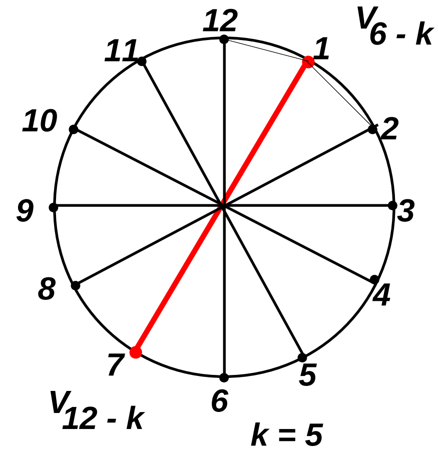
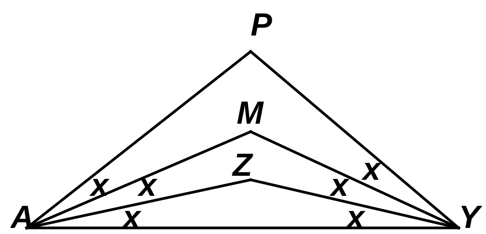
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2014 SOLUTION KEY

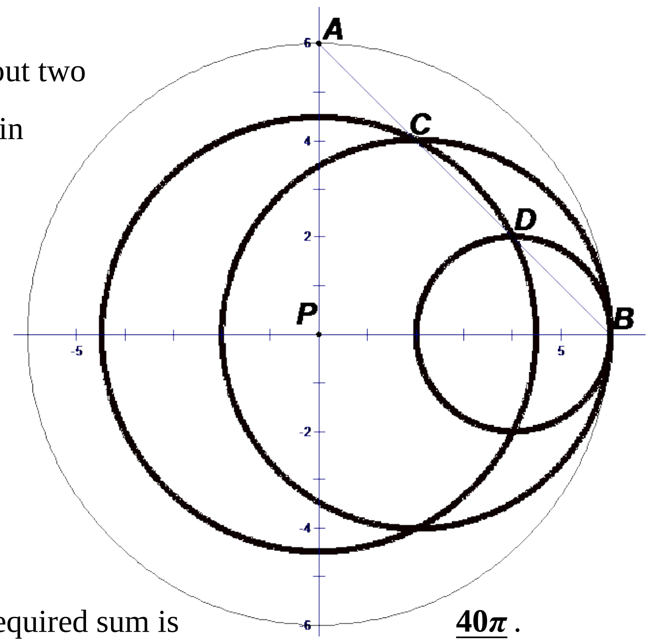


Round 5

A) Consider a clock face. 12 is opposite 6. 11 is opposite 5. 10 is opposite 4. In general, $12-k$ is opposite $6-k$.
$12-k=7 \Rightarrow k=5 \Rightarrow 6-k=1$.
Therefore, V_1 is opposite V_7 and the adjacent vertices are V_{12} and V_2, resulting in 3-letter names of either $\underline{\boldsymbol{\angle V_{12}V_1V_2}}$ or $\underline{\boldsymbol{\angle V_2V_1V_{12}}}$ for the opposite angle.



B) Let P be the vertex angle.
Examining the diagram at the right,
$m\angle P = 180 - 6x$
$m\angle M = 180 - 4x$
$m\angle Z = 180 - 2x$
Since M and Z differ by 26°, x must be 13.
Therefore, $m\angle P = 180 - 6 \cdot 13 = \underline{\mathbf{102}}°$



C) With point B fixed and A moving, C and D trace out two circles with radii $\frac{1}{3} \cdot 6 = 2$ and $\frac{2}{3} \cdot 6 = 4$, resulting in areas of 4π and 16π .

If A and B are simultaneously set in motion, we get a circle concentric with the original.
Its radius is PD (or PC).
$PB = 6 \Rightarrow AB = 6\sqrt{2} \Rightarrow DB = 2\sqrt{2}$ and $m\angle PBA = 45°$. Using the Law of Cosines on ΔDPB , we have

$$PD^2 = \left(2\sqrt{2}\right)^2 + 6^2 - 2 \cdot 2\sqrt{2} \cdot 6 \cdot \cos 45°$$
$$= 8 + 36 - 24 = 20$$

Thus, the area of the third circle is 20π and the required sum is $\underline{\mathbf{40\pi}}$.