MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 - FEBRUARY 2017
ROUND 7 TEAM QUESTIONS

ANSWERS

A) (_____ , _____) D) (_____ , _____)

B) _____ E) (_____ , _____ , _____)

C) _____ F) _____

A) The vertical and horizontal asymptotes of the function $y = \dfrac{1-2x}{x-6}$ intersect at point $P_0(a,b)$.
If this function undergoes the following 6 successive reflections in the given order.
A. across the *x*-axis
B. across the *y*-axis
C. across the horizontal line $y = k$ (where $k < 0$)
D. across the vertical line $x = h$ (where $h > 0$)
E. through the origin and, lastly,
F. across $y = x$,
the new coordinates of P_0 are $(10,-12)$. Compute the ordered pair (h,k).

B) Anne moves clockwise around a circle and Dick moves counterclockwise. Anne starts at 90° (the 3:00 position) and moves at 13° per second. Dick starts at 210° (the 8:00 position) and moves at 7° per second. They meet at *n* distinct positions on the circle (the first of which is 168°). Compute *n*.
Note: All angles are measured clockwise from 12:00.

C) Compute <u>all</u> values of *x* (in degrees), where $0° \le x < 360°$, that satisfy

$$\sin 3x + \sin x = \tan\left(Sin^{-1}\frac{2}{\sqrt{5}} \right) \cdot \cos(270° + x)$$

D) A movie theater has a maximum seating capacity of 200. At $7.50 per ticket, management estimates that 80% of its tickets will be sold for an evening show. Furthermore, for each 25¢ increase in ticket price, ticket sales will decrease by an additional 1% (below the 80% estimate), and, for each 25¢ decrease in ticket price, there will be a 2.5% increase (above the 80% estimate). Management loves to fill all the seats, but even more they like to maximize their profit. Let *T* denote ticket price which maximizes the total revenue *R* earned from the sale of tickets for an evening show. Assuming these assumptions are valid, compute (T,R).
Note: 1% decrease is taken to mean: 80%, 79%, 78%, 77%, 76%, … .
2.5% increase is taken to mean 80%, 82.5%, 85%, 87,5%, 90%., … .