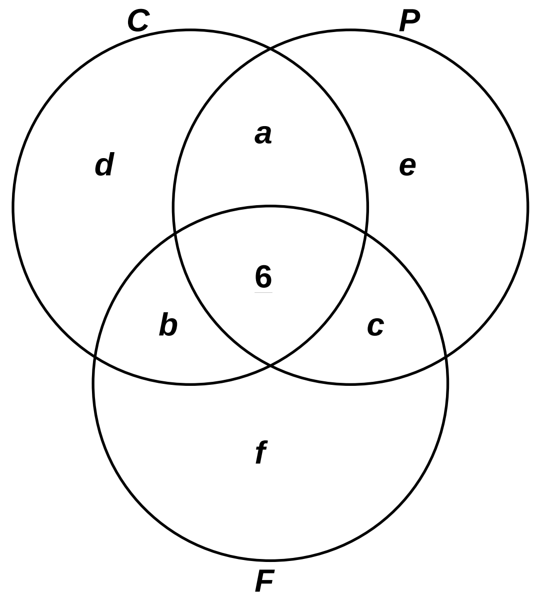
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 - FEBRUARY 2012 SOLUTION KEY

Team Round - continued



D) (A classic Venn Diagram problem)

$\frac{a+6}{b+6}=\frac{2}{4}=\frac{1}{2} \Leftrightarrow 2a+12=b+6 \Leftrightarrow b=2a+6$

$\frac{a+6}{c+6}=\frac{2}{5} \Leftrightarrow 5a+30=2c+12 \Leftrightarrow c=\frac{5a+18}{2}$

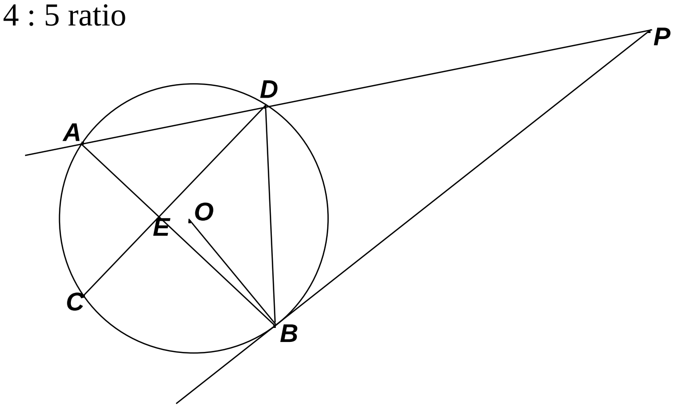
To insure that c is an integer, a must be even.
Since the total number of students involved is 116, to maximize f, we must minimize a.
Let $n(C) = 2N$, $n(P) = 4N$ and $n(F) = 5N$.
If $a = 2$, $(b, c) = (10, 14)$

$\begin{cases} d+18=2N \\ e+22=4N \\ 30+f=5N \end{cases}$ and

$(2+10+14)+d+e+f+6=116 \Leftrightarrow d+e+f=84$

Consequently,

$\Rightarrow (2N-18)+(4N-22)+(5N-30)=84 \Leftrightarrow 11N=154 \Leftrightarrow N=14 \Rightarrow f=70-30=\underline{\mathbf{40}}$.



E) Since angles AED and BED are supplementary, a 4 : 5 ratio implies $m\angle AED = 80°$ and $m\angle BED = 100°$.
Let $m\angle ADC = x°$ and $m\angle DBA = (x + 10)°$ and $m\left(\overset{\frown}{BD}\right)=y°$.

As arcs subtended by inscribed angles, $m\left(\overset{\frown}{AC}\right)=2x°$ and $m\left(\overset{\frown}{AD}\right)=(2x+20)°$.

As a leftover arc, $m\left(\overset{\frown}{BC}\right)=(340-4x-y)°$

As an angle formed by intersecting chords,

$m\angle BED=\frac{1}{2}(2x+y)=100 \Leftrightarrow 2x+y=200$

As an angle formed by a tangent and a secant line $m\angle P=\frac{1}{2}\left((340-2x-y)-y\right)=5$

$\Leftrightarrow 340-2x-2y=10$

$\Leftrightarrow x+y=165$

Thus, $x = 35$, $y = 130 \Rightarrow m\left(\overset{\frown}{AD}\right)=90° \Rightarrow m\left(\overset{\frown}{ADB}\right)=220°$

If $\overleftrightarrow{BO}$ intersects the circle in point X, then $m\left(\overset{\frown}{AX}\right)=220°-180°=40°$.

As an inscribed angle, $m\angle EBO = \underline{\mathbf{20}}°$.