MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2006 SOLUTION KEY

Team Round - continued

C) Method 1: Hammer and Tongs (Brute Force)
Consider $2x^2 + xy - 6y^2 + 7y - 2 = 0$ a quadratic equation in x, namely
$Ax^2 + Bx + C = 0 \leftrightarrow 2x^2 + yx + (-6y^2 + 7y - 2) = 0 \rightarrow A = 2, B = y$ and $C = -6y^2 + 7y - 2$

Applying the quadratic formula, $x = \dfrac{-y \pm \sqrt{y^2 - 4(2)(-6y^2 + 7y - 2)}}{4} = \dfrac{-y \pm \sqrt{49y^2 - 56y + 16}}{4}$

$= \dfrac{-y \pm \sqrt{(7y-4)^2}}{4} = \dfrac{-y \pm (7y-4)}{4}$

$\rightarrow x = \dfrac{6y-4}{4} = \dfrac{3y-2}{2} \rightarrow 2x - 3y + 2 = 0$ or

$x = \dfrac{-8y+4}{4} = -2y + 1 \rightarrow x + 2y - 1 = 0$

Thus, the equation in factored form is: $(2x - 3y + 2)(x + 2y - 1) = 0$

Solving $\begin{cases} 2x - 3y + 2 = 0 \\ x + 2y - 1 = 0 \end{cases} \rightarrow \begin{cases} 2x - 3y + 2 = 0 \\ -2x - 4y + 2 = 0 \end{cases} \rightarrow -7y + 4 = 0 \rightarrow (x, y) =$ <u>**(-1/7, 4/7)**</u>.

Method 2: Indeterminant Coefficients (Guess and Check)

Suppose $2x^2 + xy - 6y^2 + 7y - 2$ factors to $(2x + ay + 2)(x + by - 1)$ for some constants a and b.
Multiplying out the trinomials leads to the linear equations $-a + 2b = 7$ and $a + 2b = 1$
and $(a, b) = (-3, 2)$ producing the factors $(2x - 3y + 2)(x + 2y - 1)$, as above.

But what would have happened if we assumed a factorization of $(2x + ay - 2)(x + by + 1)$?
Multiplying out these trinomials leads to the linear equations $a - 2b = 7$ and $a + 2b = 1$
and $(a, b) = (4, -1.5)$ producing the factors $(2x + 4y - 2)(x - 1.5y + 1)$. At first glance this appears to be a different factorization; however, taking out a factor of 2 from the first factor and distributing it through the second produces the same factors as before.

Note: The sum of the coefficients in the original polynomial is $2 + 1 - 6 + 7 - 2 = +2$
Compare this with the product of the sum of the coefficients in each factor!
$(2 - 3 + 2)(1 + 2 - 1) = (1)(2) = +2$
This is always true! Check it out.