MASSACHUSETTS MATHEMATICS LEAGUE
FEBRUARY 2004
ROUND 7: TEAM QUESTIONS

ANSWERS

A) 3 D) 703

B) 305 E) 3

C) $(\sqrt{2}-\sqrt{6})/4$ F) 36

A) If $f(x) = 2x^2 - 17x + 24$ and $f(x + a) = 2x^2 - 5x - 9$, calculate the value of a.

$2(x+a)^2 - 17(x+a) + 24 = 2x^2 - 5x - 9$

$2(x^2+2xa+a^2) - 17(x+a) + 24$ so $4a - 17 = -5$, $a = 3$

B) Determine the 142nd positive integer divisible by three or five.

300 is the 100th div by 3, and the 60th div by 5, but the 20th div by 15. So 300 is the $100+60-20 = 140$th div by 3 or 5.

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C) Express $\cos^2 \frac{7\pi}{24} - \sin^2 \frac{7\pi}{24}$ in simple radical form.

$= \cos \frac{7\pi}{12} = \cos 105^\circ = \cos(60^\circ + 45^\circ) = \frac{1}{2}\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}-\sqrt{6}}{4}$

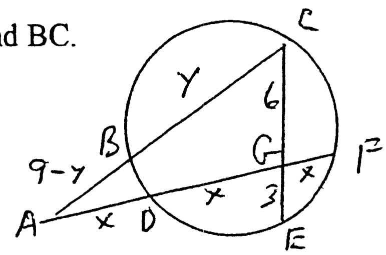
D) The hundred's digit of a three-digit number is one more than twice the units digit, and the tens digit is three less than the units digit. If the digits were reversed, the number obtained would be 396 less than the original number. Find the original number.

$h = 2u+1$, $T = u-3$, $100(2u+1) + 10(u-3) + u = 100u + 10(u-3) + (2u+1) + 396$

$200u + 100 + u = 102u + 397$

$99u = 297$, $u = 3$; $T = 0$, $h = 7$

E) In the figure, AC = 9, GC = 6, GE = 3, and AD = DG = GF. Find BC.



$x^2 = 3\cdot 6 = 18$, $x = 3\sqrt{2}$

$9(9-y) = 3\sqrt{2}(9\sqrt{2}) = 54$

$9 - y = 6$, $y = 3$

F) An infinite geometric series has a sum of 54 while the sum of the first three terms is 52. What is the first term?

$\frac{a}{1-r} = 54$, $a + ar + ar^2 = 52$, so $54(1-r)(1+r+r^2) = 52$,

$54(1-r^3) = 52$, $54r^3 = 2$, $r^3 = \frac{1}{27}$, $r = \frac{1}{3}$, $a = \frac{54}{1}\cdot\frac{2}{3} = 36$