Team Round – continued

F) At each vertex of the enclosed regular polygon, you must have three angles, two from the surrounding regular polygons and one the enclosed regular polygon. Thus, for the square-octagon, (360 – 90)/2 = 135 gives the measure of the <u>interior</u> angle of the surrounding regular polygons and the exterior angles are 45°. Since 360 is divisible by 45, we have a solution! $360/n = 45 \rightarrow n = 8$ (i.e. the surrounding regular polygons are octagons). The results of repeating this scenario for different values of m are shown in the following chart. There are only 3 other possibilities.

enclosed polygon			surrounding polygons		
sides	interior	interior	exterior	# sides	
	angle	(360 - *int*)/2	180 - θ	360/*E*	
m	*int*	θ	*E*	***n***	**(*m*, *n*)**
3	60	150	30	12	**(3, 12)**
4	90	135	45	8	(4, 8)
5	108	126	54	*reject*	
6	120	120	60	6	**(6, 6)**
7	***REJECT***				
8	135	112.5	67.5	*reject*	
9	140	110	70	*reject*	
10	144	108	72	5	**(10, 5)**

How do we know there are no more ordered pairs awaiting discovery?

The interior angle in an m-sided regular polygon measures $\frac{180(m-2)}{m}°$.

Algebraically representing the technique described above to find n given m we have:

$$n = \frac{360}{E} = \frac{360}{180 - \left(\dfrac{360 - \dfrac{180(m-2)}{m}}{2}\right)}$$

Carefully simplifying this expression,

$$\frac{360}{180 - \left(\dfrac{360 - \dfrac{180(m-2)}{m}}{2}\right)} = \frac{2}{1 - \left(\dfrac{2 - \dfrac{(m-2)}{m}}{2}\right)} = \frac{2}{1 - (1 - \dfrac{m-2}{2m})} = \frac{4m}{m-2}$$

Using long division, we have $n = 4 + \frac{8}{m-2}$ and, for $m > 10$,

$(m – 2)$ will never be a divisor of 8 and the search stops.