MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2017 SOLUTION KEY

Round 6

A) Cross multiplying, $\frac{1}{T}=\frac{2T-1}{15}\Leftrightarrow 2T^2-T=15$, provided $T\neq 0$.

$2T^2-T-15=(2T+5)(T-3)=0\Rightarrow T=\underline{\mathbf{-\frac{5}{2}}}$, $\cancel{3}$ (non-integer values only)

B) $3KK_{(8)}=3\left(8^2\right)+K\left(8^1\right)+K\left(8^0\right)=192+9K$

$4KJ_{(7)}=4\left(7^2\right)+K\left(7^1\right)+J\left(7^0\right)=7K+J+196$

Equating, $2K-J=4$

Since J must be a digit in base 7, $0\le J\le 6$

Since K must be a digit in both base 7 and base 8, we also have $0\le K\le 6$.

Thus, $(K,J)=(2,0),(3,2),(4,4),(5,6)$

$2+3+4+5=14_{(10)}=\underline{\mathbf{13}}_{(11)}$

C) $2x-\frac{1}{x}=\frac{1}{6}\Rightarrow\frac{2x^2-1}{x}=\frac{1}{6}$

Cross multiplying, $12x^2-6=x\Leftrightarrow 12x^2-x-6=(4x-3)(3x+2)=0$

$x=\frac{3}{4},-\frac{2}{3}$ and $a<b\Rightarrow(a,b)=\left(-\frac{2}{3},\frac{3}{4}\right)$.

Since $a-b<0$, $ab<0$, and $\frac{a}{b}<0$, the winner is $a+b=\frac{3}{4}-\frac{2}{3}=\frac{9-8}{12}=\underline{\mathbf{\frac{1}{12}}}$.