MASSACHUSETTS MATHEMATICS
CONTEST 6 - MARCH 2013 SOLUTION KEY

Team Round

A) $\begin{bmatrix} 3 & 2 \\ 1 & k \end{bmatrix}\begin{bmatrix} 4 & -1 \\ 2 & k-3 \end{bmatrix} = \begin{bmatrix} 3\cdot 4+2\cdot 2 & 3\cdot -1+2(k-3) \\ 1\cdot 4+2k & 1\cdot -1+k(k-3) \end{bmatrix} = \begin{bmatrix} 16 & 2k-9 \\ 2k+4 & k^2-3k-1 \end{bmatrix}$

Taking the determinant,

$16\left(k^2-3k-1\right)-(2k+4)(2k-9)=16k^2-48k-16-4k^2+10k+36=12k^2-38k+20=60$

Therefore,

$12k^2-38k-40=2(6k^2-19k-20)=2(k-4)(6k+5)=0$

Thus, $k = \underline{\mathbf{4, -\frac{5}{6}}}$.

Alternative solution:

Invoking the theorem $\det(AB)=\det(A)\cdot\det(B)$, we have

$\det\begin{bmatrix} 3 & 2 \\ 1 & k \end{bmatrix} = 3k-2,\ \det\begin{bmatrix} 4 & -1 \\ 2 & k-3 \end{bmatrix} = 4k-12+2=4k-10$

Then: $(3k-2)(4k-10)=60 \Rightarrow 12k^2-38k-40=0$ and the same result follows.

It is left to <u>you</u> as an exercise to prove the theorem that

for all 2 x 2 matrices A and B, $\det(AB)=\det(A)\cdot\det(B)$.

Is this true for any square matrices?

Prove (or disprove) your contention.

Send your proof or counterexamples to olson.re@gmail.com .

The best write-ups will be included in the solution set of the next contest.

Everyone likes to see themselves in print, except certain babies who don't like cash!