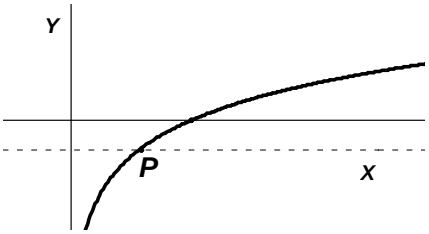
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2015 SOLUTION KEY

Round 4

A) $\log_3(3x) = 9 \Rightarrow 3x = 3^9 \Rightarrow x = 3^8$, $3y = 81 \Rightarrow y = 27$ Thus, $\frac{y}{x} = \frac{3^3}{3^8} = \frac{1}{3^5} = \frac{1}{243} \Rightarrow$ **<u>1 : 243</u>**.

B) Consider the horizontal line $y = k$. For any value of k, it intersects $y = \log_8 x$ exactly once at point P. We require the distance from P to the y-axis to be 0.25, but this is simply the x-coordinate of the point P. Thus, we have

$k = \log_8(0.25) \Leftrightarrow 8^k = 2^{-2} \Rightarrow k = -\frac{2}{3} \Rightarrow P\underline{\left(\mathbf{\frac{1}{4}}, -\mathbf{\frac{2}{3}}\right)}$.



C) Suppose $\log_8 49 = N$. Then: $8^N = 49 \Leftrightarrow 2^{3N} = 7^2$ Taking the log of both sides,

$3N\log 2 = 2\log 7 \Rightarrow N = \frac{2}{3}\cdot\frac{\log 7}{\log 2} \Rightarrow \boxed{\log_8 49 = \frac{2}{3}\log_2 7}$.

$W = \log_{14} 0.125 = \log_{14}\frac{1}{8} = \log_{14} 2^{-3} = -3\log_{14} 2 = -3\cdot\frac{\log 2}{\log 14} = \frac{-3\log 2}{\log 2 + \log 7} = \frac{-3}{1+\frac{\log 7}{\log 2}} = \frac{-3}{1+\log_2 7}$

Cross multiplying, $W + W\log_2 7 = -3 \Rightarrow \log_2 7 = \frac{-3-W}{W}$

Substituting, $\log_8 49 = \frac{2}{3}\left(\frac{-3-W}{W}\right) = -\frac{2}{3}\left(\frac{W+3}{W}\right) \Rightarrow (m,b) = \underline{\left(-\mathbf{\frac{2}{3}}, \mathbf{3}\right)}$.