MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2017 SOLUTION KEY

Round 1

A) Adding the first two equations, we have $4z = 2020 \Rightarrow z = 505$.
Adding the last two equations, we have $3y = 3 \Rightarrow y = 1$
$\therefore (x, y, z) = \underline{(-1, 1, 505)}$.

B) If the entries in any two rows (or any two columns) of a matrix are proportional, then the determinant of that matrix will be zero. Comparing rows with row 2 (and columns with column 1), we note that the constants in column 1 are proportional to the corresponding constants in column 3. Specifically, multiplying the entries in column 3 by –2 produces the entries in column 1. Therefore, $k = \underline{-23.2}$.

$$\begin{vmatrix} 14 & 3 & -7 & 8 \\ k & 5 & 11.6 & 3 \\ -6 & 9 & 3 & 5 \\ .8 & .6 & -.4 & -.3 \end{vmatrix}$$

C) Assume Dick walks a mile in T minutes and Marty takes $(T+9)$ minutes.

Rate x Time = Distance $\Rightarrow \begin{cases} (1) & R_D T = 1 \\ (2) & R_M (T+9) = 1 \end{cases}$ (Note: Rates are in miles <u>per minute</u>)

To walk 5 miles, Marty takes $\dfrac{5}{R_M}$ minutes.

To walk 8 miles, Dick takes $\dfrac{8}{R_D}$ minutes.

$\dfrac{5}{R_M} = \dfrac{8}{R_D} \Leftrightarrow R_D = \dfrac{8}{5} R_M$ (3)

Substituting for R_D in (1) $\Rightarrow \left(\dfrac{8}{5} R_M \right) T = 1 \Rightarrow R_M T = \dfrac{5}{8}$

Expanding (2) and substituting for $R_M T$, $\dfrac{5}{8} + 9R_M = 1 \Rightarrow 9R_M = \dfrac{3}{8} \Rightarrow R_M = \dfrac{1}{24}$ miles per

minute $\Rightarrow \dfrac{1}{24} \cdot 60 = \underline{2.5}$ miles per hour.