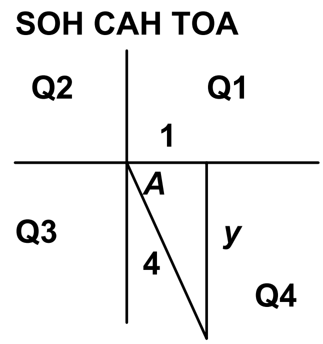
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2006 SOLUTION KEY

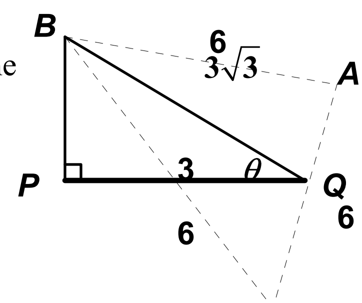
Round 5

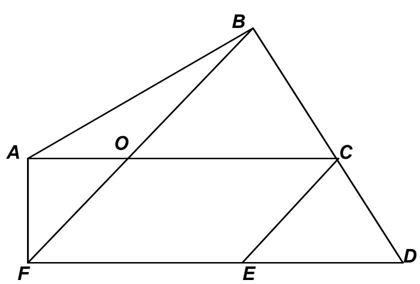
A) $\angle A$ must be located in quadrant 4, since the cosine is positive and the tangent is negative. $y^2 = 16 - 1 = 15$ and $y < 0 \rightarrow y = -\sqrt{15}$
Using cofunction identities (or complementary angle relationships),
$\sin(90 - A) \cdot \cos(90 - A) = \cos(A) \cdot \sin(A) = \dfrac{\sin A}{4} = \underline{\dfrac{\mathbf{-\sqrt{15}}}{\mathbf{16}}}$

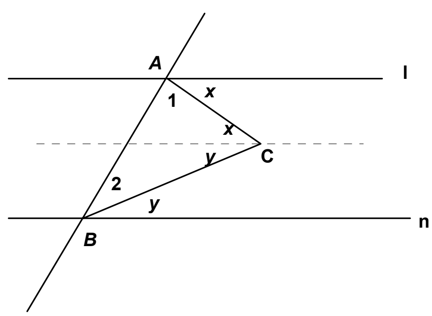


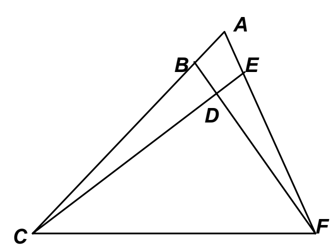
B) Since $\sin^2\theta + \cos^2\theta = 1$,
$$\cos^2(\tfrac{\pi}{9}) + \cos^2(\tfrac{2\pi}{9}) + \cos^2(\tfrac{3\pi}{9}) + \cos^2(\tfrac{4\pi}{9}) \;+$$
$$\sin^2(\tfrac{\pi}{9}) + \sin^2(\tfrac{2\pi}{9}) + \sin^2(\tfrac{3\pi}{9}) + \sin^2(\tfrac{4\pi}{9}) \;= 4$$
and $\cos^2(\tfrac{\pi}{9}) + \cos^2(\tfrac{2\pi}{9}) + \cos^2(\tfrac{3\pi}{9}) + \cos^2(\tfrac{4\pi}{9}) = 4 - \dfrac{a}{b} = \underline{\dfrac{\mathbf{4b - a}}{\mathbf{b}}}$

C) The slant height of the pyramid is the altitude from B in each equilateral triangle. Let P denote the center of the square base and Q the foot of one of these altitudes. Then $AB = 6 \rightarrow BQ = 3\sqrt{3}$ and $PQ = 3$
Using the Pythagorean Theorem, $BP^2 = 27 - 9 = 18 \rightarrow BP = 3\sqrt{2}$
Thus, the angle formed by a face with the base of the pyramid is $\angle\theta$ as indicated in the diagram at the right. $\sin(\theta) = \dfrac{3\sqrt{2}}{3\sqrt{3}} = \boxed{\dfrac{\mathbf{\sqrt{6}}}{\mathbf{3}}}$









Round 6

A) Since ΔCED is an isosceles triangle with a base angle of 69°, its vertex angle CED has a measure of 42°. Thus, $\overline{OC}$ is both parallel and congruent to $\overline{FE}$, forcing $OCEF$ to be a parallelogram. Since opposite angles in a parallelogram are congruent, m$\angle OFE$ = 42° $\rightarrow$ m$\angle AFO = 93 - 42 = 51°$. Finally, in ΔFAB, x = m$\angle FAB = 180 - (20 + 51) =$ **<u>109</u>**

B) 4(m$\angle$1 + m$\angle$1) = 180 $\rightarrow$ m$\angle C$ = 135°. Draw a line thru point C parallel to n.
Since alternate interior angles of parallel lines are congruent, $x + y = 135 \rightarrow y =$ **<u>135 - *x*</u>**

C) Since $\Delta ACE \cong \Delta AFB$ (by SAA), $AC = AF$ and ΔACF is isosceles w/base CF.
$BC = 6 \rightarrow EF = 6 \rightarrow AE = 2 \rightarrow AC = 8$
Thus, $8 = CF/2 + 6 \rightarrow CF =$ **<u>4</u>**