MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2011 SOLUTION KEY

Round 1

A) Knowing that $f^{-1}(a)=b \Leftrightarrow f(b)=a$, we need not bother finding $f^{-1}(x)$.

Solving $4x + 5 = 3 \rightarrow x = -\frac{1}{2}$. Thus, $f\left(-\frac{1}{2}\right)=3 \Leftrightarrow f^{-1}(3)=-\frac{1}{2}$.

$f(-1) = 1$. Thus, $f^{-1}(3)+g(f(-1)) = -\frac{1}{2}+g(1) = -\frac{1}{2}+6+1-4=$ **<u>2.5</u>** $\left(2\frac{1}{2} \text{ or } \frac{5}{2}\right)$

B) $f(x) = 4x - 1 \rightarrow f^{-1}(x)=\frac{x+1}{4}$. $g(t) = 3 - 2t \rightarrow g^{-1}(t)=\frac{t-3}{-2}=\frac{3-t}{2}$

$$f\left(g^{-1}(2a)\right)=f\left(\frac{3-2a}{2}\right)=4\left(\frac{3-2a}{2}\right)-1=6-4a-1=5-4a$$

$$g\left(f^{-1}(2-a)\right)=g\left(\frac{(2-a)+1}{4}\right)=g\left(\frac{3-a}{4}\right)=3-2\left(\frac{3-a}{4}\right)=\frac{6}{2}-\frac{3-a}{2}=\frac{3+a}{2}$$

Equating, $5-4a=\frac{3+a}{2} \rightarrow 10 - 8a = 3 + a \rightarrow a = \underline{\boldsymbol{\frac{7}{9}}}$

C) Solution #1: (Brute Force - Find the 4 roots, plug and chug.)
The possible integer roots are factors of 12, the constant term. Testing by synthetic substitution:

$$\begin{array}{r|rrrrr}
 & 3 & -8 & -11 & 28 & -12 \\
\hline
1 & 3 & -5 & -16 & 12 & 0 \\
\hline
-2 & 3 & -11 & 6 & 0 &
\end{array}$$

, we discover two integer roots, 1 and –2 and the remaining roots can be determined by factoring the quotient $3x^2 - 11x + 6 = (3x - 2)(x - 3) = 0 \rightarrow 3, \frac{2}{3}$.

Let $(A, B, C, D) = \left(1,-2,3,\frac{2}{3}\right)$.

$$(1+A)(1+B)(1+C)(1+D) = 2\cdot -1\cdot 4\cdot\frac{5}{3} = \underline{\boldsymbol{-\frac{40}{3}}}$$

This method depends on being able to <u>*factor the given expression*</u>. This is <u>not</u> always possible.
Ex: Try $f(x) = 2x^4 - 3x^3 + 5x^2 - 7x + 11$.
This polynomial does not factor over the integers. With a graphing calculator you could approximate the four zeros, plug values into the expression and <u>approximate</u> the product. However, the computations would be extremely messy. How can this be avoided??

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