MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2008 SOLUTION KEY

Team Round – continued

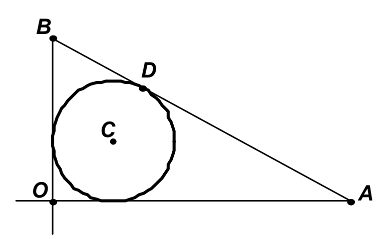
C) If the sum of two primes is odd, then one of the primes must be 2.
Therefore, $h = 2 \rightarrow k = 7$ (or vice versa)

$$\begin{cases} P(x) = Ax^2 + Bx + C \\ P(2) = 7 \\ P(7) = 2 \end{cases} \rightarrow \begin{cases} 4A + 2B + C = 7 \\ 49A + 7B + C = 2 \end{cases} \rightarrow 45A + 5B = -5 \rightarrow 9A + B = -1$$

$\rightarrow B = -1 - 9A$
Substituting, $4A + 2(-1 - 9A) + C = 7 \rightarrow -14A - 2 + C = 7 \rightarrow C = 14A + 9$
Since $A > 0$, the minimum value of C occurs when $A = 1$, i.e. $C =$ **<u>23</u>**.

D) A, B and C have been ringing every $\frac{5}{2}, \frac{10}{3}$ and $\frac{25}{6}$ minutes respectively, or in terms of a common denominator - $\frac{15}{6}, \frac{20}{6}$ and $\frac{25}{6}$ minutes.
The Least common multiple of (15, 20, 25) = 300
Thus, the bells ring together every 300/6 = 50 minutes.
The number of minutes since midnight is 14(60) + 15 = 855.
855/50 = 17.1 → **<u>17</u>** times
Note: 855 – 50(1), 855 – 50(2), … 855 – 50(17) = 5
The first simultaneous ringing occurred at 12:05 AM

E) $AB = 30$. The radius of circle $C = ½(24 + 18 - 30) = 6$
Thus, C is located at (6, 6). The equation of $\overline{AB}$ (m = -18/24 = -3/4): $3x + 4y = 72$
$\overline{CD} \perp \overline{AB}$, $m_{CD} = 4/3 \rightarrow$ Eqtn$_{CD}$: $4x - 3y = 6$



Using Cramer's rule: $x = \dfrac{-240}{-25} = \mathbf{\underline{9.6}}$ and $y = \dfrac{18-288}{-25} = \mathbf{\underline{10.8}}$

Note: Let $M(0, 6)$ and $N(6,0)$ Then area (ΔMND) =

$$\frac{1}{2}\begin{vmatrix} 0 & 6 \\ 6 & 0 \\ 9.6 & 10.8 \\ 0 & 6 \end{vmatrix} = \frac{64.8 + 57.6 - 36}{2} = 43.2$$

Distance from D to $\overline{MN}$: d((9.6, 10.8), $x + y = 6$) = $\dfrac{20.4 - 6}{\sqrt{2}} = 7.2\sqrt{2}$

Area(ΔMND) = $\dfrac{1}{2} \cdot 6\sqrt{2} \cdot 7.2\sqrt{2} = 43.2$
