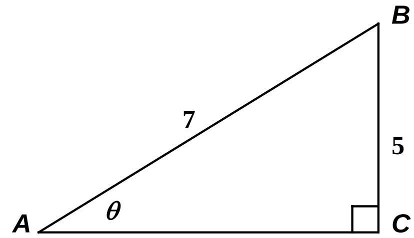
# MASSACHUSETTS MATHEMATICS LEAGUE
# CONTEST 5 – FEBRUARY 2013 SOLUTION KEY

## Round 3

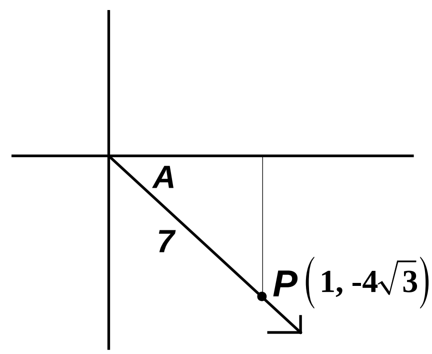



A) Let $\theta = Arc\sin\left(\frac{5}{7}\right)$. This is equivalent to $\sin\theta = \frac{5}{7}$ <u>and</u>

$0 < \theta < \frac{\pi}{2}$ (quadrant 1), since the argument was positive.

$AC = \sqrt{49-25} = 2\sqrt{6}$

Thus, $\tan\theta = \frac{5}{2\sqrt{6}} = \frac{5\sqrt{6}}{12}$ or, equivalently, $\theta = Arc\tan\left(\frac{5\sqrt{6}}{12}\right)$ and $k =$ $\underline{\mathbf{\frac{5\sqrt{6}}{12}}}$ .




B) The range of $y = Arc\tan(x)$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Since

$A$ is in quadrant 4 $\left(\tan A = -4\sqrt{3}\right)$, $\pi + A$ must

be in quadrant 2.

The sine in quadrant 2 is positive $\Rightarrow$ $\underline{\mathbf{+\frac{4\sqrt{3}}{7}}}$ .

C) Since $\begin{cases} x = 9\cos^4\theta \\ y = 9\sin^4\theta \end{cases}$, it is clear that both $x$ and $y$ must be nonnegative.

For real numbers $x$ and $y$, $\sqrt{x} = x^{\frac{1}{2}}$ and $\sqrt{y} = y^{\frac{1}{2}}$ denote nonnegative numbers.
Taking the square root and summing, we have

$$x^{\frac{1}{2}} + y^{\frac{1}{2}} = 3\cos^2\theta + 3\sin^2\theta = 3\left(\cos^2\theta + \sin^2\theta\right) = 3\cdot 1 = 3$$

$$\Rightarrow y^{\frac{1}{2}} = 3 - x^{\frac{1}{2}} \Rightarrow y = \underline{\left(\mathbf{3 - x^{\frac{1}{2}}}\right)^{\mathbf{2}}} = \underline{\mathbf{9 - 2\sqrt{x} + x}}$$

What does the graph of this function look like?
Compare your graph with the graph is at the end of this solution key.
Carefully consider the accompanying commentary.

The tail to the right of point $A$ is extraneous for our pair of parametric equations, since the value of $x$ could not be greater than 9.

$\begin{cases} x = 9\cos^4\theta \\ y = 9\sin^4\theta \end{cases}$, where $0 \le \theta < 2\pi$ and $y = \left(3 - x^{\frac{1}{2}}\right)^2$

are equivalent only over $0 \le x \le 9$.

For a real-valued function $y = \left(3 - x^{\frac{1}{2}}\right)^2$,

the only restriction on the domain is $x \ge 0$ and the graph would continue to the right.