MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2011 SOLUTION KEY

Round 5

A) Given: $4x - 3y = 12$
If $x = 2y$, then $8y - 3y = 12$ and y is not an integer.
If $y = 2x$, then $4x - 6x = -2x = 12 \Rightarrow (x, y) =$ **$\underline{(-6, -12)}$**.

B) $\frac{5}{24} = \frac{x}{180} \Rightarrow x = \frac{5(180)}{24} = \frac{5(15)}{2} = 37.5\%$ or $\frac{3}{8}$ used $\Rightarrow \frac{5}{8}$ remains in tank
$\frac{5}{8}(4.0) =$ **$\underline{2.5}$**

$\underline{FYI}$: LP turns into a gas and it's the gas that burns when we are grilling. As the gas is burned, the bottom of the tank sweats and the sweat line gives us a way of determining how full the tank is after we have been using it.

C) $\frac{x+z}{x-z} = -2011 \Leftrightarrow \frac{(x+z)y}{(x-z)y} = \frac{xy+yz}{xy-yz} = -2011$
$\Rightarrow \frac{2010+yz}{2010-yz} = -2011 \Leftrightarrow -2011(2010-yz) = 2010+yz$
$\Rightarrow 2010yz = 2011(2010) + 2010 = 2010(2011 + 1) = 2010(2012) \Rightarrow yz =$ **$\underline{2012}$**.

Generalization: You should verify that if $\begin{cases} xy = k \\ \frac{z+x}{z-x} = k+1 \end{cases}$, then $yz = k + 2$.