MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2012 SOLUTION KEY

Round 3

A) $a+b-c \div d \cdot x = 1 \Leftrightarrow a+b-\frac{cx}{d}=1 \Leftrightarrow \frac{cx}{d}=a+b-1 \Leftrightarrow x = \underline{\boldsymbol{\frac{d(a+b-1)}{c}}}$ or $\underline{\boldsymbol{\frac{ad+bd-d}{c}}}$

Equivalent expressions are acceptable (switching the order of the terms in the numerator or the order of the factors in the products in the numerator) as long as the expression contains exactly one minus sign.

B) The sum of the terms in the numerator is $\frac{n(n+1)}{2}$.

If you were unfamiliar with this result, here's how it was derived.
Let S denote the sum of the terms in the numerator of the left hand expression.
Reversing the order of the sum, of course, has no effect.

$$\begin{cases} S = 1 + \quad 2 \quad + \quad 3 \quad + \ldots + n \\ S = n + (n-1) + (n-2) + \ldots + 1 \end{cases}$$

But notice that in each of the n columns the sum is $(n + 1)$.

Thus, adding, $2S = (n + 1)n \Rightarrow S = \frac{n(n+1)}{2}$.

This means the equation simplifies to $\frac{n+1}{2} = 2012 \Rightarrow n =$ **<u>4023</u>**.

C) Let (D, Q) denote the number of dimes and quarters respectively in the collection.

$10D + 25Q = 4975 \Rightarrow 2D + 5Q = 995 \Rightarrow Q = \frac{995-2D}{5} = 199 - \frac{2}{5}D$

The smallest value of D is 5, resulting in $(D, Q) = (5, 197)$

For every 5 dimes added, 2 fewer quarters are required $\Rightarrow$ (10, 195), (15, 193), …, (495, 1)

Thus, the maximum number of coins required is 496, the minimum is 202 and $n = \frac{495}{5} = 99$.

$\Rightarrow$ **<u>(99, 202, 496)</u>**