MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2014 SOLUTION KEY

Round 3

A) $\cos\left(Sin^{-1}(-.5)\right)+\cot\left(Tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)\right)+Cos^{-1}\left(\cos\frac{\pi}{6}\right)$

Let $A=Sin^{-1}(-0.5)$, $B=Tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$ and $C=Cos^{-1}\left(\cos\frac{\pi}{6}\right)$. Then:

$\sin A=-0.5$ and $-\frac{\pi}{2}<A<0\ (Q4)\ \Rightarrow\ A=-\frac{\pi}{6}$

$\tan B=-\frac{\sqrt{3}}{3}$ and $-\frac{\pi}{2}<B<0\ (Q4)\ \Rightarrow\ B=-\frac{\pi}{6}$

Cos^{-1} and $\cos$ are inverse functions **and** $\frac{\pi}{6}$ **is in the domain of** $Cos^{-1}\ \Rightarrow\ C=\frac{\pi}{6}$

Thus, $\cos\left(Sin^{-1}(-.5)\right)+\cot\left(Tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)\right)+Cos^{-1}\left(\cos\frac{\pi}{6}\right)=\cos\left(-\frac{\pi}{6}\right)+\cot\left(-\frac{\pi}{6}\right)+\frac{\pi}{6}$

$=\cos\left(-\frac{\pi}{6}\right)+\cot\left(-\frac{\pi}{6}\right)+\frac{\pi}{6}=\frac{\sqrt{3}}{2}-\sqrt{3}+\frac{\pi}{6}=\frac{\pi-3\sqrt{3}}{6}\Rightarrow(A,B)=\underline{\boldsymbol{\left(3\sqrt{3},6\right)}}$.

B) $\cos 290^\circ=\cos\left(360^\circ-70^\circ\right)=\cos\left(-70^\circ\right)=\cos\left(70^\circ\right)$

Expanding using $\cos(A+B)=\cos A\cos B-\sin A\sin B$,

$\cos\left(x+150^\circ\right)+\cos\left(x+30^\circ\right)=\left(\cos x\cos 150^\circ-\sin x\sin 150^\circ\right)+\left(\cos x\cos 30^\circ-\sin x\sin 30^\circ\right)$

$=\left(\cos x\cdot-\frac{\sqrt{3}}{2}-\sin x\cdot\frac{1}{2}\right)+\left(\cos x\cdot\frac{\sqrt{3}}{2}-\sin x\cdot\frac{1}{2}\right)=-\sin x$

Thus, we have $\sin x=-\cos 70^\circ=-\sin 20^\circ$.

We require related angles in quadrants 3 and 4, $\begin{cases}180^\circ+20^\circ\Rightarrow \underline{\mathbf{200^\circ}}\\ 360^\circ-20^\circ\Rightarrow \underline{\mathbf{340^\circ}}\end{cases}$.

C) Let $B=Sin^{-1}\left(-\frac{4}{5}\right)$.

$\sin\left(2B+\frac{\pi}{2}\right)=\sin 2B\cos\left(\frac{\pi}{2}\right)+\cos 2B\sin\left(\frac{\pi}{2}\right)=\left(\sin 2B\right)(0)+\left(\cos 2B\right)(1)=\cos 2B$

$=1-2\sin^2 B=1-2\left(-\frac{4}{5}\right)^2=\underline{\boldsymbol{-\frac{7}{25}}}$.