MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2016 SOLUTION KEY

Round 3

A) $3\sin^2 x + 2\cos x + 2 = 0$

$\Leftrightarrow 3\left(1-\cos^2 x\right)+2\cos x + 2 = 0$

$\Leftrightarrow 3\cos^2 x - 2\cos x - 5 = \left(3\cos x - 5\right)\left(\cos x + 1\right) = 0$

$\Rightarrow \cos x = \cancel{\frac{5}{3}}, -1 \Rightarrow x = \underline{\boldsymbol{\pi}}$.

B) $\cos x = 0 \Rightarrow x = 90° + 180n \Rightarrow 90°, 270°, \ldots$

$2\sin x + 1 = 0 \Rightarrow \sin x = -\frac{1}{2} \Rightarrow x = \begin{cases} 210° + 360n \\ 330° + 360n \end{cases} \Rightarrow$ 210°, 570°, … or 330°, 690°, …

Arranging in order of increasing magnitude, 90°, 210°, 270°, 330°, 450°, 570°, 690°, … , we see the minimum value of k is **<u>330</u>**.

C) Converting to sine functions only,

$\sin\left(3\theta - 160°\right) = \cos\left(150° - 2\theta\right) \Leftrightarrow \sin\left(3\theta - 160°\right) = \sin\left(90° - \left(150° - 2\theta\right)\right) = \sin\left(2\theta - 60°\right)$

Noting that $\sin A = \sin B \Rightarrow A = B + n\left(360°\right)$ or $A = 180° - B + n\left(360°\right)$, we examine 2 cases.

Case 1: $3\theta - 160° = 2\theta - 60° + n\left(360°\right) \Rightarrow \theta = 100° + \cancel{n\left(360°\right)}$

(We ignore the co-terminal factor, since we want the <u>smallest</u> positive value.)

Case 2: $3\theta - 160° = 180° - \left(2\theta - 60°\right) + n\left(360°\right)$

$\Rightarrow 5\theta = \left(160 + 180 + 60\right)° + n\left(360°\right) = 400° + n\left(360°\right)$

$\Rightarrow \theta = 80° + 72n$. [80° is NOT the smallest.]

$n = -1$ gives us the <u>smallest</u> positive solution $\theta = \underline{\mathbf{8}}°$.

A Check: For $\theta = 8$,

$\sin\left(3\theta - 160\right) = \sin\left(-136°\right) = -\sin\left(136°\right) = -\sin\left(180° - 44°\right) = -\sin 44°$.

$\cos(150° - 2\theta) = \cos\left(134°\right) = \cos\left(180° - 46°\right) = -\cos\left(46°\right) = -\sin\left(90° - 46°\right) = -\sin 44°$.

You should be able to give a reason for each of the above equalities. For example, the sine is an odd function, or, the cosine of an angle equals the sine of its complement, etc.