

**MASSACHUSETTS MATHEMATICS LEAGUE  
CONTEST 6 - MARCH 2012 SOLUTION KEY**

**Team Round**

- F) To generate a memorable license plate we must pick 6 of the 9 nonzero digits and put them in increasing or decreasing order. This can be done in  $2\binom{9}{6} = 2(84) = 168$  ways.

Let  $x$  denote the # of “memorable” plates to be cross checked.

| Left most digit: | Possible Plates   | Count    |
|------------------|---|----------|
| <u>4</u>         | <b>56 789</b>   | <b>1</b> |
| <u>3</u>         | <b>45 678 / 679 / 689 / 789</b><br><b>46 789</b><br><b>56 789</b> | <b>6</b> |

Thus,  $2(1 + 6 + k) = 168 \Rightarrow k + 7 = 84 \Rightarrow k = 77$ , the number of strictly increasing memorable plates beginning with 1 or 2, which have not been specifically enumerated.

$$\Rightarrow x = 168 - 77 = \underline{\mathbf{91}}$$

**FYI:** [Here’s the enumerated list for those who must see to believe.]

|          |   |           |
|----------|---|-----------|
| <u>2</u> | <b>34 567 / 568 / 569 / 578 / 579 / 589 / 678 / 679 / 689 / 789</b><br><b>35 678 / 679 / 689 / 789</b><br><b>36 789</b><br><b>45 678 / 679 / 689 / 789</b><br><b>46 789</b><br><b>56 789</b>  | <b>21</b> |
| <u>1</u> | <b>23 <math>\Rightarrow</math> 456 457 458 459 467 468 469 478 479 489</b><br><b>567 568 569 578 579 589</b><br><b>678 679 689</b><br><b>789 <math>\Rightarrow</math> 20</b><br><b>24 <math>\Rightarrow</math> 10      45 <math>\Rightarrow</math> 4</b><br><b>25 <math>\Rightarrow</math> 4      46 <math>\Rightarrow</math> 1</b><br><b>26 <math>\Rightarrow</math> 1      56 <math>\Rightarrow</math> 1</b><br><b>34 <math>\Rightarrow</math> 10</b><br><b>35 <math>\Rightarrow</math> 4</b><br><b>36 <math>\Rightarrow</math> 1</b> | <b>56</b> |