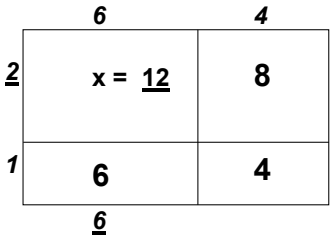
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2010 SOLUTION KEY

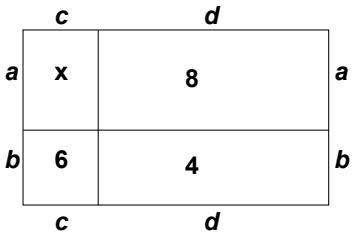
Round 6 - continued

C) The arrangement at the right produces $x = \underline{12}$.
No other arrangements produce integer dimensions.
If interested, read on to see why this is the case.

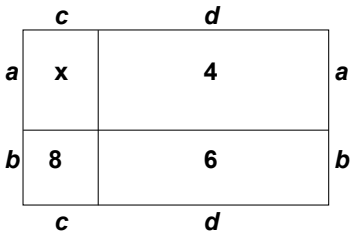


There are 6 possible arrangements, namely the rectangle with area 4, 6 or 8 is opposite the rectangle with area x and the other two may be interchanged.

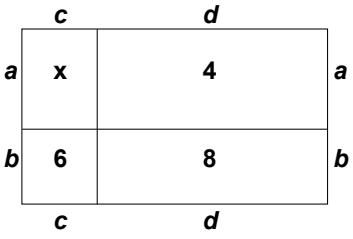
Case 1



Case 2



Case 3



In the following explanation, "|" means such that.

Case 1: $(b, d) \mid bd = 4$
(2, 2) → $a = 4, c = 3, x = 12$
(1, 4) → $a = 2, c = 6, x = \underline{12}$
(4, 1) → c fractional, i.e. $\frac{6}{4}$ - rejected

Case 2: $(a, d) \mid ad = 4$
(2, 2), (4, 1), (1, 4)
c fractional - all rejected

Case 3: $(a, d) \mid ad = 4$
(1, 4) → $b = 2, c = 3, x = \underline{3}$

Case 4: (Case 1 - interchange 6 and 8) $x = 3(4)$ or $2(6) = 12$
The only change is the orientation. The order of the areas (x - 6 - 4 - 8) is counterclockwise (CCW) in case 1 and clockwise (CW) in case 4.

In similar fashion, cases 5 and 6 are CW versions of cases 2 and 3 and introduce no additional solutions. Thus, there are only two answers.