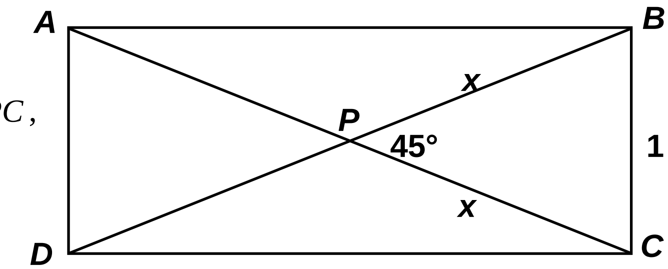
**MASSACHUSETTS MATHEMATICS LEAGUE**
**CONTEST 3 - DECEMBER 2016 SOLUTION KEY**

## Team Round




A) Solution #1:
Let $PB = PC = x$. Using the Law of Cosines on $\Delta BPC$,

$$x^2 + x^2 - 2x^2\cos 45° = 1 \Rightarrow \left(2-\sqrt{2}\right)x^2 = 1$$

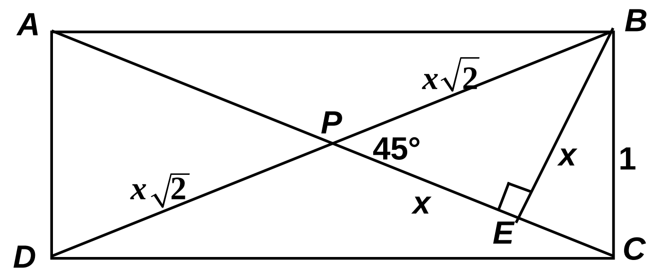
$$\Rightarrow x^2 = \frac{1}{2-\sqrt{2}} = \frac{2+\sqrt{2}}{2}$$

$$\Rightarrow BD = 2x = 2\sqrt{\frac{2+\sqrt{2}}{2}}$$

$$DC^2 = BD^2 - 1 = 4\left(\frac{2+\sqrt{2}}{2}\right) - 1 = 3 + 2\sqrt{2} = \left(1+\sqrt{2}\right)^2$$

$$\Rightarrow \text{Perimeter} = 2\left(1+\sqrt{2}\right) + 2 = \mathbf{4+2\sqrt{2}} \Rightarrow (s,t,r) = \underline{\mathbf{(4,2,2)}}.$$

Solution #2:




Drop a perpendicular from $B$ to $\overline{AC}$

$$EC = x\sqrt{2} - x = x\left(\sqrt{2}-1\right)$$

In $\Delta BEC$,

$$x^2 + \left(\sqrt{2}-1\right)^2 x^2 = 1^2 \Rightarrow x^2\left(1+2+1-2\sqrt{2}\right) = 1$$

$$\Rightarrow x^2 = \frac{1}{4-2\sqrt{2}} = \frac{4+2\sqrt{2}}{8} = \frac{2+\sqrt{2}}{4}$$

$$BD = 2x\sqrt{2} \Rightarrow BD^2 = 8x^2 = 8\left(\frac{2+\sqrt{2}}{4}\right) = 4+2\sqrt{2}$$

But $DC^2 = BD^2 - 1 = 4 + 2\sqrt{2} - 1 = 3 + 2\sqrt{2}$ and the same result follows.
Note that the perimeter is numerically equal to $BD^2$ !!

Solution #3

Note that $m\angle BDC = \frac{1}{2} m\angle BPC$. So, using $\tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta}$, we have

$$\tan\left(\angle BDC\right) = \frac{\sin 45°}{1+\cos 45°} = \frac{\sqrt{2}/2}{1+\sqrt{2}/2} = \frac{1}{1+\sqrt{2}} \Rightarrow DC = 1+\sqrt{2}$$ and the same result follows.