MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 – NOVEMBER 2011 SOLUTION KEY

Round 1

A) $\sqrt{-18}\cdot\sqrt{-8}=i\sqrt{18}\cdot i\sqrt{8}=i^2\sqrt{144}=\underline{\mathbf{-12}}$

B) If $z^{-1}=3-4i$, then $z=\dfrac{1}{3-4i}\cdot\dfrac{3+4i}{3+4i}=\dfrac{3+4i}{9+16}=\dfrac{3}{25}+\dfrac{4}{25}i$

Since $x > 0$ and $y > 0$, P is located in quadrant $\underline{\mathbf{1}}$.

C) $\sum_{k=0}^{k=3}\left(1-\sqrt{3}i\right)^k=\left(1-\sqrt{3}i\right)^0+\left(1-\sqrt{3}i\right)^1+\left(1-\sqrt{3}i\right)^2+\left(1-\sqrt{3}i\right)^3$

$\left(1-\sqrt{3}i\right)^2=1-2\sqrt{3}i+3i^2=-2-2\sqrt{3}i$

The first three terms evaluate easily as $1+\left(1-\sqrt{3}i\right)+\left(-2-2\sqrt{3}i\right)=-3\sqrt{3}i$.

The last term evaluates as follows:

$\left(1-\sqrt{3}i\right)^3=\left(1-\sqrt{3}i\right)^2\cdot\left(1-\sqrt{3}i\right)^1=\left(-2-2\sqrt{3}i\right)\left(1-\sqrt{3}i\right)=-2+2\sqrt{3}-2\sqrt{3}-6=-8$

If you recalled that the three cubes roots of –8 are –2, $-1+\sqrt{3}$ and $1-\sqrt{3}$, this expansion was unnecessary. Thus, the required sum is $\underline{\mathbf{-8-3\sqrt{3}i}}$.