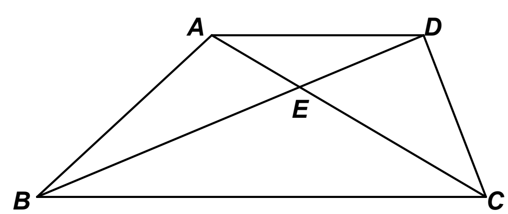
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2010 SOLUTION KEY

Round 4 - continued

B) Rewrite equation as $y^2 + (k^2 - 5k - 6)y - 7 = 0$
To have roots that are numerically equal and opposite in sign B must be 0.
[Then the roots of $Ax^2 + Bx + C = 0$ would be $\dfrac{-B \pm \sqrt{B^2 - 4AC}}{2A} = \dfrac{\pm\sqrt{-4AC}}{2A}$]
Therefore, $k^2 - 5k - 6 = (k + 1)(k - 6) = 0 \rightarrow k = -1, 6$

C) Vertex at $(1, -3)$ and a vertical axis of symmetry $\rightarrow$ equation of the parabola must be of the form $(y+3) = a(x-1)^2$
Substituting in the equation of the line ($2x - y + 7 = 0$), $x = -2 \rightarrow y = 3$.
Substituting in the equation of the parabola, $6 = 9a \rightarrow a = 2/3$
Expanding, $y = -3 + \frac{2}{3}(x-1)^2 \rightarrow C = -3 + \frac{2}{3} = \underline{\mathbf{-\frac{7}{3}}}$

Alternate solution (longer, but more straightforward):
$P(-2, 3)$, $Q(7, 21)$ and $V(1, -3)$
Substituting in the quadratic $y = Ax^2 + Bx + C$, $\begin{cases} (1) & 49A + 7B + C = 21 \\ (2) & 4A - 2B + C = 3 \\ (3) & A + B + C = -3 \end{cases}$
$(1) - (2) \rightarrow 45A + 9B = 18 \rightarrow 5A + B = 2$
$(2) - (3) \rightarrow 3A - 3B = 6 \rightarrow A - B = 2$
Adding, $6A = 4 \rightarrow (A, B) = \left(\frac{2}{3}, -\frac{4}{3}\right)$ Substituting in (3),
$C = -3 - \frac{2}{3} + \frac{4}{3} = \underline{\mathbf{-\frac{7}{3}}}$



Round 5

A) Let K denote the area of ΔADE.
$\Delta ADE \sim \Delta CBE \rightarrow \frac{AE}{CE} = \frac{4}{10} = \frac{2}{5} \rightarrow \frac{area(\Delta ADE)}{area(\Delta CBE)} = \left(\frac{2}{5}\right)^2 \rightarrow \frac{4}{25} = \frac{K}{50} \rightarrow K = \underline{\mathbf{8}}$