MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2016 SOLUTION KEY

Round 2

A) $x^2+49=x+69 \Leftrightarrow x^2-x-20=(x-5)(x+4)=0 \Rightarrow x=5 \Rightarrow \underline{\mathbf{25}}$

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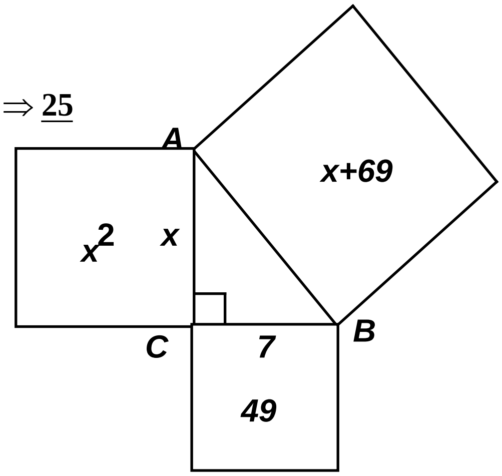

B) $48^2+\cancel{x^2}=(x+18)^2=\cancel{x^2}+36x+18^2 \Rightarrow x=\dfrac{48^2-18^2}{36}=\dfrac{(48+18)(48-18)}{36}=\dfrac{66\cdot 30}{36}=11\cdot 5=55$.

Thus, the perimeter is $48+55+73=\underline{\mathbf{176}}$.

C) Let AG denote the required altitude and the length of the side of the square be $2x$.
Then $R = 4x^2 -$ area($\Delta ADF + \Delta FCE + \Delta ABE$)

$= 4x^2 - x^2 - x^2/2 - x^2 = 3x^2/2 \Rightarrow x^2=\dfrac{2R}{3} \Rightarrow x=\sqrt{\dfrac{2R}{3}}$

Since $EF= x\sqrt{2}$, we have $\dfrac{1}{2}\cdot x\sqrt{2}\cdot AG=R$.

Substituting for x, $\dfrac{1}{2}\cdot\sqrt{\dfrac{2R}{3}}\cdot\sqrt{2}\cdot AG=R \Rightarrow \sqrt{\dfrac{R}{3}}\cdot AG=R$

$\Rightarrow AG = R\cdot\sqrt{\dfrac{3}{R}}=\sqrt{\dfrac{3R^2}{R}}=\underline{\sqrt{\mathbf{3R}}}$ (since $R \neq 0$).