MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2008 SOLUTION KEY

Round 1

A) $f\left(\frac{x}{x+1}\right) = \frac{\frac{x}{x+1}}{\frac{x}{x+1}+1} = \frac{x}{x+(x+1)} = \underline{\boldsymbol{\frac{x}{2x+1}}}$

B) The expression $(x + 2)(3 – x)$ is:

a) positive for $-2 < x < 3$
b) negative for $x < -2$ or $x > 3$
c) zero for $x = -2$ or $+3$

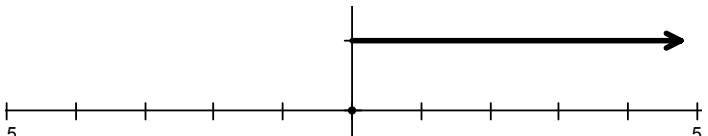
The stated domain includes only cases b) and c).

b)→ $x\left(\frac{1+(-1)}{2}\right) = 0$

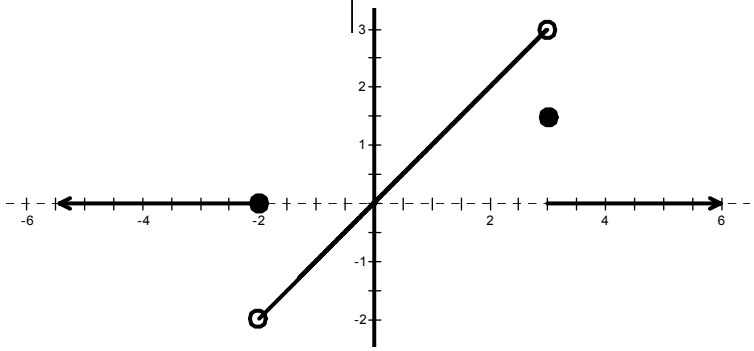
c) for $x = -2$ → $-2\left(\frac{1+0}{2}\right) = -1$ c) for $x = 3$ → $3\left(\frac{1+0}{2}\right) = 3/2$

and the range is **<u>{0, -1, 3/2}</u>**

The graph of sgn(x) is:



The graph of $f(x)$ over <u>all</u> reals is:



C) After t seconds, $(40 – 4t)(30 – 3t) = 108 + (4 + 2t)(3 + 6t)$ → $1200 – 240t + 12t^2 = 120 + 30t + 12t^2$
At this time, the sum of their perimeters is $2(70 – 7t) + 2(7 + 8t) = 154 + 2t$ → $154 + 8 =$ **<u>162</u>** feet