

Changes to original questions:

5C) The original problem did not have a diagram.

Chord \overline{AB} is 2 units from the center of a circle whose diameter is 14.

Chord \overline{PQ} is perpendicular to chord \overline{AB} , intersecting in common point R .

If $m\angle PBA = 60^\circ$ and $AR : BR = 2 : 1$, compute QR .

I thought the given conditions could produce only one diagram and I was wrong.

The following appeal made by David Fink (Acton Boxborough) was accepted.

The official answer was accepted as well as “No Answer” (due to inconsistent conditions).

Note that David’s diagram is totally consistent with the given information.

Here is David’s argument:

Since the distance of a chord from the center of a circle is measured along a perpendicular, AOM is a right triangle and

$AM = \sqrt{7^2 - 2^2} = 3\sqrt{5}$. We were given that $AR : BR = 2 : 1$, so $AB = 6\sqrt{5}$, $AR = 4\sqrt{5}$, $BR = 2\sqrt{5}$ and $MR = \sqrt{5}$.

Since $MRNO$ must be a rectangle, $NP = NQ = \sqrt{7^2 - 5} = 2\sqrt{11}$.

Therefore, $\boxed{PR = 2\sqrt{11} + 2}$.

BUT since $\triangle BPR$ is a $30 - 90 - 90$ right triangle,

$\boxed{PR = BR\sqrt{3} = 2\sqrt{15}}$

What a predicament! $2\sqrt{11} + 2 = 2\sqrt{15}$

Could this be???

Dividing by 2, only if $\sqrt{11} + 1 = \sqrt{15}$

Squaring both sides, only if $12 + 2\sqrt{11} = 15 \Leftrightarrow 2\sqrt{11} = 3$

Squaring again, only if $44 = 9$

I’m sure this isn’t true, so we have two different lengths for the same segment, an impossibility!

Therefore, the given conditions are inconsistent.

No answer is possible.

