MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2014 SOLUTION KEY

Round 6

A) $\begin{cases} (1)\ \ w = 5 - x \\ (2)\ \ x = 6y + 1 \ \text{ Substituting for } x \text{ in (1), } w = 5 - (6y+1) \Rightarrow (4)\ \ w = -6y + 4 \\ (3)\ \ y = -\dfrac{2}{3}z \end{cases}$

Substituting in (4) for y, $w = -6\left(-\dfrac{2}{3}z\right) + 4 = 4z + 4 \Rightarrow$ $z =$ $\underline{\boldsymbol{\dfrac{w-4}{4}}}$ (or equivalent).

B) $\dfrac{f(x,y)}{f(y,x)} = \dfrac{x + \frac{1}{y}}{y + \frac{1}{x}} = \dfrac{\frac{xy+1}{y}}{\frac{xy+1}{x}} = \dfrac{x}{y} = \dfrac{2}{3} \Rightarrow 3x = 2y$

$\begin{cases} x - y = 4 \\ 3x = 2y \end{cases} \Rightarrow 3x - 3y = 12 \Rightarrow 2y - 3y = 12 \Rightarrow y = -12 \Rightarrow (x, y) =$ **$\underline{(-8, -12)}$**

C) Given: x $= 5x - 2$, $\boxed{x} = \dfrac{x+a}{b}$

$\boxed{x}$ = $\boxed{x}$ $\Leftrightarrow\ 5\left(\dfrac{x+a}{b}\right) - 2 = \dfrac{5x - 2 + a}{b}$

$\Leftrightarrow\ \dfrac{5x + 5a - 2b}{b} = \dfrac{5x - 2 + a}{b}$

For nonzero values of b, this is only true if $5a - 2b = -2 + a \Leftrightarrow 4a = 2b - 2 \Leftrightarrow b = 2a + 1$.
To maximize $a + b$, we take the maximum value of b. $b = 2a + 1 = 9 \Rightarrow a = 4 \Rightarrow a + b =$ $\underline{\mathbf{13}}$.

Check: For $(a,b) = (4,9)$, both expressions evaluate to $\dfrac{5x+2}{9}$.