MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2016 SOLUTION KEY

Round 6

A) $\begin{cases} \dfrac{A}{3}+\dfrac{B}{4}=10 \\ 3A+4B=34 \end{cases} \Rightarrow \begin{cases} 4A+3B=120 \\ 3A+4B=34 \end{cases} \Rightarrow 7A+7B=154 \Rightarrow A+B=\underline{\mathbf{22}}$.

It was not necessary to first solve for A and B separately!!

B) Grouping the first terms in each trinomial, we view each trinomial as a binomial and we have the product of a sum and a difference (which is equivalent to the difference of perfect squares!)

$\left(\sqrt{3}+\sqrt{5}+\sqrt{15}\right)\left(\sqrt{3}+\sqrt{5}-\sqrt{15}\right)=\left[\left(\sqrt{3}+\sqrt{5}\right)+\sqrt{15}\right]\left[\left(\sqrt{3}+\sqrt{5}\right)-\sqrt{15}\right]$

$\left(\sqrt{3}+\sqrt{5}\right)^2-\left(\sqrt{15}\right)^2=3+2\sqrt{15}+5-15=-7+2\sqrt{15}\Rightarrow \underline{\mathbf{(-7,2,15)}}$.

C) Let P and M denote the number of Peach and Mango packets purchased. Let $T=P+M$.

$P=2.6M \Rightarrow \dfrac{P}{M}=\dfrac{13}{5}$ and $\dfrac{P-50}{M+50}=\dfrac{2}{1} \Rightarrow P=2M+150$

Substituting, $\dfrac{2M+150}{M}=\dfrac{13}{5} \Rightarrow 10M+750=13M \Rightarrow M=250,\ P=650 \Rightarrow T=\underline{\mathbf{900}}$.