MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2011 SOLUTION KEY

Team Round - continued

C) (3, 6) / $y = mx + b$ → $b = 6 - 3m$
The y-intercept is at $(0, b) = (0, 6 - 3m)$.
The x-intercept as at $\left(-\frac{b}{m}, 0\right) = \left(-\frac{6-3m}{m}, 0\right)$
The area of the first quadrant triangle is given by $\frac{1}{2}(6-3m)\cdot\left(-\frac{6-3m}{m}\right) = 100$
→ $(6 - 3m)^2 = -200m$ → $9m^2 + 164m + 36 = (9m + 2)(m + 18) = 0$ → $m = -2/9, -18$
If θ denotes the angle of inclination, then $m = \tan\theta$
Negative slopes correspond to <u>obtuse</u> angles of inclination.
$\theta = \pi - Arc\tan(18)$ or $\pi - Arc\tan\left(\frac{2}{9}\right)$ Since $y = \text{Arctan}(x)$ is a strictly increasing function, $Arc\tan(18) > Arc\tan\left(\frac{2}{9}\right)$ and the smaller of these is $\underline{\boldsymbol{\pi - Arc\tan(18)}}$

D) Regroup the terms on the left side as follows:
$\left((x+2)(x+3)\right)\left((x+1)(x+4)\right) = \left(x^2+5x+6\right)\left(x^2+5x+4\right) = 8$
If $A = (x^2 + 5x)$, then the equation becomes $(A+6)(A+4) = 8$.
→ $A^2 + 10A + 16 = (A+2)(A+8) = 0$ → $A = -2, -8$
Substituting and applying the quadratic formula,
$x^2 + 5x + 2 = 0$ → $x = \underline{\boldsymbol{\frac{-5 \pm \sqrt{17}}{2}}}$ and $x^2 + 5x + 8 = 0$ → $b^2 - 4ac = 25 - 32 < 0$ (rejected)

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