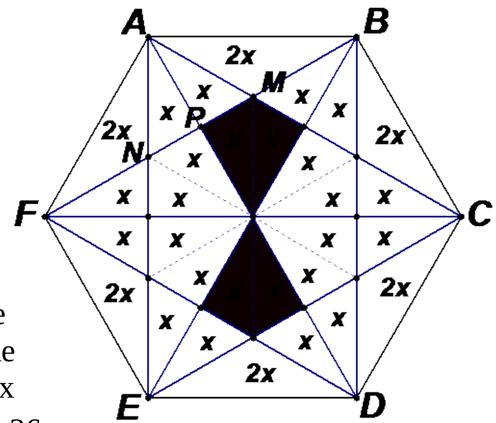
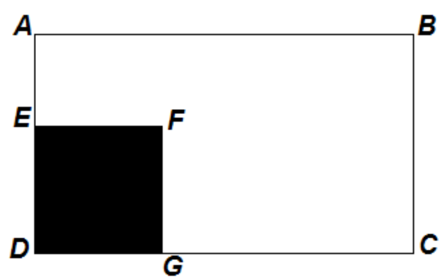
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2016 SOLUTION KEY

Round 3



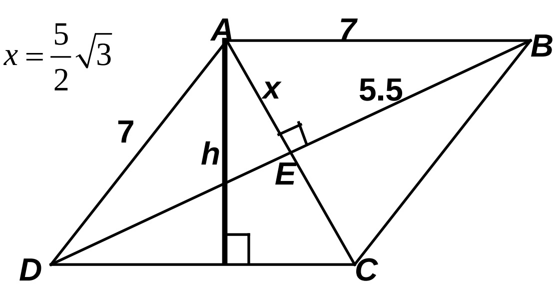
A) Clearly, the regions marked with x's are congruent and congruent regions have the same area. Therefore, let x denote the <u>area</u> of each of these regions. ΔFAN and ΔMAN are not congruent, but they do have the same area. ($FN = NM$ and $\overline{AP}$ is a common altitude when these sides are taken to be the bases of the two triangles.) The area of hexagon $ABCDEF$ equals the sum of an inner hexagon, six equilateral triangles and six congruent obtuse triangles, namely, $12x+12x+6(2x)=36x$.
The required ratio is 4 : (36 -- 4) = <u>**1 : 8**</u>.

B) Given: $DEFG$ is a square of side x. $ABCD$ is a rectangle with sides AB = 18 and BC = 14.



$$\frac{18\cdot 14-x^2}{x^2}=\frac{9}{7}\Rightarrow 7\cdot 18\cdot 14-7x^2=9x^2$$
$$\Rightarrow x^2=\frac{7\cdot 18\cdot 14}{16}=\frac{4\cdot 9\cdot 49}{16}$$
$$\Rightarrow x=\frac{3\cdot 7}{2}=\underline{\mathbf{\frac{21}{2}}}$$ or (<u>**10.5**</u>).

C) Since the diagonals of a rhombus are perpendicular,
we have $x^2+(5.5)^2=7^2\Rightarrow x^2=49-30.25=18\frac{3}{4}=\frac{75}{4}\Rightarrow x=\frac{5}{2}\sqrt{3}$



Thus, the <u>short</u> diagonal has length $5\sqrt{3}$.
Note that $5\sqrt{3}<11$ (since $5^2\cdot 3<11^2$).
Invoking the area formulas for any rhombus, we have
$$7h=\frac{1}{2}\cdot 11\cdot 5\sqrt{3}\Rightarrow h=\underline{\mathbf{\frac{55\sqrt{3}}{14}}}.$$