Round 4

A)
$$126 = 6 \cdot 21 = 2 \cdot 3^2 \cdot 7$$
$$132 = 11 \cdot 12 = 2^2 \cdot 3 \cdot 11$$
The common prime factors are 2 and 3, so the common factors are 1, 2, 3 and 6
$\Rightarrow$ **4** common factors.

B) $x^3 y - xy^3 = xy\left(x^2 - y^2\right) = 12$

$\quad x^3 y^2 - x^2 y^3 = x^2 y^2 \left(x - y\right) = 15$

$\quad \dfrac{xy\left(x^2 - y^2\right)}{x^2 y^2 \left(x - y\right)} = \dfrac{12}{15} \rightarrow \dfrac{x+y}{xy} = \dfrac{1}{x} + \dfrac{1}{y} = \dfrac{\mathbf{4}}{\mathbf{5}}$

C) $(x+1)(x-6)(x+3) - (x+1)^2(x-2) = (x+1)\left(\left(x^2 - 3x - 18\right) - \left(x^2 - x - 2\right)\right)$

$\quad = (x+1)(-2x - 16) = -2(x+1)(x+8) > 0$

Dividing through by -2, $(x+1)(x+8) < 0$

The critical values are -1 and -8.

Both factors are negative for $x < -8$, positive for $x > -1$ and in between -1 and -8, they have opposite signs. Therefore, the product is negative for $\mathbf{-8 < x < -1}$.