MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2015
ROUND 7 TEAM QUESTIONS
ANSWERS

A) _____ D) **(_____ , _____ , _____)**

B) _____ E) _____

C) _____ F) _____

A) There are four ordered pairs (x, y) that satisfy $\begin{cases} x^2 - xy + y^2 = 7 \\ \frac{4}{x} + 3y = 1 \end{cases}$.

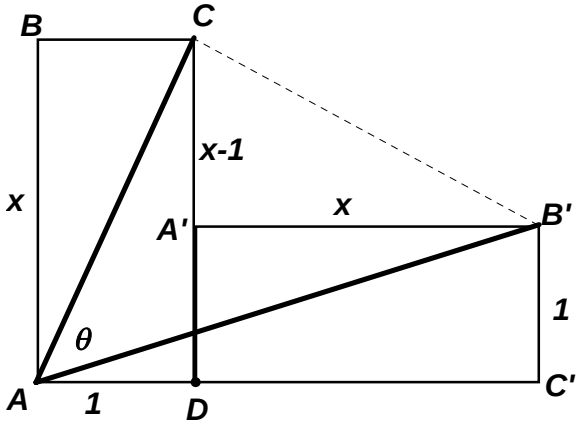
Let x_1, x_2, x_3, x_4 denote the x-coordinates of these 4 ordered pairs.

Let a, b, c, d denote 4 integers, where $b < c < d$.

$x_1 = a$, but x_2, x_3, x_4 are irrational numbers satisfying the inequalities $\begin{cases} b < x_2 < b+1 \\ c < x_3 < c+1 \\ d < x_4 < d+1 \end{cases}$. Compute ad.

B) Compute the two integer values of x for which $4^{2x+a} = 8^{5-bx}$, if $a : b = 2 : 1$

C) $ABCD$ is a 1 by x rectangle, where $x > 1$ is an integer. $ABCD$ is rotated 90° clockwise about point D to a new position. Compute $B'C$ for which $m\angle CAB'$ is closest to 60°.



D) For nonzero real constants a and b, the linear equations $y = ax + b$ and $\frac{x}{a} - \frac{y}{b} = 1$ intersect.

P, the point of intersection, is not on the line $y = x$, if $a = \underline{p}$, or if $a \neq p$ and $b \neq \underline{q}a^2 + \underline{r}a$, for constants p, q and r. Compute the ordered triple (p, q, r).

E) The lengths of two sides of a parallelogram $ABCD$ are x and $x + c$, where x and c are both positive integers. The lengths of the diagonals are $x + 3$ and $x + 5$. Find all possible perimeters of parallelogram $ABCD$.

F) Urn #1 contains 1 white, 2 red and 3 blue balls. Urn #2 contains 4 white, 4 red and 2 blue balls. The balls are indistinguishable except for color.
Two balls are drawn simultaneously from urn #1 and added to urn #2.
After the draw from urn #1, there are x white balls, y red balls and z blue balls remaining in urn #1 and $x \neq y, y \neq z$, and $x \neq z$.
Two balls are now simultaneously drawn from urn #2.
Compute the probability that these balls will be the same color.