MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 – MARCH 2010 SOLUTION KEY

Round 3

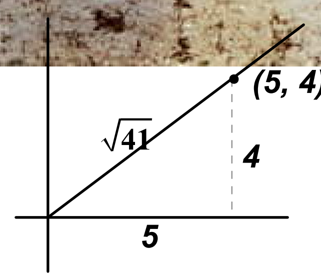
A) Rather than evaluating by direct substitution, let's simplify the expression and show that, if the expression is defined, the product P is invariant, i.e. has the same value, namely 1.
Using reduction formulas, θ is a reference value (angle) in quadrant 1 and when we see $(90 - \theta)$ – think Q1, $(90 + \theta)$ – think Q2, $(270 - \theta)$ – think Q3, $(270 + \theta)$ – think Q4
The mneumonic ASTC identifies the <u>sign</u> of the trig functions in quadrants 1 – 4.

Simplifying the lefthand side, we have $\sin\theta\cos\theta(\cot\theta)(-\tan\theta)(\csc\theta)(-\sec\theta)$
Regrouping, $(\sin\theta\cdot\csc\theta)(\cos\theta\cdot-\sec\theta)(\cot\theta\cdot-\tan\theta) = 1\cdot-1\cdot-1 = \underline{\mathbf{1}}$.
Thus, $\cot(180 - \theta) = -\cot(\theta) = 1$ ➔ θ belongs to 45° family in quadrants 2 and 4 ➔ <u>**135°, 315°**</u>.

As a 13 year old Boy Scout – 7' 4"

B) Let s denote the length of Shorty's shadow at 2:00. Then:
$\frac{x}{s} = \tan 60^\circ$ ➔ $x = s\sqrt{3}$.
At 4:00 $\frac{x}{s+10} = \tan 30^\circ = \frac{1}{\sqrt{3}}$ ➔ $x\sqrt{3} = s + 10$.
Multiplying by $\sqrt{3}$, $3x = s\sqrt{3} + 10\sqrt{3}$ ➔ $3x = x + 10\sqrt{3}$
➔ $x = \underline{\mathbf{5\sqrt{3}}}$.
Note: Wadlow was a real person and his actual maximum adult height was 8 feet 11.1 inch.
He died at the age of 22 and is acknowledged to be the modern world's tallest man.
Goto http://www.maniacworld.com/worlds_tallest_man.htm to learn more or Google <u>Robert Wadlow</u>.

C) Since $\begin{cases} \sin A = \dfrac{k}{\sqrt{41}} \\ \cos A = \dfrac{k+1}{\sqrt{41}} \end{cases}$, $\frac{k^2}{41} + \frac{(k+1)^2}{41} = 1$.



$2k^2 + 2k = 40$ ➔ $k^2 - k - 20 = (k+5)(k-4) = 0$. $k > 0$ ➔ $k = 4$.
Simplifying, $\sec\left(A - \frac{5\pi}{2}\right)\cdot\cos(A - 5\pi) =$

$$\sec\left(A - \frac{\pi}{2}\right)\cdot\cos(A-\pi) = \sec\left(\frac{\pi}{2} - A\right)\cdot\cos(\pi - A) = \csc A\cdot-\cos A = -\frac{\cos A}{\sin A} = -\cot A \text{ ➔ } \underline{-\frac{5}{4}}.$$