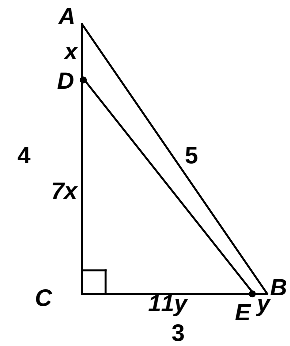
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 – NOVEMBER 2013 SOLUTION KEY



Round 3

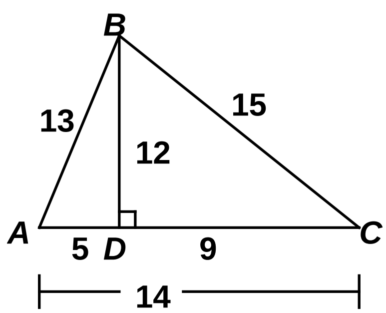
A) $AC = 4$, $AD = x \Rightarrow 8x = 4 \Rightarrow DC = \frac{7}{2}$, $BE = y \Rightarrow 12y = 3 \Rightarrow CE = \frac{11}{4}$

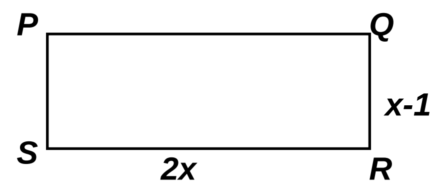
Thus, the required area is $\frac{1}{2}\cdot\frac{7}{2}\cdot\frac{11}{4} = \underline{\mathbf{\frac{77}{16}}}$ (or $\underline{\mathbf{4\frac{13}{16}}}$ or **<u>4.8125</u>**).

B) Altitude $\overline{BD}$ subdivides ΔABC into two special triangles with sides 5-12-13 and 9-12-15

Thus, its area is $\frac{1}{2}\cdot 12\cdot 14 = 84$ and, equating areas, we have

$2x(x-1) = 84$
$\Rightarrow x^2 - x - 42 = 0$
$\Rightarrow (x-7)(x+6) = 0$.
$\Rightarrow x = \underline{7}$





C) Per = $5x + 3y + 11 = 152 \Rightarrow 5x = 141 - 3y$

In ΔBCF, $x^2 + y^2 = (x+1)^2 \Rightarrow 2x = y^2 - 1$

Substituting, $\frac{5}{2}(y^2 - 1) = 141 - 3y \Rightarrow 5y^2 - 5 = 282 - 6y \Rightarrow 5y^2 + 6y - 287 = 0$

$\Rightarrow (5y+41)(y-7) = 0 \Rightarrow y = 7 \Rightarrow x = 24$

$\Rightarrow$ Area(square $ABFE$) = **<u>576</u>**.