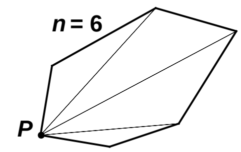
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2011 SOLUTION KEY

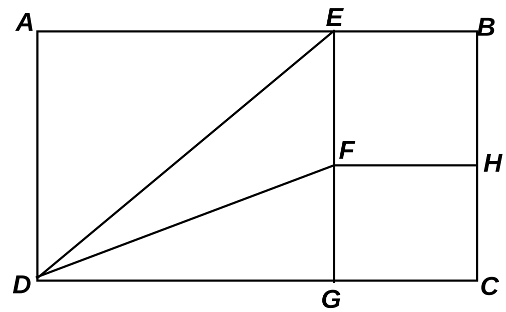
Round 6

A) Since the interior and exterior angles are supplementary, $44k + 1k = 180 \Rightarrow k = 4$.
An exterior angle of 4° $\Rightarrow$ there must be 360/4 = 90 sides.
In an n-sided polygon the number of diagonals from each vertex is $(n - 3) \Rightarrow$ **<u>87</u>** diagonals.

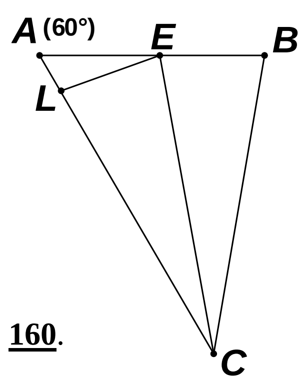


<u>Recall</u>: In the formula for the number of diagonals in a polygon with n sides, namely $d = \dfrac{n(n-3)}{2}$, n denoted the number of vertices from which a diagonal could start, $n - 3$ denoted the number of vertices to which a diagonal could be drawn and division by 2 was necessary to avoid counting each diagonal twice.

B) Let $FG = HC = x$. Then $16 : (8 + 2x) = 8 : 7 \Rightarrow x = 3$
$DF = 6$ and $FG = 3 \Rightarrow m\angle FDG = 30$, $m\angle DFE =$ **<u>120</u>**°



C) $CL = CE = CB \Rightarrow$ both ΔCLE and ΔCEB are isosceles.
$BE = EL \Rightarrow \Delta CLE \cong \Delta CEB$ (SSS)
Let $m\angle ECL = m\angle ECB = x$.
Let $m\angle CLE = m\angle CEL = m\angle CLB = m\angle LCB = y$.
Then: $4y + 2x = 360 \Rightarrow x + 2y = 180$
In ΔACE, $m\angle ACE = 180 - (60 + x) = 120 - x$
$\Rightarrow (120 - x) + y = 180$ or $y = 60 + x$.
Substituting, $x + 2(60 + x) = 180 \Rightarrow 3x = 60 \Rightarrow x = 20$, $y = 80$, $m\angle BEL =$ **<u>160</u>**.



Alternative Solution (Norm Swanson)
Draw $\overline{BL}$. $BELC$ is a kite. Let $m\angle CLB = m\angle CBL = x$° and $m\angle ELB = m\angle EBL = y$°. Since ΔCLE is isosceles, $m\angle CEL = (x + y)$°.
As an exterior angle of ΔALE, $m\angle BEL = 60 + (180 - (x + y))$.
Thus, $2(x + y) = 240 - (x + y) \Rightarrow x + y = 80 \Rightarrow m\angle BEL =$ **<u>160</u>**°.