MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2015 SOLUTION KEY

Round 4

A) For each 20 minute period, my speeds were 60 mph, 90 mph and 45 mph, during which I travelled 20 miles, 30 miles and 15 miles. Therefore, I travelled a total of 65 miles in the hour, for an average speed of **<u>65</u>** mph.

B) Since the maximum value of the function is 6 for $x = 2$, the graph of the function must pass through the point $P(2, 6) \Rightarrow y = ax^2 + bx + c$ must be equivalent to $y = a(x-2)^2 + 6$ and Since the *y*-intercept is 4, the function passes through the point $Q(0, 4)$

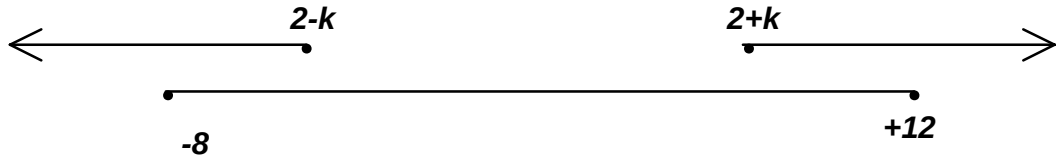
$\Rightarrow 4 = a(0-2)^2 + 6 = 4a + 6 \Rightarrow a = -\frac{1}{2}$

$y = -\frac{1}{2}(x-2)^2 + 6 = -\frac{1}{2}x^2 + 2x + 4 \Rightarrow (a,b,c) = \underline{\left(-\frac{\mathbf{1}}{\mathbf{2}}, \mathbf{2}, \mathbf{4}\right)}.$

C) $k \le |x-2| \le 10 \Rightarrow |x-2| \le 10$ and $|x-2| \ge k$

$|x-2| \le 10 \Rightarrow -10 \le x-2 \le +10 \Rightarrow -8 \le x \le 12$ (outer limits)

$|x-2| \ge k \Rightarrow x-2 \le -k$ or $x-2 \le +k \Rightarrow x \le 2-k$ or $x \le k+2$ (inner limits)



Therefore, the solution in general consists of two segments of length $10 - k$.
There will be the same number of solution on both segments, i.e. 5 each.
The five on the right must be 8, 9, 10, 11 and 12. Thus, k = **<u>6</u>**.