MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 – NOVEMBER 2014 SOLUTION KEY

E) Let $m = 3j$ and $n = 3k$ for positive integers *j* and *k*.

$$\frac{n\pi}{3} + \frac{m\pi}{6} = \frac{\pi}{6}(2n+m) = \frac{\pi}{6}(6j+3k) = \frac{\pi}{2}(2j+k)$$

Since *j* and *k* are positive integers, so is $2j+k$.

Can we get all non-coterminal multiples of $\frac{\pi}{2}$?

Yes! $2j+k$ produces all and only quadrantal values

$(j,k) = (1,1) \Rightarrow \frac{3\pi}{2}$ and $\sin\left(\frac{3\pi}{2}\right) = \underline{\mathbf{-1}}$

$(j,k) = (1,2) \Rightarrow \frac{4\pi}{2}$ and $\sin(2\pi) = \underline{\mathbf{0}}$

$(j,k) = (2,1) \Rightarrow \frac{5\pi}{2}$ and $\sin\left(\frac{5\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right) = \underline{+\mathbf{1}}$

$(j,k) = (2,2) \Rightarrow \frac{6\pi}{2}$ and $\sin(3\pi) = \sin(\pi) = 0$

F) Let (A,B,C) denote the interior angles with measures $(x, y, 3x-2y)$.

Triangle Sum $\Rightarrow$ $4x - y = 180 \Rightarrow y = 4x - 180$

Substituting in $x + y < 120$, $5x < 300 \Rightarrow x < 60$.

Thus, our starting point is $x = 59 \Rightarrow (59, 56, 180 - 115 = 65)$

Decreasing *x* by 1, decreases *y* by 4, and consequently the third interior angle will increase by 5.
We must stop when the largest angle *C* becomes a right angle.
Possible measures of *C* are 65, 70, 75, 80 and 85, implying we have **5** possible *x*-values.