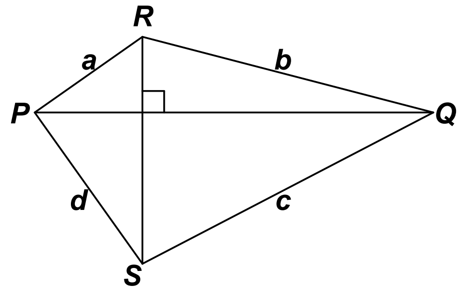
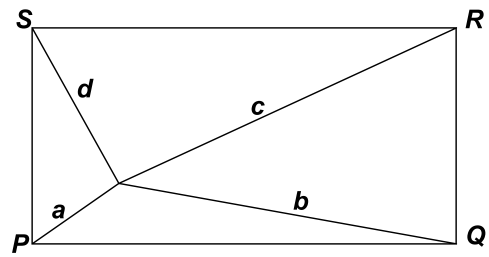
Round 2 Pythagorean Theorem - CAVEATS
Here are the two nice relations derived from applying the Pythagorean Theorem mentioned last year. If you tried justifying them to yourself, compare the following with your results.
These two results may be used <u>in any future contests</u>.
Ignore them at your own peril!

#1

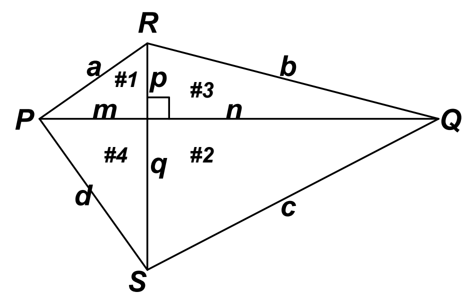


#2



#1: If $\overline{PQ} \perp \overline{RS}$, then $a^2 + c^2 = b^2 + d^2$
In **any quadrilateral with perpendicular diagonals**, the sums of the squares of the opposite sides are equal.

(Of course, this is true in any square, in any rhombus and in any kite, but it's not very useful in these cases. It is, however, very useful in those cases when neither diagonal bisects the other diagonal! Such is the case in the diagram below. It could be a trapezoid, but in general, it's just got 4 sides.)



The proof:
Rt Δs 1, 2: $a^2 = m^2 + p^2,\ c^2 = q^2 + n^2$
Rt Δs 3, 4: $b^2 = p^2 + n^2,\ d^2 = m^2 + q^2$

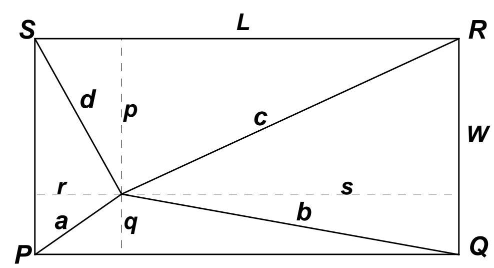
Adding, we have the required result:
$a^2 + c^2 = m^2 + p^2 + q^2 + n^2 = b^2 + d^2$

Q.E.D. (or … That's all folks!)

#2 IF $PQRS$ is a rectangle, $a^2 + c^2 = b^2 + d^2$
From any <u>interior</u> point in a rectangle, the sums of the squares of the distances to opposite vertices are equal.

The proof:
$a^2 = r^2 + q^2, b^2 = q^2 + s^2, c^2 = p^2 + s^2$ and $d^2 = r^2 + s^2$
Adding $a^2 + c^2 = r^2 + q^2 + p^2 + s^2 = b^2 + d^2$

Q.E.D. (or … That's all folks!)***



Hmmmm? – I wonder if it's true for <u>exterior</u> points, or points <u>on</u> the rectangle? How about other quadrilaterals?
Something to ponder in your spare time.

*** This abbreviation is for the Latin "quod erat demonstratum" which translates literally to "which was to be proven". It was commonly used at the end of proofs in surviving Latin translations of Euclid's 13 volume geometric masterpiece 'The Elements'. (ISBN: 978-0-7607-6312-4)