MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2008 SOLUTION KEY

Round 5

A) Given: $\begin{cases} y = 10 - 3t \\ x = 4t + 1 \end{cases}$ $\quad x = -3 \rightarrow t = -1$

Substituting in the 1st equation, $y = 13$. Thus, $\dfrac{y}{t} = \underline{\mathbf{-13}}$

B) $A = \dfrac{k\sqrt{B}}{C^3} \rightarrow 36 = \dfrac{k\sqrt{9}}{2^3} \rightarrow k = 36(8)/3 = 96$

$(A, C) = (8, 4) \rightarrow 8 = \dfrac{96\sqrt{B}}{4^3} \rightarrow \sqrt{B} = \dfrac{8(64)}{96} = \dfrac{2^9}{2^5 \cdot 3} = \dfrac{2^4}{3} \rightarrow B = \underline{\mathbf{\dfrac{256}{9}}}$

C) We need to find $\dfrac{y}{z}$ since $\dfrac{z}{y+z} = \dfrac{1}{\dfrac{y}{z}+1}$ (***).

From the given equations, we have $z = 2x + 2y$ and $y = 3x + 3z$.
Thus, $z = 2x + 2(3x + 3z) = 8x + 6z$ or $-5z = 8x$
Also $y = 3x + 3(2x + 2y) = 9x + 6y$ or $-5y = 9x$

So $\dfrac{y}{z} + 1 = \dfrac{\frac{y}{x}}{\frac{z}{x}} + 1 = \dfrac{-9/5}{-8/5} + 1 = \dfrac{9}{8} + 1 = \dfrac{17}{8}$

Substituting in (***), $\dfrac{z}{y+z} = \underline{\mathbf{\dfrac{8}{17}}}$

Notice that if the values of the two given expressions were reversed, the answer would be $\dfrac{9}{17}$ and $\dfrac{8}{17} + \dfrac{9}{17} = 1$.