

**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2016 SOLUTION KEY**

Team Round - continued

- F) Let D be the origin, \overline{DC} lie on the positive x -axis, and \overline{DA} lie on the positive y -axis.

R and S (and P and Q) are images across $y = x$, so we need only find the coordinates of one point and the coordinates of the other is found by simply interchanging the coordinates.

Clearly, the x -coordinate of points P and R is 2 and the y -coordinate of points Q and S is 2 as well.

The equation of the arc \widehat{APQC} is $x^2 + y^2 = 16$.

$$x = 2 \Rightarrow y = 2\sqrt{3}$$

Therefore, the coordinates are $P(2, 2\sqrt{3})$, $Q(2\sqrt{3}, 2)$.

$$TP + PV = TV \Leftrightarrow TP + 2\sqrt{3} = 4 \Rightarrow TP = 4 - 2\sqrt{3}$$

$$TP = RV \Rightarrow R(2, 4 - 2\sqrt{3}), S(4 - 2\sqrt{3}, 2)$$

$$PR = 2\sqrt{3} - (4 - 2\sqrt{3}) = 4\sqrt{3} - 4$$

$$PQ = \frac{PR}{\sqrt{2}} = \frac{4(\sqrt{3} - 1)}{\sqrt{2}} = 2\sqrt{2}(\sqrt{3} - 1)$$

$$\Rightarrow \text{Perimeter} = 4PQ = 8\sqrt{2}(\sqrt{3} - 1) = 8(\sqrt{6} - \sqrt{2}) \Rightarrow (a, b, c) = \underline{(8, 6, 2)}.$$

