MASSACHUSETTS MATHEMATICS LEAGUE
OCTOBER 2003
ROUND 2: PYTHAGOREAN RELATIONS

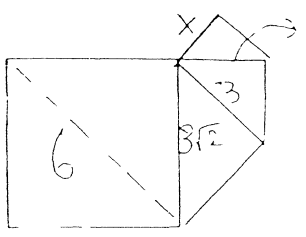
ANSWERS

A) 5

B) $7\sqrt{6}$

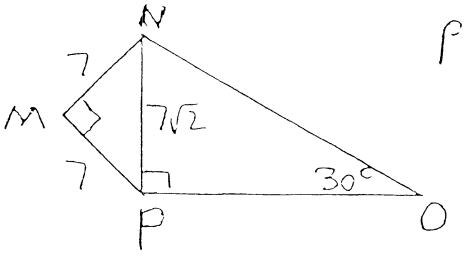
C) $2\sqrt{13}$

A) Diagonal of square ABCD is 6 and three 45-45-90 triangles adjoin it on the right in succession. The length of the leg labeled x is the reduced fraction a/b. Find the value of a+b.



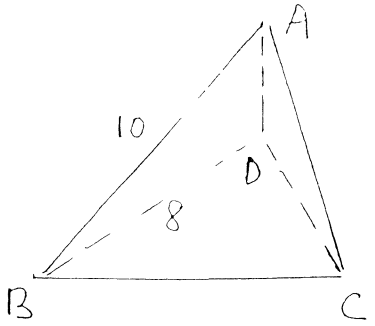
$x = \frac{3}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{3}{2}$, ANS $3+2=5$

B) In NOPM, MN = MP = 7, $\angle NMP = \angle NPO = 90^0$, and $\angle O = 30^0$. Calculate PO in simplified radical form.



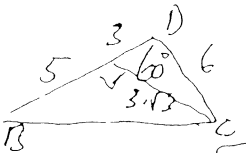
$PO = 7\sqrt{2} \cdot \sqrt{3} = 7\sqrt{6}$

C) In tetrahedron A-BCD, $\overline{AD} \perp$ *plane BCD*, AB = 10, BD = 8, $\angle CAD = \angle DCA$, and $\angle BDC = 60^0$. Calculate BC in simplified radical form.



$AD = \sqrt{10^2 - 8^2} = 6$

$DC = AD = 6$



$BC = \sqrt{5^2 + (3\sqrt{3})^2} = \sqrt{25+27} = \sqrt{52}$