MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2014
ROUND 7 TEAM QUESTIONS

ANSWERS

A) _____ D) _____

B) _____ E) (_____ , _____)

C) _____ F) (_____ , _____ , _____ , _____)

A) The equation $\begin{vmatrix} 2 & 4 & 8 \\ 0 & 8 & 4 \\ 0 & 1 & 2 \end{vmatrix} x^3 + \begin{vmatrix} 9 & 7 \\ -4 & 5 \end{vmatrix} x^2 - \begin{vmatrix} 7 & 10 \\ -3 & x \end{vmatrix} = 0$ has three real roots r_1, r_2, and r_3 and r_1 is a negative integer and $r_2 > r_3$. Compute the determinant $\begin{vmatrix} r_1 & r_3 \\ -r_2 & r_2 + r_3 \end{vmatrix}$.

B) Given: $Q = \dfrac{1^2 + 2^2 + 3^2 + ...n^2}{1 + 2 + 3 + ... + n}$

For how many positive integer values of *n* is *Q* an integer <u>and</u> $Q \le 2014$?

C) Compute the <u>integer</u> value of *A* for which $2Tan^{-1}(.4) + Tan^{-1}\left(\dfrac{1}{A}\right) = \dfrac{\pi}{4}$.

D)

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

1	5	9	13
2	6	10	14
3	7	11	15
4	8	12	16

The grid on the left is converted to the grid on the right by a series of additions and subtractions as follows (each of the 8 constants are nonnegative integers):
A is added to each entry in row 1 (top)
B is added to each entry in row 2
C is added to each entry in row 3
D is added to each entry in row 4
P is subtracted from each entry in column 1 (left)
Q is subtracted from each entry in column 2
R is subtracted from each entry in column 3
S is subtracted from each entry in column 4
Compute the <u>minimum</u> value of $A + B + C + D + P + Q + R + S$.