MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2008 SOLUTION KEY

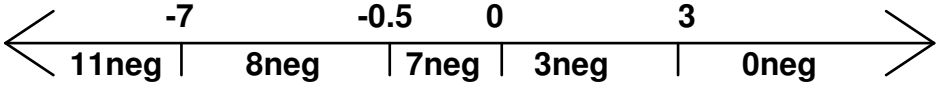
Round 5

A) $|2x - 5| \le 13 \rightarrow -13 \le 2x - 5 \le 13 \rightarrow -4 \le x \le 9 \rightarrow \dfrac{-4+9}{2} = \underline{\mathbf{2.5}}$

B) $|\,101 - 8x\,| > 27$ is equivalent to $101 - 8x < -27$ OR $101 - 8x > +27$
$\Leftrightarrow 128 < 8x$ or $78 > 8x \Leftrightarrow 16 < x$ or $37/4 < x \Leftrightarrow x > 16$ or $x < 9.25$
Thus, the integer non-solutions are 10, 11, …, 16 and the sum is $7(10) + (6\cdot7)/2 = \underline{\mathbf{91}}$.

C) In factored form, the left-hand side of the inequality is $\dfrac{(x+7)^3\left(x+\frac{1}{2}\right)x^4}{(x-3)^3}$.

In this quotient, we are dealing with 11 "factors". The critical values are −7, −1/2, 0 and 3. As we move along the number line from left to right, each factor takes on negative values, zero and then positive values. This chart summarizes the polarity (i.e. sign: + / −) of the 11 "factors":



Thus, the given expression is negative (or zero) where there are an odd number of negative factors, i.e. for $\boldsymbol{x \le -7}$ or $\boldsymbol{-\frac{1}{2} \le x < 3}$