MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2011 SOLUTION KEY

Round 3

A) $\sin x = \cos 110° \rightarrow \sin x = -\cos 70° = -\sin 20°$

Thus, x belongs to the 20° family and must have a negative sine value; therefore, it must be located in quadrant 3 or 4 → **<u>200, 340</u>**.

B) $\sin 3A = -\frac{1}{2} \rightarrow 3A = \frac{7\pi}{6} + 2k\pi \rightarrow 12A = \frac{14\pi}{3} + 8k\pi$ and $6A = \frac{7\pi}{3} + 4k\pi$, where k is an integer.

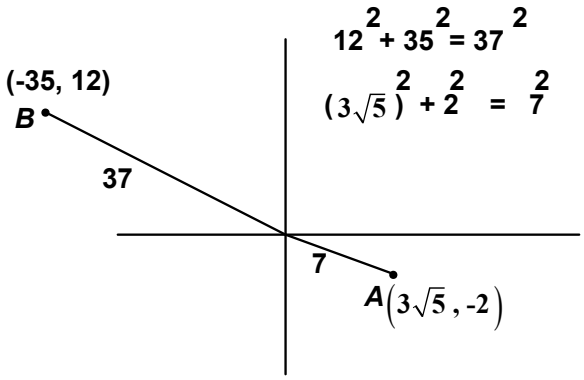
Since $\cos(x + 2k\pi) = \cos(x)$, we get:

$$\cos\left(\frac{14\pi}{3}\right)\cos\left(\frac{7\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right)\cos\left(\frac{\pi}{3}\right) = \left(-\frac{1}{2}\right)\left(\frac{1}{2}\right) = \underline{-\frac{1}{4}}$$

C) $Arc\tan\left(\frac{-2}{3\sqrt{5}}\right)$ denotes a value in quadrant 4.

$Arc\cos\left(-\frac{35}{37}\right)$ denotes a value in quadrant 2.

$$-\frac{2}{7} + \left(-\frac{12}{35}\right) = \frac{-10-12}{35} = \underline{-\frac{22}{35}}$$



Round 4

A) Let X denote the weight of the original mixture.

Black: $\frac{X}{4} + 4 = \frac{1}{3}(X + 4) \rightarrow 3X + 48 = 4X + 16 \rightarrow X = \underline{32}$ kg

B) Inverse variation → $y = \frac{k}{\sqrt{x}}$ Substituting $(x, y) = (25, 4) \rightarrow k = 20$.

Thus, $100 = \frac{20}{\sqrt{x}} \rightarrow \sqrt{x} = \frac{1}{5} \rightarrow x = \underline{\frac{1}{25}}$

C) The painting of the house takes 63 painter-days. Therefore, in one day, the initial 6 painters can complete $\frac{6}{63}^{\text{th}}$ of the job, whereas when all 10 painters are on the job they complete $\frac{10}{63}^{\text{th}}$ of the job. Let x denote the number of days the crew of 10 painters work. Then:

$\left(\frac{10}{63}\right)x + \left(\frac{6}{63}\right)3 = 1 \rightarrow 10x + 18 = 63 \rightarrow x = 4.5$ and the total time required is

$3 + 4.5 = \underline{7.5}$ days