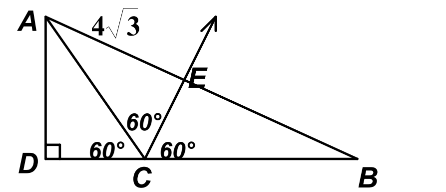
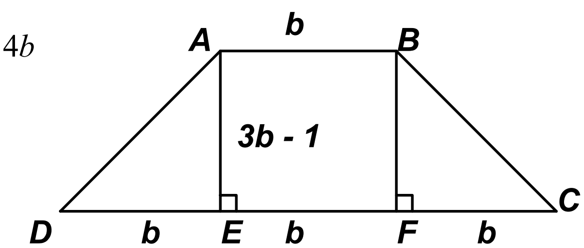
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2007 SOLUTION KEY

Round 1

A) ΔABC is a 30-60-90 triangle. Draw altitude $\overline{CE}$ from C to $\overline{AB}$. $\overline{AB}$ must be the base in isosceles triangle ABC. Therefore, E must also be a midpoint of $\overline{AB}$ and ΔACE must also be a 30-60-90 triangle congruent to ΔACD → $AD = \underline{4\sqrt{3}}$ ($ADCE$ is a kite)



B) $A = 840 = \frac{h}{2}(b_1 + b2) = (3b-1)(4b)/2$ → $1680 = 12b^2 - 4b$
→ $3b^2 - b - 420 = (3b + 35)(b - 12) = 0$ → $b = 12$
ΔADE, ΔBCF are 12 – 35 – 37 right triangles
→ $Per = 74 + 48 = 122$, $AE = 35$ → **<u>122 : 35</u>**

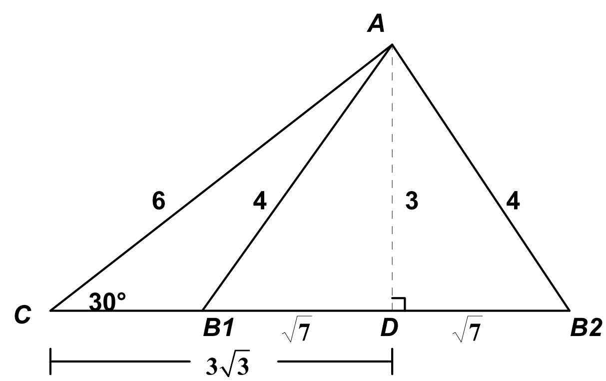


C) This is the ambiguous case, where we have information about two sides and the <u>non</u>-included angle. In general, there could be 0, 1 or 2 possible solutions. In this problem there are two solutions.

Using the law of sine, $\frac{AB}{\sin C} = \frac{AC}{\sin B}$ → $\frac{4}{.5} = \frac{6}{\sin B}$

→ $\sin B = ¾$ → $\cos B = \pm\frac{\sqrt{7}}{4}$

Dropping an altitude from A creates a 30-60-90 triangle ACD → $\overline{AD}$, the side opposite 30°, must have length 3 and $\overline{CD}$, the side opposite 60°, must have length $3\sqrt{3}$.



Clearly, referring to the diagram, the negative cosine value is associated with an obtuse angle ($\angle B$ in ΔACB_1) and the positive cosine value is associated with the acute angle ($\angle B$ in ΔACB_2)
Thus, $BC = \underline{\mathbf{\sqrt{7} + 3\sqrt{3}} \text{ or } \mathbf{3\sqrt{3} - \sqrt{7}}}$.

Alternate solution: Using only 30 – 60 – 90 right triangles (2 diagrams are possible)