

**MASSACHUSETTS MATHEMATICS LEAGUE  
CONTEST 6 - MARCH 2009 SOLUTION KEY**

**Team Round**

F) Let  $(H, M)$  denote Hits and Misses from the free throw line.

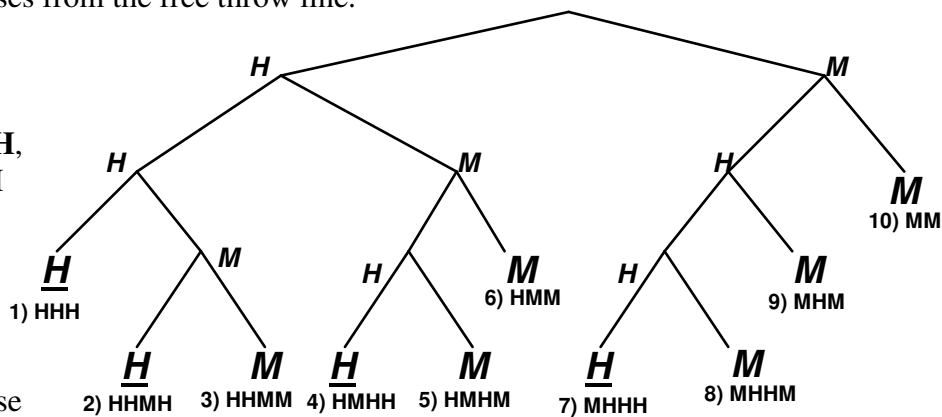
# Free Throws

2 (1) **MM**

3 (3) **HHH**, **MHM**, **HMM**

4 (6) **HHMH**, **HMHH**, **MHHH**,  
**HHMM**, **HMHM**, **MHHM**

The 10 cases where Martha hits 3 or misses 2 are summarized in the diagram at the right.



The underlined cases denote those cases where she has hit 3 before she missed 2.

Going left,  $P(\text{Hit}) = \frac{3}{4}$ . Going right,  $P(\text{Miss}) = \frac{1}{4}$ .

Thus, success occurs in 4 out of the 10 cases and the required probability is

$$\frac{\left(\frac{3}{4}\right)^3 + 3\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)}{\left(\frac{3}{4}\right)^3 + 3\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right) + 3\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)^2 + 2\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^2} = \frac{\frac{27}{64} + \frac{81}{256}}{\frac{27}{64} + \frac{81}{256} + \frac{27}{256} + \frac{6}{64} + \frac{1}{16}}$$

$$= \frac{108 + 81}{108 + 81 + 27 + 24 + 16} = \frac{189}{256}$$

Note that the denominators in the first line above are each equal to 1, so simply evaluating the numerators gives us the same result.