MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2013 SOLUTION KEY

Round 5

A) $\dfrac{|n|}{n} = \pm 1$ depending on whether n is positive or negative.

$\displaystyle\sum_{n=-1}^{n=2013} y = -1 + c + 2013(1) = 2012 + c = 0 \Rightarrow c =$ **$\underline{-2012}$**.

B) $|2x+1| > |x-5| \Leftrightarrow \sqrt{(2x+1)^2} > \sqrt{(x-5)^2}$

If $A > B$ and $A, B \geq 0$, then $A^2 > B^2$. Since each radical represents a nonnegative quantity, we can square both sides.

$(2x+1)^2 > (x-5)^2 \Leftrightarrow 4x^2 + 4x + 1 > x^2 - 10x + 25$

$3x^2 + 14x - 24 = (3x-4)(x+6) > 0$

Both factors are positive for $x > \dfrac{4}{3}$ and both factors are negative for $x < -6$.

Therefore, we have $\underline{\boldsymbol{x < -6 \text{ or } x > \frac{4}{3}}}$.

Alternate Solution: $2x+1 > -x+5 \Rightarrow 3x > 4 \Rightarrow x > \dfrac{4}{3}$ or $-2x-1 > -x+5 \Rightarrow -6 > x$

C) For $\dfrac{60}{13} < x < 5$, each of the fractions is positive.

For this sequence of fractions to be in increasing order the sequence of denominators must be in decreasing order, i.e. $x + 5 > 13x - 60$ and $13x - 60 > 5 - x$

$\Rightarrow 12x < 65$ and $14x > 65 \Rightarrow \dfrac{65}{14} < x < \dfrac{65}{12}$

Since both conditions must hold, we must take the intersection of the two intervals.

Clearly, $\dfrac{65}{12} > 5$, but which is larger $\dfrac{60}{13}$ or $\dfrac{65}{14}$.

We can decide by cross multiplying and comparing the products.

[Note: For any positive numbers a, b, c and d, $\dfrac{a}{b} > \dfrac{c}{d} \leftrightarrow ad > bc$.]

$60(14) = 840$ and $65(13) = 5(13)^2 = 5(169) = 845 \Rightarrow \dfrac{65}{14} > \dfrac{60}{13} \Rightarrow$ $\underline{\boldsymbol{\frac{65}{14} < x < 5}}$