Team Round

A) $\begin{cases} (1)\ x + y + z = n \\ (2)\ 2x - y - z = n + 1 \\ (3)\ x + 2y - 2z = n + 2 \end{cases}$

Subtracting the first two equations, $\boxed{x - 2y - 2z = 1}$.

Subtracting the last two equations, $\boxed{-x + 3y - z = 1}$.

Adding the boxed equations, $y - 3z = 2 \Rightarrow y = 2 + 3z$

Substituting in the first boxed equation, $x - 4 - 6z - 2z = 1 \Rightarrow x = 5 + 8z$

For both of the z-expressions for x and y, as z increases, the values of x and y increases also.

Thus, $x + y = 7 + 11z > 100 \Rightarrow z > \dfrac{93}{11} \Rightarrow z_{min} = 9 \Rightarrow (x_{min}, y_{min}) = (77, 29)$

Substituting in equation (1) above, $n = 77 + 29 + 9 = 115$.

$\Rightarrow (x, y, n) = \mathbf{(77, 29, 115)}$.

Check: $\begin{cases} 77 + 29 + 9 = 115 \\ 154 - 29 - 9 = 116 \\ 77 + 58 - 18 = 117 \end{cases}$

B) Adding the given equations, we have $x + y = 2t \Rightarrow t = \dfrac{x + y}{2}$

Substituting in the first equation, $x = \dfrac{x + y}{2} + \dfrac{2}{x + y} \Rightarrow$

$2x(x + y) = (x + y)^2 + 4 \Leftrightarrow 2x^2 + \cancel{2xy} = x^2 + \cancel{2xy} + y^2 + 4$

$\Rightarrow x^2 - y^2 = 4 \Leftrightarrow y = \pm\sqrt{x^2 - 4}$

Thus, $y = \pm\sqrt{\dfrac{17}{4} - 4} = \pm\sqrt{\dfrac{1}{4}} = \underline{\pm\dfrac{1}{2}}$.

Alternately, square both equations and subtract.

$\begin{cases} x^2 = t^2 + 2 + t^{-2} \\ y^2 = t^2 - 2 + t^{-2} \end{cases} \Rightarrow x^2 - y^2 = 4$

Substituting, $y^2 = \dfrac{17}{4} - 4 = \dfrac{1}{4} \Rightarrow y = \underline{\pm\dfrac{1}{2}}$.