MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2011 SOLUTION KEY

Team Round - continued

C) Let x and y denote the number of postcard and first-class stamps respectively.
$19x + 29y = 19x + 29(40 - x) = 920 \Rightarrow -10x = 920 - 1160 \Rightarrow x = 24, y = 16$
Current cost 24(28) + 16(44) = 672 + 704 = 1376¢ = **<u>$13.76</u>**

D) By long division, $N = \dfrac{10x}{x+10} = 10 - \dfrac{100}{x+10}$
Clearly, N is an integer if and only if $x + 10$ is a factor of 100.
We must consider both positive and negative factors of 100.
100 has 9 positive and 9 negative divisors. [±(1, 2, 4, 5, 10, 20, 25, 50, 100)]
Positive factors of 100 are obtained by letting $x = -9, -8, -6, -5, 0, 10, 15, 40$ and 90.
N is positive for the last four x-values. Thus, N = <u>*5, 6, 8* and *9*</u>, resulting in a total of <u>28</u>.

Negative factors of 100 are obtained by letting $x = -11, -12, -14, -15, -20, -30, -35, -60$ and -110.
We get 9 values for N: <u>*110, 60, 35, 30, 20, 15, 14, 12* and *11*</u>. A total of <u>307</u>.
Note that except for the sign, the list of x-values read backwards is the list of N-values.
The smallest N-value in our list is 5.
Could N assume an integer value smaller than this, say 4?
$N = 4 \Rightarrow \dfrac{10x}{x+10} = 4 \Rightarrow 10x = 4x + 40$, which is not solvable for <u>integer</u> x.
Similarly, N = 3, 2 and 1 fail.
Our double check confirms that the 13 N-values we found are the only possible integer ones.
Their sum is **<u>335</u>**.