MASSACHUSETTS MATHEMATICS LEAGUE CONTEST 2 - NOVEMBER 2016 ROUND 1 COMPLEX NUMBERS (No Trig)

ANSWERS

|--|

A) Compute the minimum positive integer A for which $i^A \cdot i^{2A} \cdot i^{4A} \cdot i^{8A} \cdot i^{16A} = i$ is true.

B) Let
$$z = 0.1 + 6i$$
. If $a = \frac{1}{z + \overline{z}}$ and $bi = z - \overline{z}$, compute $|a + bi|$.

Recall:

If z = a + bi, then the conjugate of z (written \overline{z}), is defined as a - bi.

$$|a+bi| = \sqrt{a^2 + b^2}$$
, the distance from the origin to the point $P(a, b)$ in the complex plane.

C) One of the two possible values of $(3+4i)^{\frac{3}{2}}$ may be written as A+Bi, where A and B are positive integers. Compute the ordered pair (A,B).