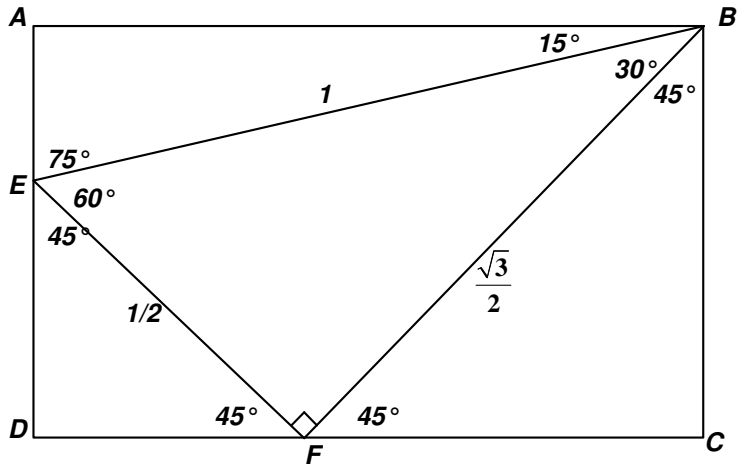
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2008 Notes

Ever need to explain how the sin(15°) is computed to a student with limited experience with trig formulas, i.e. no experience with formulas like sin($A \pm B$)?

Thanks to Mary Beth McGinn for the following gem.

Consider rectangle $ABCD$ with an embedded 30 – 60 – 90 right triangle having sides as indicated.



$$FC = BC = AD = \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} = \frac{\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{6}}{4}$$

$$DE = DF = \frac{1}{2} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$$

$$AB = DF + FC = \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$AE = AD - DE = \frac{\sqrt{6}-\sqrt{2}}{4}$$

Now we have the <u>exact</u> lengths of the 3 sides in ΔABE, the 15 – 75 – 90 right triangle.

Using only the basic definitions of sine and cosine (SOH·CAH·TOA)

$$\mathbf{\sin 15° = \cos 75° = \frac{\sqrt{6}-\sqrt{2}}{4}} \quad \text{and} \quad \mathbf{\sin 75° = \cos 15° = \frac{\sqrt{6}+\sqrt{2}}{4}}$$