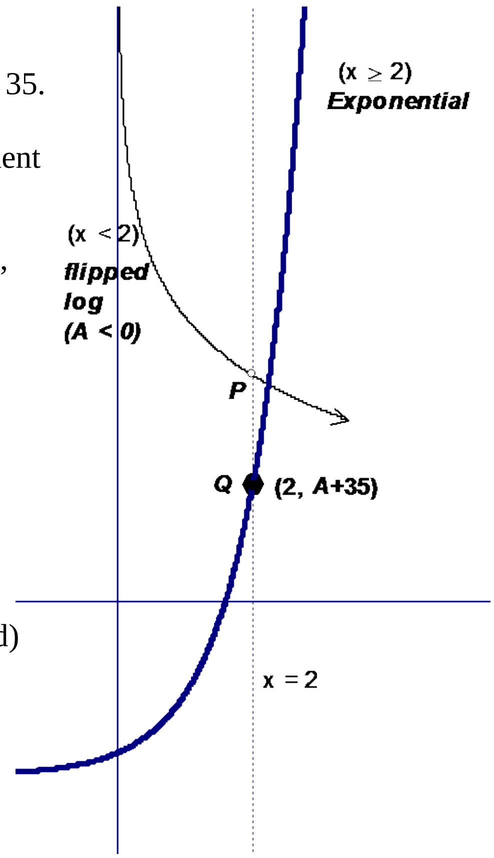
Team Round - continued



D) For $x = 2$, the top level of the function rule applies and $f(2) = A + 35$. The piecewise function is defined by the logarithmic component to the left of the vertical line $x = 2$ and by the exponential component to the right. As x approaches 2 from the left, $f(x)$ approaches

$A\log_4(2) + B = \dfrac{A}{2} + B$. If the function is to be continuous at $x = 2$,

then these function values must be equal, namely $A + 35 = \dfrac{A}{2} + B$.

Combining with $A + B = 17$, we have

$$\frac{A}{2} + (17 - A) = A + 35 \Rightarrow A + 34 - 2A = 2A + 70 \Rightarrow A = -12$$

Thus, $(A, B) =$ **<u>(–12, 29)</u>**.
Graphically, since $A < 0$, the logarithmic piece is flipped.
Each piece is itself continuous, but there is a gap at $x = 2$
To close the gap the logarithmic piece must be translated (dropped) so that point P (the hole) coincides with the endpoint Q.

E) Given: $x = 2$, $y < 0$, $z > 0$ and $\dfrac{x+y}{z} = \dfrac{y+z}{x} = \dfrac{x+z}{y}$

Substituting for x, we have $\dfrac{2+y}{z} = \dfrac{y+z}{2} = \dfrac{2+z}{y}$

Cross multiplying the first two fractions, $4 + 2y = yz + z^2 \Leftrightarrow z^2 + yz - 2(y+2) = 0$

Since this is a quadratic equation in z, using the QF,

$$z = \frac{-y \pm \sqrt{y^2 + 8(y+2)}}{2} = \frac{-y \pm \sqrt{(y+4)^2}}{2} = \frac{-y \pm (y+4)}{2}$$

Thus, $z = 2, -(y + 2)$.

Case 1:

$z = 2 \Rightarrow \dfrac{2+y}{2} = \dfrac{y+2}{2} = \dfrac{2+2}{y} = \dfrac{4}{y}$ which is satisfied if $y^2 + 2y - 8 = (y+4)(y-2) = 0$

$\Rightarrow y = 2, -4 \Rightarrow (x, y, z) =$ $\cancel{(2,2,2)}$ or (2,–4,2) The first solution is rejected, since $y > 0$.

Case 2:

$z = -(y + 2) > 0 \Rightarrow y < -2$ and solution must be of the form $(2, y, -(y + 2))$.
Picking y as large as possible, we have (2, –3, 1).
Thus, the maximum value of y is **<u>–3</u>**.