MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 – NOVEMBER 2012 SOLUTION KEY

Round 6

A) Let $m\angle M = m\angle A = m\angle G = a°$ and $m\angle I = m\angle C = b°$.
Then: $3a + 2b = 3(180) = 540$
$b = 1 \Rightarrow 538/3$ fails
$b = 2 \Rightarrow 536/3$ fails
$b = \underline{\mathbf{3}} \Rightarrow 534/3 = 178$ Bingo!

B) The minute hand makes a complete revolution (or turns through 360°) in 1 hour, whereas the hour hand takes 12 hours to make a complete revolution.
Since the minute hand turns 12 times faster than the hour hand, in one minute the minute hand turns through $\frac{360°}{60} = 6°$ and the hour hand turns through $\frac{1}{2}°$.
At 4:21, the minute hand has turned though 126° (6° per minute measured from the top of the hour). The hour hand has turned $\frac{1}{12}$ as far, namely through $\frac{1}{12}(126°) = 10.5°$ or (130.5° measured from the top of the hour). Thus, the minute hand has passed the hour hand and the degree measure of angle between the hands is $130.5 - 126 = \underline{\mathbf{4.5}}$.

C) Draw a line through S parallel to $\overleftrightarrow{TA}$. Two pairs of alternate interior angles are formed, one pair measuring $2x°$ and the other pair measuring $3x°$. Thus, $m\angle S = 5x$. Similarly, $m\angle L = 7x$.
Since the other 4 angles are supplements of the marked angles, they have measures of $180 – x$, $180 – 2x$, $180 – 3x$ and $180 – 6x$.
The largest angle must be either $\angle L$ or $\angle TAL$.
To guarantee $\angle L$ is the largest, we require $7x > 180 – x$ or $x > 22.5$
Since $m\angle L = 7x < 180$ and x is an integer, $x \le 25$.
Thus, we must examine angle measures for x = 23, 24 and 25.
Let x = 23. The 6 angles in hexagon *POSTAL* (in increasing order) measure
(P) <u>42°</u>, (O)111°, (S) 115°, (T) 134°, (A) 157° and (L) <u>161°</u>$\Rightarrow$ 203°.
For the other possible values of x, the smallest angle will be P ($180 – 6x$) and the largest will be L ($7x$). As x increases by 1, $m\angle P$ decreases by 6 and $m\angle Q$ increases by 7, changing the net total by +1, producing additional totals of 204 and 205.
The required sum is 30 + 175 = $\underline{\mathbf{205}}$.