

**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2012 SOLUTION KEY**

Team Round - continued

F) We require that $\frac{n(n-3)}{2} < \frac{180(n-2)}{n}$.

Since $n > 0$, we can cross multiply. $n^2(n-3) < 360(n-2) \Leftrightarrow n^3 - 3n^2 - 360n + 720 < 0$

Using direct or synthetic substitution, we want the smallest n that satisfies the inequality.

$$\begin{array}{r|rrrr} 1 & -3 & -360 & 720 \\ \hline 20 & 1 & 17 & -20 & > 0 \text{ (20 sides fails)} \end{array}$$

$$19 \mid 1 \quad 16 \quad -56 \quad < 0 \text{ (**19** sides works)}$$

Check: 19 sides: $\frac{19(16)}{2} = 152$ diagonals / $\frac{180(17)}{19} = 161^+$ degrees $(152 < 161^+)$

20 sides: $\frac{20(17)}{2} = 170$ diagonals / $\frac{180(18)}{20} = 162$ degrees $(170 \not< 162)$