MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2017 SOLUTION KEY

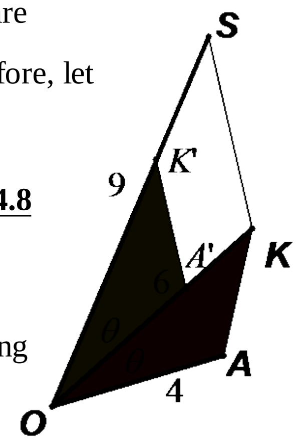
Round 5

A) Since $\angle SOK \cong \angle KOA$ and the lengths of the sides that include these angles are proportional, namely, $\left(\frac{6}{4}=\frac{9}{6}\right)$, $\Delta SOK \sim \Delta KOA$ by SAS and $\frac{KS}{AK}=\frac{3}{2}$ Therefore, let $KA=2x$ and $KS=3x$.

The perimeter of $SOAK$ is $4+9+5x=21 \Rightarrow 5x=8 \Rightarrow x=\frac{8}{5} \Rightarrow KS=$ $\underline{\mathbf{\frac{24}{5}}}$ or **4.8**

Why does SAS~ work?

Pivot (rotate) ΔKOA about point O through an angle θ and $\overline{A'K'} \parallel \overline{SK}$, resulting the more common scenario where similar triangles are formed when a line passes through two sides of a triangle parallel to the third side.



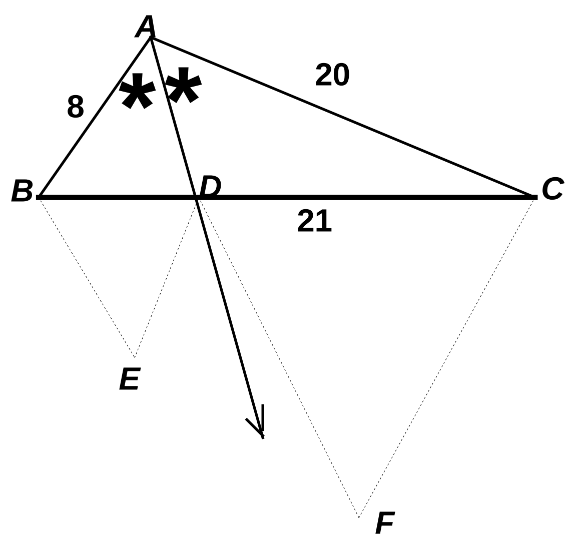
B) $(x-2)+(2x)+(2x+1)=49 \Rightarrow x=10$

$\Rightarrow (AB, AC, BC)=(8,20,21)$

According to the angle bisector theorem,

$\frac{BD}{DC}=\frac{AB}{AC}=\frac{8}{20}=\frac{2}{5}$.

Without determining the lengths of $\overline{BD}$ and $\overline{DC}$, we know the required ratio is 4 : 25 which gives us $K+J=\underline{\mathbf{29}}$.



C) Since the 3-4-5 triangle has an area of 6, the sides of T are scaled by a factor of $\frac{1}{\sqrt{6}}$. Inscribing T in a circle means that the hypotenuse of T will be the diameter of the circle.

Thus, the area of the circle is $\pi r^2=\pi\left(\frac{1}{2}\cdot\frac{5}{\sqrt{6}}\right)^2=$ $\underline{\mathbf{\frac{25\pi}{24}}}$.