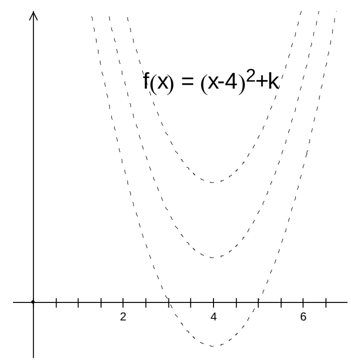
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2017 SOLUTION KEY

Round 1

A) $P(4,15)$, $Q(-12,-5)$. The slope of the line is $m=\dfrac{15-(-5)}{4-(-12)}=\dfrac{20}{16}=\dfrac{5}{4}$.

Therefore, a change of +4 in *x* produces a change of +5 in *y*.
Starting at *Q*, we have $(-12+4,-5+5)=(-8,0)\Rightarrow h=\underline{\mathbf{-8}}$.



B) Given: $y=(x-4)^2+k$

By inspection, the vertex is at $V(4,k)$. By letting $x=0$, $B(0,16+k)$.

Therefore, $$BV=\sqrt{(4-0)^2+(k-(16+k))^2}=\sqrt{16+(-16)^2}=\sqrt{2^4+2^8}$$
$$=\sqrt{2^4(1+2^4)}=\underline{\mathbf{4\sqrt{17}}}.$$

C) Since the center of C_1 must lie on $\mathscr{L}$,

$m=\dfrac{3}{4}$, $\Delta x=+4$, $\Delta y=+3$.

$P(-2,-9)\to C_1(2,-6)\to R(2+4,-6+3)=\boxed{R(6,-3)}$

$r_2=40\Rightarrow PQ=80=16\cdot 5$

Using a slope of $\dfrac{3}{4}$ and *similar* triangles,

$16(4\text{-}3\text{-}5)\Rightarrow 64\text{-}48\text{-}80\Rightarrow$

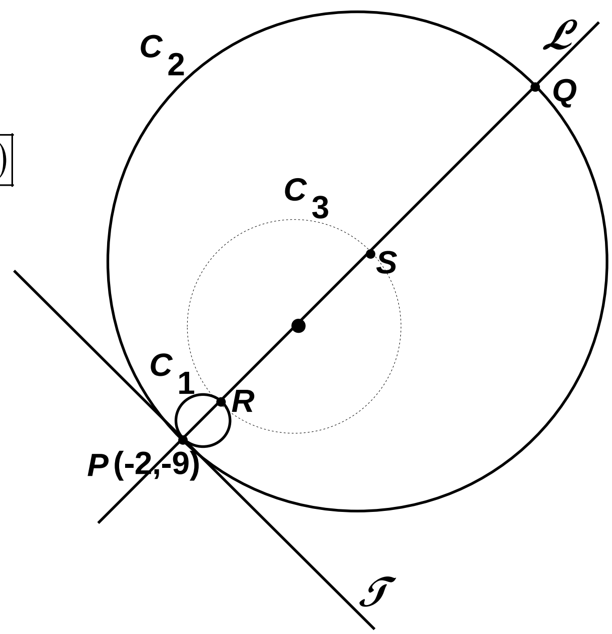
$(-2+64,-9+48)=Q(62,39)$

S is the midpoint of $\overline{PQ}\Rightarrow$

$\left(\dfrac{-2+62}{2},\dfrac{-9+39}{2}\right)=\boxed{S(30,15)}$

The center of C_3 is the midpoint of $\overline{RS}\Rightarrow$

$\left(\dfrac{6+30}{2},\dfrac{-3+15}{2}\right)=C_3(18,6)$



The radius is the distance from $(18,6)$ to $(30,15)$ which is $\sqrt{12^2+9^2}=\sqrt{225}=15$.

Thus, the required ordered triple is $\underline{\mathbf{(18,6,225)}}$.