Round 2

A) Without calculating A, B, C or D or even factoring the trinomial!

Since $(Ax+B)(Cx+D) = ACx^2 + (AD+BC)x + BD$, equating coefficients, we have

$AC = 24$ and $BD = 100$. Thus, $ABCD = \underline{\textbf{2400}}$.

Note: $24x^2 + 175x + 100$ factors as $(8x+5)(3x+20)$ and

$8 \cdot 5 \cdot 3 \cdot 20 = (8 \cdot 3)(5 \cdot 20) = 24(100) = 2400$

B) $(x-3)^3 + 2(x-3)^2 = 8x - 24$

$\Leftrightarrow (x-3)^3 + 2(x-3)^2 = 8(x-3)$

$\Leftrightarrow (x-3)^3 + 2(x-3)^2 - 8(x-3) = 0$

Let $a = (x-3)$ and factor out the common binomial term.

$a(a^2 + 2a - 8) = a(a+4)(a-2) = 0 \Rightarrow a = 0, -4, 2 \Rightarrow x = \underline{\textbf{3}, \textbf{-1}, \textbf{5}}$ (in any order).

C) To avoid division by zero, note that $x \neq \pm\dfrac{3}{2}$

$\dfrac{x}{2x+3} - \dfrac{x+1}{3-2x} - \dfrac{20x}{8x^2-18} = 0 \Leftrightarrow \dfrac{x}{2x+3} + \dfrac{x+1}{2x-3} - \dfrac{10x}{(2x+3)(2x-3)} = 0$

Therefore, the least common denominator is $(2x+3)(2x-3)$.

Multiplying through by the LCD, we have $x(2x-3) + (x+1)(2x+3) - 10x = 0$

$\Leftrightarrow 2x^2 - 3x + 2x^2 + 5x + 3 - 10x = 0$

$\Leftrightarrow 4x^2 - 8x + 3 = (2x-1)(2x-3) = 0 \Rightarrow x = \underline{\dfrac{\textbf{1}}{\textbf{2}}}, \cancel{\dfrac{3}{2}}$