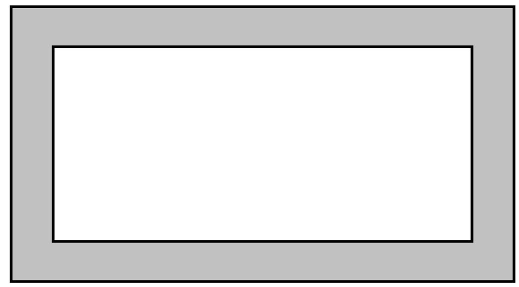
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2015 SOLUTION KEY

Round 6

A) $(W+6)(L+6)-WL=6W+6L+36=210 \Leftrightarrow W+L+6=35 \Leftrightarrow W+L=29$

Since $W>L$, the rectangle is not a square.
$\Rightarrow (W,L)=(28,1),\ (27,2),\ (26,3),\ ...,\ (15,14)$
a total of **<u>14</u>** ordered pairs.



B) $10^4-5^4=(10^2+5^2)(10^2-5^2)=125(10+5)(10-5)=5^3\cdot(3\cdot5)\cdot5=3^1\cdot5^5$

This product has $(1+1)(5+1)=12$ factors.

1 is not a multiple of 5 and $1=5^0$ is not a nonzero power of 5, but all other factors (except 3) are multiples of 5. Therefore, we have exactly **<u>10</u>** factors that satisfy at least one of the given conditions. Note that the factors like 5, 25, 125, 625 and 3125 satisfy both criterion.

C) $(1204)_8=8^3+2\cdot8^2+4=2^9+2^7+4=512+132=644$

$(245)_6=2\cdot6^2+4\cdot6+5=72+29=101$

Since $444_5+1=1000_5$, $444_5=(1000)_5-1=5^3-1=125-1=124$

$644+101-124=$ **<u>621</u>**