

**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2011
ROUND 7 TEAM QUESTIONS**

ANSWERS

- A) _____ D) $y =$ _____
- B) _____ E) _____ sq. units
- C) _____ F) (_____ , _____)

******* NO CALCULATORS ON THIS ROUND *******

- A) There are four squares with sides parallel to the x - and y -axes that have a common vertex at $P(3, 4)$ and whose sides have length 5. What is the sum of the distances from the four vertices of the square formed by the union of the four original squares to the line $y = x$?

- B) Factor completely as the product of polynomials with integer coefficients: $x^8 + x^4 + 1$

- C) Some older textbooks define the following trigonometric functions:
$$\begin{cases} \text{vers } A = 1 - \cos A \\ \text{covers } A = 1 - \sin A \\ \text{hav } A = \frac{1}{2} \text{vers } A \end{cases}$$

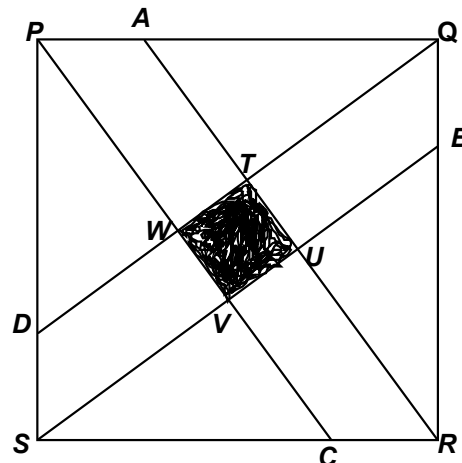
Compute all values of A , where $0 \leq A < 2\pi$ for which $\text{hav}(2A) + \frac{\text{covers}(A)}{2} = 1$.

- D) Solve for y in terms of x .

$$x^2 - xy - 6y^2 + x + 7y - 2 = 0$$

Answers must be simplified.

- E) A, B, C and D are points placed so that their distances to the endpoints of their corresponding sides of square $PQRS$ are in a 3 : 1 ratio, as indicated in the diagram at the right. ($AQ > AP$). Find the area of the shaded region, if $PQ = 8$.



- F) Let $A = \sqrt{x+37}$ and $B = \sqrt{x-N}$, where x, A, B and N are positive integers. The number of possible ordered pairs (A, B) depends on the value of N . The smallest value of N for which there are exactly three possible ordered pairs (A, B) is k . Let T denote the sum of the values of x for which this happens. Compute the ordered pair (k, T) .

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