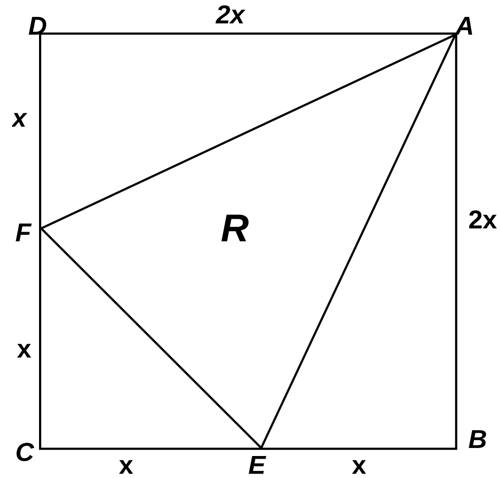
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2012 SOLUTION KEY

Round 2

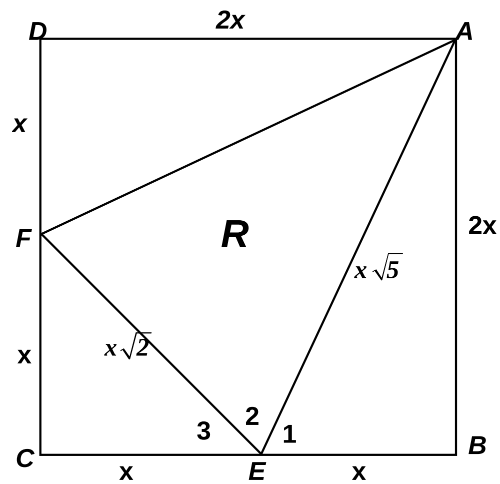
A) (8, 9) ⇒ (7,10) ⇒ (6,11) ⇒ (5, 12) which is part of the recognizable Pythagorean Triple (5, 12, 13). The hypotenuse has length **<u>13</u>**.



B) $R = 4x^2 - \text{area}(\Delta ADF + \Delta FCE + \Delta ABE)$
$= 4x^2 - x^2 - x^2/2 - x^2 = 3x^2/2$

$$\frac{area\square}{area\triangle} = \frac{4x^2}{3x^2/2} = \frac{8}{3} \Rightarrow \text{area}(ABCD) = \underline{\frac{\mathbf{8R}}{\mathbf{3}}}$$

Alternate solution using trig:



$\tan(\angle 1) = 2$

$m\angle 3 = 45°$

Thus, $Tan^{-1}(2) + m\angle 2 + 45 = 180$

and $m\angle 2 = 135 - Tan^{-1}(2)$

Using the expansion for $\tan(A - B)$,

$$\tan(\angle 2) = \frac{-1-2}{1+(-1)(2)} = \frac{-3}{-1} = 3 \Rightarrow$$

$$\sin(\angle 2) = \frac{3}{\sqrt{10}}$$

$$\text{Area}(\Delta) = \frac{1}{2}ab\sin\theta = \frac{1}{2} \cdot x\sqrt{2} \cdot x\sqrt{5} \cdot \frac{3}{\sqrt{10}} = \frac{3}{2}x^2$$

$$\Rightarrow \frac{R}{area\square} = \frac{3x^2/2}{4x^2} = \frac{3}{8} \Rightarrow \text{area}(ABCD) = \underline{\frac{\mathbf{8R}}{\mathbf{3}}}$$

C) Let $10x$ denote the distance traveled by the bus in 2 hours. Then:

$$500^2 = (10x)^2 + (10x+340)^2$$

Scale the linear dimensions by a factor of 10 to avoid excessive computations.

$\Rightarrow 50^2 = (x)^2 + (x+34)^2$

$\Rightarrow 2500 = 2x^2 + 68x + 1156 \Rightarrow 2x^2 + 68x - 1344 = 0$

$\Rightarrow x^2 - 34x - 672 = 0 \qquad 672 = 2^5 \cdot 3 \cdot 7 \Rightarrow 14(48) \Rightarrow (x - 14)(x + 48) = 0$

$\Rightarrow x = 14$. Thus, the bus traveled 140 miles in 2 hours and the plane traveled 480 miles in 3 hours. $\Rightarrow (P, B) =$ **<u>(160, 70)</u>**