MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2009 SOLUTION KEY

Team Round

B) $N-24\sqrt{2}$ and $A-B\sqrt{2}$ must be positive.
Squaring both sides, $N-24\sqrt{2}=(A^2+2B^2)-2AB\sqrt{2}$
$\rightarrow \begin{cases} N=A^2+2B^2 \\ AB=12 \end{cases}$
There are only 6 possible ordered pairs:

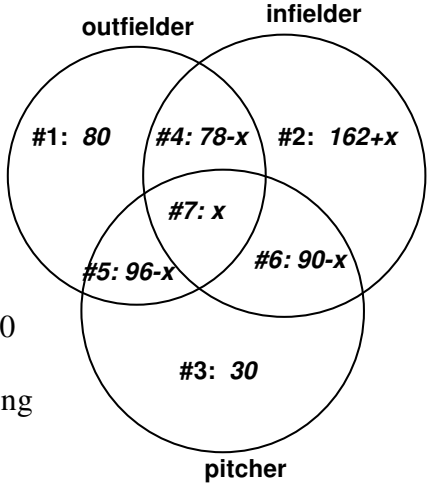
(A, B)	$N\ (A^2 + 2B^2)$
(1, 12)	1 + 288 = 289
(2, 6)	4 + 72 = 76
(3, 4)	9 + 32 = 41
(4, 3)	16 + 18 = 34
(6, 2)	36 + 8 = **44**
(12, 1)	144 + 2 = **146**

The first 4 ordered pairs are rejected since $A-B\sqrt{2}$ is negative.
For the last two ordered pairs, both radical expressions are positive. → N = **44, 146**

C) Since the equation defining the folium is unchanged when x is replaced by y (and vice versa), the graph is symmetric to the line $y = x$. Thus, point P lies on this line.
$\begin{cases} x^3+y^3-3xy=0 \\ y=x \end{cases} \rightarrow 2x^3-3x^2=0 \rightarrow x^2(2x-3)=0 \rightarrow x=0,\ 3/2$

The distance from $\left(\frac{3}{2},\frac{3}{2}\right)$ to $x + y + 1 = 0$ is given by $\frac{\left|1\left(\frac{3}{2}\right)+1\left(\frac{3}{2}\right)+1\right|}{\sqrt{1^2+1^2}} = \frac{4}{\sqrt{2}} = \mathbf{2\sqrt{2}}$

D) Let x denote the number of voters who voted for all three players.
The voters who followed the voting instructions either voted for an outfielder and did not vote for a pitcher (#1 and #4) – the IF part - or voted for a pitcher and did not vote for an outfielder (#3 and #6) - the ONLY IF part. The given information can be summarized in the Venn diagram at the right.



Thus, $80 + (78 - x) + (162 + x) + (96 - x) + x + (90 - x) + 30 = 500$
→ $536 - x = 500$ → $x = 36$
Therefore, the number of voters who correctly deciphered the voting instructions: $(80 + 42) + (30 + 54)$ = **206**