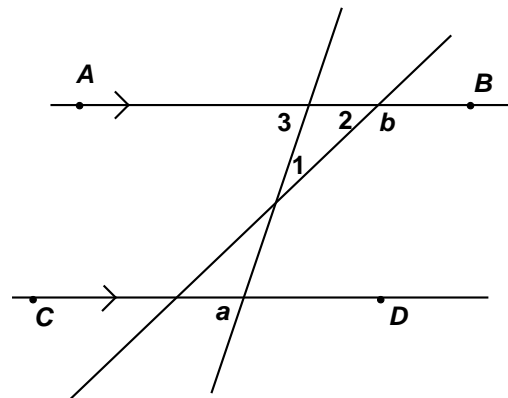


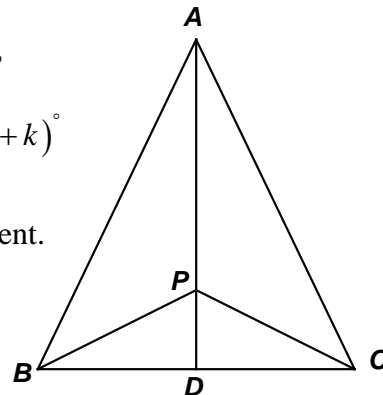
**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 – NOVEMBER 2011 SOLUTION KEY**

Round 6

- A) Let $m\angle 2 = x^\circ$ and $m\angle 1 = (x + 15)^\circ$
 $m\angle 3 = 73^\circ \Rightarrow (x + 15) + x + 107 = 180 \Rightarrow x = 29$
 $\Rightarrow m\angle 1 = 44^\circ, m\angle 2 = 29^\circ \Rightarrow b = 151$
 Since a is vertical to the alternate interior angle of $\angle 3$,
 $a = 73$. Thus, $b - a = 151 - 73 = \underline{78^\circ}$.



- B) In isosceles $\triangle ABC$, a vertex angle of $(2k)^\circ$ leaves base angles of $(90 - k)^\circ$
 Thus, $m\angle ABP = \left(45 - \frac{k}{2}\right)^\circ \Rightarrow m\angle BPA = \left(135 - \frac{k}{2}\right)^\circ$ and $m\angle BPC = (90 + k)^\circ$
 The required difference is $\left(135 - \frac{k}{2}\right)^\circ - (90 + k)^\circ = \underline{\left(45 - \frac{3k}{2}\right)^\circ}$ or equivalent.



- C) Case 1 - Given angles are the base angles:
 $x + 5 = 2x - 30 \Rightarrow x = 35$
 \Rightarrow base angles 40° , vertex angle 100°

 Case 2 - Both base angles measure $(x + 5)^\circ$:
 $2(x + 5) + 2x - 30 = 180 \Rightarrow 4x = 200 \Rightarrow x = 50$
 \Rightarrow vertex angle $= 2(50) - 30 = 70^\circ$

 Case 3 - Both base angles measure $(2x - 30)^\circ$:
 $2(2x - 30) + (x + 5) = 180 \Rightarrow 5x = 235 \Rightarrow x = 47$
 \Rightarrow vertex angle $= 47 + 5 = 52^\circ$
 Thus, the required sum is 222°.