

**MASSACHUSETTS MATHEMATICS LEAGUE**  
**JANUARY 2004**  
**ROUND 1: ANALYTIC GEOMETRY**

**ANSWERS**

A)  $\underline{4x + 5y = -7}$

B)  $\underline{2\sqrt{6}}$

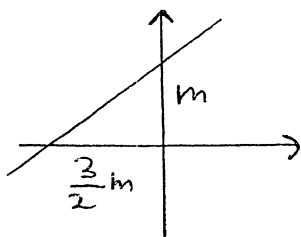
C)  $\underline{x^2 + y^2 = 117}$

A) Find the equation of the line of centers of the circles  $x^2 + y^2 + 6x - 2y + 1 = 0$ , and  $2x^2 + 2y^2 - 8x + 12y - 24 = 0$ . Write the equation in  $ax + by = c$  form.

$$C_1 = (-3, 1); C_2 = (2, -3), \quad m = \frac{-3 - 1}{2 + 3} = -\frac{4}{5}$$

$$4x + 5y = -12 + 5 = -7$$

B) A triangle with area 18 is formed by the axes and a line with slope  $\frac{2}{3}$  which has a positive y-intercept. Calculate in simple radical form, the value of this positive y-intercept.



$$A = \frac{1}{2} \cdot \frac{3}{2} m \cdot m = 18$$

$$\frac{1}{4} m^2 = 6, \quad m^2 = 24$$

$$m = 2\sqrt{6}$$

C) Find the equation of the circle with center at the origin which is tangent to the line  $2x + 3y = 39$

$$2x + 3y = 39$$

$$2x + 3y = 0$$

$$r = \frac{39 - 0}{\sqrt{2^2 + 3^2}} = \frac{39}{\sqrt{13}} = 3\sqrt{13}. \quad \text{ANS } x^2 + y^2 = 117$$