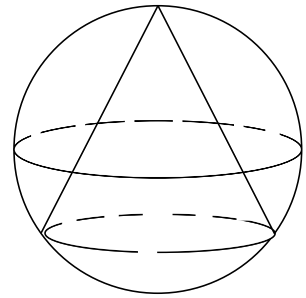
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2009 SOLUTION KEY

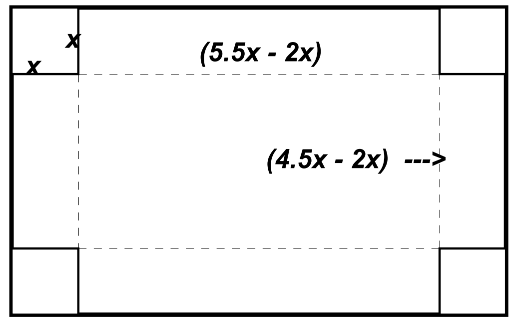


Round 1

A) $\dfrac{V_{sphere}}{V_{cone}} = \dfrac{\frac{4}{3}\pi(5)^3}{\frac{\pi}{3}(3)^2(4+5)} = \dfrac{4(125)}{81}$ ➔ **500 : 81**

B) The volume of the box is $x\left(\dfrac{9x}{2}-2x\right)\left(\dfrac{11x}{2}-2x\right)$

$= x\cdot\dfrac{5x}{2}\cdot\dfrac{7x}{2} = \dfrac{35x^3}{4} = 560 = 35(16)$ ➔ $x^3 = 64$ ➔ $x = 4$.
Thus, the dimensions of the sheet of cardboard are 18 x 22 ➔ Per = 2(18 + 22) = **80**



C) Let the side of the base be $2t$.
Let V_1 and v_1 denote the volumes of the original and smaller pyramids respectively.
Then the perimeter of the base is $8t$ and altitude of the pyramid is $3t$.

$V = \frac{1}{3}(2t)^2\cdot 3t = 108$ ➔ $4t^3 = 108$ ➔ $t = 3$

Computing the slant height (l) of the original pyramid,

$3^2 + 9^2 = 90$ ➔ $l = 3\sqrt{10}$

Thus, the linear dimensions of the pyramids are in 3 : 1 ratio and their volumes are is a 27 : 1 ratio.

$$\frac{V_1}{v_1} = \frac{108}{27} = \underline{\mathbf{4}}$$