

**MASSACHUSETTS MATHEMATICS LEAGUE  
CONTEST 1 - OCTOBER 2016 SOLUTION KEY**

**Team Round**

C) A typical equation might be  $\frac{x+y+z}{3} + v + w = 21 \Leftrightarrow 3v + 3w + x + y + z = 63$

This results in a system of 10 equations in 5 unknowns:

$$\begin{cases} 3v + 3w + x + y + z = 63 \\ 3v + 3x + w + y + z = 69 \\ 3v + 3y + w + x + z = 75 \\ 3v + 3z + w + x + y = 81 \\ 3w + 3x + v + y + z = 75 \\ 3w + 3y + v + x + z = 81 \\ 3w + 3z + v + x + y = 87 \\ 3x + 3y + v + w + z = 87 \\ 3x + 3z + v + w + y = 93 \\ 3y + 3z + v + w + x = 99 \end{cases}$$

Adding the 10 equations and dividing by 18, we get  $v + w + x + y + z = 45$ .

Subtracting this equation from the first six equations above, we get:

$$\begin{cases} (1) v + w = 9 \\ (2) v + x = 12 \\ (3) v + y = 15 \\ (4) v + z = 18 \\ (5) w + x = 15 \\ (6) w + y = 18 \end{cases}$$

Subtracting (2) – (1), we get  $x - w = 3$  and solving with (5),  $2x = 18 \Rightarrow x = 9, w = 6$ ,  
 $v = 3, y = 12, z = 15$

Thus,  $(v_1, v_2, v_3, v_4, v_5) = \underline{(3, 6, 9, 12, 15)}$ .