

**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2016 SOLUTION KEY**

Round 3

A) $A = 90 + 360n < 2016 \Rightarrow 450, 810, 1170, 1530, 1890$ - 5 values.

B) $\sin(4x) = \cos(2x) \Leftrightarrow 2\sin(2x)\cos(2x) - \cos(2x) = 0 \Leftrightarrow \cos(2x)(2\sin 2x - 1) = 0$
 $\Rightarrow \cos(2x) = 0$ or $\sin(2x) = \frac{1}{2}$.

1st condition $\Rightarrow 2x = \frac{\pi}{2} + n\pi \Rightarrow x = \frac{\pi}{4} + \frac{n\pi}{2} = \frac{\pi(2n+1)}{4}$ and $n = -1, 0 \Rightarrow \underline{\pm \frac{\pi}{4}}$.

2nd condition $\Rightarrow 2x = \frac{\pi}{6} + 2n\pi \Rightarrow x = \frac{\pi}{12} + n\pi = \frac{\pi(12n+1)}{12}$ and $n = 0 \Rightarrow \underline{\frac{\pi}{12}}$.

$2x = \frac{5\pi}{6} + 2n\pi \Rightarrow x = \frac{5\pi}{12} + n\pi = \frac{\pi(12n+5)}{12}$ and $n = 0 \Rightarrow \underline{\frac{5\pi}{12}}$.

C) $A\left(\frac{1}{2}, \cos^{-1}\left(\frac{1}{2}\right)\right) = \left(\frac{1}{2}, \frac{\pi}{3}\right)$, since $\cos \frac{\pi}{3} = \frac{1}{2}$.

\overline{AD} has equation $y = \frac{\pi}{3}$. \overline{BC} has slope $-\frac{\pi}{2}$ and

equation $y - 0 = -\frac{\pi}{2}(x - 1) \Leftrightarrow y = \frac{\pi}{2}(1 - x)$

$\frac{\pi}{3} = \frac{\pi}{2}(1 - x) \Rightarrow x = \frac{1}{3} \Rightarrow AD = \frac{1}{2} - \frac{1}{3} = \underline{\frac{1}{6}}$.

