MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2015 SOLUTION KEY

Round 5

A) Given: $x+2=y+1=a$, $\dfrac{x}{y}=\dfrac{a}{b}$ and $x+y=6$.

$x+y=6 \Leftrightarrow \left(a-2\right)+\left(a-1\right)=6 \Rightarrow a=4.5$, $x=2.5$, $y=3.5$

$\dfrac{2.5}{3.5}=\dfrac{4.5}{b} \Leftrightarrow \dfrac{5}{7}=\dfrac{9}{2b} \Rightarrow 10b=63 \Rightarrow b=\underline{\mathbf{\dfrac{63}{10}}}$ or $\underline{\mathbf{6.3}}$.

B) $\dfrac{2n^2+13n-24}{2n^3-8n}=\dfrac{(2n-3)(n+8)}{2n(n+2)(n-2)}$.

The ratio is zero when the numerator is zero, namely when $n=\dfrac{3}{2}$ or -8 .

The ratio is undefined when the denominator is zero, namely when $n=0$ or ± 2.

Therefore, $(K,J)=\underline{\left(\mathbf{5,-8}\right)}$.

C) Given: $\dfrac{3a+7}{b+2}=\dfrac{5}{6}$ and $b=ka$ Substituting for b and cross multiplying, we have

$18a+42=5ka+10 \Rightarrow a=\dfrac{32}{5k-18}$ and $5k-18$ must be a factor of 32. Thus,

$5k-18=1,2,4,8,16$ or $32 \Rightarrow 5k=19,20,22,26,34$ or 50.

The only possible integer values of k are 4 or $\cancel{10}$ (10 is rejected since $k=10 \Rightarrow a=1$.)

$k=4 \Rightarrow a=32/2=16$ and $b=4a \Rightarrow$ $(a,b)=\underline{\left(\mathbf{16,64}\right)}$.