Round Six:

A. $2 = z^2 + z$ has solutions of $z = 1$ and $z = -2$. Since $y = z(x) = 2z$, $y = 2$ or -4

B. Slope $4c/2 = -0.5$ so $c = -0.25$. Translation shows (0, -3c) is also a point so y-intercept is 0.75.

C. Ages in 1996 were s and 7s; three years ago (2002) ages were s+6 and 7s+6. If $3(s+6) = 7s+6$ then $s = 3$ and Joe is 18 years older. Sue turns 18 in 2011.

Team Round:

A. The angle bisector divides AC into the ratio BC:BA or 13:5 so $x = \dfrac{(1)(13)+(-8)(5)}{5+13} = -1.5$ and $y = \dfrac{(2)(13)+(11)(5)}{5+13} = 4.5$

B. $x^3(x^{4n} + 4x^{2n} + 16) = x^3(x^{4n} + 8x^{2n} + 16 - 4x^{2n}) = x^3[(x^{2n}+4)^2 - (2x^n)^2] =$ $x^3(x^{2n} + 2x^n + 4)(x^{2n} - 2x^n + 4)$

C. Since $\cos(a-b) - \cos(a+b) = 2\sin(a)\sin(b)$, $\cos x - \cos 3x + \sin 2x =$ $\cos(2x-x) - \cos(2x+x) + \sin 2x = 2\sin 2x \sin x + \sin 2x = (2\sin x + 1)\cdot\sin 2x$ so $n = 2$ and *foo* is sin. Thus $n + foo(\pi/3n) = 2 + \sin(\pi/6) = 2.5$

D. If AD=6 and AB=x and FB=y then $6^2 + (x-y)^2 = y^2$ so $y = \dfrac{36+x^2}{2x}$ and the area of the triangles not in the shared region is $6(x-y) = 6x - \dfrac{108+3x^2}{x}$ which is also $6x - 45$ so $\dfrac{108+3x^2}{x} = 45$ so $3x^2 - 45x + 108 = 0$ and x=3 or 12. Since x>6, the answer is **12**

E. Let AD = x By angle bisector thm, $x/25 = 39 - x/40 \rightarrow x = 15$. By Stewart's theorem*, $25^2 \cdot 24 + 40^2 \cdot 15 = BD^2 \cdot 39 + 39 \cdot 15 \cdot 24 \rightarrow BD^2 = 640$ so $BD = 8\sqrt{10}$ (or drop altitude to AC solve system to get h=24 and altitude divides the 15 into 7 and 8 making BD hypotenuse w. legs 24, 8). Triangle BAE is isos. so AE= 25 By AA, $\Delta ADE \sim \Delta CDB$ w ratio 5:8 so $DE = 5\sqrt{10}$ Thus, per=$15+25+5\sqrt{10}$

F. One solution is x =2; divide by $(x-2)$ to get $x^2 + 2x + 4 = ax + b$. Solutions must be either 1&3 $(x^2 - 4x + 3) = 0$ so a=6,b=1 or 3&4$(x^2 - 7x + 12) = 0$ so a=9,b=–8

*Stewart's Theorem (Matthew Stewart, 1717-1785 Scottish Mathematician): In any triangle with sides a, b, c and any segment m from C dividing c into x and y, $a^2x + b^2y = m^2c + cxy$