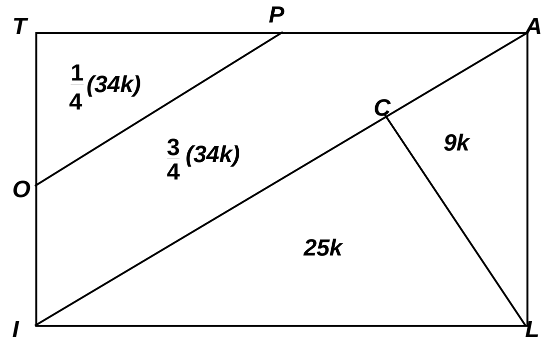
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2013 SOLUTION KEY

Team Round - continued



E) The area of rectangle *LATI* is 15.
The diagram contains a blizzard of similar triangles, namely $\Delta TOP \sim \Delta TIA \sim \Delta LAI \sim \Delta CAL \sim \Delta CLI$ and a pair of congruent triangle, namely $\Delta TIA \cong \Delta LAI$.
For $\Delta CAL \sim \Delta CLI$, the ratio of areas is
$\left(\frac{LA}{LI}\right)^2 = \left(\frac{3}{5}\right)^2 = \frac{9}{25}$. Thus, we know that the area of ΔTIA may also be represented as $34k$.
Since $\frac{TO}{TI} = \frac{1}{2}$, the areas of triangles *TOP* and *TIA* are in a 1: 4 ratio.
Multiplying through by 2, the required ratio is #1 : #4 : #3 : #2 = **17 : 18 : 50 : 51**.

F) Consider the chart at the right, where consecutive integers are listed 8 per row.

1	2	3	4	***5***	6	7	*8*
9	***10***	11	12	13	14	***15***	*16*
17	18	19	***20***	21	22	23	*24*
25	26	27	28	29	***30***	31	*32*
33	34	***35***	36	37	38	39	***40***
~~41~~	~~42~~	~~43~~	~~44~~	~~45~~	~~46~~	~~47~~	~~48~~
~~49~~	~~50~~	~~51~~	~~52~~	~~53~~	~~54~~	~~55~~	~~56~~
~~57~~	~~58~~	~~59~~	~~60~~	~~61~~	~~62~~	~~63~~	~~64~~

In the top row, dividing by 8, these entries leave all possible remainders for division by 8, namely 1,2,3,4,5,6,7 and 0.
In each column, division by 8 leaves the same remainder, but division by 5 leaves a different remainder, since 8 and 5 are relatively prime. Only five remainders are possible for division by 5 and each column contains these five remainders, just "shuffled" into a different order.
For example, in column 1, the remainders of division by 5 are 1, 4, 2, 0, 3; in column 2, remainders = 2, 0, 3, 1, 4; in column 3, remainders = 3, 1, 4, 2, 0
As soon as a multiple of 5 is reached in each column, the next and subsequent entries can be written as a (linear) combination of 5 and 8. For example, in column 1, $33 = 5 \cdot 5 + 1 \cdot 8$;
in column 6, $46 = 6 \cdot 5 + 2 \cdot 8$ - etc.
The only chicken nugget purchases possible in the first row of the chart are 5 and 8.
Each entry in the rightmost column is a quantity (Q) that may be purchased as each is a multiple of 8-piece nuggets. In the leftmost 7 columns, the numbers above the underlined entry can not be expressed as a (linear) combination of 5 and 8, namely as $Q = 5x + 8y$.
(Notice that all entries after 40 can be written as a linear combination.)
Therefore, the largest possible number of nuggets that may not be purchased is **27**.
Note also that $27 = 8 \cdot 5 - (8 + 5)$.
It is left for you to verify that the maximum value is always $AB - (A + B)$, whenever A and B are relatively prime.