MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2016 SOLUTION KEY

Round 4

A) $\log_2 x + \log_2 \frac{1}{4} = \frac{3}{2}\log_2 25 \Leftrightarrow \log_2 \frac{x}{4} = \log_2\left(25^{\frac{3}{2}}\right) = \log_2 125 \Rightarrow \frac{x}{4} = 125 \Rightarrow x = \underline{\mathbf{500}}$.

B) $\log_3\left(\log_2\left(\log_2 x\right)\right) = 1 \Rightarrow \log_2\left(\log_2 x\right) = 3^1 \Rightarrow \log_2 x = 2^3 = 8 \Rightarrow x = 256$

$\left(\log_4 256\right)^{\frac{1}{2}} \cdot \log_2 256 = \sqrt{4} \cdot 8 = \underline{\mathbf{16}}$.

C) $x^3 - x = x(x+1)(x-1) \neq 0 \Rightarrow x \neq 0, \pm 1$

$$\frac{x^3+1}{x^3-x} = \frac{\cancel{(x+1)}\left(x^2-x+1\right)}{x\cancel{(x+1)}(x-1)} = \frac{\left(x-\frac{1}{2}\right)^2+\frac{3}{4}}{x(x-1)}$$

As a real-valued function, $y = \log_{10}\left(\frac{x^3+1}{x^3-x}\right)$ must have a positive argument.

$\Rightarrow \frac{\left(x-\frac{1}{2}\right)^2+\frac{3}{4}}{x(x-1)} > 0$. Since the numerator is always positive, the denominator determines the sign of the quotient. For $x < 0$ or $x > 1$, both factors in the denominator have the same parity (i.e. both are positive or both are negative) and the quotient will be positive.

Thus, the domain (*which must exclude* -1) is $\underline{\boldsymbol{x < -1,\ -1 < x < 0,\ x > 1}}$.

Interval notation is also acceptable:

$(-\infty,-1),\ (-1,0),\ (1,\infty)$

Commas may be replaced by "or"s. Also accept $x < 0,\ x > 1\ (x \neq -1)$ or , since "and"s are evaluated before "or"s, $x < 0$ and $(x \neq -1)$ or $x > 1$.