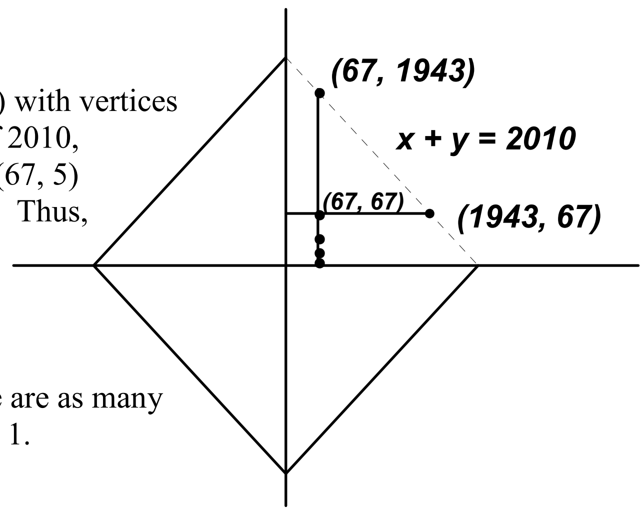
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2010 SOLUTION KEY

Round 5

A) Clearly, for $x > 0$ there is no solution.
For $x < 0$, the equation is equivalent to $x + 2011 = -x$.
Thus, $2x = -2011 \Rightarrow x = -1005.5 \Rightarrow N = \underline{\mathbf{-1006}}$

B) $\left|x^2 - 3\right| < 2 \Rightarrow -2 < x^2 - 3 < +2 \Rightarrow x^2 - 3 > -2$ and $x^2 - 3 < +2$
$\Rightarrow x^2 - 1 > 0$ and $x^2 - 5 < 0 \Rightarrow$ outside ± 1 and inside $\pm\sqrt{5}$
i.e. $\underline{\mathbf{-\sqrt{5} < x < -1}}$ or $\underline{\mathbf{1 < x < \sqrt{5}}}$



C) The given inequality defines a diamond (actually a square) with vertices at $(\pm 2010, 0)$ and $(0, \pm 2010)$. There are 4 prime factors of 2010, namely 2, 3, 5 and 67. Clearly, the points (67, 2), (67, 3), (67, 5) and (67, 67) are in the first quadrant and inside the region. Thus, there are 4(4) = 16 points in quadrant 1 satisfying the requirements and **<u>64</u>** in total.

Note: If (x, y) is a solution of $|x| + |y| \le 2010$,
$(-x, y)$, $(x, -y)$ and $(-x, -y)$ are also solutions; hence, there are as many solutions in quadrants 2, 3 and 4 as there were in quadrant 1.

Round 6

A) The decimal equivalent of my lucky number is computed as follows:

$$1101_2 = 1\left(2^3\right) + 1\left(2^2\right) + 0\left(2^1\right) + 1\left(2^0\right) = 8 + 4 + 1 = 13_{10}$$

Converting to base 3, $13 = 9 + 3 + 1 = 1\left(3^2\right) + 1\left(3^1\right) + 1\left(3^0\right) = \underline{\mathbf{111_3}}$

B) $\left((2*3)\circ 4\right) = 2^3 \circ 4 = 8 \circ 4 = (8+4)^2 = 144$
$\left((4\circ 3)*2\right) = 7^2 * 2 = 49 * 2 = 49^2$
Thus, $\left((2*3)\circ 4\right) \$ \left((4\circ 3)*2\right) = 144 \$ 49^2 = \sqrt{144(49^2)} = 12(49) = \underline{\mathbf{588}}$

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