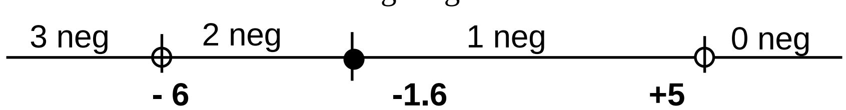
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2015 SOLUTION KEY

Round 5

A) For integer values of k, $0 < |x| < k \Leftrightarrow 0 < |x| \le k-1$. As long as $k-1$ is positive, there are $k-1$ positive solutions and $k-1$ negative solutions. $2(k-1) = 20 \Rightarrow k = \underline{\mathbf{11}}$.

B) $\left| x - |2x+3| \right| = 1 - x$
Since any solution requires that the right hand side of the equation be nonnegative, we have a <u>pre-condition</u> that $1 - x \ge 0 \Leftrightarrow x \le 1$.
Case 1: $x - |2x+3| = +(1-x) \Rightarrow |2x+3| = 2x - 1$
Now we have $\frac{1}{2} \le x \le 1 \Rightarrow 1 \le 2x \le 2 \Rightarrow \begin{cases} 4 \le 2x+3 \le 5 \\ 0 \le 2x-1 \le 1 \end{cases}$
Since these intervals do not overlap, this case produces no solutions.
Case 2: $x - |2x+3| = -(1-x) \Rightarrow |2x+3| = 1$
$\Rightarrow 2x + 3 = \pm 1 \Leftrightarrow x = \underline{\mathbf{-1, -2}}$ (Since both of these values satisfy the pre-condition.)

C) $\dfrac{2}{x+6} - \dfrac{3}{5-x} \ge 0 \Leftrightarrow \dfrac{2}{x+6} + \dfrac{3}{x-5} \ge 0 \Leftrightarrow \dfrac{2(x-5)+3(x+6)}{(x+6)(x-5)} \ge 0 \Leftrightarrow \dfrac{(5x+8)}{(x+6)(x-5)} \ge 0$
The critical values are –6, –8/5 and +5.
The sign of the quotient depends of how many of the three binomials return a positive value.
This information is summarized in the following diagram.



Thus, we have $\underline{\mathbf{-6 < x \le -\frac{8}{5}} \text{ or } \mathbf{x > 5}}$.