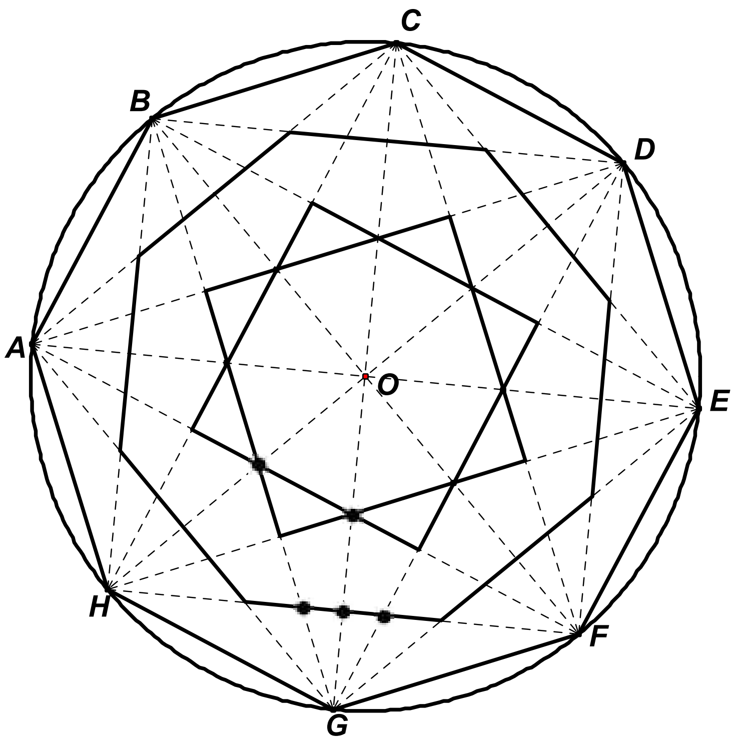
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2010 SOLUTION KEY

E) When my son is x years old, I will be $(x + 21)$ years old.
In a certain number of years, say n years, my age and my son's age are in a 3 : 2 ratio.
$\dfrac{x+21+n}{x+n}=\dfrac{3}{2}$ ➔ $x + n = 42$. If $n = 25$, then $x = 17$ and, on that birthday, our ages are
$g = 25$, $s = 42$ and $f = 63$. On that birthday, my son is 17 years older than his daughter and I am 38 years older than my granddaughter. This condition is invariant, as long as we are all alive.
k years after my granddaughter is born our ages total 100 ➔ $k + (17 + k) + (38 + k) = 100$ ➔ $k = 15$
Thus, $(g, s, f) =$ **<u>(15, 32, 53)</u>**.



F) The minimum number of points of intersection will occur in a regular octagon. Points A through H are excluded. The inner octagon contains 3 points of intersection on each side, plus the 8 vertices, for a total of 32. The two intersecting squares contain 16 additional points of intersection, two on each side and the 8 vertices. Adding the center point, we have a <u>minimum</u>, $m = 49$.

To maximize the number of points of intersection, we must examine the points where more than two lines intersect, i.e. the 8 points where the two squares intersect as well as the center point. At each of the 8 points there are three intersecting lines which could have determined 3 points, instead of a single point of concurrency. This would add an additional $24 - 8 = 16$ points.

At the center point there are four intersecting lines which could have determined $\binom{4}{2} = 6$ points, instead of a single point of concurrency. This would add $6 - 1 = 5$ additional points of intersection. Thus, the <u>maximum</u> $M = 49 + 16 + 5 = 70$. $(M, m) =$ **<u>(70, 49)</u>**

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