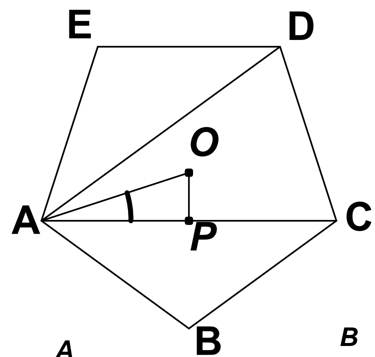
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2010 SOLUTION KEY

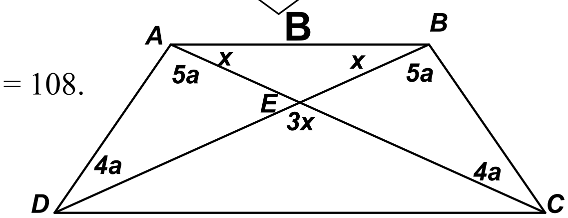
Round 6



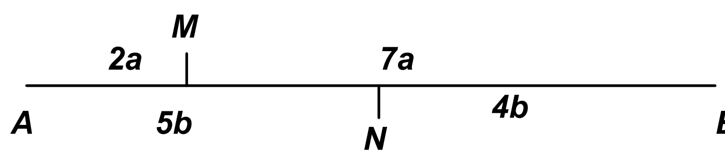
A) m$\angle E$ = 108° ➔ m$\angle EAD$ = 36° (Base angle of isosceles ΔEAD)
Likewise, m$\angle BAC$ = 36° ➔ m$\angle DAC$ = 36°

m$\angle OAP = \dfrac{1}{2}$ m$\angle DAC$ = 18°

$\dfrac{360}{n} = 18$ ➔ n = **<u>20</u>**.

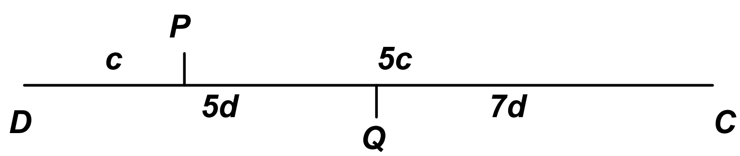


B) In ΔAEB, we have $x + x + 3x = 180$ ➔ $x = 36$ ➔ m$\angle DEC$ = 108.
Since $\angle DEC$ is an exterior angle of ΔAED, $9a = 108$
➔ $a = 12$ ➔ m$\angle ADE$ = m$\angle BCE$ = **<u>48</u>**°.



C) The fact that $\dfrac{AM}{MB} = \dfrac{2}{7}$ and $\dfrac{AN}{NB} = \dfrac{5}{4}$ is shown in the diagram at the right.

Clearly, $9a = 9b$ and $a = b$.



The fact that $\dfrac{DP}{PC} = \dfrac{1}{5}$ and $\dfrac{DQ}{QC} = \dfrac{5}{7}$ is shown in the diagram at the right.
Clearly, $6c = 12d$ and $c = 2d$.

As opposite sides of a parallelogram, $AB = CD$ ➔ $9a = 12d$ ➔ $\dfrac{a}{d} = \dfrac{4}{3}$

Thus, the required ratio is $\dfrac{MB}{PC} = \dfrac{7a}{5c} = \dfrac{7a}{10d} = \dfrac{7}{10} \cdot \dfrac{4}{3} = \underline{\mathbf{\dfrac{14}{15}}}$