MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2013
ROUND 7 TEAM QUESTIONS

ANSWERS

A) _____ D) (_____ , _____)

B) (_____ , _____ , _____) E) (_____ , _____)

C) _____ F) _____

******* NO CALCULATORS IN THIS ROUND *******

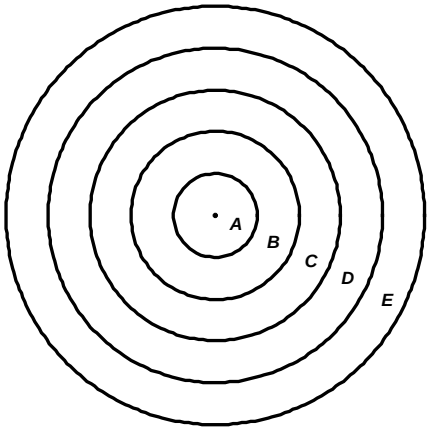
A) Let $A = \begin{bmatrix} 3 & 2 \\ 1 & k \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -1 \\ 2 & k-3 \end{bmatrix}$. Compute <u>all</u> values of k for which $\det(AB) = 60$.

B) The radical expression $\sqrt{18+\sqrt{35}}$ may be expressed in the form $\frac{x+\sqrt{y}}{\sqrt{z}}$, where x, y and z are positive integers. Compute the ordered triple (x, y, z) for which $x + y + z$ is a minimum.

C) Given: $\begin{cases} x = 3\sin(t)+1 \\ y = 2\cos(t)-5 \end{cases}$ for $0 \le t \le \pi$.
Let (X_M, Y_M) denote the maximum values of x and y respectively.
Let (X_m, Y_m) denote the minimum values of x and y respectively.
Compute $X_M Y_M - X_m Y_m$.

D) A record of better than 0.700 is quite an achievement in the modern-era of major league baseball. The best winning percentage of 0.716 belongs to the 2001 Seattle Mariners (116 wins – 46 losses). Suppose our local club's record is currently 72 wins and 38 losses. If we play k more games and lose <u>at least</u> 10 of them, compute the ordered pair (g, W), where g is the <u>minimum</u> value of k for which our winning percentage is <u>over</u> 0.700 and W is our total number of wins for the season.

E) The diagram at the right is a dartboard of concentric circles of radii 1, 2, 3, 4, and 5. I threw a dart and its location is determined by the point of intersection of two perpendicular chords of the largest circle whose lengths are 8 and 9. The lengths of the segments on the shorter chord are in a 25 : 7 ratio. The lengths of the segments on the longer chord are in a 5:1 ratio. Let k denote the region in which the dart landed. Let d denote the exact distance to the <u>nearest</u> circle. Compute the ordered pair (k, d).



F) Compute the coefficient of x^9 in the expansion of $\left(x^2 + x - 1\right)^6$.