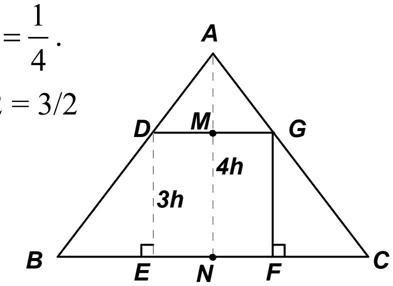
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2011 SOLUTION KEY

Round 5 - continued

B) Alternate solution #1

Drop a perpendicular from A to $\overline{BC}$, intersecting $\overline{DG}$ and $\overline{BC}$ at M and N respectively.

$\frac{\text{area}(BDGC)}{\text{area}(ABCD)} = \frac{15}{16} \rightarrow \frac{area(\Delta ADG)}{area(\Delta ABC)} = \frac{1}{16}$. $\Delta ADG \sim \Delta ABC \rightarrow \frac{AD}{AB} = \frac{AM}{AN} = \frac{1}{4}$.



Let $AD = 1$ and $AM = h$. Then: $DG = EF = 1$, $AB = BC = 4$, $BE = (4 - 1)/2 = 3/2$ and $DE = MN = 3h$.

$$\frac{area(\Delta ABC)}{area(\Delta BED)} = \frac{\frac{1}{2}BC\cdot AN}{\frac{1}{2}BE\cdot DE} = \frac{4\cdot 4h}{\frac{3}{2}\cdot 3h} = \frac{16}{9/2} = \underline{\mathbf{32:9}}.$$

Alternate solution #2 (Norm Swanson): Let $BE = FC = 6$ and $EF = 4$. $\rightarrow \dfrac{\frac{1}{2}4h\cdot 16}{\frac{1}{2}3h\cdot 6} = \underline{\mathbf{\dfrac{32}{9}}}$

C) In a regular hexagon (with side s), the lengths of the diagonals are either $2s$ or $\sqrt{3}s$.

$$\text{Area}(A) = 6\left(\frac{(s_A)^2\sqrt{3}}{4}\right) = \frac{9\sqrt{3}}{4} \rightarrow s_A = \frac{\sqrt{6}}{2}\text{ ; long diag}_B = 4\sqrt{6} \rightarrow s_B = 2\sqrt{6}$$

Thus, $s_C = \frac{\sqrt{6}}{2}\cdot\sqrt{3} = \frac{3}{2}\sqrt{2}$ and $s_D = 2\sqrt{6}\cdot\sqrt{3} = 6\sqrt{2}$

The sum of the areas is $\frac{3}{2}\sqrt{3}\left(\frac{9}{4}\cdot 2 + 36\cdot 2\right) = \frac{3}{2}\sqrt{3}\left(\frac{153}{2}\right) = \underline{\mathbf{\frac{459}{4}\sqrt{3}}}$.

Round 6

A) Substituting for x we get $4t + 1 + y + 3t = 0$ or $y + 7t = -1$. Subtracting $y + 3t = 11$, we get $4t = -12$ or $t = -3$. Substituting, we get $x = -11$ and $y = 20 \rightarrow$ **<u>(–11, 20, –3)</u>**

B) Convert the repeating decimal to a ratio of integers as follows:
$$\begin{array}{r} 100N = 16.\overline{6} \\ 10N = 1.\overline{6} \\ \hline 90N = 15 \end{array} \rightarrow N = \frac{15}{90} = \frac{1}{6}$$

Thus, $$\frac{\frac{17}{100} - \frac{1}{6}}{\frac{1}{6}} = \frac{\frac{102-100}{600}}{\frac{1}{6}} = \frac{2}{600}\cdot\frac{6}{1} = \frac{2}{100} = \underline{\mathbf{2\%}}.$$

C) Let n, d and q denote the number of nickels, dimes and quarters respectively. Then:

$$\begin{cases} 5n + 10d + 25q = 650 \\ 10d = 4(5n) \\ n + d + q = 50 \end{cases} \rightarrow \begin{cases} n + 2d + 5q = 130 \\ d = 2n \\ q = 50 - n - d = 50 - 3n \end{cases}$$ Then: $n + 2(2n) + 5(50 - 3n) = 130$

$\rightarrow 250 - 10n = 130 \rightarrow n = 12 \rightarrow q = \underline{\mathbf{14}}$.