MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2012 SOLUTION KEY

<u>Addendum</u>
Team A) continued

The triangularization process:

$$\begin{bmatrix} 1 & 7 & 5 & 12 \\ 2 & 9 & 4 & 20 \\ 6 & A & 3 & 19 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 7 & 5 & 12 \\ 0 & -5 & -6 & -4 \\ 6 & A & 3 & 19 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 7 & 5 & 12 \\ 0 & 5 & 6 & 4 \\ 6 & A & 3 & 19 \end{bmatrix}$$

row 2 replaced by row 2 − 2(row 1)
Then row 2 multiplied by −1

$$\Rightarrow \begin{bmatrix} 1 & 5 & 7 & 12 \\ 0 & 6 & 5 & 4 \\ 6 & 3 & A & 19 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 5 & 7 & 12 \\ 0 & 6 & 5 & 4 \\ 0 & -27 & A-42 & -53 \end{bmatrix}$$

columns 2 and 3 interchanged
Then row 3 replaced by row 3 − 6(row 1)

$$\begin{bmatrix} 1 & 5 & 7 & 12 \\ 0 & 6 & 5 & 4 \\ 0 & 0 & 2A-39 & -70 \end{bmatrix}$$

row 3 replaced by 9(row2) + 2(row3)

The equivalent system: $\begin{cases} x+5z+7y=12 \\ 6z+5y=4 \\ (2A-39)y=-70 \end{cases} \Rightarrow y=\dfrac{-70}{2A-39}$

Smallest possible positive value of $A = 2$, $y = \dfrac{-70}{-35} = 2$

Backtracking (substituting for y in 2nd equation), $6z + 10 = 4 \Rightarrow z = -1$
Backtracking (substituting for x and y in 1st equation), $x + 5(-1) + 7(2) = 12 \Rightarrow x = 3$
Thus, (x, y, z) = **<u>(3, 2, –1)</u>**.