MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2012
ROUND 7 TEAM QUESTIONS

ANSWERS

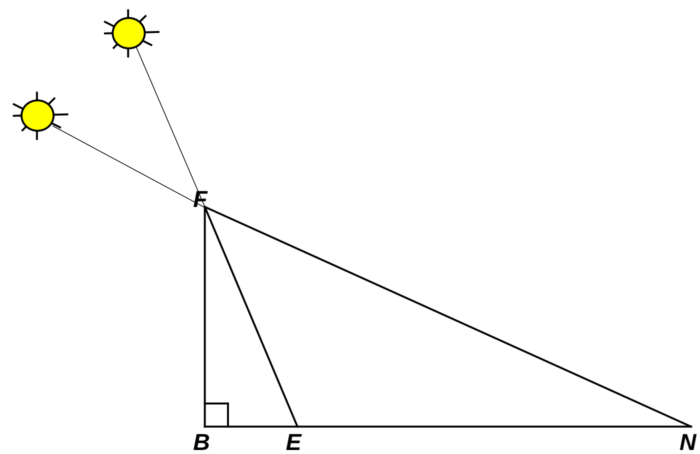
A) ____________________________ D) (__________ , ___________)

B) _______________________ft. E) ____________________________

C) (____ , ____ , ____ , ____) F) (______ , _______ , ______)

A) A rectangular block of ice cream has dimensions n inches , $(n-1)$ inches and $(n-2)$ inches, for $n \geq 4$. It is completely covered with chocolate (like a Klondike Bar). Suppose it is then cut up into one inch cubes by making cuts parallel to the faces. Let A be the number of cubes with 1 or 3 faces covered with chocolate. Let B be the number of cubes with 0 or 2 faces covered with chocolate. Compute all values of n for which $A = B$.

B) At 9:00 AM, a 12-foot flagpole casts a shadow 68 feet longer than it will at 11:30 AM. If the perimeter of ΔFEN is 153, compute the length (in feet) of the shadow at 11:30.



C) Solve the following system of equations

$$\begin{cases} 64A+16B+4C+D=204 \\ 27A+9B+3C+D=104 \\ 8A+4B+2C+D=46 \\ A+B+C+D=18 \end{cases}$$

Express your answer as the ordered 4-tuple (A, B, C, D).

D) Ted Williams was the last Major League Baseball player to bat over 0.400 for an entire season. Through the next to the last day of the season he had 179 hits in 448 trips to the plate for an average of 0.39995. This would have rounded to 0.400 and he could have sat out the last day and backed into the history books. He chose to play on the last day in both games of a doubleheader.
If he had gone hitless in x plate appearances, his average would have dropped below 0.393.
Let x be the minimum number of plate appearances for which this would happen.
He actually got h hits in x plate appearances and finished the season with an average of 0.4057 which put him in the record books at 0.406. Determine the ordered pair (h, x).

E) Solve for x: $\sqrt{|x+4|}+\sqrt{|x-1|}=\sqrt{|x-4|}$

F) The sum $\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+...+\frac{1999}{2000!}$ may be expressed in simplified form as $A-\frac{B}{C!}$, where A, B and C are positive integers. Compute the ordered triple (A, B, C), where the sum $A + B + C$ is a minimum.