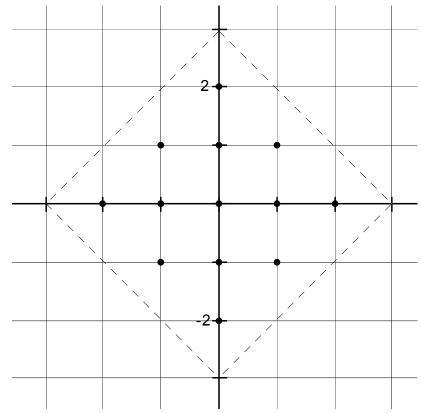
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2007 SOLUTION KEY

Team Round – continued



E) The regions bounded by $|x - 2007| + |y + 2008| = A$ and $|x| + |y| = A$ contain the same number of lattice points.
The latter is a square (diamond) with vertices at $(\pm A, 0)$ and $(0, \pm A)$.
The diagram at the right illustrates the lattice points for $A = 3$.
The number of lattice points is $2(1 + 3) + 5 = 13$.

Method #1: Pattern appears to be:
Considering the first A consecutive odd integers, the number of lattice points is found by doubling the sum of the first $A - 1$ odd integers, and then adding the Ath.

$2[1 + 3 + 5 + \ldots + (2A - 3)] + (2A - 1) = 1985$
Applying sum formulas for arithmetic sequences,

$$2\left[\frac{A-1}{2}(1+2k-3)\right]+(2A-1)=1985 \rightarrow (A-1)(2A-2)+(2A-1)$$

$$\rightarrow 2(A-1)^2 + 2A - 1 = 2A^2 - 2A - 1984 = 0 \rightarrow A^2 - A - 992 = 0 \rightarrow A(A-1) = 992 \rightarrow A = \underline{\mathbf{32}}$$

Method #2
Construct a chart for small values of A and look for a pattern.
Note that the number of lattice pts $= 2A^2 - 2A + 1$.

A	***#LPs***	
1	1	2(1)(0)+1
2	5	2(2)(1)+1
3	13	2(3)(2)+1
4	25	2(4)(3)+1
5	41	2(5)(4)+1
	2A(A-1) + 1	

F) With 4 people (A, B, C, D) and 4 phones (a, b, c, d),
examine the $4! = 24$ permutations of *abcd*.
Any that start with a are eliminated.
Starting with *b*, ~~*bacd*~~, *badc*, ~~*bcad*~~, *bcda*, bdac, ~~*bdca*~~ → 3 total mismatches
Starting with *c*, ~~*cabd*~~, *cadb*, ~~*cbad*, *cbda*~~, *cdab*, *cdba* → 3 total mismatches
Starting with d, *dabc*, ~~*dacb*, *dbac*, *dbca*~~, *dcab*, *dcba* → 3 total mismatches
Thus, for 4 phones, there are $3(3) = 9$ total mismatches → P(total mismatch) $= 9/24 = \underline{\mathbf{\frac{3}{8}}}$