MASSACHUSETTS MATHEMATICS LEAGUE CONTEST 2 - NOVEMBER 2007 SOLUTION KEY

Round 5

A) =
$$1(-1)(1)^2 \left(\frac{\sqrt{2}}{2}\right)^3 = \frac{\sqrt{2}}{4}$$

B)
$$\sin^2(120) = \tan(225) \cdot \cot(45) - \sin^2(x) \rightarrow \frac{3}{4} = 1 - \sin^2 x \rightarrow \sin x = \pm \frac{1}{2} \rightarrow \underline{30^\circ, 150^\circ, 210^\circ, 330^\circ}$$

C)
$$\frac{\tan x + \sqrt{3}}{1 + \sqrt{3}\cot x} = 1 \Rightarrow \tan x - 1 + \sqrt{3}(1 - \cot x) = 0 \Rightarrow \tan^2 x - \tan x - \sqrt{3}(\tan x - 1) = 0$$

 $\Rightarrow (\tan x - 1)(\tan x + \sqrt{3}) = 0 \Rightarrow x = 45^{\circ}, 225^{\circ}$ [120°, 300° are extraneous]