MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2009 SOLUTION KEY

Round 5

A) The number of diagonals is given by $d = \dfrac{n(n-3)}{2}$.

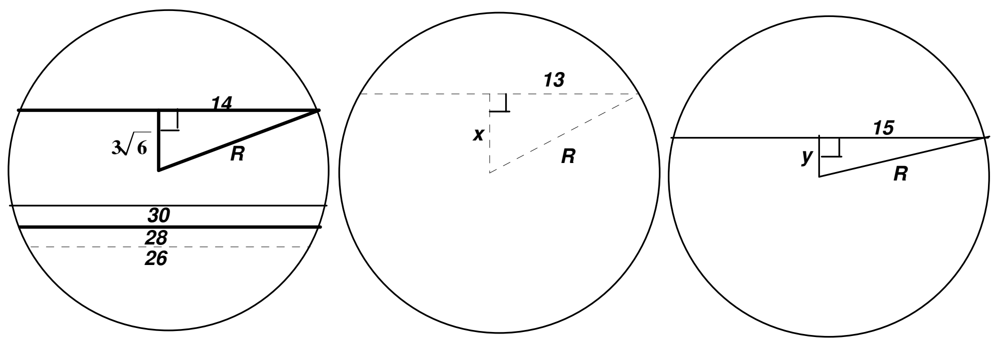
For $n = 15$, $d = 90$, but for $n = 16$, $d = 104$. Thus, the polygon must have at least 16 sides.

The measure of each angle is given by $A = \dfrac{180(n-2)}{n}$.

For $n = 16$ and 17, A is not an integer, but for $n = 18$, $A = 160°$.
Thus, the minimum number of sides is **<u>18</u>**.

B) Since the distance to a chord is measured along a radius drawn perpendicular to the chord and any perpendicular radius bisects the chord to which it is drawn, the radius of circle O may be determined from $14^2 + \left(3\sqrt{6}\right)^2 = 196 + 54 = 250 = R^2$. Then:

$x^2 + 13^2 = 250$
→ $x = 9$
$y^2 + 15^2 = 250$
→ $y = 5$
Therefore, the distance between the chords <u>on the same side</u> of a diameter is $9 - 5 =$ **<u>4</u>**



C) 360/12 = 30° → vertex angle of each of the 12 isosceles triangles comprising the 12-gon. Dealing with 30°-75°-75° triangles (OAB, OBC, etc) appears difficult, but draw an altitude from the vertex of a base angle to the opposite leg ($\overline{CP}$ in ΔOBC) and a 30°-60°-90° triangle is created, ΔOCP
→ $CP = x/2$. Now, using $\overline{OB}$ as the base and $\overline{CP}$ as the height, the area may be easily determined.
Area $= 12(\frac{1}{2} \cdot x \cdot \frac{x}{2}) = 972$ → $x^2 = 324$ → $x = 18$
$\overline{AC}$ and $\overline{CE}$ represent sides of the inscribed hexagon and they clearly have lengths 18. A hexagon of side 18 is comprised of 6 equilateral triangles of side 18. Thus, the area of the inscribed hexagon is $6(\frac{18^2}{4}\sqrt{3}) =$ <u>$\mathbf{486\sqrt{3}}$</u>.