MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2008 SOLUTION KEY

Team Round

A) Let the unspecified entries in the square be denoted *a*, *b*, *c*, *d* and *e*.

Specifically, the square is $\begin{array}{|ccc|}\hline x & 27 & d \\ 15 & b & e \\ a & c & 21 \\ \hline\end{array}$

→

$$\begin{cases} \text{diagonals: } x+b+21=a+b+d \Rightarrow a=x-d+21 \\ \text{col1,row1: } x+15+a=x+27+d \Rightarrow d=a-12 \Rightarrow a=\dfrac{x+33}{2} \text{ and } d=\dfrac{x+9}{2} \\ \text{col1,row3: } x+15+a=a+c+21 \Rightarrow c=x-6 \\ \text{row1,col3: } x+27+d=d+e+21 \Rightarrow e=x+6 \\ \text{row2,col3: } 15+b+e=d+e+21 \to b-d=6 \to b-(a-12)=6 \to b=a-6 \Rightarrow b=\dfrac{x+21}{2} \end{cases}$$

Thus, the magic square becomes $\begin{array}{|ccc|}\hline x & 27 & \dfrac{x+9}{2} \\ 15 & \dfrac{x+21}{2} & x+6 \\ \dfrac{x+33}{2} & x-6 & 21 \\ \hline\end{array}$

Since $c = x - 6$ must represent a positive integer,

the minimum possible *x*-value is <u>**7**</u> → $\begin{array}{|ccc|}\hline 7 & 27 & 8 \\ 15 & 14 & 13 \\ 20 & 1 & 21 \\ \hline\end{array}$

B) $P = C^{\frac{1}{3}+\frac{1}{6}+\frac{1}{12}+\frac{1}{24}+\dots}$ The exponent is an infinite geometric progression.

The sum is given by $\dfrac{a}{1-r} = \dfrac{\frac{1}{3}}{1-\frac{1}{2}} = \dfrac{2}{3}$ → $P = C^{\frac{2}{3}}$

Thus, *C* must be a perfect cube to insure that *P* is an integer. Let $C = x^3$, where $x > 10^2$.

$N = P - 10^4 = C^{\frac{2}{3}} - 10^4 = \left(x^3\right)^{\frac{2}{3}} - 10^4 = x^2 - 10^4 > 10^3$ and the minimum *x* is 101.

$101^2 = 10201$, … $104^2 = 10816$ fail, but $105^2 = 11025$ → $N =$ <u>**1025**</u>