Round Six:

A. HW done = 4 + 2 + 1 + …. + a geometric progression of 180 terms with r = ½. The difference between the sum of 180 terms and the sum of an infinite sequence is considerably less than 1 minute, so use $a/(1 - r)$ → $4/(1 - ½) = 8$ hrs = 480 min

B. $a_{2006} = a_{2000} + 6d = 2(a_{2000} + 4d)$, so $a_{2000} = -2d$, while $a_{2000} + 6d = 500 + 3a_{2000}$ so $a_{2000} = 3d - 250$. Thus, $-5d = -250$ → $d = 50$, $a_{2000} = -100$ and $a_{2005} = -100 + 5(50) = 150$.

C. $a + ar + ar^2 = 296$, while $a/(1 - r) = 512$. $a = 296/(1 + r + r^2) = 512(1 - r)$, so $296/512 = (1 + r + r^2)(1 - r) = 1 - r^3$ → $r^3 = 1 - 296/512 = 216/512$ → $r = 3/4$ and $a = 128$.

Team Round:

A. $f^{-1}(x) = \dfrac{1-3x}{2-x}$ so if $f(x) \cdot f^{-1}(x) = \dfrac{2x-1}{x-3} \cdot \dfrac{1-3x}{2-x} = \dfrac{-6x^2+5x-1}{-x^2+5x-6} = 1$ then $5x^2 = 5$, so $x = \pm 1$.

B. Smallest such pair is 3 and 5. Largest such pair is 59 and 61. (Note that twin primes with three or more digits either share the most significant digit or the smaller has 9 as both its one and tens digit) Sum is 128.

C. Draw a right triangle to find $\tan(B) = 3/5$. $\tan A \cdot \tan B \cdot \tan C = \tan A + \tan B + \tan C$ gives $\tan C = -1$ OR use tangent sum identity to find $\tan(A + B) = 1$.

D. I am x years old now. "Then" my mother was $2x$ and my brother $x - 12$. Thus, $2x - (x - 12) = 26$ → $x = 14$. I am 14 and my brother is 22 now. My mother is 26 + 22 = 48 now and thus, 34 when I was born.

E. (See diagram below) $CD = AC(CB)/IC = 12$; $CG = .5(18) - 6 = 3$. Since ΔCGJ is a 30-60-90, $CJ = 2\sqrt{3}$, so $JE = .5(22) - 4 - CJ$.
$GF = GJ + 2(JE) = \sqrt{3} + 2(7 - 2\sqrt{3}) = 14 - 3\sqrt{3}$, while $EF = \sqrt{3}JE = 7\sqrt{3} - 6$
Difference: $20 - 10\sqrt{3}$

F. $\sqrt{20(3j+2)} = 3k + 2$. Square and simplify to $20j = 3k^2 + 4k - 12$ which must be a multiple of 4, so k is even. (WHY?)
If we substitute $k = 2n$ we have $3n^2 + 2n - 3 = 5j$.
Trial and error yields $n \cong 3 \bmod 5$ or $n = 3, 8, 13, \ldots$
→ $k = 6, 16, \ldots$ Since $k > 6$, $k = 16$ and $j = 41$.