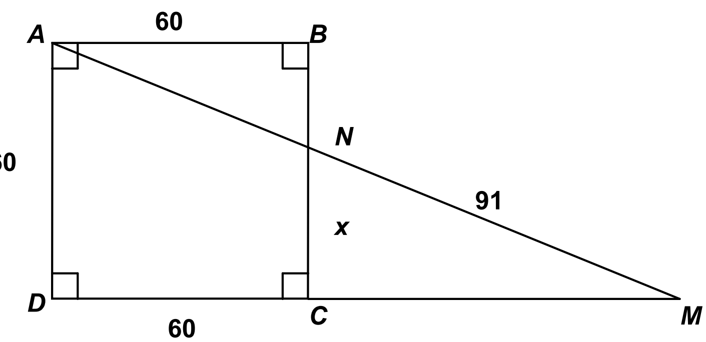
Team Round - continued

E) Solution #3 (Norm Swanson – Hamilton-Wenham/M.I.T.)



Since $\Delta ABN \sim \Delta MCN$, let $BN = kx$, then

$x + kx = 60 \rightarrow x = \dfrac{60}{k+1}$ and $CM = \dfrac{60}{k}$

By the Pythagorean Theorem, we have

$\left(\dfrac{60}{k}\right)^2 + \left(\dfrac{60}{k+1}\right)^2 = 91^2$.

Since 91 = 7(13), let $r = \dfrac{60}{7}$ and rewrite the equation as $\left(\dfrac{r}{k}\right)^2 + \left(\dfrac{r}{k+1}\right)^2 = 13^2$. Recalling that 5-12-13 is a pythagorean triple, we try $\dfrac{r}{k} = 12$ and $\dfrac{r}{k+1} = 5$ (the larger the denominator, the smaller the fraction).

Since $x = \dfrac{60}{k+1}$, solving for $k + 1$ will give us x.

$\dfrac{r}{k+1} = 5 \rightarrow \dfrac{60/7}{k+1} = 5 \rightarrow k+1 = \dfrac{12}{7}$. Thus, $x = \dfrac{60}{12/7}$ = **35**.