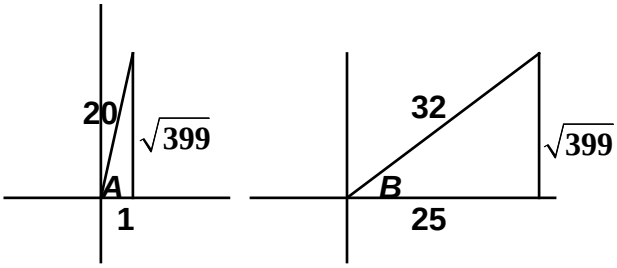
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 - FEBRUARY 2013 SOLUTION KEY

Team Round - continued

C) Rewrite $Arc\cos\left(\frac{25}{32}\right)+Arc\cos(x)=Arc\cos\left(\frac{1}{20}\right)$ as

$Arc\cos(x)=Arc\cos\left(\frac{1}{20}\right)-Arc\cos\left(\frac{25}{32}\right)$ and let $A = Arc\cos\left(\frac{1}{20}\right)$ and $B = Arc\cos\left(\frac{25}{32}\right)$. Then, taking the cosine of both sides,

$$x = \cos(A-B) = \cos A\cos B+\sin A\sin B=\frac{1}{20}\cdot\frac{25}{32}+\frac{\sqrt{399}}{20}\cdot\frac{\sqrt{399}}{32}=\frac{424}{20\cdot 32}=\underline{\mathbf{\frac{53}{80}}}$$



D) Suppose in 2021 there are n stamps on the sheet costing c cents each.
In 2022, the sheet consists of $(n + 8)$ stamps costing $(c + 4)$ cents each.

Then: $\begin{cases} nc = 672 \\ (n+8)(c+4)=1200 \end{cases} \Rightarrow 4n+8c=1200-32-nc=1200-704=496$ or $n=124-2c$

$(124-2c)c=672 \Rightarrow 2c^2-124c+672=2\left(c^2-62c+336\right)=2(c-6)(c-56)=0 \Rightarrow c=\cancel{6},56$

Thus, in 2021, there were 12 stamps on the sheet, costing 56¢ each.
(112 stamps at 6¢ is rejected, since 112 > 50.)
Thus, the cost of a FOREVER stamp in 2022 is 60¢, **14**¢ more than in 2013.