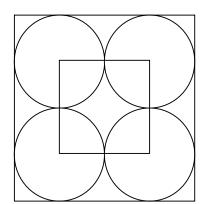
## MASSACHUSETTS MATHEMATICS LEAGUE CONTEST 6 - MARCH 2008 SOLUTION KEY

## Round 5

A) 
$$AB = 10$$
,  $\triangle ADE \sim \triangle ABC \Rightarrow \frac{AD}{AB} = \frac{DE}{BC} \Rightarrow \frac{4}{10} = \frac{8}{BC}$   
 $\Rightarrow 4(BC) = 8(10) \Rightarrow BC = 20$ 

B) 
$$4r^2 - \pi r^2 = 12 - 3\pi \rightarrow r^2 = 3 \rightarrow \text{Area}_{\text{small square}} = 12$$
 and  $A_{\text{large square}} = 48 \rightarrow \text{required area} = \underline{36}$ 



C) AF = FC = 8. The measure of an interior angle of a regular octagon is  $\frac{6(180)}{8} = 135^{\circ}$ .

m
$$\angle EFC = \text{m} \angle GFA = 45^\circ$$
, so m $\angle AFC = 45^\circ$  also.  
Draw  $\overline{AJ} \perp \overline{FC}$ . Then  $\triangle AFJ$  is  $45^\circ$ - $45^\circ$ - $90^\circ$  and  $AJ = 4\sqrt{2}$   
Area of  $\triangle AFC = \frac{1}{2}(FC)(AJ) = \frac{1}{2}(8)(4\sqrt{2}) = 16\sqrt{2}$ 

Alternate method: Area(
$$\triangle AFC$$
) =  $\frac{1}{2}(AF)(FC)\sin(\angle AFC)$   
=  $\frac{1}{2}(8)(8)\sin(45^\circ) = 32 \cdot \frac{\sqrt{2}}{2} = \underline{16\sqrt{2}}$ 

