MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2009 SOLUTION KEY

Team Round – continued

F) $3-\dfrac{1}{\frac{1}{A}+\frac{1}{x}}=\dfrac{1}{3} \rightarrow \dfrac{8}{3}=\dfrac{1}{\frac{1}{A}+\frac{1}{x}}$

$\rightarrow \dfrac{8}{3}=\dfrac{Ax}{x+A} \rightarrow 8x+8A=3Ax$

$\rightarrow 8A=x(3A-8) \rightarrow x=\dfrac{8A}{3A-8}=\dfrac{8}{3-\frac{8}{A}}$

A = 1, 2, 3, 4 5, 6, … → x = –8/5, –8, **<u>24</u>**, 8, 40/7, 24/5…

Alternate solution:

After getting $x=\dfrac{8A}{3A-8}$. If x is an integer then so is $3x$.

Since $3x=\dfrac{24A}{3A-8}$, by long division, $3x=8+\dfrac{64}{3A-8}$

Clearly, the values of $3x$ increase until the fractional component on the right hand side becomes negative, i.e. when $A < 3$, and thereafter they decrease. $A = 3 \rightarrow 3x=8+\dfrac{64}{1}=72 \rightarrow x=$ **<u>24</u>**.