MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2014
ROUND 7 TEAM QUESTIONS

ANSWERS

A) _____ D) _____

B) _____ E) _____ : _____

C) _____ F) _____

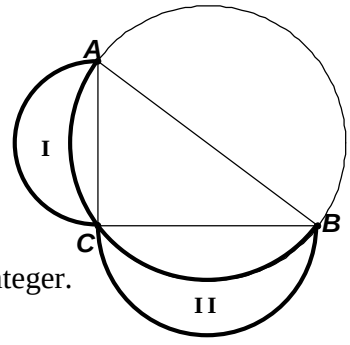
A) In ΔABC, $AB = 8$, $BC = 6$, and D is on $\overline{AB}$ so that $CD = \sqrt{10}$, and $AC = AD$. Compute AC.

B) Two positive reduced fractions $\frac{a}{b}$ and $\frac{c}{d}$, where a, b, c and d are integers and $b \neq d$, have a sum of $\frac{5}{19}$. Compute the minimum sum $b+d$.

C) Circle O is tangent to both $7x + y = 8$ and $x - y = 4$. The radius of circle O is $\sqrt{2}$. The center of circle O is (h, k). Compute all possible values of $h+k$.

D) Suppose P and Q are positive integers and that the point $R\left(4^{2P-Q}, \log_2(P+2Q)\right)$ denotes an ordered pair of positive integers. Compute the smallest possible sum $P + Q$ for which R lies on the line $y = x$.

E) A lune is a region bounded by arcs of circles with different radii. The circumcircle of a right triangle with sides of 3, 4 and 5 and semi-circles drawn on the legs in the exterior of the triangle form a pair of lunes. Compute the ratio of the sum of the areas of the lunes (I + II) to the area of the triangle.



F) Regular polygon P has m sides and interior angles of $x°$, where x is an integer. Regular polygon Q has n sides and interior angles of $(x + 2)°$. Compute all possible values of $n-m$.