MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2013 SOLUTION KEY

Round 3

A) Multiplying by 60, $\frac{2x}{15}+\frac{y}{4}=1 \Leftrightarrow 8x+15y=60 \Leftrightarrow y=-\frac{8}{15}x+4$

Since the equation is now in $y=mx+b$ form, the required order pair is $\underline{\left(-\frac{\mathbf{8}}{\mathbf{15}},\mathbf{4}\right)}$.

B) $4\left(x^2-3x+\underline{\frac{9}{4}}\right)+4\left(y^2+5y+\underline{\frac{25}{4}}\right)=-18+9+25 \Leftrightarrow \left(x-\frac{3}{2}\right)^2+\left(y+\frac{5}{2}\right)^2=4$

$\Rightarrow$ Center: $\left(\frac{3}{2},-\frac{5}{2}\right)$ and radius 2 $\Rightarrow \left(\frac{3}{2}\pm 2,-\frac{5}{2}\right) \Rightarrow \underline{\boldsymbol{A}\left(-\frac{\mathbf{1}}{\mathbf{2}},-\frac{\mathbf{5}}{\mathbf{2}},\right)\boldsymbol{B}\left(\frac{\mathbf{7}}{\mathbf{2}},-\frac{\mathbf{5}}{\mathbf{2}},\right)}$

C) The equation of line $\mathscr{L}$ is $(y+2)=-\frac{1}{2}(x-13) \Leftrightarrow x+2y=9$

A radius through the center drawn to the point of contact will have slope +2.
Its equation is $(y+2)=2(x-3) \Leftrightarrow 2x-y=8$
The point of tangency T is the intersection of these two lines.

$\begin{cases} x+2y=9 \\ 2x-y=8 \end{cases} \Rightarrow \begin{cases} x+2y=9 \\ 4x-2y=16 \end{cases} \Rightarrow 5x=25 \Rightarrow (x,y)=(5,2)$

The radius of this circle is the distance from C to T, namely $\sqrt{(2+2)^2+(5-3)^2}=\sqrt{20}$.

Therefore, the required equation is $\underline{\left(\boldsymbol{x}-\mathbf{3}\right)^{\mathbf{2}}+\left(\boldsymbol{y}+\mathbf{2}\right)^{\mathbf{2}}=\mathbf{20}}$.

Alternately, the point-to-line distance formula could have been used to find the radius.

The distance from $(3,-2)$ to $x+2y-9=0$ is $\frac{|3\cdot 1+(-2)\cdot 2+(-9)|}{\sqrt{1^2+2^2}}=\frac{10}{\sqrt{5}}=2\sqrt{5}$ which is the radius of the given circle, resulting in the same equation as above.