MASSACHUSETTS MATHEMATICS LEAGUE CONTEST 2 – NOVEMBER 2008 SOLUTION KEY

Round 5

A)
$$4(\sin\theta + 1) = 3\left(\frac{1}{\sin\theta}\right) \rightarrow 4\sin^2\theta + 4\sin\theta - 3 = (2\sin\theta - 1)(2\sin\theta + 3) = 0$$

 $\sin\theta = -3/2$ is impossible
 $\sin\theta = 1/2 \rightarrow \frac{\pi}{6}, \frac{5\pi}{6}$

B)
$$\cos x = \frac{\cot(1035^\circ) \cdot \tan(135^\circ)}{\sec^2(-45^\circ)} = \frac{\cot(315^\circ) \cdot \tan(135^\circ)}{\sec^2(45^\circ)} = \frac{-1 \cdot -1}{2} = \frac{1}{2} \rightarrow 60^\circ \text{ family } \rightarrow \underline{\pm 60}$$

C) =
$$([\sqrt{3} - 1][\sqrt{3} + 1])^{70} = 2^{70} = (\sqrt{2})^{140} \rightarrow |b| = \sqrt{2}$$

Note: The equation $b^{140} = (\sqrt{2})^{140}$ has 140 roots, only two of which are real, but the absolute value of all of them is $\sqrt{2}$.