MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 – NOVEMBER 2010 SOLUTION KEY

Round 1

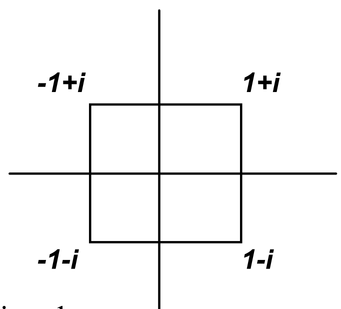
A) $\left(\frac{1-i}{1+i}\right)^{2010} = \left(\frac{1-i}{1+i}\cdot\frac{1-i}{1-i}\right)^{2010} = \left(\frac{(1-i)^2}{1-i^2}\right)^{2010} = \left(\frac{-2i}{2}\right)^{2010} = (-1)^{2010}\cdot i^{2008}\cdot i^2 = 1\cdot 1\cdot -1 = \underline{\mathbf{-1}}$

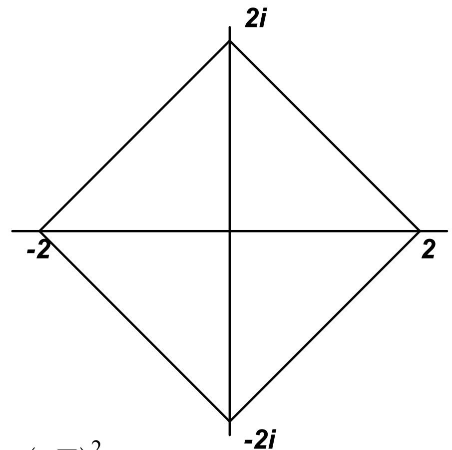
If you know DeMoirve's theorem, you might want to use it to formulate an alternative solution for comparison.

B) Equating the real and imaginary coefficients, $\begin{cases} x^2-x-5=1 \\ y^2-7y+3=-7 \end{cases}$ ➔

$\begin{cases} x^2-x-6=(x-3)(x+2)=0 \\ y^2-7y+10=(y-2)(y-5)=0 \end{cases}$ ➔ $x = 3, -2$ and $y = 2, 5$ ➔ (3, 5), (–2, 5), (3, 2), (–2, 2) ➔ **<u>(–2, 5)</u>**

C) $(1+i)\cdot\begin{cases}(1+i)\\(-1+i)\\(-1-i)\\(1-i)\end{cases} = \begin{cases}2i\\-2\\-2i\\2\end{cases}$





The new figure is a square with side $2\sqrt{2}$, so the area is **<u>8</u>**.

Alternate solution: The area of the original square is $2^2 = 4$, and multiplying the vertices by $(1 + i)$ rotates the square 45° and expands each side by a factor of $|1 + i| = \sqrt{2}$. Therefore, the new square will have area $4\left(\sqrt{2}\right)^2 = \underline{\mathbf{8}}$.

Round 2

A) $\frac{5280/4 \text{ feet}}{60 \ \cancel{\text{min}}}\cdot 48 \ \cancel{\text{min}} = \frac{\frac{5280}{4}(4)}{5} = \frac{5280}{5} = \underline{\mathbf{1056}}$ feet

B) $\begin{cases}(1)\ \ 10t+u=7(t+u)\\(2)\ \ (10t+u)(t+u)=567\end{cases}$

(1) ➔ $t = 2u$

Substituting for $10t + u$ in (2), $7(t+u)^2 = 567$ ➔ $(t+u)^2 = 81$ ➔ $t + u = 3u = 9$

➔ $u = 3, t = 6$ ➔ **<u>63</u>**

C) Discounts of 16 2/3% ➔ 1/6 off, 12 1/2% ➔ 1/8 off and 4% ➔ 1/25 off

Merchant A's price $\frac{5}{6}\cdot\frac{7}{8}\cdot\frac{24}{25} = \frac{7}{10} = 70\%$ of list (or 30% off list).

Merchant B's price: (.92)(.9)(.85) = .7038 = 70.38% of list (or 29.62% off list).

Thus, merchant A has the best price by 0.38% (less than 1%)

➔ $\frac{0.38}{100}(6000) = 0.38(60) = 22.8$ ➔ **<u>$22.80</u>** .

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