MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2017 SOLUTION KEY

Round 5

A) According to the triangle inequality, $AC < AB + BC \Leftrightarrow AC < (x+3)+(2x-1) = 3x+2$.
To make x as small as possible, we must make AC as large as possible.
The maximum possible *integer* value of AC is $3x+1$.
Thus, $(x+3)+(2x-1)+(3x+1) = 51 \Rightarrow 6x = 51-3 = 48 \Rightarrow x = \underline{\mathbf{8}}$

B) $\dfrac{n(n-3)}{2} \Rightarrow$ 12-gon: 54 diagonals, 18-gon: 135 diagonals , P (n-gon):
$n(n-3) = 2(54+135) = 378$
$n = 20$ is too small, since 20(17) = 340 < 378, but $21(18) = 378$ and P has **<u>21</u>** sides.

C) Given: $DC = 3x + y$, $AD = 6x$, $AC = 45$ (inches)
$y = \dfrac{3}{2}x$
Applying the Pythagorean Theorem,
$(3x+y)^2 + (6x)^2 = 45^2$. Since $DC = 4.5x$, we have
$(4.5x)^2 + (6x)^2 = 45^2$. Multiplying through by 4,
$(9x)^2 + (12x)^2 = 4 \cdot 45^2 \Rightarrow x^2 = \dfrac{4 \cdot 45^2}{81+144} \Rightarrow x = \dfrac{2 \cdot 45}{15} = 6,\ y = 9$.
Thus, the area is $27 \cdot 36 = \underline{\mathbf{972}}$.