MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2008 SOLUTION KEY

Team Round – F) continued

The longer list!

Here's the distribution of those 773 positive reduced fractions with denominators ≤ 50.

Less than N and relatively prime to N

N	**RP(N)**	N	**RP(N)**
		26	**12**
2	**1**	27	**18**
3	**2**	28	**12**
4	**2**	29	**28**
5	**4**	30	**8**
6	**2**	31	**30**
7	**6**	32	**16**
8	**4**	33	**20**
9	**6**	34	**16**
10	**4**	35	**24**
11	**10**	36	**12**
12	**4**	37	**36**
13	**12**	38	**18**
14	**6**	39	**24**
15	**8**	40	**16**
16	**8**	41	**40**
17	**16**	42	**12**
18	**6**	43	**42**
19	**18**	44	**20**
20	**8**	45	**24**
21	**12**	46	**22**
22	**10**	47	**46**
23	**22**	48	**16**
24	**8**	49	**42**
25	**20**	50	**20**
	199		*574*
		Total:	**773**

Some usual facts for computing the number of positive integers less than N that are relatively prime to N, i.e. RP(N):

If N is prime, RP(N) = $N - 1$
If A and B are relatively prime, RP(AB) = RP(A) · RP(B)

The usual notation for RP(N) is $\phi(N)$ and it is referred to as the Euler ϕ function (pronounced "fee") after the master mathematician Leonard Euler (1707 – 1783).
Perhaps you can 'discover' a formula for computing the values in the table above!

When ordered from smallest to largest the 495th fraction is 23/36.
Code the algorithm suggested above and check it out!