MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2012 SOLUTION KEY

Round 6

A) $8x+3-7x=7x-(4x+1) \Rightarrow 2x=4 \Rightarrow x=2$
AP: 9, 14, 19, …
$a = 9$, $d = 5$ and $n = 30$ $S_{30} = \frac{30}{2}\left(2(9)+(30-1)5\right)=15(18+29\cdot 5) = 15(163) =$ **2445**

B) The first three terms of the AP are $(x + 2)$, $(4x + 14)$ and $(12x + 6)$.
The common difference $d = (4x + 14) - (x + 2) = (12x + 6) - (4x + 14) \Rightarrow 3x + 12 = 8x - 8$
$\Rightarrow x = 4$ and $d = 24$. The AP is 6, 30, 54, … and the GP is 6, 18, 54, … .
The required ratio is $(54 + 48) : (54\cdot 9) = 102 : 486 =$ **17 : 81**.

C) t_n generates a geometric sequence, where $r = -\frac{2}{3}$ and $a = -2$.
Since $|r| < 1$, the infinite geometric series converges to $\frac{a}{1-r}=\frac{-2}{1+\frac{2}{3}}=-\frac{6}{5}$.

$B_3=(1-i)^1+(1-i)^2+(1-i)^3=1-i+(-1-2i+1)+(-2i-2)=-1-5i$.

$\frac{A}{B_3}=-\frac{6}{5}\cdot\frac{1}{-1-5i}\cdot\frac{-1+5i}{-1+5i}=\frac{6-30i}{5(26)}=\mathbf{\frac{3-15i}{65}}$.