MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 – MARCH 2008 SOLUTION KEY

Round 2

A) Suppose $N = 4^{y}$.
$N = 2(64)^{-2/3} = 2(4)^{-2} = 2^{-3} = 4^{y} = 2^{2y} \rightarrow 2y = -3 \rightarrow y =$ **-3/2**

B) $$= \left(\sqrt{4 + x^2 - 2 + \frac{1}{x^2}}\right)\left(\frac{x^2+1}{3x}\right)^{-1} = \left(\sqrt{x^2 + 2 + \frac{1}{x^2}}\right)\left(\frac{3x}{x^2+1}\right) = \left(\sqrt{(x + x^{-1})^2}\right)\left(\frac{3x}{x^2+1}\right)$$
$$= \left(\sqrt{\frac{\left(x^2+1\right)^2}{x^2}}\right)\cdot\left(\frac{3x}{x^2+1}\right) = \frac{x^2+1}{|x|}\cdot\frac{3x}{x^2+1} = \frac{3x}{|x|} = \underline{\mathbf{\pm 3}}$$

C) In order to extract the square roots in both the numerator and the denominator, the radicands must be perfect squares. Therefore, let $49 - 8\sqrt{3} = \left(A + B\sqrt{3}\right)^2$ and $21 + 12\sqrt{3} = \left(C + D\sqrt{3}\right)^2$.
Multiplying out and equating rational and irrational parts we have,
$\begin{cases} A^2 + 3B^2 = 49 \\ AB = 4 \end{cases} \rightarrow (A, B) = (-1, 4)$ and $\begin{cases} C^2 + 3D^2 = 21 \\ CD = 6 \end{cases} \rightarrow (C, D) = (3, 2)$

Thus, $\dfrac{\sqrt{49 - 8\sqrt{3}}}{\sqrt{21 + 12\sqrt{3}}} = \dfrac{-1 + 4\sqrt{3}}{3 + 2\sqrt{3}}\cdot\dfrac{3 - 2\sqrt{3}}{3 - 2\sqrt{3}} = \dfrac{-3 + 2\sqrt{3} + 12\sqrt{3} - 24}{-3} = \dfrac{27 - 14\sqrt{3}}{3} \rightarrow$ **(27, 14)**