MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2016 SOLUTION KEY

Round 3

A) Since the center of C_1 is at (6, 4) and $r^2 = 5\frac{1}{16} = \frac{81}{16}$, we have the radius is $\frac{9}{4} = 2.25$

Q(8.25, 6.25) and M(6,1.75).

Thus, rectangle *HORN* is 1.75 x 8.25, resulting in a perimeter of 2(1.75 + 8.25) = **<u>20</u>**.

B) $m_{\mathcal{L}} = \frac{8-1}{17+4} = \frac{1}{3}$.

The equation of $\mathcal{L}$ is $x - 3y = -7$.

The equation of the perpendicular is $(y-0) = -3(x-3)$

$\Rightarrow 3x + y = 9$ ***.

Solving simultaneously,

$3x + y = 9$

$\underline{x - 3y = -7}$

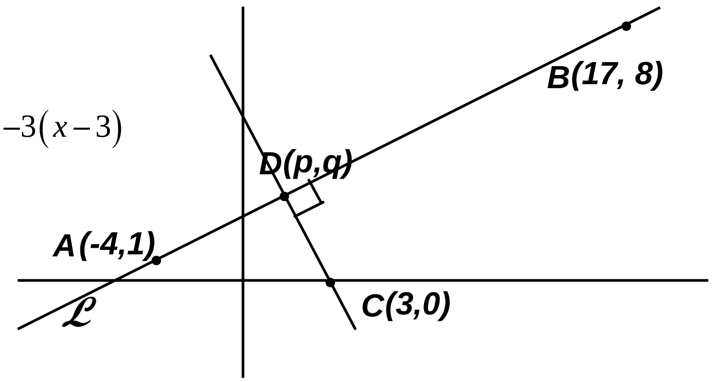
$4x - 2y = 2$

$2x - y = 1$***

Adding,

$5x = 10 \Rightarrow x = 2, y = 3$

Thus, $(p,q) = \underline{\mathbf{(2,3)}}$.



C) The given circle is origin-centered with radius 26. Thus, we are looking at two concentric origin-centered circles of radii 25 and 26. The equations of the required unit circles must be of the form $(x-h)^2 + (y-k)^2 = 1$, where $h^2 + k^2 = 25^2$.

This suggests two possible Pythagorean Triples: 7-24-25 and 5(3-4-5) = (15-20-25)

For each triple there are 8 possibilities: two in each of the 4 quadrants, as the coordinates of the center are swapped and the signs are changed from (+,+) to (-,+), (-,-) and (+,-).

We also must consider $(\pm 25, 0)$ and $(0, \pm 25)$.

Therefore, the center can be located at **<u>20</u>** different lattice points.