MASSACHUSETTS MATHEMATICS LEAGUE
DECEMBER 2004 BRIEF SOLUTIONS

Round One:

A. Since $\tan(x)<0$ we are in fourth quadrant with right triangle having sides of 11,5, and $4\sqrt{6}$ so $\csc(x) = \dfrac{-11\sqrt{6}}{24}$.

B. Law of Cosines: $PK^2 = 16^2 + 18^2 - 2(16)(18)(0.75) = 148$ so $PK=2\sqrt{37}$

C. Pythagorus gives $AE=\sqrt{15}$ Law of Sines gives BE= $\sin \angle D$ (BD) / $\sin \angle BED$. Since $\sin \angle CEA = 7/8$, BE = 0.5 (91)(8/7) = 52

Round Two:

A. 26,390=2x5x7x13x29 so 2x13 + 5x7 + 29 = 90.

B. The desired numbers must be the squares of primes so we have 2^2=4 up to 19^2=361 or 8 such numbers

C. $8n^3 - 2197 = (2n)^3 - 13^3 = (2n - 13)(4n^2 + 26n + 169)$ which is prime only if one of the factors is one. Thus n=7 and the number is 547.

Round Three:

A. (61-1)/3=20; (45-0)/3=15 Find (1+n(20) ,0+ n(15)) for n=1 and 2.

B. OQ = |b|; OP = |–b/2| ; so |b| times |–b/2| = 200, thus $b^2 = 400$

C. Perpendicular through origin is $y = -2x$; system solves to $\left(\dfrac{-k}{5}, \dfrac{2k}{5}\right)$

Round Four:

A. $x(x-6) = 16$ so $x = 8$ or $x = -2$ Since –2 has no log the solution is $x = \mathbf{8}$.

B. Use log properties simplify to $3 - 5 = x^2 - 3x$ to $0 = (x-1)(x-2)$

C. Divide $3/8 = a b^3$ by $½ = a b$ to get $¾=b^2$ so $b=\sqrt{3}/2$, $a=\sqrt{3}/3$

Round Five:

A. Scale by 0.5 for thickness, $(0.5)^2$ for depth, 2.0 for length. Net scaling 0.25

B. If area DECB=x, area ABC=2.5x and area ADE=1.5x so similar triangles have ratio $\sqrt{0.6}$ and AD=$\left(\sqrt{0.6}\right)82.175 = 63.6524... \approx 63.652$

C. 1/6 + (1/6 + 1/x) + 204/60 (1/6) =1 so x =10. 5/6=T(1/6 + 1/10) so T=25/8

Round Six:

A. Number sides consecutively. Sum of even sides = sum of odd sides so
2 + 8 + 10 = 5 + AB + CD thus AB + CD = 15

B. If $\angle B=x$, $\angle EDF=2x$, $\angle GEF=3x$ etc $\angle A=7x$ so sumΔABC gives x=12. Since $\angle 1=6x$, $\angle 2=4x$, and $\angle 3=2x$, their sum is 144.

C. Ratio of angles is (180–360/n) / (360/n) simplifies to (n–2)/n. Solving (n/2)(n–3) = 20.9 (n–2)/n so $10n^2 - 30n = 209n - 418$ so n = 22 (or 1.9)