MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 - FEBRUARY 2016
ROUND 7 TEAM QUESTIONS
ANSWERS

A) _____ D) (_____ , _____)

B) C = _____ S = { _____ } E) (_____ , _____ , _____ , _____ , _____)

C) _____ F) _____

A) Given: $f(x)=\dfrac{2x-3}{x+c}$, where $0<c<1$.

Let P be the x-intercept of $y=f(x)$.

Let Q be the y-intercept of $y=f(x)$.

Let R be the intersection of the vertical and horizontal asymptotes of $y=f(x)$.

Compute the unique value of c for which the area of ΔPQR is 6 units^2.

B) Start with any <u>two-digit</u> positive integer N.

Let C = 0

Repeat

Reverse the digits of N to form the integer M.

Let T = (N + M) mod 100. [i.e. Add N and M, divide by 100 and save the remainder. Call it T.]

Let N = T and increase C by 1.

until T = 99.

Amazingly, no matter what the starting value is, this is never an infinite loop. Every starting value sooner or later produces 99. For example, $11 \Rightarrow 22 \Rightarrow 44 \Rightarrow 88 \Rightarrow 76 \Rightarrow 43 \Rightarrow 77 \Rightarrow 54 \Rightarrow 99$.

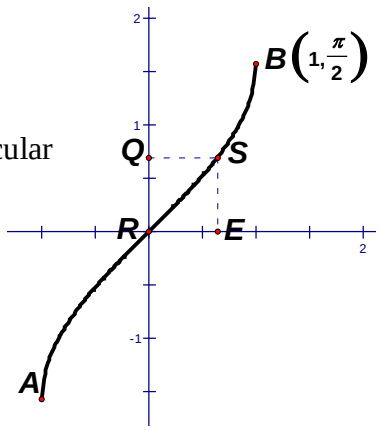
We say that 11 has a cycle of 8, since, for $N=11$, $C=8$. (The loop has been executed 8 times.)

Determine C, the longest cycle and S, the set of all N-values with cycle C.

C) At the right is a graph of the principal inverse sine function.

The domain is $-1\le x\le 1$ and its range is $-\dfrac{\pi}{2}\le y\le +\dfrac{\pi}{2}$.

Point $S(x,y)=\left(x, Sin^{-1}(x)\right)$ is a point on the graph. $\overline{SQ}$ is perpendicular to the y-axis and $\overline{SE}$ is perpendicular to the x-axis.



$SQRE$ can never actually be a square, that is, the ratio $\dfrac{y}{x}=\dfrac{SE}{SQ}$ can never be exactly 1, but this ratio can be made arbitrarily close to 1 by relocating point S along the curve.

If $Sin^{-1}(x)=\dfrac{3}{5}$, compute $\dfrac{x}{y}$ to the nearest 0.001,

using the identity $\sin x = x-\dfrac{x^3}{3!}+\dfrac{x^5}{5!}-\dfrac{x^7}{7!}+\ldots+(-1)^{n+1}\dfrac{x^{2n-1}}{(2n-1)!}+\ldots$.

It is necessary to use only the first <u>three</u> terms to approximate the ratio to the nearest 0.001.