MASSACHUSETTS MATHEMATICS LEAGUE
JANUARY 2005 BRIEF SOLUTIONS

Round One:

A. Given points A, B, C consider ABC?, ACB? and CAB? Thus covering all of the six permutations

B. Expanding #2 and subtracting #1 gives 10x+10y=60. Substituting into #1 for y gives a quadratic in x, x=2 or 4. Note #2 requires first quad solutions.

C. Factoring the first three terms gets us to (3x+2y+?)(2x–3y+?) whereas factoring the terms $6x^2 - 29x + 28$ gives (3x – 4)(2x – 7) so we see the complete factoring is (3x + 2y – 4)(2x – 3y – 7)so the intersection is **(2, –1)**

Round Two:

A. (32-2x)(40-2x)=560 becomes $x^2 - 36x + 180 = 0$. x=30 or 6. Nearest edge is 6. Playground is 20 X 28 so perimeter is 96 ft.

B. $2x = \dfrac{2-x-x^2}{2+x}$ becomes $2x = \dfrac{(2+x)(1-x)}{2+x}$ Since $x \neq -2$, 2x = 1 – x so x=1/3

C. Factoring each side: $(x+1)(x^2 - x + 1) = (x+1)(x+3)$ so x = – 1 or $(x^2 - x + 1) = (x+3)$ which by quad formula gives $x = 1 \pm \sqrt{3}$

Round Three:

A. Only first two factors have zeroes so x = π/6, π/3 , 2π/3, 5π/6

B. Factors to $(\cot z - 1)(\cot^2 z - 3) = 0$ so z = 45, 225 or z = 30, 150, 210, 330 sum is 990, average 165.

C. (sin θ = 0.35) has solutions that are supplementary so sum to π; (tan θ = 0.80) means (cot θ = 1.25) and with (tan θ = 1.25) gives complementary first quadrant solutions and third quad solutions for sum of π/2 + (2π + π/2) and (sec θ = 1.70) gives solutions summing to 2π so final sum is 6π.

Round Four:

A. Use the quadratic formula or factor as $(\sqrt{6}x + 2)(x - \sqrt{6}) = 0$ so $x = \dfrac{-2}{\sqrt{6}} = \dfrac{-\sqrt{6}}{3}$ or $x = \sqrt{6}$

B. For integer coefficients use $(2x+1)(cx-3) = 2cx^2 + (c-6)x - 3 = ax^2 - 2x - b$ so b=3 and c–6 = –2 so c = 4 and a = 2c so a = 8.

C. View the expression as 1 – x and notice $x = \dfrac{1}{4-x}$ so $x^2 - 4x + 1 = 0$ and $x = 2 + \sqrt{3}$ so $1 - x = -1 + \sqrt{3}$ and the answer is **2**

Round Five:

A. The larger triangle is 9-12-15 so its area is 54. The smaller triangle is scaled by 2/3 so it's area is 4/9 the larger triangle; thus the trapezoid is 5/9 of 54 or **30**

B. By AA, ΔADE ~ ΔACB. Note carefully the order of the vertices. Thus, 5/(12 + 6) = 12/(x + 5) → x = 38.2

C. If the smaller hexagon has area A the larger has area 16/9 A. so sum is 25/9A. In the smaller hexagon the altitude of one of the 6 equilateral triangles is $2\sqrt{3}$ so the triangle's area is $4\sqrt{3}$ and the hexagon's area = $24\sqrt{3}$ so sum is $\dfrac{200\sqrt{3}}{3}$