Round 5

A) $42 = \dfrac{1}{2}(7x - 3x) \rightarrow x = 21 \rightarrow BC = 360 - 10(21) = \underline{\textbf{150°}}$

B) Let $(x, y) = (CE, DE)$. Thus, $CD = x + y$.

Applying the product chord theorem. $2(10 - 2) = 36 = xy$
As integer factors of 36, the possible ordered pairs are:
$(x, y) = (1, 36), (2, 18), (3, 12), (4, 9)$ and $(6, 6)$ - or vice versa.
Since $CD < 20$, the last three ordered pairs give us
$CD = \underline{\textbf{15, 13 or 12}}$.

C) Let $BD = DE = EC = CP = x$. Using the tangent-secant
relationship $AP^2 = CP \cdot BP$. Substituting,
$108 = x(4x) \rightarrow x^2 = 27 \rightarrow x = 3\sqrt{3}$

Thus, the required area is $\pi\left(\dfrac{9\sqrt{3}}{2}\right)^2 - \pi\left(\dfrac{3\sqrt{3}}{2}\right)^2$

$= \pi\left(\dfrac{81 \cdot 3 - 9 \cdot 3}{4}\right) = \pi\left(\dfrac{3 \cdot 72}{4}\right) = \underline{\textbf{54}\pi}$