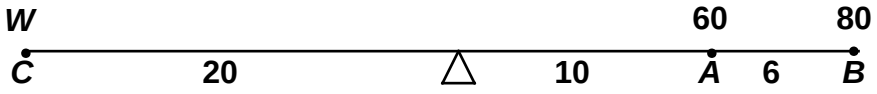
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2013 SOLUTION KEY

Round 5

A) The heavier child should sit farther from the pivot point.
60(10) + 80(10 + 6) = $W(20) \Rightarrow 600 + 1280 = 1880 = 20W \Rightarrow W =$ **94**.
Note: If the heavier child sits closer to the pivot point on the right side, we would have
60(10 + 6) + 80(10) = $W(20) \Rightarrow 960 + 800 \Rightarrow W = 48 + 40 = 88$, a smaller value.



B) $y_1 = 3x + 8$ and $y_2 = \dfrac{3}{x} \Rightarrow$

$x(3x+8) = 3 \Leftrightarrow 3x^2 + 8x - 3 = (3x-1)(x+3) = 0 \Rightarrow x = \dfrac{1}{3}, -3 \Rightarrow A\left(\dfrac{1}{3}, 9\right),\ B(-3,-1)$

Applying the midpoint formula, $M\left(\dfrac{\frac{1}{3}+(-3)}{2}, \dfrac{9+(-1)}{2}\right) = \underline{\left(-\dfrac{\mathbf{4}}{\mathbf{3}}, \mathbf{4}\right)}$.

C) His overall average rate is the harmonic average of his rates $\left(\dfrac{2r_1r_2}{r_1+r_2}\right)$ - NOT the arithmetic average $\left(\dfrac{r_1+r_2}{2}\right)$. A good topic of discussion with your teammates/coach!

Since r denotes his average return rate, we have

$\dfrac{2rR}{r+R} = \dfrac{3}{4}R = \dfrac{3R}{4}$ Cross multiplying, $8rR = 3Rr + 3R^2$. Dividing through by R,

$8r = 3r + 3R \Rightarrow 5r = 3R$ or $r = \underline{\dfrac{\mathbf{3R}}{\mathbf{5}}}$.

(Acceptable alternative forms are listed on the answer key.)