MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2006
ROUND 7 TEAM QUESTIONS

ANSWERS

A) _____ D) _____

B) _____mph E) _____

C) _____ F) _____

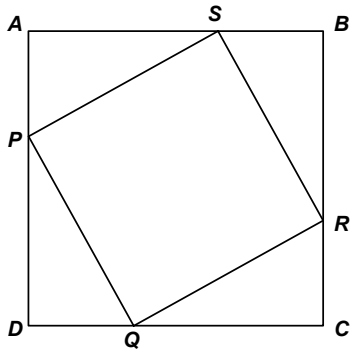
A) Given: $z = 3 + 4i$

For some complex constant c, $\bar{z} + \frac{1}{z} = \sqrt{z} + c$.

If $\sqrt{z}$ denotes a complex number in quadrant 1, determine the value of c.
Note: $\bar{z}$ denotes the conjugate of z.

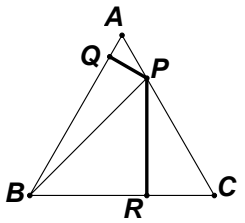
B) The upstream rate of an amateur kayaker is 80% of his downstream rate. If he kayaks 6 miles upstream and drops off some maps and immediately returns to his original starting point downstream in a total of 3 hours, determine the kayak's rate in still water. Assume no loss of time in dropping off the maps and turning around.

C) Squares $ABCD$ and $PQRS$ have areas 144 and 128 respectively. Determine $SA^2 + SB^2 + SC^2 + SD^2$.



D) Factor the polynomial $-x^{10} + x^4 + x - x^7$ over the integers.

E) Given: ΔABC is equilateral with side of length 6
$m\angle PBC = 45°$, $\overline{PQ} \perp \overline{AB}$ and $\overline{PR} \perp \overline{BC}$. Determine $PQ + PR$.



F) A regular polygon of m sides is exactly enclosed by m regular polygons of n sides each, as illustrated for $m = 4$ and $n = 8$.
Specify all <u>other</u> ordered pairs (m, n) for which this statement is true?