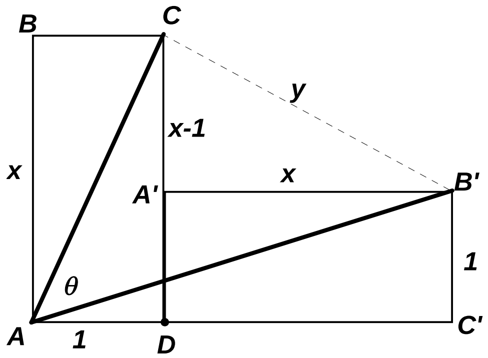
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2015 SOLUTION KEY

Team Round - continued



C) Using the Pythagorean Theorem,

$y^2 = x^2 + (x-1)^2$

$(AC)^2 = x^2 + 1$

$(AB')^2 = (x+1)^2 + 1$

Using the law of cosines on $\Delta AB'C$,

$y^2 = \left(x^2+1\right) + \left((x+1)^2+1\right) - 2\sqrt{\left(x^2+1\right)\left((x+1)^2+1\right)}\cos 60°$

$2x^2 - 2x + 1 = 2x^2 + 2x + 3 - \sqrt{\left(x^2+1\right)\left((x+1)^2+1\right)}$

Transposing,

$\sqrt{\left(x^2+1\right)\left((x+1)^2+1\right)} = 4x + 2$

$\left(x^2+1\right)\left(x^2+2x+2\right) = \left(4x+2\right)^2$

$x^4 + 2x^3 + 3x^2 + 2x + 2 = 16x^2 + 16x + 4$

$x^4 + 2x^3 - 13x^2 - 14x - 2 = 0$

Using synthetic substitution with integer values of $x > 1$, we look for the closest value to zero.

	1	2	-13	-14	-2
2	1	4	-5	-24	-50
3	1	5	2	-8	-26
4	1	6	11	30	118

Clearly, $x = 3$ produces the angle closest to 60° and $B'C = \underline{\sqrt{\mathbf{13}}}$.