MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2011 SOLUTION KEY

Round 4

A) Given: $Q = \dfrac{10+3x}{6-x}$ The value of x must be less than 6 for the value of the fractional expression to be positive. Testing $x = \underline{\mathbf{5}} \rightarrow Q = 25$, $x = \underline{\mathbf{4}} \rightarrow Q = 11$, $x = \underline{\mathbf{2}} \rightarrow Q = 4$
$x = 1, 3$ fail! Where's the 4th value?!
There was no requirement that x had to be positive. $x = \underline{\mathbf{-1}} \rightarrow Q = +1$

B) Using the Venn diagram to the right

$100 = x + z + 40 + 12$

$120 = x + y + 12 + 40$

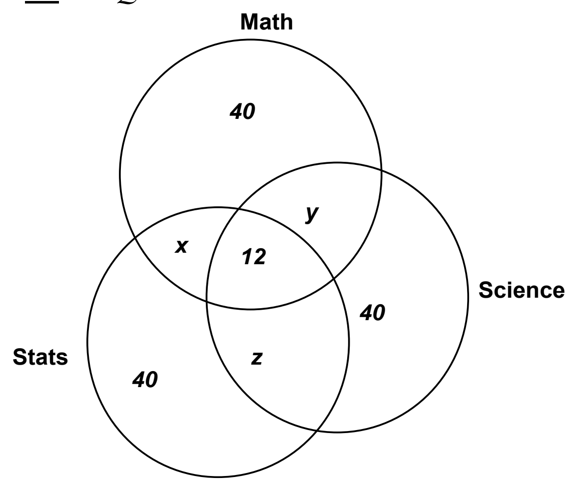
$130 = y + z + 40 + 12$

Then:

$48 = x + z$

$68 = x + y$

$78 = y + z$



Solving, $x = 19$, $y = 49$ and $z = 29$
Thus, the number of students belonging to no club is:
$1000 - (40 + 40 + 40 + 12 + 19 + 49 + 29) = 1000 - 229 = \underline{\mathbf{771}}$

C) $$\frac{1-4x^{-2}}{8-x^{-3}} \div \frac{x+2}{\dfrac{4}{x^{-2}}+\dfrac{2}{x^{-1}}+1} = \frac{\left(x^3-4x\right)\left(4x^2+2x+1\right)}{\left(8x^3-1\right)(x+2)} = \frac{x\left(x^2-4\right)\left(4x^2+2x+1\right)}{(2x-1)\left(4x^2+2x+1\right)(x+2)} = \underline{\boldsymbol{\frac{x(x-2)}{2x-1}}}$$

Note: $8x^3-1$ is factored as $A^3 - B^3 = (A-B)\left(A^2 + AB - B^2\right)$, where $A = 2x$ and $B = 1$.

Round 5

A) The vertex angle measures either $(3x)°$ or $(6x)°$.
In the first case, $15x = 180 \rightarrow x = 12$ and $\angle$s measure 36, 72 and 72
In the second case, $12x = 180 \rightarrow x = 15$ and $\angle$s measure 90, 45 and 45
Thus, the smallest possible interior angle measures $\underline{\mathbf{36}}$°.

B) The shortest distance is measured along the altitude. Let the length of the altitude in ΔPQR be h units. The sides of ΔPQR have lengths $3x$, $4x$ and $5x$, where x represents the proportionality constant. The area of ΔPQR is $\frac{1}{2}(3x)(4x)$ or $\frac{1}{2}h(5x)$. Equating, $h = \frac{12}{5}x$ or $x = \frac{5h}{12}$.

Perimeter $= 12x = 12\left(\frac{5h}{12}\right) = \underline{\mathbf{5}\boldsymbol{h}}$.

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