MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2009 SOLUTION KEY

Round 4

A) Cross multiplying, $\dfrac{x^2-10x+12}{10x-x^2-28}=\dfrac{1}{3}$ ➔ $3x^2-30x+36=10x-x^2-28$

➔ $4x^2-40x+64=4(x^2-10x+16)=4(x-2)(x-8)=0$ ➔ $x=$ **<u>2, 8</u>**

B) $5x^2+4x-x^3-20=\left(5x^2-x^3\right)+\left(4x-20\right)=x^2\left(5-x\right)-4(5-x)=0$ ➔ $\left(x^2-4\right)(5-x)=0$

➔ $x=$ **<u>±2, 5</u>**

C) $x^{24}-x^8-256x^{16}+256 = \left(x^{24}-256x^{16}\right)-\left(x^8-256\right)=\left(x^{16}-1\right)\left(x^8-256\right)=$

$\left(x^8+1\right)\left(\boldsymbol{x^8-1}\right)\left(x^4+16\right)\left(\boldsymbol{x^4-16}\right) = \left(x^8+1\right)\left(x^4+1\right)\left(\boldsymbol{x^4-1}\right)\left(x^4+16\right)\left(x^2+4\right)\left(\boldsymbol{x^2-4}\right) =$

$\left(x^8+1\right)\left(x^4+1\right)\left(x^2+1\right)(x+1)(x-1)\left(x^4+16\right)\left(x^2+4\right)\left(x+2\right)\left(x-2\right)$ ➔ $N=$ **<u>9</u>**

Round 5

A) $= (\sqrt{3}+1)^3 = (\sqrt{3}+1)\ (\sqrt{3}+1)^2 = (\sqrt{3}+1)\ (2\sqrt{3}+4) = 10+6\sqrt{3}$ ➔ $(A, B, C) =$ **<u>(10, 6, 3)</u>**

B) $2\tan(x) = 3\cot(x) - 1$ ➔ $2\tan^2(x) = 3 - \tan(x)$

➔ $2\tan^2(x) + \tan(x) - 3 = (2\tan x + 3)(\tan x - 1)$ ➔ $\tan(x) = \underline{\boldsymbol{-\dfrac{3}{2}}}$

(1 is extraneous since x would be special, i.e. it would belong to the 45° family.)

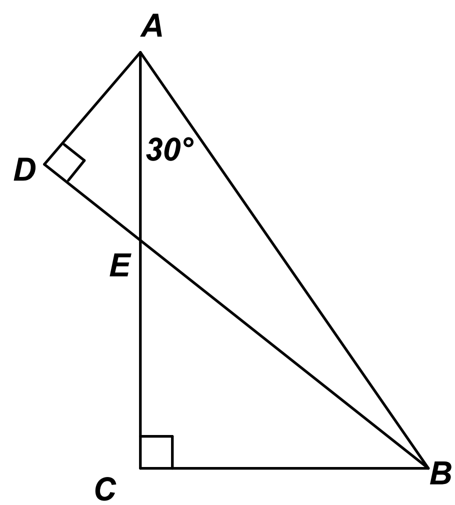
C) $m\angle CBA = 60°$, $BC = 2$, $AC = 2\sqrt{3}$

$m\angle CBE = m\angle CEB = m\angle AED = 45°$

$BE = 2\sqrt{2}$, $AE = 2\sqrt{3}-2$ and

$DE = \dfrac{2\sqrt{3}-2}{\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}}=\dfrac{2\sqrt{6}-2\sqrt{2}}{2}=\sqrt{6}-\sqrt{2}$

Thus, $BD = BE + ED = 2\sqrt{2}+\left(\sqrt{6}-\sqrt{2}\right) = \underline{\boldsymbol{\sqrt{6}+\sqrt{2}}}$



Alternate solution

In right ΔBAD, $\dfrac{BD}{AB}=\cos\left(\angle DBA\right)$.

$m\angle DBA = 60° - 45° = 15°$

Thus, $BD = 4\cos(15°) = 4\cos(45° - 30°) = 4\left(\cos 45°\cos 30°+\sin 45°\sin 30°\right)$

$= 4\left(\dfrac{\sqrt{2}}{2}\cdot\dfrac{\sqrt{3}}{2}+\dfrac{\sqrt{2}}{2}\cdot\dfrac{1}{2}\right)=\underline{\boldsymbol{\sqrt{6}+\sqrt{2}}}$