MASSACHUSETTS MATHEMATICS LEAGUE
MARCH 2004
ROUND 3: POLYNOMIAL FUNCTIONS

ANSWERS

A) -7

B) $-2, 3/2, \frac{-1 \pm i\sqrt{3}}{2}$

C) $-23/6$

A) Given $P(x) = 2x^3 - 5x^2 + ax + b$. Calculate the value of a + b if $P(-1) = 8$, and the remainder when $P(x)$ is divided by $x - 2$ is -22.

$P(-1) = -2 - 5 - a + b = 8, \quad -a + b = 15$

$P(2) = 16 - 20 + 2a + b = -22, \quad 2a + b = -18$

$3a = -33, \quad a = -11$

$a + b = (2a + b) - a = -18 - (-11) = -7$

B) Solve for x: $2x^4 + 3x^3 - 3x^2 - 5x - 6 = 0$

-2	2	-1	-1	-3	0
$3/2$	2	2	2	0	

$\frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm i\sqrt{3}}{2}$

C) If r, s, and t are the roots of $3x^3 + 7x^2 - 4 = 0$, calculate the value of

$\frac{1}{r-1} + \frac{1}{s-1} + \frac{1}{t-1}$ as a reduced fraction

(1)

	3	7	0	-4
1	3	10	10	(6)
1	3	13	(23)	
1	3	16		
1	3			

ANS $-\frac{23}{6}$

(2) ORIG roots $-1, -2, \frac{2}{3}$

roots $-1 = -2, -3, -\frac{1}{3}$

recip. are $-\frac{1}{2}, -\frac{1}{3}, -3$

Sum $= \frac{-3 - 2 - 18}{6} = -\frac{23}{6}$