MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2011
ROUND 2 ARITHMETIC/NUMBER THEORY

ANSWERS

A) ______________________________

B) ______________________________

C) ______________________ base (–2)

A) The difference between two primes is 45. Compute the sum of these two primes.

B) The Gregorian calendar is now used virtually everywhere in the secular world.
A non-century year is a leap year (366 days) if and only if it is divisible by 4.
A century year is a leap year if and only if it is divisible by 400.
To the nearest integer, how many weeks (7 days) were there in the 400-year cycle from 1600 through 1999 inclusive, using the Gregorian calendar?

C) Consider integers written in "base (–2)", instead of the customary base (10).
In base (–2), suppose the allowable digits are only 0 and 1.
For integers, the place values are $(-2)^0$, $(-2)^1$, $(-2)^2$, etc., instead of $(10)^0$, $(10)^1$, $(10)^2$, etc.
Here are some examples:
In base (–2), $5_{(10)}$ is expressed as $101_{(-2)}$ and $3_{(10)}$ is expressed as $111_{(-2)}$.
All base (–2) representations of base (10) integers are unique.

Express $10101_{(-2)} + 11010_{(-2)}$ in base (–2).