MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2006 SOLUTION KEY

Round 1

A) $i(2 + 3i)(1 - 4i) = i(2 - 8i + 3i - 12i^2) = i(2 - 5i + 12) =$ **<u>5 + 14i</u>**

B) Let $\sqrt{2i} = \sqrt{0+2i} = \sqrt{(a+bi)^2} = \sqrt{(a^2-b^2)+(2ab)i} \rightarrow a^2 - b^2 = 0$ and $ab = 1$
Thus, $b > 0 \rightarrow b = 1$ and $a = 1 \rightarrow$ **<u>$1 + i$</u>**

C) $(1-i\sqrt{3})^2 = -2 - 2i\sqrt{3} = -2(1 + i\sqrt{3})$
$(1-i\sqrt{3})^4 = [-2(1 + i\sqrt{3})]^2 = 4(-2 + 2i\sqrt{3}) = -8(1 - i\sqrt{3})$
$(1-i\sqrt{3})^8 = [-8(1 - i\sqrt{3})]^2 = 64(-2 - 2i\sqrt{3}) = -128(1 + i\sqrt{3})$
Thus, the sum is $(-2 - 8 - 128) + (-2 + 8 - 128)i\sqrt{3} =$ **<u>$-138-(122\sqrt{3})i$</u>**

Round 2

A) $4^2 + 3\cdot7 - 8^{-2}\cdot\frac{2^8}{5\cdot9}\div\frac{1}{4\cdot3}\cdot6^2 = 16 + 21 - \frac{1}{2^6}\cdot\frac{2^8}{5\cdot3^2}\cdot2^2\cdot3\cdot2^2\cdot3^2 = 37 - \frac{2^6\cdot3}{5} = 37 - \frac{192}{5}$
$= 37 - 38.4 =$ **<u>-1.4</u>**

B) Let $a = \left(\frac{1+x}{2}\right)$ Think $a^2 - 3a - 18 = 0 \rightarrow (a - 6)(a + 3) = 0 \rightarrow a = 6$ or -3
Substituting for a, $1 + x = 12$ or $-6 \rightarrow x =$ **<u>11 or -7</u>**

C)

	Now	In year of my birth
Me	x	0
Ramanujan	$2x + 1$	$x + 1$

According to the chart, $x + (2x + 1) = 3x + 1 = 178 \rightarrow x = 59 \rightarrow 2006 - 59 =$ **<u>1947</u>**