MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2015 SOLUTION KEY

Round 6

A) The probability of {H,H,H,T,T} is $\frac{5!}{3!2!}\left(\frac{1}{2}\right)^5=\frac{10}{32}$.

The probability of {H,T,T,T,T} is $\frac{5!}{1!4!}\left(\frac{1}{2}\right)^5=\frac{5}{32}$.

Since there is no overlap, the probability of either/or is $\underline{\mathbf{\frac{15}{32}}}$.

B) Using the 3rd row of Pascal's triangle (1-3-3-1), in the expansion of $\left(4x^2+k\right)^3$, the x^2-term is

$3\cdot\left(4x^2\right)^1\cdot k^2=432x^2\Rightarrow 12k^2=432\Rightarrow k^2=36\Rightarrow k=\pm 6$.

However, the constant term is negative, so k = $\underline{\mathbf{-6}}$ only.
The same results could have been obtained by cubing the binomial by brute force.

C)

$$\sqrt{1+x}=\left(1+x\right)^{\frac{1}{2}}=1+\frac{\frac{1}{2}}{1!}x+\frac{\frac{1}{2}\cdot-\frac{1}{2}}{2!}x^2+\frac{\frac{1}{2}\cdot-\frac{1}{2}\cdot-\frac{3}{2}}{3!}x^3+\frac{\frac{1}{2}\cdot-\frac{1}{2}\cdot-\frac{3}{2}\cdot-\frac{5}{2}}{4!}x^4=1+\frac{1}{2}x-\frac{1}{8}x^2+\boxed{\frac{\frac{3}{8}}{6}x^3+\frac{-\frac{15}{16}}{24}x^4}$$

$$\Rightarrow\left(A,B\right)=\underline{\left(\mathbf{\frac{1}{16}},\mathbf{\frac{5}{128}}\right)}$$