**MASSACHUSETTS MATHEMATICS LEAGUE**
**CONTEST 1 - OCTOBER 2007**
**ROUND 7 TEAM QUESTIONS**

**ANSWERS**

A) _____ D) _____

B) _____ E) _____

C) ( _____ , _____ ) F) _____

A) An equilateral triangle with side $s$ and its inscribed circle are rotated around one of the axes of symmetry of the equilateral triangle generating a cone and a sphere. Determine a simplified expression in terms of $s$ for the volume of the region inside the cone and outside the sphere.

B) In a right triangle with sides of integer length, the hypotenuse is 1 unit longer than the longer leg. Determine the lengths of the sides of such a right triangle if the perimeter is less than 1000, but as large as possible.

C) A number of pencils were purchased for 30¢. If the price per dozen were lowered by 27¢, then 3 more pencils could have been bought for 30¢. If $Q$ denotes the number of pencils that could be bought for 30¢ at the lower unit price $P$ (in cents), determine the ordered pair $(Q, P)$.

D) Let the solution of the equation $\dfrac{1}{1+\dfrac{2}{x+3}} = 4 - 0.\overline{5}$ be $x = \dfrac{A}{B}$, a reduced fraction, where $A$ and $B$ denote integers and $A < B$. Determine the smallest positive integer $n$ such that $\dfrac{A+n}{B}$ is an integer.

E) The region satisfying the inequality $|x - 2007| + |y + 2008| < A$ contains 1985 lattice points, i.e. points with integer coordinates. Determine the integer $A$ for which this statement is true.

F) Four students enter a room. Each student has a cell phone. The students put all their cell phones into the same pile. When the students leave, each student is given a cell phone. What is the probability that none of the students will receive their own cell phone?