MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2006 SOLUTION KEY

Round 4

A) Note that A and B, the roots of the quadratic equation, are each <u>positive</u> numbers
$(\frac{22 \pm \sqrt{22^2 - 12(27)}}{6})$ and we can let $x = \log_3 A + \log_3 B = \log_3(AB)$
But AB, the product of the roots of the quadratic, is given by the constant term divided by the lead coefficient → $27/3 = 9$
Thus, $x = \log_3(9) =$ **<u>2</u>**.

B) $8^a = 2^{3a} = 45$ and $2^b = 7.5$ or $2^{b+1} = 15$. Dividing, $2^{3a-(b+1)} = 3$
→ $3a - b - 1 = \log_2 3 = \frac{1}{\log_3 2} = c$ → $b =$ **<u>$3a - 1 - c$</u>**

C) $(2x - 3)^2 > 0$ for all x except 3/2. The critical points in the numerator of the argument $(x - 1)(x + 4)$ are 1 and -4. The product is positive for $x < -4$ or $x > 1$ and negative in between. Since the log of zero or negative values is undefined, the domain is restricted to $x < -4$ or $x > 1$ (excluding $x = 3/2$).

Round 5

A) Let x, $x + 2$ and $x + 4$ denote the three consecutive odd integers. Then the next three larger consecutive even integers are $x + 5$, $x + 7$ and $x + 9$.
$(3x + 6) : (3x + 21) = 3 : 4$ → $(x + 2) : (x + 7) = 3 : 4$ → $4x + 8 = 3x + 21$ → $x = 13$
$(13 + 15 + 17) + (18 + 20 + 22) = 45 + 60 =$ **<u>105</u>**

B) $(.84P + .54F)/(P + F) = .78$ → $6P = 24F$ → $P = 4F$
Part of group that passed $= P/(P + F) = 4F/(4F + F) =$ **<u>4/5</u>**

C) Substituting for x, y, z and w in $x = kyz/(w^2)$ → $k = 5$.
Let $w = n$, $x = 2n$, $y = 3n$ and $z = 4n$ → $(2n)(n^2) = 5(3n)(4n)$ → $n = 30$ and $wx^2 = 4n^3$
→ $4 \cdot 30^3 = 4(27000) =$ **<u>108,000</u>**