MASSACHUSETTS MATHEMATICS LEAGUE
JANUARY 2004
ROUND 3: TRIG EQUATIONS
NON-CALCULATOR

ANSWERS

A) $30^\circ, 150^\circ$

B) $2\pi/3, 4\pi/3$

C) $\pi/3, 2\pi/3, 4\pi/3, 5\pi/3$

A) Solve for $0^\circ \le \theta < 360^\circ$, $\sin\theta + \csc\theta = 2.5$

$\sin\theta = \frac{1}{2}$, $\csc\theta = 2$

$\theta = 30^\circ, 150^\circ$

B) Solve for $0 \le x < 2\pi$, $\dfrac{2\sin^2 x}{1-\cos x} = 1$

$2\sin^2 x = 1 - \cos x$

$2(1-\cos^2 x) = 1 - \cos x$

$2 - 2\cos^2 x = 1 - \cos x$

$2\cos^2 x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

$\cos x = -\frac{1}{2}$, $\cos x = 1$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}, 0$, 0 is ext. Since $1-\cos x \neq 0$

C) Solve for $0 \le x < 2\pi$, $\tan 2x = -3\cot x$

$\dfrac{2\tan x}{1-\tan^2 x} = \dfrac{-3}{\tan x}$, $2\tan^2 x = -3 + 3\tan^2 x$

$\tan^2 x = 3$

$\tan x = \pm\sqrt{3}$ $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$