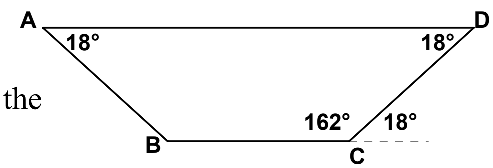
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2006 SOLUTION KEY

Round 6

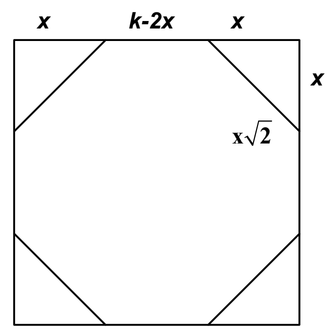
A) Since the diagonals of a rhombus are perpendicular and bisect each other, ΔDEC is a right triangle with legs of length 20 and 21. Using the Pythagorean Theorem (or a common Pythagorean triple), the side of the rhombus is 29. Thus, the required ratio is $\frac{1}{2}\cdot 20\cdot 21 : 4\cdot 29$ → **<u>105 : 58</u>**.

B) Refer to the 4 consecutive vertices A_i, A_{i+1}, A_{i+2} and A_{i+3} as A, B, C and D respectively. Since the other pair of 162° base angles are each interior angles of the regular polygon, the exterior angles measure 18°.



Thus, $\frac{360}{k} = 18$ → $k =$ **<u>20</u>.**

C) Method 1:



$k - 2x = x\sqrt{2}$ → $x(2+\sqrt{2}) = k$ → $x = \frac{k}{2+\sqrt{2}} = \frac{k(2-\sqrt{2})}{2}$

Thus, per(square) = $4k$ and per(octagon) = $8x\sqrt{2} = 4k(2-\sqrt{2})\sqrt{2}$
$= 8k(\sqrt{2}-1)$ and the positive difference is $4k - (8k(\sqrt{2}-1))$
$= 12k - 8k\sqrt{2} = \boxed{\mathbf{4k(3-2\sqrt{2})}}$.

Method 2:
The triangles in the 4 corners are 45 – 45 – 90 triangles. If the sides of these triangles were 1, 1 and , $\sqrt{2}$, then the side of the square would be $2+\sqrt{2}$, the perimeter of the square would be $4(2+\sqrt{2})$ and the perimeter of the octagon would be $8\sqrt{2}$. Since the square has the larger perimeter, the positive difference is $8 - 4\sqrt{2}$. Applying a scale factor of $\frac{k}{2+\sqrt{2}}$ makes the side of the square k. Thus, the positive difference is $(8 - 4\sqrt{2})\cdot\frac{k}{2+\sqrt{2}} =$

$$4(2-\sqrt{2})\cdot\frac{k}{2+\sqrt{2}}\cdot\frac{2-\sqrt{2}}{2-\sqrt{2}} = \frac{4k(2-\sqrt{2})^2}{2} = 2k(4-4\sqrt{2}+2) = \boxed{\mathbf{4k(3-2\sqrt{2})}}.$$