MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2015 SOLUTION KEY

Round 2

A) Let x denote the # of blue marbles replaced by a red marble.
Initially, $(R,W)=(16,7)$. After replacement, $R:W=2:1$.
$16+x=2\left(7+\left(13-x\right)\right)=40-2x\Rightarrow 3x=24\Rightarrow x=\underline{\mathbf{8}}$.

B) $3x+8y=101\Rightarrow x=\dfrac{101-8y}{3}=33-2y+2\left(\dfrac{1-y}{3}\right)$

The slope of this line is $\dfrac{-3}{8}$ or $\dfrac{3}{-8}$ and clearly, for $y = 1$, the value of x will be an integer, namely, $x = 33 – 2 + 2(0) = 31$.
Increasing y by 3 will decrease x by 8, changing the sum by –5.
Thus, (31, 1) $\Rightarrow$ **32** (23, 4) $\Rightarrow$ **27** (15,7) $\Rightarrow$ **22** (7, 10) $\Rightarrow$ **17**
For any other ordered pairs, either x or y is negative, so these are the only possible ordered pairs.

C) For the first 2 miles Sara's rate was $\dfrac{2}{\frac{15}{60}}=8$ mph.

Solution #1: If her overall average rate was 25% faster than her average rate for the first 2 miles, then she averaged $\dfrac{5}{4}\cdot 8=10$ mph for the 5 mile race.
Thus, 2 miles @ 8 mph plus 3 miles @ x mph = 5 miles at 10 mph.
$\dfrac{2}{8}+\dfrac{3}{x}=\dfrac{5}{10}\Leftrightarrow\dfrac{3}{x}=\dfrac{1}{4}\Rightarrow x=\underline{\mathbf{12}}$

Solution #2: Let the time to complete the last 3 miles be A minutes.
$R=\dfrac{D}{T}=\dfrac{5}{\frac{15+A}{60}}=8\left(\dfrac{5}{4}\right)=10\Leftrightarrow\dfrac{300}{15+A}=10\Rightarrow A=15\Rightarrow$ 5 min per mile $\Leftrightarrow$ **12** mph

Solution #3: $8\cdot\dfrac{3}{2}=\underline{\mathbf{12}}$ HUH??!

Rate · Time = Distance $\Rightarrow R=\dfrac{D}{T}$ Let r and x denote the rates for the 2 mile and 3 mile legs respectively. Then: $R=\dfrac{5}{\frac{2}{r}+\frac{3}{x}}=\dfrac{5}{4}r\Rightarrow\dfrac{1}{\frac{2x+3r}{rx}}=\dfrac{r}{4}\Rightarrow\dfrac{\cancel{x}x}{2x+3r}=\dfrac{\cancel{r}}{4}$

Cross multiplying, $4x=2x+3r\Rightarrow x=\dfrac{3}{2}r$