MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2007 SOLUTION KEY

Round 4

A) $\left(\log_3 x\right)^2 - 6\log_3 x - 7 = \left(\log_3 x + 1\right)\left(\log_3 x - 7\right) = 0 \rightarrow \log_3 x = -1, +7 \rightarrow x = \underline{\mathbf{\frac{1}{3}, 2187}}$

B) $a = \log_{36}(8) \rightarrow 2a = \log_6(8)$

If $N = \log_{216}(48)$, then $3N = \log_{\sqrt[3]{216}}(48) = \log_6(48) = \log_6(6 \cdot 8) = 1 + \log_6(8) = 1 + 2a$

Thus, $N = \underline{\mathbf{\frac{2a+1}{3}}}$

C) $3^{3\log_3 x + 1} - 2^{2\log_2 x} = 2x^4 \rightarrow 3x^3 - x^2 = 2x^4 \rightarrow x^2(2x^2 - 3x + 1) = x^2(x - 1)(2x - 1) = 0$

$\rightarrow x = 0$ (extraneous), $\underline{\mathbf{1, \frac{1}{2}}}$