Round 1

A) Either knowing common Pythagorean Triples or using the Pythagorean Theorem, the hypotenuse has length 13. Therefore, $\cos B + \cos C = \dfrac{5}{13} + \dfrac{12}{13} = \underline{\dfrac{\mathbf{17}}{\mathbf{13}}}$

B) By the law of Sines, $\dfrac{\sin 30°}{12} = \dfrac{1}{24} = \dfrac{\sin C}{k}$

$\Rightarrow \sin C = \dfrac{k}{24} = \dfrac{k}{2^3 \cdot 3}$

Only fractions with denominations containing exclusively powers of 2, powers of 5 or products thereof have terminating decimal representations.
Therefore, k must be a positive multiple of 3 and less than or equal to 24.
$\Rightarrow$ 3, 6, 9, 12, 15, 18, 21, 24 $\Rightarrow$ **108**.

C) Using the Law of Cosines, $k^2 = 64 + 16 - 2 \cdot 4 \cdot 8 \cos A = 80 - 64 \cos A$.

Solving for cos$A$, $\cos A = \dfrac{80 - k^2}{64}$.

Applying the Triangle Inequality to ΔBAD, the minimum value of k is 5 and the maximum value is 11. Substituting for k, $\cos A = \dfrac{80-25}{64} = \dfrac{55}{64}, \dfrac{80-121}{64} = \dfrac{-41}{64}$

Thus, the positive difference is $\dfrac{96}{64} = \underline{\dfrac{\mathbf{3}}{\mathbf{2}}}$.