MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2012 SOLUTION KEY

Team Round

A) Examining complex fractions with increasingly more instances of *i*, starting with $k = 1$, look for a pattern.

1 i: $\boxed{\frac{1}{i}} = \frac{1}{i}\cdot\frac{i}{i} = -1i = -\frac{1}{1}i$

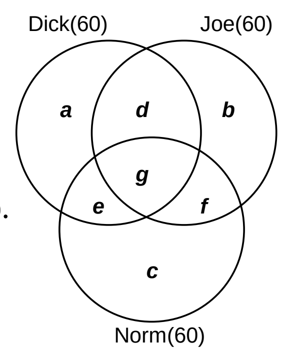
2 i s: $\frac{1}{i-\boxed{\frac{1}{i}}} = \frac{1}{i-(-1i)} = \frac{1}{2i} = -\frac{1}{2}i$

3 i s: $\frac{1}{i-\boxed{\frac{1}{i-\frac{1}{i}}}} = \frac{1}{i-\left(-\frac{1}{2}i\right)} = \frac{1}{\frac{3}{2}i} = -\frac{2}{3}i$

4 i s: $\frac{1}{i-\boxed{\frac{1}{i-\frac{1}{i-\frac{1}{i}}}}} = \frac{1}{i-\left(-\frac{2}{3}i\right)} = \frac{1}{\frac{5}{3}i} = -\frac{3}{5}i$

Each time the denominator of the current term becomes the numerator of the next term and the denominator of the next term is the sum of the numerator and the denominator of the previous term. Shades of the Fibonacci sequence, 1, 1, 2, 3, 5, 8, 13, …!
Continue the sequence until we reach a perfect square, 21, 34, 55, 89, ***144***, 233
Thus, the 12th term (the complex fraction with 12 is) will be $-\frac{144}{233}i$;
hence, (k, A, B) = **(12, 144, 233)**.

B) Consider the Venn diagram at the right.
The circles contain the questions the mathletes answered correctly.
a, *b* and *c*: answered correctly by exactly 1 mathlete (i.e. HARD questions).
d, *e*, and *f* were answered questions correctly by exactly 2 mathletes.
g were questions answered correctly by all 3 mathletes (i.e. EASY questions).
Knowing they each answered 60 questions correctly, we have



$$\begin{cases} a+d+e+g=60 \\ b+d+f+g=60 \Rightarrow (a+b+c)+2(d+e+f)+3g=180 \\ c+e+f+g=60 \end{cases}$$

Thus, HARD + 2(d + e + f) + 3EASY = 180
Rearranging the terms, we have $(a+b+...+g)+(d+e+f)+2g=180$
Since the first sum represents all the questions, we have 100 + (d + e + f) + 2EASY = 180 or (d + e + f) = 80 – 2EASY
Substituting, HARD + 2(80 – 2EASY) + 3EASY =180 $\Rightarrow$ HARD – EASY = 20 $\Rightarrow k =$ **20**.