MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2008 SOLUTION KEY

Round 2

A) The prime factorization of N is of the form $2^x \cdot 3^a \cdot 5^b \cdot ...$ or $2^x \cdot P$, where P denotes a product of all powers of odd primes. Clearly, $A = P$, since an odd product requires all odd factors. Similarly, since 1 is a factor of every integer, 1 must be the <u>only</u> odd factor of B, implying B is a power of 2. Thus, $B = 2^x$ and $AB = 2^x \cdot P = N$
Thus, without bothering to factor N, $AB =$ **<u>158400</u>**

B) $24^3 \cdot 120^2 \cdot 441 = 2^{9+6} \cdot 3^{3+2+2} \cdot 5^2 \cdot 7^2 = 2^{15} \cdot 3^7 \cdot 5^2 \cdot 7^2$
$28^A \cdot 15^B \cdot 12^C \cdot 2^D = 2^{2A+2C+D} \cdot 3^{B+C} \cdot 5^B \cdot 7^A$
Thus, $A = B = 2$, $2 + C = 7$ and $2A + 2C + D = 15 \rightarrow (A, B, C, D) =$ **<u>(2, 2, 5, 1)</u>**

C) The base 3 place values are: 1, 3, 9, 27, 81, 243, ….
Since 243 is the smallest power of 3 greater than 211, the representation will have 6 digits.
$243 - 27 = 216 \rightarrow$ (ACB _ _ _)
$216 - 9 = 207 \rightarrow$ ($ACBB$ _ _)
$207 + (3 + 1) = 211$
Thus, $211_{10} =$ ***<u>ACBBAA</u>***$_3$