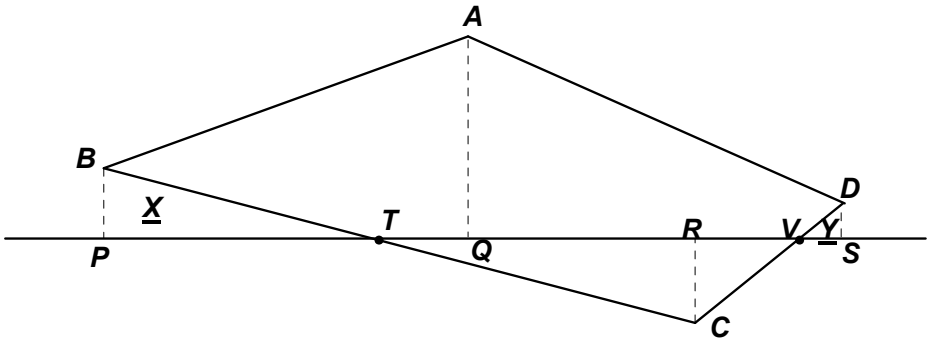
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2010 SOLUTION KEY

Team Round

C) Compared to $ABCD$, the trapezoids $ABPQ$ and $ADSQ$ combined include some regions that should be excluded and exclude some regions that should be included. ΔBPT and ΔDSV should be excluded, while ΔCRT and ΔCRV should be included.



Since $\Delta BPT \sim \Delta CRT$ and $BP : CR = 4 : 5$, Area(ΔBPT) : Area(ΔCRT) = 16 : 25.
Since $\Delta DSV \sim \Delta CRV$ and $DS : CR = 2 : 5$, Area(ΔDSV) : Area(ΔCRV) = 4 : 25.
Let (X, Y) denote the areas of ΔBPT and ΔDSV respectively. Then

$$\text{Area}(ABCD) = \text{Area}(ABPQ) + \text{Area}(ADSQ) - \underline{X} - \underline{Y} + \frac{25}{16}\underline{X} + \frac{25}{4}\underline{Y}$$

$$= \text{Area}(ABPQ) + \text{Area}(ADSQ) + \frac{9}{16}\underline{X} + \frac{21}{4}\underline{Y} \rightarrow (a, b) = \underline{\left(-\frac{\mathbf{9}}{\mathbf{16}}, -\frac{\mathbf{21}}{\mathbf{4}}\right)}.$$