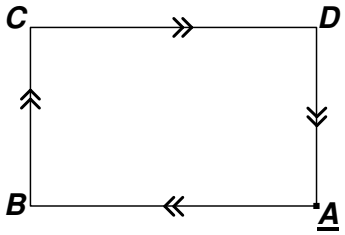
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2008 SOLUTION KEY

Team Round - continued

C) Let $AB = 3x$ and $BC = 2x$. Then $10x = 1320 \rightarrow AB = 396$ and $BC = 264$



Let $T_{AB} = t$ denote the time needed to travel from A to B.

Given: $v_{AB} : v_{BC} : v_{CD} : v_{DA} = 1 : 2 : 4 : 40$, $2AB = 3BC$

$BC = \frac{2}{3}AB$ and $v_{BC} : v_{AB} = 2 : 1 \rightarrow 2 \cdot T_{BC} = 1 \cdot \frac{2}{3} \cdot T_{AB} \rightarrow T_{BC} = t/3$

$CD = AB$ and $v_{CD} : v_{AB} = 4 : 1 \rightarrow 4 \cdot T_{CD} = 1 \cdot t \rightarrow T_{CD} = t/4$

$DA = BC$ and $v_{DA} : v_{BC} = 40 : 2 \rightarrow 40 \cdot T_{DA} = 2 \cdot T_{BC} = 2(t/3) \rightarrow T_{DA} = t/60$

Summing the times, $96t/60 = 8t/5 = 12 \rightarrow t = 7.5$

Thus, $V_{AB} = 396/7.5 = 52.8 \rightarrow V_{DA} = 40(52.8) =$ **2112** feet/min

Alternate solution:

Let $AB = 3x$ and $BC = 2x$. Then $10x = 1320 \rightarrow AB = 396$ and $BC = 264$

Now let r denote the rate from A to B (solving for $40r$). $\frac{396}{r} + \frac{264}{2r} + \frac{396}{4r} + \frac{264}{40r} = 12$

$\rightarrow 396(40) + 264(20) + 396(10) + 264 = 12(40r)$

$\rightarrow 3960(5) + 264(21) = 12(40r)$

$\rightarrow 330(5) + 22(21) = 40r = 1650 + 462 =$ **2112** feet/min

D) Let w denote the total number of games won. Then $\frac{w}{82} > \frac{4}{5} \rightarrow w > \frac{328}{5} = 65.6 \rightarrow w \geq 66$

Since the team has already won 45 games, during the remainder of the season they must win at least 21 games out of the remaining 24 games [82 – (45 + 13)] Since $B > A$, winning 24 out of 24 is rejected, leaving 3 possibilities: 21… 23 out of 24 $\rightarrow$ $(A, B) = (23, 24)$ (11, 12) or (7, 8)

Alternate solution:

There are $nB = 24$ games remaining $\rightarrow$

$B = 1, 2, 3, 4, 6, 8, 12$ or 24

$\frac{45 + An}{82} > \frac{4}{5} \rightarrow 5An > 328 - 225 = 103$

$\rightarrow An > 20.6 \rightarrow A \geq \left[\frac{20.6}{n}\right]$

The chart at the right summarizes the possibilities:

5 out of every 6 → record 65/82 (0.793-)
7 out of every 8 → record 66/82 (0.804+)
11 out of every 12 → record 67/82 (0.817+)
23 out of every 24 → 68/82 (0.829+)

B	n	A	Verdict
1	24	**>0**	rejected
2	12	**>1**	rejected
3	8	**>2**	rejected
4	6	**>3**	rejected
6	4	**>5**	rejected
8	3	**>6**	7
12	2	**>10**	11
24	1	**>20**	23

E) The region defined by the system of inequalities is illustrated at the right. It can be shown that any maximum (or minimum) value occurs at a vertex on the boundary of the region. Evaluating the expression we have, $A(0, 0)$: 2008, $B(4, 2)$: 2024, $C(6, 9)$: 2020 and $D(0, 12)$ 1984. Thus, the maximum value is **2024**.