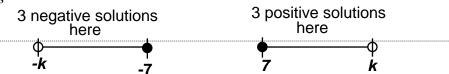
MASSACHUSETTS MATHEMATICS LEAGUE CONTEST 1 - OCTOBER 2016 SOLUTION KEY

Round 5

A) Graphically, the solution set is



$$k = 10 \Longrightarrow -7, -8, -9, 7, 8, 9$$

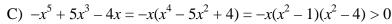
B) Clearly, $x \le 7$, otherwise the right side of the equation is negative and the result would be extraneous.

$$\begin{aligned} &\left|3x+1\right| = \begin{cases} 3x+1 \\ -\left(3x+1\right) \end{aligned} \text{ depending on whether } x \ge -\frac{1}{3} \text{ or } x < -\frac{1}{3} \text{ .} \end{aligned}$$

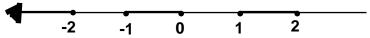
$$\text{For } x \ge -\frac{1}{3}, \ 3x+1 \le 7-x \Rightarrow x \le \frac{3}{2} \Rightarrow -\frac{1}{3} \le x \le \frac{3}{2}$$

$$\text{For } x < -\frac{1}{3}, \ -3x-1 \le 7-x \Rightarrow x \ge -4 \Rightarrow -4 \le x < -\frac{1}{3}$$

$$\Rightarrow -\mathbf{4} \le x \le \frac{3}{2}.$$



Multiplying by -1 and factoring, x(x + 1)(x - 1)(x + 2)(x - 2) < 0, we have critical points at x = -2, -1, 0, 1 and 2. At the extreme left, all 5 factors are negative. As we move from left to right, every time a critical point is passed one more factor becomes positive.



So the sign of the product alternates negative and positive. In the highlighted regions, the product is negative, i.e. x < -2 or -1 < x < 0 or 1 < x < 2.