MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 - FEBRUARY 2010
ROUND 6 ALG 2: SEQUENCES AND SERIES

******* NO CALCULATORS IN THIS ROUND *******

ANSWERS

A) _____

B) (_____ , _____ , _____ , _____)

C) _____

A) The sequence $\{a_n\}$ has $a_1 = \frac{1\cdot 2}{3\cdot 4}$, $a_2 = \frac{3\cdot 4}{5\cdot 6}$, $a_3 = \frac{5\cdot 6}{7\cdot 8}$. Compute $a_{12}\cdot a_{13}$.

B) Consider the following sequence of ordered pairs:

$$t_1 = \left(3\cdot 5,\ 2\cdot 3^2\right),\ t_2 = \left(5\cdot 7,\ 2^2\cdot 3\right),\ t_3 = \left(7\cdot 9,\ 2^3\cdot 1\right),\ t_4 = \left(9\cdot 11,\ 2^4\cdot \frac{1}{3}\right),\ \ldots$$

The 15th term can be written in the form $\left(A\cdot B, 2^x\cdot 3^y\right)$.
Compute the ordered quadruple (A, B, x, y).

C) AB, $3AB$, $18A$ form an increasing geometric progression.
A^3, $A + B + 1$, B form a decreasing arithmetic progression.
If A, B, C and D are real numbers and $\left(A + Bi\right)^3 = C + Di$, where $i = \sqrt{-1}$,
compute $\frac{C}{D}$.