MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2015 SOLUTION KEY

Round 6

A) $f(0) = 4 = c_0$
$f(1) = 3 = 4 + c_1 + c_2$
$f(2) = 2 = 4 + 2c_1 + 4c_2$
Thus, $c_0 = 4$, $c_1 = -1$ and $c_2 = 0$ and $f(x) =$ **4 – x** .

B) This series consists of terms of an arithmetic progression with a common difference of 3, starting with $3a - 5$. Thus, the sequence is $3a-5,\ 3a-2,\ 3a+1, 3a+4$, etc.
Trying $a = 1$, the sequence would begin with –2, 1, … . Rejected, only 1 negative.
Adjusting, let $a = -1$ and the sequence is –8, –5, –2, 1, 4 - Bingo!
The series generates 5 terms and the sum is (–8) + (–5) + (–2) + 1 + 4 = –10.
Thus, $(a, b, S) =$ **(–1, 3, –10)**.

C) For the geometric sequence $3, -\frac{9}{4}, \frac{27}{16}, \ldots,\ a = 3$ and $r = -\frac{3}{4}$.
Since $|r| < 1$, the sum of the series converges to $\frac{a}{1-r}$.
The sum of all the terms is $\frac{3}{1+\frac{3}{4}} = \frac{12}{4+3} = \frac{12}{7}$

The sum of n terms of any geometric series is given by $\frac{a - ar^n}{1-r}$. We have $\boxed{a = 3, r = -\frac{3}{4}}$.
In this case, the sum of the first 4 terms is $\frac{a\left(1-r^4\right)}{1-r}$.
Rather than "simply" computing $3 - \frac{9}{4} + \frac{27}{16} - \frac{81}{64}$ or plugging in specific values, let's simplify the formula. $\frac{a\left(1-r^4\right)}{1-r} = \frac{a\left(1-r^2\right)\left(1+r^2\right)}{1-r} = \frac{a\cancel{(1-r)}(1+r)\left(1+r^2\right)}{\cancel{1-r}} = a(1+r)\left(1+r^2\right) \Rightarrow$
$3\left(\frac{1}{4}\right)\left(\frac{25}{16}\right) = \frac{75}{64}$. The required ratio is $\frac{12}{7} \div \frac{75}{64} = \frac{\cancel{12}^{4}(64)}{7(\cancel{75}^{25})} = \frac{256}{175} \Rightarrow (A, B) =$ **(256, 175)** .