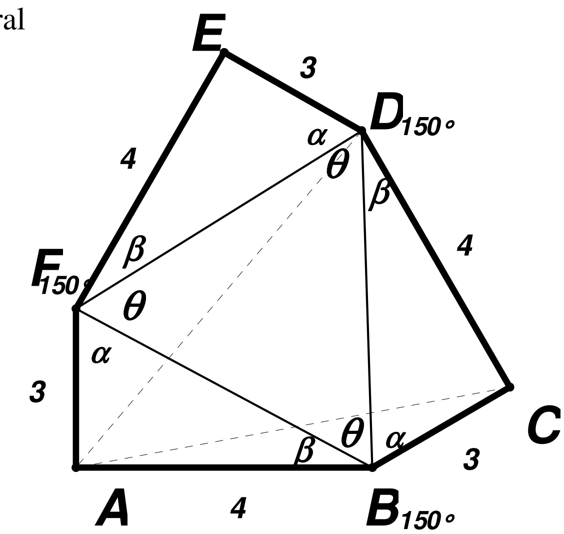
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2008 SOLUTION KEY

Team Round - continued

E) Let doubles ($2B$) = $3n$, triples ($3B$) = n and homeruns $HR = 2n$. Then:
Singles ($1B$) = $H - (2B + 3B + HR) \rightarrow 1B = 35 - (3n + n + 2n) = 35 - 6n$
Numerator = $(35 - 6n) + 2(3n) + 3(n) + 4(2n) = 35 + 11n$
Denominator = $120 - SAC - (BB + HBP) = 120 - 5 - 5 = 110$
$\dfrac{35+11n}{110} = 0.618 \rightarrow n = 2.998 \approx 3 \rightarrow 1B = 35 - 6(3) = \underline{\mathbf{17}}$

F) Clearly, $BD = DF = FB = 5$ and ΔBDF is both equilateral and equiangular ($\theta = 60°$).



Using the law of cosines in ΔABC,
$AC^2 = 3^2 + 4^2 - 2\cdot 3\cdot 4\cdot \cos 150°$
$\rightarrow AC^2 = 25 + 12\sqrt{3}$
Using the law of cosines on ΔFAD,
$AD^2 = 3^2 + 5^2 - 2\cdot 3\cdot 5\cdot \cos(\alpha + 60°)$
$= 34 - 30\cos(\alpha + 60°)$
$= 34 - 30(\cos\alpha\cos 60° - \sin\alpha\sin 60°)$
$= 34 - 30\left(\dfrac{3}{5}\cdot\dfrac{1}{2} - \dfrac{4}{5}\cdot\dfrac{\sqrt{3}}{2}\right)$
$= 34 - 30\left(\dfrac{3-4\sqrt{3}}{10}\right) = 34 - 9 + 12\sqrt{3} = 25 + 12\sqrt{3} = AC^2$
Thus, $AC^2 - AD^2 = \underline{\mathbf{0}}$

[In this version of the problem, 3-4-5 triangles were 'attached' to the sides of an equilateral triangle. Can this problem be generalized by starting with an equilateral triangle and 'attaching' other types of congruent triangles to each side?]