

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2013
ROUND 6 ALG 1: EVALUATIONS

ANSWERS

A) _____

B) _____

C) _____ : _____

A) Let $\hat{n} = (2n + 1)!$ and $a \# b = (a^b + b^a)!$

Compute $\frac{3 \# 2}{\hat{7}}$.

Recall:

$n!$ denotes a factorial.

Specifically, $n!$ is defined as the product $n \cdot (n-1) \cdot (n-2) \cdot \dots \cdot 1$ and $0! = 1$.

B) The combination padlock for my new laptop is three digits. It is factory preset at 000, but each position can be changed to any digit from 0 through 9 inclusive. How many different combinations are possible, if the sum of the three digits is 10?

C) 6 and 28 are the first two ‘perfect numbers’, i.e. the sum of the proper divisors, excluding the number itself, equals the number. ($6 = 1 + 2 + 3$ and $28 = 1 + 2 + 4 + 7 + 14$)

Numbers for which the sum of the proper divisors is less than the number are termed deficient.

Numbers for which the sum of the proper divisors is more than the number are termed abundant

Compute $D : A$, the ratio of the number of deficient numbers to the number of abundant numbers for integers strictly between the first two perfect numbers.