MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 - FEBRUARY 2012
ROUND 2 ARITHMETIC / NUMBER THEORY

ANSWERS

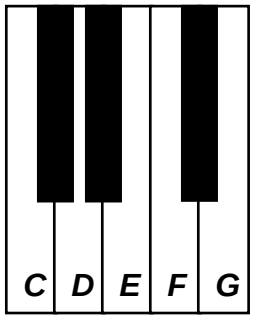
A) (_____ , _____)

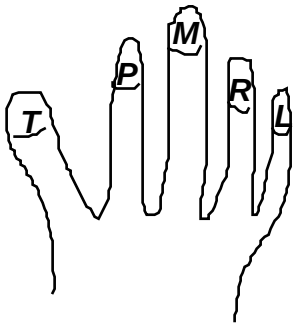
B) _____

C) (_____ , _____)

******* NO CALCULATORS ON THIS ROUND *******

A) Engelbert practiced proper finger position on the piano with his right hand. Starting with his thumb he plays middle C, followed immediately by DEFGFED, using his other fingers as follows: pointer, middle, ring, "pinky" (i.e. little), ring, middle and pointer.
Starting again with the thumb he continues to repeat the exercise (ad nauseum).
Let *F* denote the finger used –
T (thumb), P (pointer), M (middle), R (ring), L ("pinky", i.e. little).
Let *N* denote the note being played.
Determine the ordered pair (*F*, *N*) when he plays his 2012th note and gets to stop for lunch.





B) Determine all primes between 300 and 500 ending in 7 whose digit sum is a multiple of 11?

C) A spinner has 13 equally spaced positions, numbered 1 through 13 clockwise.
Pointer *A* is initially pointing at 3 and moves clockwise 7 positions every second.
Pointer *B* is initially pointing at 3 and moves counterclockwise 5 positions every second.
Pointer *C* is initially pointing at 3 and moves clockwise 2 positions every second.
Let (*a*, *b*, *c*) denote the numbers being referenced by the pointers *A*, *B* and *C* at one second intervals and $S(n) = a + b + c$, after *n* seconds have elapsed.
For example, at $n = 1$, $(a, b, c) = (10, 11, 5)$ and $S(1) = 26$.
Compute (*n*, *m*), where *m* = the maximum value of S(*n*) and *n* is as small as possible.