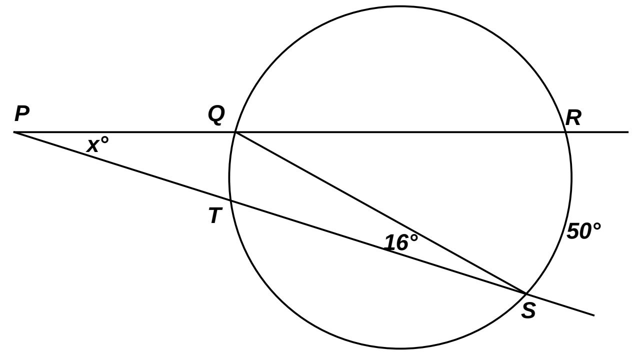
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 – MARCH 2012 SOLUTION KEY

Round 5



A) Let $\angle P = x°$

As an exterior angle of ΔPQS, $m\angle RQS = (x + 16)°$.

As an inscribed angle, $m\angle RQS = \frac{1}{2}(50) = 25°$.

Equating, $x =$ **9**°.

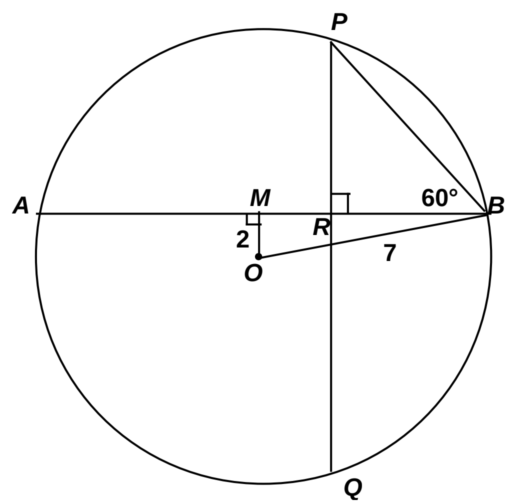
B) Let $AM = MB = a$ and $AN = ND = b$.

Then: $ABCD$ has area $4ab$.

Since $\Delta CPQ \sim \Delta CND$, $\frac{PQ}{ND} = \frac{CQ}{CD} = \frac{1}{2} \Rightarrow PQ = \frac{b}{2}$.

$PQDN$ has area $\frac{1}{2}(a)\left(b + \frac{b}{2}\right) = \frac{3ab}{4}$

Thus, the required fraction is $\frac{3ab/4}{4ab} = \mathbf{\frac{3}{16}}$.



C) Let O denote the center of the circle.

$(OM, OB) = (2, 7) \Rightarrow BM = 3\sqrt{5}$

Since a line through the center of a circle perpendicular to a chord bisects that chord, $AB = 6\sqrt{5}$.

$AR : RB = 2 : 1 \Rightarrow BR = 2\sqrt{5} \Rightarrow PR = \left(2\sqrt{5}\right)\cdot\sqrt{3} = 2\sqrt{15}$

Using the product chord theorem, $AR \cdot RB = PR \cdot RQ$.

$\left(4\sqrt{5}\right)\left(2\sqrt{5}\right) = 2\sqrt{15}\cdot RQ \Rightarrow RQ = \frac{20}{\sqrt{15}} = \mathbf{\frac{4}{3}\sqrt{15}}$