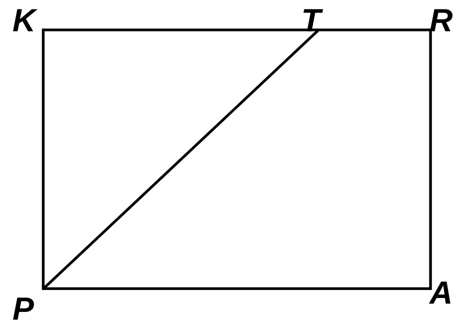
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 – NOVEMBER 2011 SOLUTION KEY

Round 3



A) Since K is a right angle, isosceles triangle PTK must be 45 – 45 – 90 and $KT = KP$.
Thus, $\overline{KR}$ must be the longer side.
$KP = RA = 3$, $KR = PA = 5 \Rightarrow TR = 2$
Use the area formula for a trapezoid directly

Area = $\frac{1}{2}3(2+5) = \frac{21}{2} = \underline{\mathbf{10.5}}$

or determine the area of the rectangle $PARK$ and subtract the area of the triangle PKT.

Area = $15 - \frac{1}{2}\cdot 3\cdot(5-3) = 15 - 4.5 = \underline{\mathbf{10.5}}$

B) $(3x)^2 + (2x)^2 = 3328 \Rightarrow x^2 = \frac{3328}{13} = 256 \Rightarrow x = 16$
Thus, the sides of the squares are 32 and 48 units.
$\overline{PQ}$ is the hypotenuse of a right triangle with a horizontal base of 80 and a vertical height of 48.
$(48, 80, PQ)$ is a Pythagorean triple. $(48, 80, PQ) = 16(3, 5, x) \Rightarrow x = \sqrt{34} \Rightarrow PQ = \underline{\mathbf{16\sqrt{34}}}$.

C) Let x denote the side of the square. The side of the larger square is $x + \frac{1}{4}x = \frac{5}{4}x$

The perimeter of the rectangle is $2(2k + 5k) = 28 \Rightarrow k = 2$.
The rectangle is 4 x 10, resulting in an area of 40.

Thus, $\left(\frac{5}{4}x\right)^2 = 40 \Rightarrow x^2 = \frac{40\cdot 4^2}{5^2} = \frac{2^2\cdot 4^2\cdot 10}{5^2} \Rightarrow x = \underline{\mathbf{\frac{8}{5}\sqrt{10}}}$.