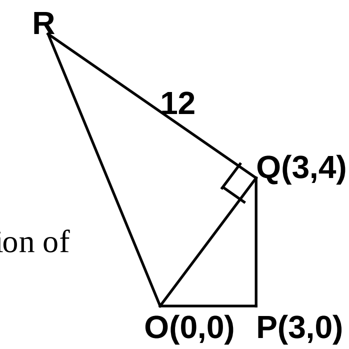
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2015 SOLUTION KEY

Team Round - continued



C) $OQ = 5 \Rightarrow OR = 13$

An equation of a origin-centered circle passing through point R is $x^2 + y^2 = 169$

Since the slope of $\overline{OQ}$ is $\frac{4}{3}$, the slope of $\overleftrightarrow{RQ}$ is $-\frac{3}{4}$ and the equation of $\overleftrightarrow{RQ}$ is $(y-4) = -\frac{3}{4}(x-3) \Leftrightarrow 3x + 4y = 25$

Solving these equations, $\begin{matrix} 9x^2 + 9y^2 = 9 \cdot 169 \\ 9x^2 = (25-4y)^2 \end{matrix} \Rightarrow (625 - 200y + 16y^2) + 9y^2 = 9 \cdot 169$

$\Rightarrow 25y^2 - 200y + 625 - 1521 = 0$

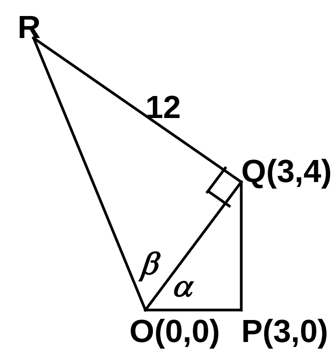
$\Rightarrow 25y^2 - 200y - 896 = 0$

$\Rightarrow y = \dfrac{200 \pm \sqrt{200^2 + 4 \cdot 25 \cdot (896)}}{50} = \dfrac{200 \pm \sqrt{100(400+896)}}{50}$

$\Rightarrow y = \dfrac{200 \pm \sqrt{10^2 \cdot 36^2}}{50} = \dfrac{200 \pm 360}{50} \Rightarrow y = \dfrac{56}{5} = 11.2,\ \cancel{-\dfrac{16}{5}}$

Substituting, $3x + 4(11.2) = 25 \Rightarrow 3x = -19.8 \Rightarrow x = -6.6$

Thus, $R\underline{(\mathbf{-6.6, 11.2})}$ or $R\underline{\left(\mathbf{-\frac{33}{5}, \frac{56}{5}}\right)}$.



Solution #2 (Norm Swanson):

Clearly, in ΔPOQ $\sin\alpha = \frac{4}{5}$ and $\cos\alpha = \frac{3}{5}$, and,

in ΔROQ, $\sin\beta = \frac{12}{13}$ and $\cos\beta = \frac{5}{13}$

Since the coordinates of point R are $(\cos(\angle ROP), \sin(\angle ROP))$, we must evaluate $\sin(\alpha + B)$ and $\cos(\alpha + B)$.

$\sin(\alpha+\beta) = \frac{4}{5} \cdot \frac{5}{13} + \frac{3}{5} \cdot \frac{12}{13} = \frac{56}{65}$ $\quad \cos(\alpha+\beta) = \frac{3}{5} \cdot \frac{5}{13} - \frac{4}{5} \cdot \frac{12}{13} = -\frac{33}{65} \Rightarrow R\underline{\left(\mathbf{-\frac{33}{65}, \frac{56}{65}}\right)}$

D) Taking log of both sides, $\begin{cases} \log m = (b+c)\log a \\ \log n = (c+a)\log b \end{cases}$

Adding, we have

$\log m + \log n = \log mn = c(\log a + \log b) + a\log b + b\log a = \log(ab)^c + \log(a^b b^a) = \log(ab)^c + 2$

$\Rightarrow \log mn = \log(ab)^c + \log 100 = \log\left(100(ab)^c\right) \Rightarrow (ab)^c = \underline{\dfrac{\mathbf{mn}}{\mathbf{100}}}$