

**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2008
ROUND 7 TEAM QUESTIONS**

ANSWERS

A) _____ D) _____

B) _____ E) _____

C) _____ F) _____

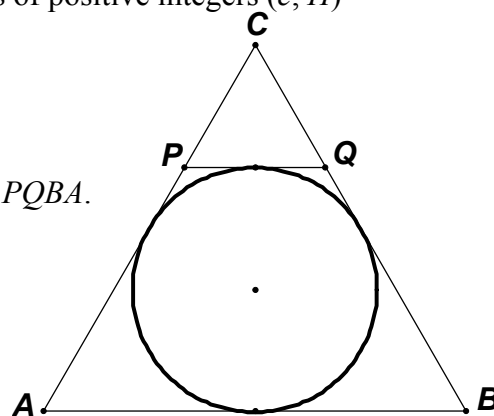
A) Given:
$$\begin{cases} f(x) = 8x \\ f(g(x)) = 27x^3 \\ f(g(h(x))) = 27x^{15} \end{cases} \quad \text{Compute: } h^{-1}(1024) \cdot g^{-1}(-1)$$

B) What is the smallest value of N for which $N!$ ends in exactly 2008 zeros?

C) Given: $\cos^{-1}(2x) - \sin^{-1}(x) = 5\pi/6$
Compute $x < 0$.

D) In still water, a motorboat travels at 18 mph.
A fisherman leaves a dock and travels downstream on a river with a constant current of c mph for an hour, turns the motor off and fishes for half an hour, floating with the current. He then immediately starts the motor and travels back upstream, returning to the dock. The upstream trip takes H hours. Determine all possible ordered pairs of positive integers (c, H) satisfying these conditions.

E) Given: $\overline{PQ} \parallel \overline{AB}$, $PQ : AB = 1 : 3$ and circle O is inscribed in trapezoid $PQBA$
Compute the ratio of the area of circle O to the area of trapezoid $PQBA$.



F) The arithmetic average of n numbers is M . If 1 is added to the first number, 2 to the second number and so on until n is added to the n^{th} number, then the average of the new numbers is A . If $M : A = 2 : 3$ and $n > 1,000,000$ then determine the minimum value of M .