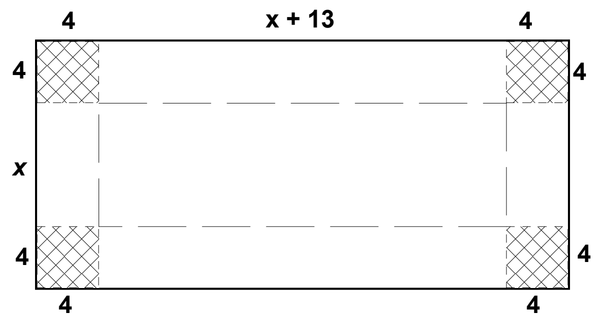
**MASSACHUSETTS MATHEMATICS LEAGUE**
**CONTEST 4 - JANUARY 2011 SOLUTION KEY**

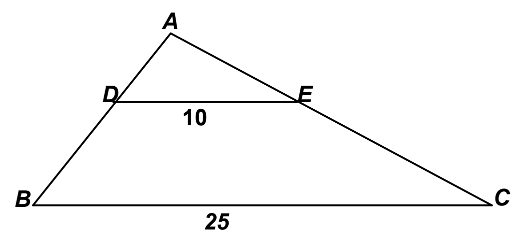
**Round 4**

A) $\left(x+1\right)\left(\dfrac{1}{x}+1\right)=1+x+\dfrac{1}{x}+1=x+\dfrac{1}{x}+2=\dfrac{13}{6}+2=\underline{\mathbf{\dfrac{25}{6}}}$



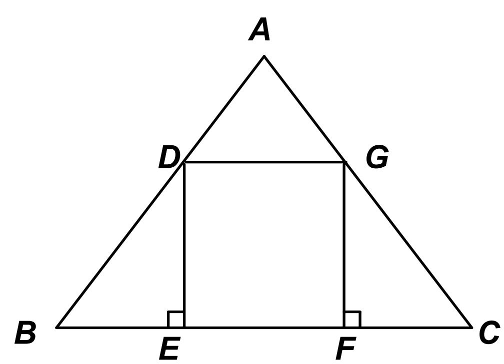

B) Let the dimensions of the cardboard be
$(x + 8)$ by $(x + 21)$
→ dimensions of the box are 4 by $x$ by $(x + 13)$
$V = 4x(x + 13) = 1200$ → $4(x^2 + 13x) - 1200 = 0$
→ $x^2 + 13x - 300 = 0$
→ $(x + 25)(x - 12) = 0$ → $x = 12$, $\cancel{-25}$
→ dimensions: 4 x 12 x 25 → sum: **<u>41</u>**.

C) To have real roots the discriminant $B^2 - 4AC$ must be nonnegative.
$\left(-m\right)^2 - 4(m+3)(1) = m^2 - 4m - 12 = (m-6)(m+2) \ge 0$
The critical points on the number line are –2 and +6.
Testing in the 3 intervals on the number line, the product is positive when $m \le -2$ or $m \ge 6$.
However, we also require that the roots be nonzero, that is
$\dfrac{m \pm \sqrt{m^2-4m-12}}{2} \ne 0$ → $m \ne \pm\sqrt{m^2-4m-12}$ → $m^2 \ne m^2 - 4m - 12$ → $4m \ne -12$ → $m \ne -3$
Therefore, the required set of $m$-values is: $\underline{\boldsymbol{m \le -2 \text{ or } m \ge 6\ \ (m \ne 3)}}$
Alternately, $m < -3$ or $-3 < m \le -2$ or $m \ge 6$
Also acceptable: $\left(-\infty,-3\right) \cup \left(-3,-2\right] \cup \left[6,\infty\right)$




**Round 5**

A) $\Delta ADE \sim \Delta ABC$ and the ratio of corresponding sides is 2 : 5.
Thus the ratio of the areas is 4 : 25.
Therefore, the ratio of the required areas is 4 : (25 – 4) = **<u>4 : 21</u>**.




B) Let $s$ denote the length of the side of equilateral triangle $ABC$
$|ABC| = \dfrac{s^2\sqrt{3}}{4}$, where $|ABC|$ denotes the area of $\Delta ABC$
Area(Trap $BDGC$) = 15/16 area ($\Delta ABC$) → $AD : AB = 1 : 4$

→ $BD = \dfrac{3}{4}s$, $BE = \dfrac{3}{8}s$ and $DE = \dfrac{3}{8}s\sqrt{3}$
Thus, $a = |BDE| = \dfrac{1}{2}\left(\dfrac{3}{8}s\right)\left(\dfrac{3}{8}s\sqrt{3}\right) = \dfrac{9}{128}\sqrt{3}s^2$

Taking the required ratio, we have $\dfrac{1/4}{9/128}$ = **<u>32 : 9</u>**