MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2011
ROUND 7 TEAM QUESTIONS

ANSWERS

A) (_____ , _____ , _____) D) _____

B) _____ E) _____

C) _____ F) (_____ , _____)

******* NO CALCULATORS ON THIS ROUND *******

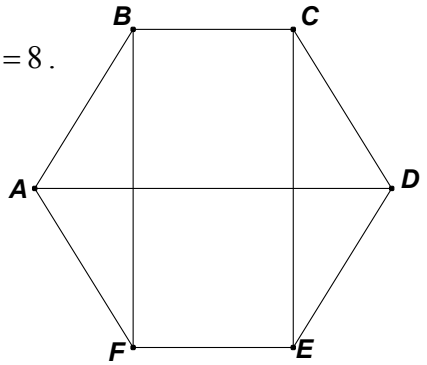
A) Suppose A and B are relatively prime integers such that $A > B > 0$ and the equation.
$A \cdot \det\left(\begin{bmatrix} x+1 & 1 \\ 1 & x+2 \end{bmatrix}\right) + B \cdot \det\left(\begin{bmatrix} x-1 & 1 \\ 1 & x-2 \end{bmatrix}\right) = 0$ has exactly one real root R.
Compute the ordered triple (A, B, R).

B) Let S be the set of positive two digit integers generated by the expression $\sqrt{1+4n}$ for integer values of n. An element of set S is randomly selected. What is the probability that the selected element is greater than 20?

C) A line with slope m passes through (3, 6) and cuts off a triangle in quadrant 1 with area 100.
Compute the smallest possible value for the angle of inclination of this line.
Give your answer (in radians) as a simplified expression in terms of a first quadrant reference value.
Use an inverse trig function which avoids the use of any radicals.
Recall: An angle of inclination is the directed angle from the positive x-axis to the line.

D) Compute the two real values of x for which $(x+1)(x+2)(x+3)(x+4) = 8$.

E) In hexagon $ABCDEF$, $AB = CD = DE = FA = 1$.
In rectangle $BCEF$, $CE = 2BC$.
If $AD = 2.2$, compute both possible lengths of $\overline{CE}$.



F) Given: $S = (A+B)^n + (C+D)^{n+1} + (E-F)^{n+2}$ for n, a positive integer > 10.
Let P denote the sum of the binomial coefficients of S divided by 16.
P can be written in the simplified form $k \cdot 2^x$, where k is a constant and x denotes an expression in terms of n. Determine the ordered pair (k, x).

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