Round 4

A) $\log_9\left(1+\frac{\sqrt{6}}{3}\right)+\log_9\left(1-\frac{\sqrt{6}}{3}\right) = \log_9\left(\left(1+\frac{\sqrt{6}}{3}\right)\cdot\left(1-\frac{\sqrt{6}}{3}\right)\right) = \log_9\left(1-\frac{6}{9}\right) = \log_9\left(\frac{1}{3}\right) = N$

$\rightarrow 9^N = 3^{2N} = \frac{1}{3} = 3^{-1} \rightarrow N = \underline{-\frac{1}{2}}$

B) $\log_4 x^3 - 2\log_{16} x + \log_{64} x^6 = 3 \rightarrow \log_4 x^3 - \log_{16} x^2 + \log_{64} x^6 = 3$

$\rightarrow \log_4 x^3 - \log_4 x + \log_4 x^2 = 3 \rightarrow \log_4 \frac{x^3 \cdot x^2}{x} = \log_4 x^4 = 3 \rightarrow x^4 = 4^3 = 64$

$\rightarrow x = \underline{+2\sqrt{2}}$ only

C) $\frac{2^x - 2^{-x}}{2} = -1.875 \rightarrow 2^x - 2^{-x} = -3.75 = -\frac{15}{4}$ Let $A = 2^x$. Then:

$A - \frac{1}{A} = -\frac{15}{4} \rightarrow 4A^2 + 15A - 4 = (4A - 1)(A + 4) = 0 \rightarrow A = 2^x = \begin{cases} \frac{1}{4} = 2^{-2} \\ -4 \text{ (rejected)} \end{cases} \rightarrow x = \underline{-2}$

Round 5

A) $\frac{\sqrt{2009}}{\sqrt{2009k}} = \frac{1}{\sqrt{k}} = \frac{\sqrt{2}}{\sqrt{7}} \rightarrow \sqrt{k} = \frac{\sqrt{7}}{\sqrt{2}} \rightarrow k = \underline{\frac{7}{2}}$

B) Since June is eating (not picking) her rate is $-1/3$.
Let T denote the time spent picking (and eating). Then:
$\frac{T}{5} + \frac{T}{7} - \frac{T}{3} = 1 \rightarrow 21T + 15T - 35T = 105 \rightarrow T = 105$

Buckets picked by mom and dad $= \frac{105}{5} + \frac{105}{7} = 21 + 15 = \underline{\mathbf{36}}$

C) $48 = \frac{kD}{\frac{D}{40}+\frac{D}{60}} = \frac{k}{\frac{1}{40}+\frac{1}{60}} = \frac{k(40)(60)}{40+60} = 24k \rightarrow k = 2$

The distance cancels out, so we can ignore D.

Thus, $R = \frac{2}{\frac{1}{8}+\frac{1}{12}} = \frac{2(8)(12)}{20} = \frac{96}{10} = \underline{\mathbf{9.6}}$