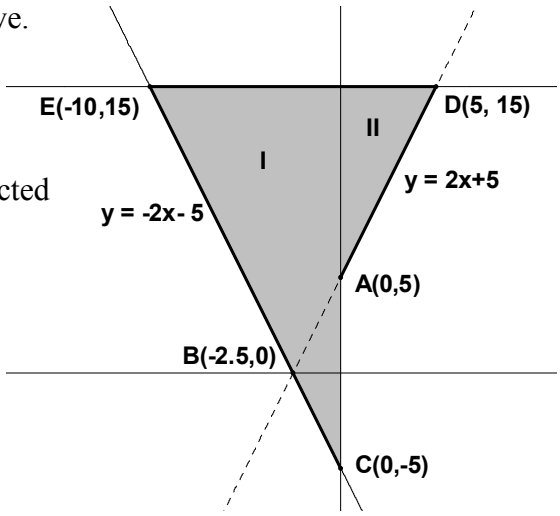
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2009 SOLUTION KEY

Team Round

E) $y = 2x + 5$ is a line with slope 2 and y-intercept at (0, 5) and x-intercept at (–2.5, 0) – the dotted line

$\frac{|x|}{x} = \pm 1$ depending on whether x is positive or negative.

The following diagram shows the region in question.



For positive x, y was positive and it stays positive.
For negative x, y changed sign, i.e. the graph was reflected across the x-axis.

The area of region I is $\frac{1}{2}(10)(20) = 100$

The area of region II is $\frac{1}{2}(5)(10) = 25$

Thus, the total area is **<u>125</u>**

F) $(10)(5)(3) = 150$ coins available.
I have 50 P quarters.

I have at least $\frac{3}{4}50 = 37.5$ ➔at least 38 D quarters.

I have at most $\frac{1}{8}\cdot 50 = 6.5$ ➔ at most 6 S quarters

Thus, minimum and maximum number of coins I have are 50 + 38 + 0 = 88 and 50 + 50 + 6 = 106

and $(m, M) = \left(\frac{88}{150}, \frac{106}{150}\right) = \left(\frac{176}{300}, \frac{212}{300}\right) = \left(\frac{176}{3}\%, \frac{212}{3}\%\right) = \underline{\left(\mathbf{58\frac{2}{3}\%, 70\frac{2}{3}\%}\right)}$