Team Round – continued

D) For $a=1$, $L_1\left(y=ax+b\right)$ and $L_2\left(\frac{x}{a}-\frac{y}{b}=1\right)$ do not intersect on $L_3\left(y=x\right)$,

since L_1 is parallel to L_3 for nonzero values of *b*. Therefore, all ordered pairs $\left(a,b\right)$, where $a=1$ must be excluded, regardless of the value of *b*, and we have $p=1$.

How about when $a\neq 1$? Substituting for *y* in the second equation,

$$\left.\begin{array}{l} y=ax+b \\ \frac{x}{a}-\frac{y}{b}=1 \end{array}\right\}\Rightarrow bx-ay=ab$$

$$\Rightarrow bx-a\left(ax+b\right)=ab$$

$\Rightarrow x\left(b-a^2\right)=2ab\Rightarrow \boxed{x=\frac{2ab}{b-a^2}}$ Substituting in the first equation for *x*,

$$y=a\left(\frac{2ab}{b-a^2}\right)+b=\frac{2a^2b+b^2-a^2b}{b-a^2}=\frac{a^2b+b^2}{b-a^2}$$

Thus, the point of intersection is $\left(\frac{2ab}{b-a^2},\frac{a^2b+b^2}{b-a^2}\right)$.

Since, we were given that L_1 and L_2 intersect, $b-a^2\neq 0$.

P can only be on L_3, if $2ab=a^2b+b^2$. Since $b\neq 0$, this simplifies to $2a=a^2+b$ or $b=-a^2+2a$.

Thus, if $b\neq -a^2+2a$, the point $\left(a,b\right)$ cannot be on L_3, and we have $\left(p,q,r\right)=\underline{\mathbf{\left(1,-1,2\right)}}$.