MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 – MARCH 2007 SOLUTION KEY

Round 1

A) Adding and substituting $x = 5$, $2ax = 20 \rightarrow$ **$\underline{a = 2}$**
Subtracting and substituting $y = 15$, $2by = -10 \rightarrow$ **$\underline{b = -1/3}$**

B) The matrix equation is equivalent to: $\begin{cases} 3x + ay = 7 \\ bx + 4y = -6 \end{cases}$. Substituting, $\begin{cases} 15 + a = 7 \\ 5b + 4 = -6 \end{cases}$

$\rightarrow (a, b) = (-8, -2)$
Evaluating the determinant, $12 - (-8)(-2) =$ **$\underline{-4}$**

C) Think of -21 as $0x^2 + 0x - 21$

As a single fraction, the left hand side is $\dfrac{A(2x^2 + 5x - 3) + B(x^2 + 3x) + C(2x^2 - x)}{2x^3 + 5x^2 - 3x}$

Re-arranging terms, $\dfrac{(2A + B + 2C)x^2 + (5A + 3B - C)x - 3A}{2x^3 + 5x^2 - 3x}$

Thus, $\begin{cases} 2A + B + 2C = 0 \\ 5A + 3B - C = 0 \\ -3A = -21 \end{cases} \rightarrow A = 7$ and $\begin{cases} B + 2C = -14 \\ 3B - C = -35 \end{cases} \rightarrow (B, C) = (-12, -1)$

and the required sum is $7 + (-12) + (-1) =$ **$\underline{-6}$**.

Round 2

A) Trial and error $\rightarrow (a, b, c, d) = (1, -2, -4, 3) \rightarrow (1)^{-2} - (-4)^3 = 1 + 64 =$ **$\underline{65}$**

B) $x - y = 4$ and $2^{4x-2y} = 2^{24x-6y} \rightarrow 2^{20x-4y} = 1 \rightarrow 20x - 4y = 0$ or $5x - y = 0$
Solving simultaneously, $4x = -4 \rightarrow$ **$\underline{x = -1}$**. Substituting back, **$\underline{y = -5}$**.

C) The radicand must represent a perfect square, call it $(a + b\sqrt{3})^2 = a^2 + 3b^2 + 2ab\sqrt{3}$
For integer values of a and b, $a^2 + 3b^2$ must represent an integer and $2ab\sqrt{3}$ a multiple of $\sqrt{3}$.
Thus, $a^2 + 3b^2 = 48$ and $ab = -12$. Clearly, a and b have opposite signs and checking out factors of 12 in the first equation produces either $(6, -2) \rightarrow \boxed{\mathbf{6 - 2\sqrt{3}}}$ which is positive or $(-6, 2) \rightarrow -6 + 2\sqrt{3}$ which is a negative value and must be rejected. Thus, **$\underline{a = 6, b = -2, c = 3}$**