**MASSACHUSETTS MATHEMATICS LEAGUE**
**CONTEST 5 - FEBRUARY 2015 SOLUTION KEY**

**Team Round - continued**

F) GS: $x, y, -27, \ldots$ AS: $x, y, 21, \ldots$

In the GS, the common ratio is $\frac{y}{x} = \frac{-27}{y} \Rightarrow y^2 = -27x$.

In the AS, the common difference is $d = y - x = 21 - y \Rightarrow x = 2y - 21$.

Substituting, $y^2 = -27(2y - 21) \Leftrightarrow y^2 + 54y - 27 \cdot 21 = 0$ or

$y^2 + 54y - 9 \cdot 63 = 0 \Leftrightarrow (y - 9)(y + 63) = 0$

$\Rightarrow y = 9,\ x = -3\ (r = -3)$ or $y = -63,\ x = -147\ \left(r = \frac{3}{7}\right)$

Thus, there are two possible pairs of sequences.

$$\begin{cases} \text{G.S. } -3, 9, -27, \boxed{81}, -243, \ldots \\ \text{A.S. } -3, 9, 21, 33, \boxed{45}, \ldots \end{cases} \Rightarrow \frac{45}{81} = \underline{\mathbf{\frac{5}{9}}}$$

$$\begin{cases} -147, -63, -27, -\frac{81}{7}, \ldots \\ -147, -63, 21, 105, 189, \ldots \end{cases} \Rightarrow 189\left(-\frac{7}{81}\right) = \underline{\mathbf{-\frac{49}{3}}}$$