etc.

Thus, the four roots are: $\underline{\boldsymbol{\sqrt{3}+i,\ -1+\sqrt{3}i,\ -\sqrt{3}-i,\ 1-\sqrt{3}i}}$

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