MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2015 SOLUTION KEY

Round 5

A) $\dfrac{\left(\sec 330^\circ \cdot \sin 240^\circ \cdot \tan 495^\circ\right)^3}{\left(\csc 120^\circ \cdot \cot 225^\circ\right)^2} = \dfrac{\left(\dfrac{2}{\sqrt{3}} \cdot \dfrac{-\sqrt{3}}{2} \cdot -1\right)^3}{\left(\dfrac{2}{\sqrt{3}} \cdot 1\right)^2} = \dfrac{1}{\frac{4}{3}} = \underline{\mathbf{\dfrac{3}{4}}}$.

B) $\sin 15^\circ \cdot \cos 30^\circ \cdot \tan 45^\circ \cdot \cot 60^\circ \cdot \sec 75^\circ \cdot \csc 90^\circ = \sin 15^\circ \cdot \dfrac{\sqrt{3}}{2} \cdot 1 \cdot \dfrac{1}{\sqrt{3}} \cdot \dfrac{1}{\cos 75^\circ} \cdot 1$

$= \dfrac{1}{2}\dfrac{\sin 15^\circ}{\cos 75^\circ} = \dfrac{1}{2}\dfrac{\sin 15^\circ}{\sin 15^\circ} = \underline{\mathbf{\dfrac{1}{2}}}$.

C) Since $\overline{AB}$ and $\overline{BD}$ are sides of a right triangle with lengths in a ratio of $1:\sqrt{3}$, ΔBAD is a 30-60-90 triangle. Similarly, ΔABC is a 30-60-90 triangle. Since $\measuredangle ECB = 120^\circ$, ΔEAC is also a 30-60-90 triangle.

Thus, $AD = 12$, $BC = 2\sqrt{3}$, $AC = 4\sqrt{3} \Rightarrow EC = 2\sqrt{3}, EA = 6$

ΔPAD is not a special right triangle, but, applying the Pythagorean Theorem, we have the last length needed. $PA^2 = 12^2 + \left(\sqrt{3}\right)^2 = 147 = 49 \cdot 3 \Rightarrow PA = 7\sqrt{3}$.

Therefore,

$AD + BC + EA + AP = 12 + 2\sqrt{3} + 6 + 7\sqrt{3} =$

$\underline{\mathbf{18 + 9\sqrt{3}}}$ or $\underline{\mathbf{9\left(2+\sqrt{3}\right)}}$.