MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2010 SOLUTION KEY

Round 4

A) Since the middle coefficient is -1, we start with $(a, b) = (1, 2)$.
$x^2 - x - n = (x + 1)(x - 2)$ which gives $n = 2$.
Thus, the absolute value of the difference between a and b must always be 1.
2(3) ➔ 6   3(4) ➔12   4(5) ➔ 20   5(6) ➔ 30   6(7) ➔ **42**   7(8) ➔ 56 (too big)

B) Given: $P = 280x^3y^2$, $\text{GCF}(P, Q) = 28x^2y^2$ and $\text{LCM}(P, Q) = 3080x^3y^3z$.
Note: Given any two integers m and n, $\boxed{mn = \text{GCF}(m, n) \cdot \text{LCM}(m, n)}$.
Ex: GCF(24, 30) = 6 and LCM(24, 30) = 120 and 24(30) = 6(120) = 720
The same principle applies to literal expressions.

$PQ = \text{GCF}(P, Q)\cdot \text{LCM}(P, Q)$ ➔ $280x^3y^2(Q) = (28x^2y^2)(3080x^3y^3z)$
➔ $x^3y^2Q = 308x^5y^5z$ ➔ $Q = \mathbf{308x^2y^3z}$.

C) Combining like terms, $8A^2 - 7AB + 13B^2 - 3W^2 - 4B^2 - 4A^2 + 19AB - 13W^2$
$= 4A^2 + 12AB + 9B^2 - 16W^2 = (2A + 3B)^2 - (4W)^2$.
As the difference of perfect squares this factors to $\mathbf{(2A + 3B - 4W)(2A + 3B + 4W)}$

Round 5

A) The numerator $\cot(45°) + 2\sin(210°)$ evaluates to $1 + 2\left(-\frac{1}{2}\right) = 0$.

Without bothering to evaluate, we note that the denominator is nonzero, since the tangent of a first quadrant angle is positive. Thus, the expression evaluates to **0**.

B) $\left(\sin 510° \cos 240° \cot^3 315° \csc\frac{11\pi}{6} \sec\left(\frac{-7\pi}{3}\right)\right)^5 = \left(\sin 150° \cos 240° \cot^3 315° \csc\frac{11\pi}{6} \sec\left(\frac{5\pi}{3}\right)\right)^5$

$= \left(\sin 30° \cdot -\cos 60° \cdot -\cot^3 45° \cdot -\csc 30° \cdot \sec 60°\right)^5 =$

$\left(\sin 30° \cdot -\csc 30° \cdot -\cos 60° \cdot \sec 60° \cdot -\cot^3 45°\right)^5 = \left((-1)(-1)(-1)^3\right) = \underline{\mathbf{-1}}$

Created with nitro PDF professional
download the free trial online at nitropdf.com/professional