MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2017
ROUND 7 TEAM QUESTIONS

ANSWERS

A) (_____ , _____ , _____) D) _____

B) _____ E) _____

C) _____ F) _____

A) Given: $\begin{cases} ax+by+cz=216 \\ bx+cy+az=60 \\ cx+ay+bz=84 \end{cases}$

If $a:b:c=1:2:3$ and $x+y+z=5$, compute the ordered triple (x,y,z).

B) Express the following as a simplified fraction with a rationalized denominator: $\dfrac{\sqrt{3+2\sqrt{2}}}{2\sqrt{1+\sqrt{2}}}$

C) Let *P* denote the infinite product $16cis\frac{\pi}{3}\cdot cis\frac{\pi}{6}\cdot cis\frac{\pi}{12}\cdot cis\frac{\pi}{24}\cdot...$, where the first term has a coefficient of 16 and the pattern of multiplying the angle by $\frac{1}{2}$ is continued forever.

$\sqrt{P}$, expressed in rectangular form, is $A + Bi$. Compute all possible ordered pairs (*A*, *B*).

D) 300 people were asked how they voted during an election.
3 candidates (*A*, *B* and *C*) were running to fill vacant positions on the Board of Selectman
The results were reported in the following curious manner:

- 54 people voted for *A* and *B*
- 66 people voted for *B* only
- 186 people voted for *A* or *B*, but not *C*
- 42 voted for *A* and *C*
- 51 voted for *B* and *C*
- 45 voted for *C* only

If each person voted for *at least* one of these three candidates, how many people voted for *B if and only if* they did not vote for *C*?

FYI:

The statement "*P* if and only if *Q*" $(P \Leftrightarrow Q)$ is a called a *bi-conditional*.

It is equivalent to "(*P*, if *Q*) or (*Q*, if *P*)", or, symbolically, $(P \Rightarrow Q) \vee (Q \Rightarrow P)$.

The bi-conditional is logically equivalent to "*P* and *Q*" or "not *P* and not *Q*".