MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2013 SOLUTION KEY

Team Round

D) $x^{14}-x^8-x^6+1=\left(x^6-1\right)\left(x^8-1\right)=\left(x^3+1\right)\left(x^3-1\right)\left(x^4+1\right)\left(x^4-1\right)$

$=\left(x+1\right)\left(x^2-x+1\right)\left(x-1\right)\left(x^2+x+1\right)\left(x^4+1\right)\left(x^2+1\right)\left(x+1\right)\left(x-1\right)$

Thus, the sum of the factors is $x^4+3x^2+4x+4 \Rightarrow$ **(1, 3, 4, 4)**.

E) $$\frac{\cot 2x\cdot\cot x+1}{\cot x-\cot 2x}=\frac{\frac{1-\tan^2 x}{2\tan x}\cdot\frac{1}{\tan x}+1}{\frac{1}{\tan x}-\frac{1-\tan^2 x}{2\tan x}}\cdot\frac{2\tan^2 x}{2\tan^2 x}=\frac{1-\tan^2 x+2\tan^2 x}{2\tan x-\tan x\left(1-\tan^2 x\right)}$$

$$=\frac{1+\tan^2 x}{\tan x+\tan^3 x}=\frac{\cancel{1+\tan^2 x}}{\tan x\left(\cancel{1+\tan^2 x}\right)}=\frac{1}{\tan x}=\cot x$$

Thus, we have $\cot x=\tan 300°=-\tan 60°=-\cot 30°$.

The 30° family over the specified domain is $\left\{30°,150°,210°,330°\right\}$.

The solution set consists of only values in quadrants 2 and 4, where the cotangent takes on a negative value, namely, x = **150°, 330°** .

F) Since the interior and exterior angles in a regular polygon with n sides are given by $\frac{180(n-2)}{n}$ and $\frac{360}{n}$ respectively, the given ratios translate to $\frac{n-2}{2}=\frac{11}{q}$ and $\frac{2}{m-2}=\frac{1}{11}$.

Thus, $q=\frac{22}{n-2}$ and m = 24 and the exterior angles must be 15°.

The required ratio is $\frac{\frac{180(n-2)}{n}}{15}=12\left(1-\frac{2}{n}\right)=12-\frac{24}{n}$ and

n must be a factor of 24 ($\geq$ 3 of course).

Thus, n = 3, 4, 6, 8, 12 and 24 are under consideration and only 3, 4 and 24 produce integer values of q. Therefore, (n, q) = **(3, 22), (4, 11) and (24, 1)**.