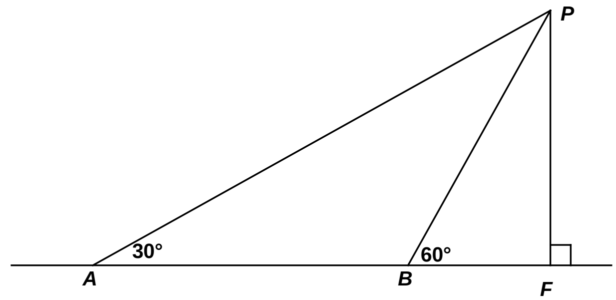
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2017 SOLUTION KEY



Round 3

A) $BF = 10000$. Let $PF = k$ and $AB = x$.

In ΔPBF, $\tan 60° = \sqrt{3} = \dfrac{k}{10{,}000} \Rightarrow k = 10000\sqrt{3}$.

In ΔPAF,

$$\tan 30° = \frac{\cancel{\sqrt{3}}}{3} = \frac{k}{10{,}000 + x} = \frac{10{,}000\cancel{\sqrt{3}}}{10{,}000 + x} \Rightarrow x + 10{,}000 = 30{,}000 \Rightarrow x = \underline{\mathbf{20{,}000}}$$

B) $(\tan x - i\sec x)(\tan x + i\sec x) = \tan^2 x + \sec^2 x = 2\tan^2 x + 1 = 7$

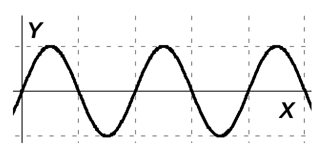
$\Rightarrow \tan x = \pm\sqrt{3} \Rightarrow x = \underline{\boldsymbol{\frac{\pi}{3}, \frac{2\pi}{3}}}$

C) The graph of $y = \sin\left(\frac{5}{2}x\right)$ has a period of $\dfrac{2\pi}{5/2} = \dfrac{4\pi}{5}$

Thus, over the interval $0 \le x < 2\pi$, there are $\dfrac{2\pi}{\frac{4\pi}{5}} = 2 \cdot \dfrac{5}{4} = 2.5$ cycles of the sine function.

$|3y - 2| = 3 \Leftrightarrow 3y - 2 = \pm 3 \Leftrightarrow y = \dfrac{2 \pm 3}{3} = -\dfrac{1}{3}, \dfrac{5}{3}$

Since $-1 \le \sin\frac{5}{2}x \le 1$, $y = \frac{5}{3}$ never intersects the sine graph.



$y = -\frac{1}{3}$ intersects the graph of $y = \sin\left(\frac{5}{2}x\right)$ twice over a complete cycle.
Therefore, there are **4** points of intersection, since the last "half-cycle" is above the x-axis.