MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2009 SOLUTION KEY

Round 4

A) $x^2 - 3A^2B^2 = 2ABx \rightarrow x^2 - 2ABx - 3A^2B^2 = (x - 3AB)(x + AB) = 0$
$\rightarrow x = 3AB$ or $-AB$ Thus, $x = \underline{\boldsymbol{-AB}}$ only.

B) $\begin{cases} 9 = 9A + 12 + B \\ -7 = 25A - 20 + B \end{cases} \rightarrow -16 = 16A - 32 \rightarrow A = 1$ and $B = -12$
Thus, the expression $x^2 - 4x - 12$ is equivalent to $(x - 2)^2 - 16$.
The minimum value of $\underline{\boldsymbol{-16}}$ occurs when $x = 2$.

C) Re-arranging the terms of $3x^2 - 6x + xy + 4y - 2y^2 = 0$, we have $3x^2 + xy - 2y^2 - 2(3x - 2y) = 0$
$\rightarrow (3x - 2y)(x + y) - 2(3x - 2y) = 0$
$\rightarrow (3x - 2y)(x + y - 2) = 0 \rightarrow 3x - 2y = 0$ or $x + y - 2 = 0$
$\rightarrow y = \underline{\boldsymbol{\frac{3x}{2}, 2 - x}}$