

MASSACHUSETTS MATHEMATICS LEAGUE

FEBRUARY 2004

ROUND 6: SEQUENCES & SERIES

ANSWERS

A) -10

B) ± 9

C) $(-\frac{15}{2}, \frac{4}{3}, -\frac{27}{2})$

A) In an arithmetic sequence of ten terms, the tenth term is 14, and their sum is 5. Find the second term.

$$a_1 + 9d = 14, \quad 5(2a_1 + 9d) = 5 \quad \text{so} \quad a_1 + 9d = 14$$

$$\underline{2a_1 + 9d = 1} \quad a_1 = -13$$

$$-13 + 9d = 14, \quad 9d = 27, \quad d = 3.$$

$$a_2 = a_1 + d = -13 + 3 = -10$$

B) The second term of a geometric sequence is 12, and the sixth term is $1024/27$. Find the first term.

$$12r^4 = \frac{1024}{27}, \quad r^4 = \frac{1024}{12 \cdot 27} = \frac{256}{81}, \quad r = \pm \frac{4}{3}$$

$$a_1 = \frac{a_2}{r} = \frac{12}{\pm \frac{4}{3}} = \pm \frac{12}{1} \cdot \frac{3}{4} = \pm 9$$

C) The six terms $2x - 3$, t , $7 - 12y$, $x + 3$, $3y - 4$, $x + 12$ are in arithmetic sequence. Find the ordered triple (x, y, t) .

$$(x + 12) - (x + 3) = 9 = 2d, \quad d = \frac{9}{2}. \quad (3y - 4) - (7 - 12y) = 9.$$

$$15y - 11 = 9, \quad 15y = 20, \quad y = \frac{20}{15} = \frac{4}{3}, \quad a_3 = 7 - 12y = 7 - 12 \cdot \frac{4}{3} =$$

$$7 - 16 = -9. \quad a_2 = a_3 - d = -9 - \frac{9}{2} = -\frac{27}{2}, \quad a_4 = x + 3 = a_3 + d =$$

$$-9 + \frac{9}{2} = -\frac{9}{2} \quad \text{so} \quad x + 3 = -\frac{9}{2} \quad \text{and} \quad x = -3 - \frac{9}{2} = -\frac{6+9}{2} = -\frac{15}{2}.$$

$$\underline{\text{Ans}} \quad (x, y, t) = \left(-\frac{15}{2}, \frac{4}{3}, -\frac{27}{2}\right)$$