

**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2011 SOLUTION KEY**

Round 5

A) $x^2 + 10x \leq 24 \Leftrightarrow x^2 + 10x - 24 \leq 0 \Leftrightarrow (x + 12)(x - 2) \leq 0$. The critical values are $-12, +2$, which divide the number line into three regions. We require a negative or zero product. Testing a value in each region, only the region between -12 and 2 inclusive satisfies the required condition \Rightarrow $-12 \leq x \leq 2$.

B) $-2x - 1 < 3x - 11 < 2x + 1 \Rightarrow -2x - 1 < 3x - 11$ and $3x - 11 < 2x + 1$
 $\Rightarrow 10 < 5x$ and $x < 12 \Rightarrow$ $2 < x < 12$.

C) $y + |x| > 8 \Rightarrow y < 8 - |x| \Rightarrow \begin{cases} y = x + 8 \\ y = -x + 8 \end{cases}$

$$|x + 1| < 7 \Rightarrow -7 < x + 1 < 7 \Rightarrow -8 < x < 6$$

$y \geq 0$ restricts us to above the y -axis.

Examining the graphs of the related equations, we see the required region is the interior of a polygon, comprised of a triangle in quadrant 2 and a trapezoid in quadrant 1 between the y -axis and the vertical line $x = 6$.

Since $y = -x + 8$ and $x = 6$ intersect at $(6, 2)$, we have the

$$\frac{1}{2} \cdot 8 \cdot 8 + \frac{1}{2} \cdot 6 \cdot (2 + 8) = 32 + 30 = \underline{62}$$

necessary dimensions to find the area of each region.

