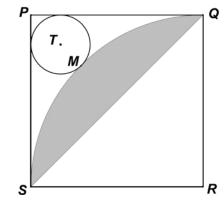
## MASSACHUSETTS MATHEMATICS LEAGUE CONTEST 5 - FEBRUARY 2013 SOLUTION KEY

## **Team Round - continued**

E) The shaded region plus the region bounded by  $\triangle SQR$  is a quarter circle with area  $\frac{\pi(6)^2}{4} = 9\pi$ . Thus, the area of the shaded region is  $9\pi - \frac{1}{2} \cdot 6 \cdot 6 = 9(\pi - 2)$ .



Draw diagonal  $\overline{PR}$ , intersecting  $\overline{SQ}$  at point O.

$$PR = 6\sqrt{2} \Rightarrow PO = 3\sqrt{2}$$
.

Since points T and M lie on  $\overline{PR}$ ,  $PT + TM + MO = 3\sqrt{2}$  (\*\*\*).

If r denotes the radius of circle T,  $PT = r\sqrt{2}$  (since P, T and the points of tangency on the square form a small square with side r). Therefore, (\*\*\*) is equivalent to:

$$r\sqrt{2} + r + (6 - 3\sqrt{2}) = 3\sqrt{2}$$
  $(MO = RM - RO)$ 

$$\Leftrightarrow r(\sqrt{2}+1)=6(\sqrt{2}-1)$$

$$\Leftrightarrow r = \frac{6(\sqrt{2} - 1)}{\sqrt{2} + 1} = 6(\sqrt{2} - 1)^2 = 6(3 - 2\sqrt{2})$$

Finally, the required ratio is  $\frac{\pi r^2}{9(\pi - 2)} = \frac{\pi \cdot 36 \cdot \left(3 - 2\sqrt{2}\right)^2}{9(\pi - 2)} = \frac{\pi \left(68 - 48\sqrt{2}\right)}{\pi - 2}$  and

$$(A, B) = (68 - 48\sqrt{2}, 2) \text{ or } (4(17 - 12\sqrt{2}), 2)$$

 $4(\sqrt{2}-1)^4$  is **not** an acceptable as the computed value of *A*.