MASSACHUSETTS MATHEMATICS LEAGUE
OCTOBER 2003
ROUND 4: FRACTIONS & MIXED NUMBERS
NON-CALCULATOR

ANSWERS

A) -1

B) $-10/-8$, $5/5$

C) $-111/44$

A) If $\frac{1}{a(b+1)} + \frac{1}{b(a+1)} = \frac{1}{(a+1)(b+1)}$, what is the value of $\frac{1}{a} + \frac{1}{b}$?

$1(a+1) + a(b+1) = ab$ $\quad$ So $\frac{1}{a} + \frac{1}{b} = \frac{b+a}{ab} = -1$

$a + 1 + ab + a = ab$

$1 + a = -ba$

$\frac{b+a}{ba} = -1$

B) The numerator of a fraction is two less than the denominator. When both the numerator and the denominator are increased by five, the result is 4/3 of the original fraction. What was the original fraction?

$\frac{x-2}{x} = \text{orig fract}$ $\quad \frac{x+3}{x+5} = \frac{4(x-2)}{3x}$ $\quad x^2 + 3x - 40 = 0$

$(x+8)(x-5) = 0$

$3x(x+3) = (4x-8)(x+5)$ $\quad x = -8, \; x = 5$

$3x^2 + 9x = 4x^2 + 12x - 40$ $\quad$ ANS $\frac{-10}{-8}, \frac{3}{5}$

C) If $\frac{x}{y} = \frac{3y}{2y}$ and $\frac{x}{y} + \frac{2}{3}$, what is the value of $\frac{3x}{4y}$ expressed as a fraction?

$\frac{x-3y}{x+2y} \quad \frac{14}{3}$ $\quad 3x - 9y = 14x + 28y$

$-11x = 37y$

$\frac{x}{y} = -\frac{37}{11}, \quad \frac{3x}{4y} = -\frac{111}{44}$