MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 - FEBRUARY 2017 SOLUTION KEY

Team Round - continued

F)

- if a term is even, the next term is half the current term
- if a term is odd, the next term is 1 more than the current term

Following this algorithm, starting with a power of 2, we eventually produce the alternating sequence 2, 1, 2, 1, … .
Starting with 32, we have 32 - 16 - 8 - 4 - 2 - 1, alternating 2 - 1 thereafter.
The 5th term starts the repetition and $k = 5$, as required.
Let $t_1 = n$.
$n > 32$ will clearly require longer to settle into the 2-1 repetition.
For $n \le 4$, 3 - 4 - 2 - 1 is the longest sequence and $k = 3$ is rejected.
Here are the sequences where $k = 5$:
For $n \le 8$, 5 - 6 - 3 - 4 - 2 - 1 ~~6,3,4,2,1~~ $(k = 4)$ ~~7,8,4,2,1~~ $(k = 4)$ ~~8,4,2,1~~ $(k = 3)$
For $n \le 16$, ~~16,8,4,2,1~~ $(k = 4)$ 15 - 16 - 8 - 4 - 2 - 1, 14 - 7 - 8 - 4 - 2 - 1, 12 - 6 - 3 - 4 - 2 - 1
It is left to you to verify that $n = 9,10,11,13$ fail and that for $17 \le n \le 31$, $k \ge 6$ before the sequence settles into a 2 -1 repetition.
Thus, there are 5 possibilities: **5, 12, 14, 15, 32**

There are several instances of $k = 6$
For example, start with 28.
Note that if $k = 6$ for these sequences, starting with 27 or 29 will settle for $k = 7$.
What patterns do you see?