MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2007 SOLUTION KEY

Round 6

A) Assume x ounces of the 40% solution are required.
Then $0.40x + 0.15(30 - x) = 0.25(30) \rightarrow 40x + 15(30 - x) = 25\ (30)$
$\rightarrow 25x = 750 - 450 = 300 \rightarrow x =$ **12**

B) Cross multiplying, $|x||x + 5| = 6 \rightarrow |x(x + 5)| = 6 \rightarrow x^2 + 5x = \pm 6$
$x^2 + 5x + 6 = (x + 3)(x + 2) = 0 \rightarrow x = -3, -2$
$x^2 + 5x - 6 = (x + 6)(x - 1) = 0 \rightarrow x = -6, 1 \rightarrow$ sum = **-10**.

C) $a^b = 1$ if and only if

Case 1: ($a = 1$) $x^2 - 7x + 11 = 1 \rightarrow x^2 - 7x + 10 = (x - 2)(x - 5) = 0 \rightarrow x = 2, 5$
Case 2: ($b = 0$ (and $a \neq 0$)) $2x^2 + 11x - 6 = 0 \rightarrow (2x - 1)(x + 6) = 0$
Since any real number raised to the zero power is 1, except 0,
$x = 1/2$ (checks since $(1/2)^2 - 7(1/2) + 11 \neq 0$) and
$x = -6$ (checks since $(-6)^2 - 7(-6) + 11 \neq 0$)
Case 3: ($a = -1$ and b is an even integer) $x^2 - 7x + 11 = -1 \rightarrow x^2 - 7x + 12 = (x - 3)(x - 4) = 0$
$x = 3$ (fails, since $2(3)^2 + 11(3) - 6 = 45$ which is not an even integer exponent)
$x = 4$ (checks, since $2(4)^2 + 11(4) - 6 = 70$ which is an even integer exponent)

Thus, the product of the solutions is $2(5)(1/2)(-6)(4) =$ **-120**