MASSACHUSETTS MATHEMATICS LEAGUE
MARCH 2004
ROUND 5: GEOMETRY ANYTHING

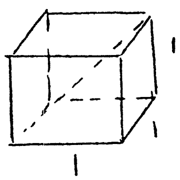
ANSWERS

A) $4\sqrt{3}/3$

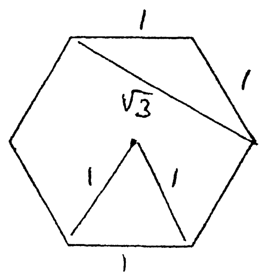
B) $3/4$

C) 10

A) The length of a diagonal of a cube is the same as the length of a shorter diagonal of a regular hexagon. The ratio of the total surface area of the cube to the area of the hexagon is A/B. Compute A/B in simplified radical form.



$d = \sqrt{3}$

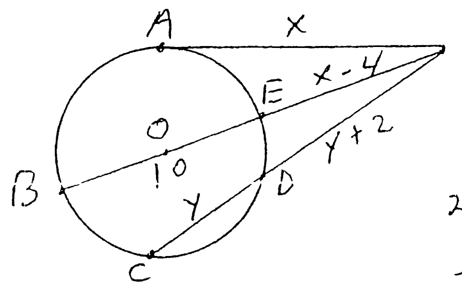


$S.A. = 6$

$A = 6 \cdot \frac{1}{4}\sqrt{3}$

$\frac{6}{\frac{6}{4}\sqrt{3}} = \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$

B) $\overline{PA}$ is tangent to circle O at A PEOB and PDC are secants to circle O. AP = PE + 4, PD = CD + 2. The circumference of circle O is 10π In simplified form, find PD/PA.



$d = 10$

$x^2 = (x-4)(x+6)$

$x^2 = x^2 + 2x - 24$

$2x = 24$

$x = 12 = PA$

$(y+2)(2y+2) = 144$

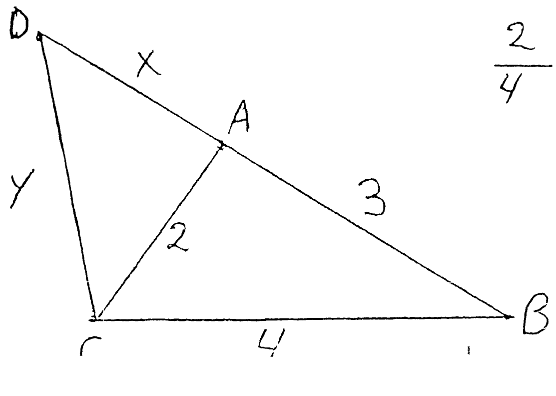
$(y+2)(y+1) = 72$

$y^2 + 3y - 70 = 0$

$(y+10)(y-7)$

$PD = 9$

C) Given ΔABC, AC = 2, AB = 3, and BC = 4 $\overline{BA}$ is extended to D so that $\Delta CAD \sim \Delta BCD$. Find the perimeter of ΔBCD.



$\frac{2}{4} = \frac{x}{y} = \frac{y}{x+3}$, $\frac{P_{\Delta CAD}}{P_{\Delta BCD}} = \frac{1}{2} =$

$\frac{x+y+2}{x+y+7}$, so $2(x+y)+4 = x+y+7$

and $x+y=3$, $P_{\Delta BCD} = 3+7 = 10$