MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2009 SOLUTION KEY

Round 4

A) If T (hours) denotes the time both machines are working together, then $\frac{T+\frac{1}{4}}{2}+\frac{T}{3}=1$.

Multiplying by 24, $12T + 3 + 8T = 24 \rightarrow T = 21/20$ hours = **<u>63</u>** minutes

B) $2008(x - 2008) = 2009(x - 2009)$

$2008x - 2008^2 = 2009x - 2009^2$

$x = 2009^2 - 2008^2 = (2009 + 2008)(2009 - 2008) =$ **<u>4017</u>**

C) $\begin{cases} N = 100h + 10t + u \\ t = u/2 \rightarrow u = 2t \\ h = u + t \end{cases}$

Thus, $h = 3t$ and $t \leq 4 \rightarrow N = 300t + 10t + 2t = 312t = 2^3 \cdot 3 \cdot 13t$

So, clearly, N is divisible by 3, 4, 6 and 13 $\rightarrow d =$ **<u>7</u>**