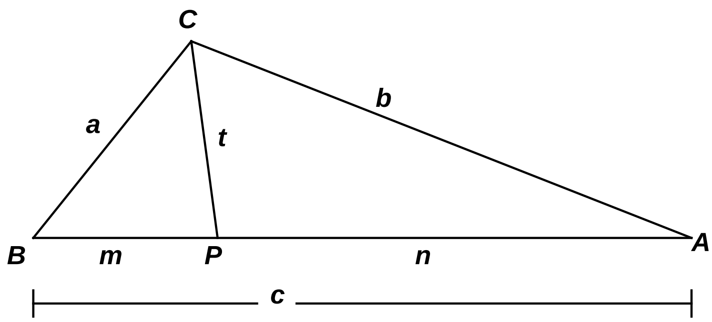
Stewart's Theorem

If a segment is drawn from the vertex of <u>any</u> triangle to <u>any</u> point on the opposite side (with lengths as indicated in the diagram below) then

$$\boxed{a^2n+b^2m=t^2c+cmn}.$$



Using Law of Cosines on ΔBPC, $a^2=t^2+m^2-2tm\cos(\measuredangle BPC)$.

Using Law of Cosines on ΔCPA, $b^2=t^2+n^2-2tn\cos(\measuredangle CPA)$.

BUT $\measuredangle BPC$ and $\measuredangle CPA$ are supplementary and

$\cos(\measuredangle CPA)=\cos(180-\measuredangle BPC)=-\cos(\measuredangle BPC)$

Therefore, the two equations become $\begin{cases} a^2=t^2+m^2-2tm\cos(\measuredangle BPC) \\ b^2=t^2+n^2+2tn\cos(\measuredangle BPC) \end{cases}$

The plan is to eliminate the last terms in each equation by multiplying the first equation by *n*, the second equation by *m*, and then adding the two equations.

$$\left(a^2n+b^2m\right)=n\left(t^2+m^2\right)+m\left(t^2+n^2\right)$$

$$\Rightarrow \left(a^2n+b^2m\right)=t^2\left(m+n\right)+\left(nm^2+mn^2\right)$$

$$\Rightarrow \left(a^2n+b^2m\right)=t^2\left(m+n\right)+mn\left(m+n\right)$$

$$\Rightarrow \boxed{a^2n+b^2m=t^2c+cmn}$$

Q.E.D. – That's all folks!

Powerful medicine indeed – when the problem involves triangles and nothing else seems to apply, try Stewart's Theorem.