

MASSACHUSETTS MATHEMATICS LEAGUE

JANUARY 2006

ROUND 7: TEAM QUESTIONS

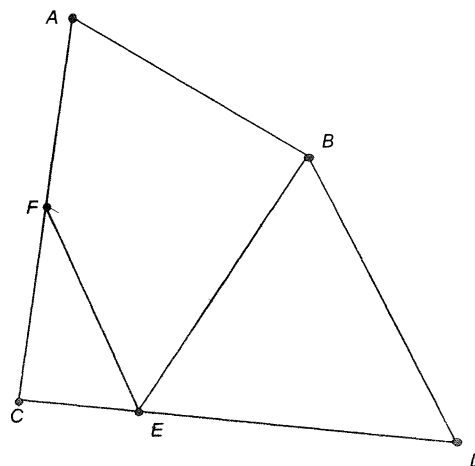
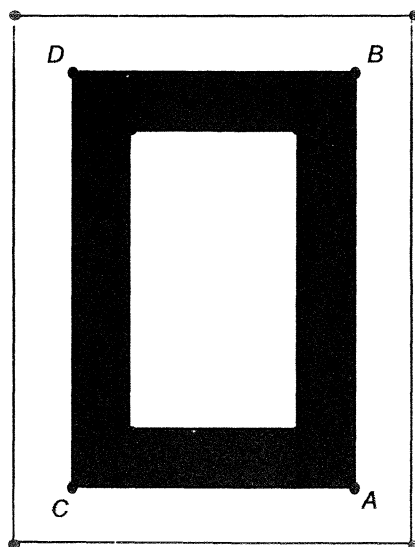
ANSWERS

A) _____ D) _____

B) _____ E) _____:

C) _____ F) _____

- A) An ellipse with foci at $(12, 16)$ and $(12, 9)$ passes through the origin. If $(w, 9)$ lies on the ellipse, find all possible values of w .
- B) Factor $4(3b+1)^4 + 3(3b+1)^2 + 1$ over the polynomials with integer coefficients.
- C) Find all pairs of complimentary angle measures x and y with $0 < x < y$ such that x is a solution to $2\cos^2(6x) = \sin(12x)$ and y is a solution to $2\sin(12y) + 2 = \cos^2(12y)$. Give your answers as ordered pairs (x, y) .
- D) The inner rectangle below is surrounded by two bands of the same uniform width. If the shaded region is one third the area of the largest rectangle and $ABDC$ is a 10 by 16 rectangle, what are both possible areas for the innermost rectangle?



- E) In the above sketch on the right $ACDB \sim ABEF$ with $AC = DC = 12$ and $AB = 9$. Find the ratio of the area of $\triangle DFC$ to the area of $\triangle DAF$ as a simplified ratio of integers.
- F) Sue and Bob each received money from Uncle Joe to buy mini music CDs; since they weren't the same age they got different amounts. SuperStore has a music club where you pay \$15 to join then pay \$2 per CD, while GrandSong charges just \$5 to join then \$6 per CD. Sue claimed her money would buy twice as many songs at one store than at the other and Bob claimed the same thing about his different amount of money! How much money did Uncle Joe send in total to Sue and Bob?