MASSACHUSETTS MATHEMATICS LEAGUE
FEBRUARY 2004
ROUND 1: ALGEBRAIC FUNCTIONS

ANSWERS

A) $-10h$

B) $5/8$

C) $-37/5$

A) If $f(x) = -2x^2 + 7x - 3$, calculate $f(3 + h) - f(3 - h)$ in terms of h.

$[-2(9+6h+h^2) + 7(3+h) - 3] - [-2(9-6h+h^2) + 7(3-h) - 3]$

$= [-18 - 12h - 2h^2 + 21 + 7h - 3] - [-18 + 12h - 2h^2 + 21 - 7h - 3]$

$= (-5h - 2h^2) - (5h - 2h^2) = -10h$

B) If $f(x) = x + 5$ and $g(x) = x^2$, solve the equation $f(g(2 - a)) = g(f(a - 3))$ for a.

$f[(2-a)^2] = g[(a-3)+5]$, $(2-a)^2 + 5 = (a+2)^2$

$4 - 4a + a^2 + 5 = a^2 + 4a + 4$

$5 = 8a$

$a = 5/8$

C) If $f(x) = 2x + 1$ and $g(x) = 3x - 2$, solve the equation $f^{-1}(f^{-1}(w)) = f(g^{-1}(w))$ for w.

$f^{-1}(x) = \frac{x-1}{2}$, $g^{-1}(x) = \frac{x+2}{3}$

$\frac{\left(\frac{w-1}{2}\right) - 1}{2} = 2\left(\frac{w+2}{3}\right) + 1$, $\frac{w-1-2}{4} = \frac{2w+4+3}{3}$,

$\frac{w-3}{4} = \frac{2w+7}{3}$, $3w - 9 = 8w + 28$

$-37 = 5w$, $w = -37/5$