MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2011 SOLUTION KEY

Round 2

A) $x^2+2x+1=10000$ → $(x+1)^2=10000=10^4$ → $x+1=\pm 10^2$ → $x=$ **99, –101**.

B) Factoring out a 4, the first and third terms are perfect squares.
$36x^2 - kx + 16 = 4\left((3x)^2 + \_\_\_ + (2)^2\right)$. Since A, B and C are positive integers,
$A(Bx-C)^2 = 4(3x-2)^2$ and $k = 4(2)(3)(2) =$ **48**.

C) $A^2-4x=x^2+4A$ → $\left(x^2+4x+4\right)=A^2-4A+4$ → $(x+2)^2=(A-2)^2$ → $x+2=\pm(A-2)$
→ $x=$ ***A – 4, –A***

Round 3

A) $\text{Tan}(5x) = 1$ → $5x = 45° + 180n$ → $x = 9° + 36n$
$n = 0, 1, 2, 3, \ldots$ → $x = 9°, 45°, 81°, 117°, \ldots$ → $k =$ **117**

B) $(\sin 2x)\left(\cos^4 x-\sin^4 x\right)=(\sin 2x)\left(\cos^2 x-\sin^2 x\right)\left(\cos^2 x+\sin^2 x\right)=\sin 2x\cos 2x=\frac{1}{2}\sin 4x=0$

→ $4x = 0 + n\pi$ → $x = \mathbf{0}, \frac{\boldsymbol{\pi}}{\mathbf{4}}, \frac{\boldsymbol{\pi}}{\mathbf{2}}, \frac{\mathbf{3}\boldsymbol{\pi}}{\mathbf{4}}$ (in any order)

C) $\sin\left(2x+\frac{\pi}{4}\right)=\cos\left(\frac{7\pi}{4}\right)$ → $\sin\left(2x+\frac{\pi}{4}\right)=+\frac{\sqrt{2}}{2}$ → $2x+\frac{\pi}{4}=\begin{cases}\frac{\pi}{4}+2n\pi\\ \frac{3\pi}{4}+2n\pi\end{cases}$ → $x=\begin{cases}n\pi\\ \frac{\pi}{4}+n\pi\end{cases}$

→ $x=\pi, 2\pi, \frac{5\pi}{4}, \frac{9\pi}{4}$ (in any order)