MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 - FEBRUARY 2016 SOLUTION KEY

Round 4

A) In the 6 rows, there must be, in some order, 1, 2, 3, 4, 5 and 6 students, for a total of 21 students. That leaves 15 empty seats when everyone is present. Thus, there would be **<u>17</u>** empty seats.

B) Let x denote the total number of shots taken over the rest of the season. Then:
$\frac{7+x}{12+x} \geq \frac{9}{10} \Leftrightarrow 70+10x \geq 108+9x \Leftrightarrow x \geq 38 \Rightarrow$ **<u>19</u>** more games.

C) Each 1 more $\Rightarrow 3n+5n+7n = N+3 \Leftrightarrow N = 15n-3$.
Each 3 less $\Rightarrow 5m+9m+13m = N-9 \Leftrightarrow N = 27m+9$.
Equating, $15n-3 = 27m+9 \Rightarrow 5n-9m = 4 \Rightarrow n = \frac{9m+4}{5}$.
$N \geq 100 \Leftrightarrow 15\left(\frac{9m+4}{5}\right)-3 \geq 100 \Leftrightarrow 27m+9 \geq 100 \Rightarrow m \geq 4$.
$m = 4 \Rightarrow n = 8 \Rightarrow (A,B,C) = \underline{\mathbf{(23,39,55)}}$.
Alternate Solution:
$(mw+1):(p+1) = 3:5 \Rightarrow 5(mw+1) = 3(p+1)$.
$(mw-3):(p-3) = 5:9 \Rightarrow 9(mw-3) = 5(p-3)$.
Solving this system of simultaneous equations, we have $(mw,p) = (23,39)$ and, therefore,
$(mw+1):(p+1):(k+1) = 24:40:56$.
Therefore, $N = A+B+C$ and $N \geq 100 \Rightarrow (A,B,C) = \underline{\mathbf{(23,39,55)}}$
for the <u>minimal</u> value of $N = 24(1)+40(1)+56(1)-3 = 117$.