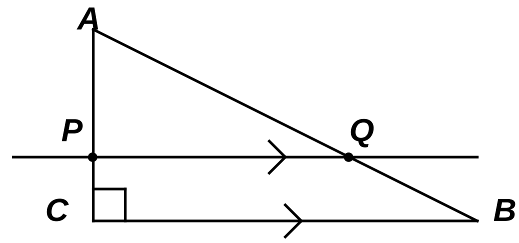
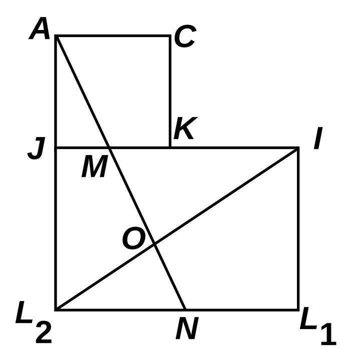
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2016 SOLUTION KEY

Round 5

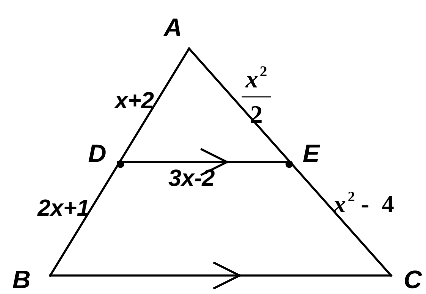


A) $AC^2 = AB^2 - BC^2 = 289 - 225 = 64 \Rightarrow AC = 8$ and
the area of ΔABC is $\frac{1}{2}\cdot 8\cdot 15 = 60$.
$AP : PC = 3 : 1 \Rightarrow AP = 6$
Since $\Delta APQ \sim \Delta ABC$, their areas are in the ratio of the square of their corresponding sides, namely $9:16$. Thus, the area of trapezoid $PQBC$ is $\frac{7}{16}(60) = \frac{7\cdot 15}{4} = \underline{\mathbf{\frac{105}{4}}}$ or $\underline{\mathbf{26.25}}$.

Alternately, $\Delta APQ \sim \Delta ABC$ with corresponding sides in a ratio of 6 : 8 or 3 : 4
Therefore, $PQ = \frac{3}{4}(15) = \frac{45}{4}$ and the area of the trapezoid is $\frac{1}{2}\cdot 2\cdot\left(\frac{45}{4}+15\right) = \underline{\mathbf{\frac{105}{4}}}$.



B) $\Delta JAM \sim \Delta L_2AN \Rightarrow \frac{L_2N}{JM} = \frac{L_2A}{JA} \Rightarrow \frac{L_2N}{1} = \frac{5}{2} \Rightarrow L_2N = 2.5$.
$\begin{cases} L_1L_2 = JK + KI = 6 \\ MK = 1 \end{cases} \Rightarrow MI = 5$.
Since $\Delta MOI \sim \Delta NOL_2$, $\frac{NO}{MO} = \frac{NL_2}{MI} = \frac{2.5}{5} \Rightarrow = \underline{\mathbf{1:2}}$.



C) Since $\Delta ADE \sim \Delta ABC$, $\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC} \Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$
$\Rightarrow \frac{x+2}{2x+1} = \frac{x^2}{2\left(x^2-4\right)} \Rightarrow 2x^3 - 16 + 4x^2 - 8x = 2x^3 + x^2$
$\Rightarrow 3x^2 - 8x - 16 = (3x+4)(x-4) = 0 \Rightarrow x = 4.$
$x = -\frac{4}{3}$ is extraneous. ($\Rightarrow BD = 2x+1 < 0$)
Thus, $AD = 6$, $DB = 9$, $DE = 10$ and $\frac{6}{6+9} = \frac{10}{BC} \Rightarrow BC = \underline{\mathbf{25}}$.