**MASSACHUSETTS MATHEMATICS LEAGUE**
**CONTEST 1 - OCTOBER 2008 SOLUTION KEY**

**Round 4**

A) Replacing $L$ and $g$ by $100L$ and $g/4$ respectively, we have $\pi\sqrt{\dfrac{100L}{g/4}} = \pi\sqrt{\dfrac{400L}{g}} = 20\pi\sqrt{\dfrac{L}{g}}$.

Thus, the effect is an **<u>increase</u>** by a factor of **<u>20</u>**.

B) Converting the times to hours, we have times of $\dfrac{2}{5}, \dfrac{1}{2}, \dfrac{5}{3}, \dfrac{25}{4}$.

We need only determine the LCM of the numerators. Clearly the LCM(1, 2, 5, 25) = 50
Each of these four fractions divides evenly into 50.
The quotients in order are: 125, 100, 30 and 8.
This means that the 125$^{\text{th}}$ ringing of bell $A$ coincides with the 100$^{\text{th}}$ ringing of $B$ which coincides with the 30$^{\text{th}}$ ringing of $C$ which coincides with the 8th ringing of $D$, 50 hours later.
Thus, 2 days plus two hours later the bells ring together (**<u>5:30 PM on Saturday</u>**)

C) Substituting, $1+\dfrac{rR}{f}=\dfrac{r}{F} \rightarrow 1+\dfrac{48R}{12F/5}=1+\dfrac{20R}{F}=\dfrac{48}{F} \rightarrow R=\left(\dfrac{48}{F}-1\right)\cdot\dfrac{F}{20} = \dfrac{48-F}{20}$

Clearly, $(F, R)$ = (**<u>8</u>**, 2) and (**<u>28</u>**, 1) are the only solutions in positive integers.

Alternate solution: $1+\dfrac{48R}{f}=\dfrac{48}{F} \Leftrightarrow 1+\dfrac{5(48R)}{5f}=\dfrac{12(48)}{12F} \Leftrightarrow 1+\dfrac{5(48R)}{12F}=\dfrac{12(48)}{12F}$

$\Leftrightarrow 12F + 5(48R) = 12(48) \Leftrightarrow F + 20R = 48$ which is a linear equation with $F$ intercept at (48, 0) and slope of $-1/20 \rightarrow$ (**<u>28</u>**, 1) and (**<u>8</u>**, 2)