

MASSACHUSETTS MATHEMATICS LEAGUE
NOVEMBER 2003
ROUND 5: TRIGONOMETRY
NON-CALCULATOR

ANSWERS

A) $\frac{(15 + 7\sqrt{3})}{6}$

B) $45^\circ, 90^\circ, 135^\circ$

C) 24

A) Evaluate and leave the result in simplest radical form.

$$\sin 30^\circ + \cos 30^\circ + \cot 225^\circ + \tan 135^\circ + \csc 120^\circ + \sec 300^\circ$$

$$\frac{1}{2} + \frac{\sqrt{3}}{2} + 1 - 1 + \frac{2}{\sqrt{3}} + 2$$

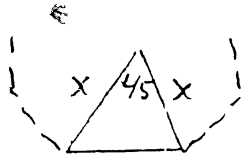
$$\frac{\sqrt{3}}{2\sqrt{3}} + \frac{3}{2\sqrt{3}} + \frac{4}{2\sqrt{3}} + \frac{4\sqrt{3}}{2\sqrt{3}} = \frac{7 + 5\sqrt{3}}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{15 + 7\sqrt{3}}{6}$$

B) Solve for $0^\circ \leq x < 180^\circ$: $\cos(x) \cot^2(x) = \cos(x)$

$$\cos x = 0 \quad \cot^2 x = 1, \quad \tan x = \pm 1$$

$$x = 90^\circ, \quad x = 45^\circ, 135^\circ$$

C) The area of a regular octagon is $288\sqrt{2}$. The circumference of its circumscribed circle is $k\pi$. What is the value of k ?



$$8 \cdot \frac{1}{2} x^2 \sin 45^\circ = 288\sqrt{2}$$

$$4x^2 \frac{\sqrt{2}}{2} = 288\sqrt{2}$$

$$2x^2 = 288, \quad x^2 = 144$$

$$x = 12$$

$$C = 2\pi x = 24\pi$$

$$k = 24$$