

MASSACHUSETTS MATHEMATICS LEAGUE
FEBRUARY 2004
ROUND 2: NUMBER THEORY

ANSWERS

A) (3, 2), (6, 4)

B) 2

C) 120

A) Given $(ABA)_9 = (BB0)_{11}$ where 0 is zero, and A and B are distinct natural numbers. Determine both possible values of A and B. Write the answers in the form (A, B).

$$81A + 9B + A = 121B + 11B$$

$$82A = 132B - 9B = 123B$$

$$2A = 3B, \quad A = 3, B = 2 \text{ or } A = 6, B = 4$$

B) Determine the units digit of $7^{2003} + 9^{2003}$.

$$\begin{array}{r|l} 7^0 & 1 \\ 7^1 & 7 \\ 7^2 & 9 \\ 7^3 & 3 \\ 7^4 & 1 \end{array}$$

$$\begin{array}{r|l} 9^0 & 1 \\ 9^1 & 9 \\ 9^2 & 1 \end{array}$$

$$4 \overline{) 2003} \quad R=3$$

$$2 \overline{) 2003} \quad R=1$$

$$3 + 9 = 12 \text{ (2)}$$

C) How many positive even divisors does $(12^3)(18^4)$ have?

$$(2^2 \cdot 3)^3 (2 \cdot 3^2)^4 = 2^6 \cdot 3^3 \cdot 2^4 \cdot 3^8 = 2^{10} \cdot 3^{11}$$

$$\# \text{ divisors} = (10+1)(11+1) = 11(12) = 132$$

$$\underline{\text{Ans}} \quad 132 - \text{odd divisors} = 132 - 12 = 120$$