

**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2011 SOLUTION KEY**

Round 6

A) $\frac{n}{2}(2+29) = 155 \rightarrow n/2 = 5 \rightarrow n = 10$. Thus, $t_{10} = 2 + 9d = 29 \rightarrow d = \underline{3}$

B) (1) $-2x - y = y - x \rightarrow x = -2y$

(2) $\frac{x-1}{-3y} = \frac{-3y}{3x}$

Substituting, $\frac{-2y-1}{-3y} = \frac{-3y}{-6y} = \frac{1}{2} \rightarrow -3y = -4y - 2 \rightarrow (x, y) = (4, -2)$

Thus, the GP is $12, 6, 3, \dots \rightarrow t_{10} = 12\left(\frac{1}{2}\right)^9 = \underline{\underline{\frac{3}{128}}}$

C) $a_{n+2} = 2a_{n+1} + a_n \rightarrow \begin{cases} a_6 = 2a_5 + a_4 \\ a_7 = 2a_6 + a_5 \end{cases} \rightarrow \begin{cases} 64 = 2a_5 + 11 \\ a_7 = 128 + a_5 \end{cases} \rightarrow a_5 = \frac{53}{2} \text{ and } a_7 = 128 + \frac{53}{2}$

Also $a_{n+2} = 2a_{n+1} + a_n \rightarrow a_n = a_{n+2} - 2a_{n+1}$

If $n = 3$, we have $a_3 = a_5 - 2a_4 = \frac{53}{2} - 2(11) = \frac{9}{2}$

Thus, $a_3 + a_7 = 128 + \frac{62}{2} = \underline{\underline{159}}$