## MASSACHUSETTS MATHEMATICS LEAGUE CONTEST 1 - OCTOBER 2006 ROUND 7 TEAM QUESTIONS

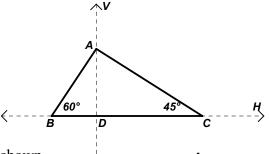
## **ANSWERS**

A) \_\_\_\_\_\_ D) \_\_\_\_

B) \_\_\_\_\_E) \_\_\_\_

C) \_\_\_\_\_\_ F) \_\_\_\_

A) Two solids are generated, one by rotating the triangle pictured below about the horizontal axis, the other by rotating about the vertical axis. If BC = 10, determine the positive difference between the volumes of the two solids. Give an exact answer.



B) An infinite sequence of right triangles is formed as shown. Determine the length of the first (smallest) hypotenuse with integer length.

C) Scientists have discovered life on Venus and have learned that Venutians can have 2, 3 or 4 legs. There are 26 Venutians in a crater with 68 legs among them. If there are at least three 4-legged Venutians for every 3-legged Venutian and at least one of each type, how many 2-legged Venutians are there?

D) How many irreducible fractions  $\frac{a}{b}$  lie between  $\frac{4}{5}$  and  $\frac{5}{6}$ , where a and b are integers and a+b=2400?

E) Find the value of A for which the distance d between the two points of intersections of the graphs of  $y = \frac{1}{2}(x+|x|)$  and |x|+|y| = A is  $2\sqrt{5}$  units.

F) Consider the following recursive definition, defined for  $\underline{any}$  integer values of n.

$$T_n = T_{n-1} + 2T_{n-2}$$
 ,where  $T_2 = 1.5$  and  $T_4 = 1.5T_0$ 

Determine the exact value of  $T_{-1}$