MASSACHUSETTS MATHEMATICS LEAGUE CONTEST 1 - OCTOBER 2011 SOLUTION KEY

Round 5

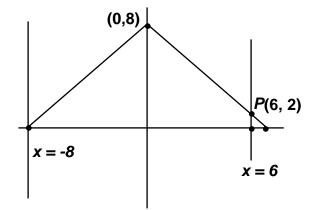
- A) $x^2 + 10x \le 24 \Leftrightarrow x^2 + 10x 24 \le 0 \Leftrightarrow (x+12)(x-2) \le 0$. The critical values are -12, +2, which divide the number line into three regions. We require a negative or zero product. Testing a value in each region, only the region between -12 and 2 inclusive satisfies the required condition $\Rightarrow -12 \le x \le 2$.
- B) $-2x 1 < 3x 11 < 2x + 1 \Rightarrow -2x 1 < 3x 11$ and $3x 11 < 2x + 1 \Rightarrow 10 < 5x$ and $x < 12 \Rightarrow 2 < x < 12$.

C)
$$y+|x| > 8 \Rightarrow y < 8-|x| \Rightarrow \begin{cases} y = x+8 \\ y = -x+8 \end{cases}$$

 $|x+1| < 7 \Rightarrow -7 < x+1 < 7 \Rightarrow -8 < x < 6$

 $y \ge 0$ restricts us to above the y-axis.

Examining the graphs of the related equations, we see the required region is the interior of a polygon, comprised of a triangle in quadrant 2 and a trapezoid in quadrant 1 between the *y*-axis and the vertical line x = 6.



Since y = -x + 8 and x = 6 intersect at (6, 2), we have the

$$\frac{1}{2} \cdot 8 \cdot 8 + \frac{1}{2} \cdot 6 \cdot (2+8) = 32 + 30 = \underline{62}$$
 necessary dimensions to find the area of each region.