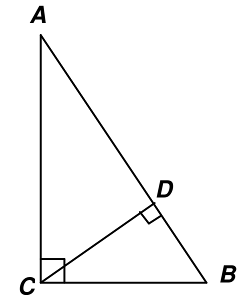
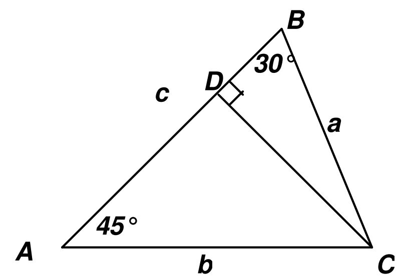
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2008 SOLUTION KEY

Round 1



A) The sides of right triangle ABC are 8 – 15 – 17.
If $m\angle A = \theta$, $m\angle ACD = m\angle CBD = 90 - \theta$.
Therefore, $\sin(\angle ACD) = \sin(\angle CBD) = \underline{\mathbf{\frac{15}{17}}}$

B) Since the largest angle is opposite the longest side and the smallest angle is opposite the shortest side, the medium sized angle is A.
According to the law of cosines, $10^2 = 9^2 + 11^2 - 2(9)(11)\cos A$
$\rightarrow \cos A = \dfrac{10^2-(9^2+11^2)}{-2\cdot 9\cdot 11} = \dfrac{102}{2\cdot 9\cdot 11} = \dfrac{17}{33} \rightarrow m + n = \underline{\mathbf{50}}$



C) As an angle in ΔABC, $\cos B = \dfrac{\sqrt{3}}{2} \rightarrow B = 30°$

and $C = (180 - 45 - 30) = 105°$

By the law of sines, $\dfrac{\sqrt{2}/2}{a} = \dfrac{1/2}{2\sqrt{2}} \rightarrow a = 4$

$a^2 + b^2 = \left(2\sqrt{2}\right)^2 + 4^2 = 24$

By the law of cosines, $c^2 = a^2 + b^2 - 2ab\cos(105°) \rightarrow c^2 = 24 - 16\sqrt{2}\cos(60° + 45°)$
$= 24 - 16\sqrt{2}\cdot\dfrac{\sqrt{2}-\sqrt{6}}{4} = 24 - 8 + 4\sqrt{12} = 16 + 8\sqrt{3}$. Thus, $a^2 + b^2 + c^2 = 24 + 16 + 8\sqrt{3} =$
$\underline{\mathbf{40 + 8\sqrt{3}}}$

Alternately, drop an altitude from C to $\overline{AB}$, creating 45-45-90 and 30-60-90 triangles.
This quickly gives $AB = 2 + \sqrt{3}$ and the same result follows.