MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2013 SOLUTION KEY

Round 6

A) Substituting $2C$ for F and solving, $C=\frac{5}{9}(2C-32)\Leftrightarrow 9C=10C-160\Leftrightarrow C=\underline{\mathbf{160}}$.

Check: $160\overset{?}{=}\frac{5}{9}(320-32)\qquad \frac{5}{9}(288)=5\cdot 32\overset{\checkmark}{=}160$

B) $3x^2+2x=1\Leftrightarrow(3x-1)(x+1)=0\Rightarrow x=\frac{1}{3},\,-1$

Substituting, $x=\frac{1}{3}\Rightarrow\frac{10}{2x-1}=-30$ and $x=-1\Rightarrow-\frac{10}{3}$. Thus, the minimum value is **<u>–30</u>**.

C) $|2x-c|<10\Leftrightarrow-10<2x-c<+10$. Isolating x, we have $\frac{c-10}{2}<x<\frac{c+10}{2}$.

For values of $c\geq 10$, there are no negative solutions.
Thus, we examine positive integer values of c < 10.

$c = 9\Rightarrow-\frac{1}{2}<x<\frac{19}{2}\Rightarrow$ 0 negative and 9 positive integer solutions.
$c = 8\Rightarrow-1<x<9\Rightarrow$ 0 negative and 8 positive integer solutions.
$c = 7\Rightarrow-\frac{3}{2}<x<\frac{17}{2}\Rightarrow$ 1 negative and 8 positive integer solutions.
Continuing,… c = **<u>3</u>** $\Rightarrow-\frac{7}{2}<x<\frac{13}{2}$ and we have 3 negative solutions,
namely x = –1, –2,–3, and 6 positive solutions, namely x = 1, …, 6.