MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2010 SOLUTION KEY

Team Round – continued

E) $\sqrt{5x+9}+\sqrt{8x+17}=2 \rightarrow \sqrt{8x+17}=2-\sqrt{5x+9}$

Squaring both sides, $8x+17=4-4\sqrt{5x+9}+5x+9=5x+13-4\sqrt{5x+9}$

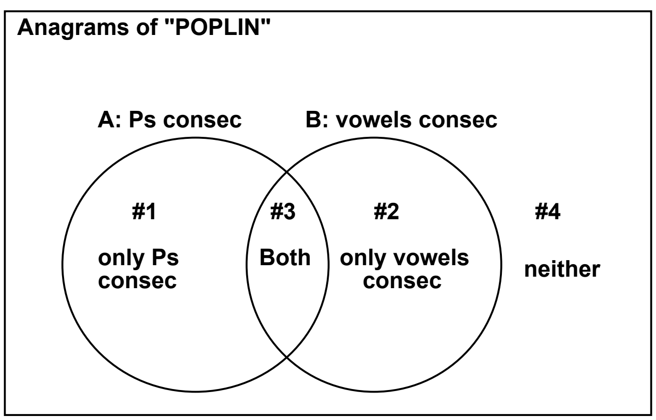
$\rightarrow 3x+4=-4\sqrt{5x+9}$ (***)

$\rightarrow 9x^2+24x+16=16(5x+9)=80x+144 \rightarrow 9x^2-56x-128=(9x+16)(x-8)=0$

$\rightarrow x = \underline{\mathbf{-\frac{16}{9}}} \left(\sqrt{\frac{-80}{9}+\frac{81}{9}}=\sqrt{\frac{1}{9}}=\frac{1}{3}, \sqrt{\frac{-128}{9}+\frac{153}{9}}=\sqrt{\frac{25}{9}}=\frac{5}{3}\right)$

(8 is extraneous $7 + 9 \neq 2$ or note from (***) above that $x \leq -\frac{4}{3}$, since both sides must be negative.)

F) Consider a Venn Diagram with two circles containing anagrams with consecutive Ps and anagrams with consecutive vowels. The intersection (region #3) contains anagrams satisfying both conditions. POPLIN would reside in region #4, BUT it is not an anagram (of itself) and, therefore, must be excluded entirely.
These 4 regions are mutually exclusive, i.e. any anagram resides in exactly one of these regions, ensuring that no anagram is missed or double counted.



Region #3: Consider the 6 letters as 4 distinct items, namely PP, OI, L and N.
They may be arranged to form 4!(2) = 48 anagrams

Circle A: Consider the 6 letters as 5 items. Namely PP, O, I, L and N
These can be arranged in 5! = 120 ways → Region #1: 120 – 48 = 72 anagrams.

Circle B: Consider the 6 letters as 5 items, OI, P, P, L and N
These can be arranged in $\frac{5!\cdot 2}{2!}$ = 120 ways → Region #2: 120 – 48 = 72 anagrams

Thus, region #4 contains 359 – (72 + 48 + 72) = 359 – 192 = **<u>167</u>** anagrams.