MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2015
ROUND 7 TEAM QUESTIONS
ANSWERS

A) ____________________________ D) (______ , ______ , ______)

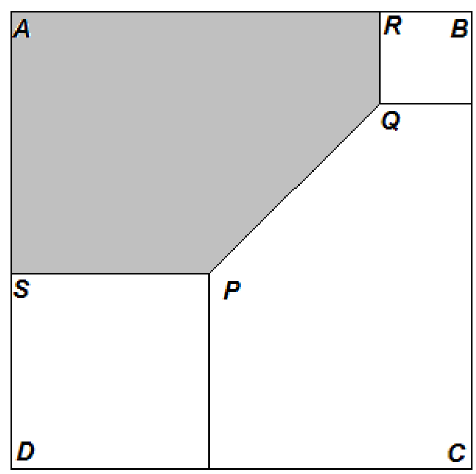
B) ____________________________ E) (__________ , __________)

C) ____________________________ F) ____________________________

A) Determine the integer value of n for which $(1+i)^{11}+(1-i)^{n}=-16+16i$.

B) One million lottery tickets are numbered 000000 through 999999.
Let A be the set of lucky lottery tickets.
A lucky lottery ticket has the form *abcxyz*, where $a+b+c=x+y+z$.
Let B be the set of unlucky lottery tickets.
An unlucky lottery ticket is defined to be one where the 6 digits sum to 27.
For 1) – 5) below, list the numbers of the true statements.
Note $N(X)$ denotes the number of elements in set X.

1) $N(B)>N(A)$
2) $N(B)<N(A)$
3) $N(B)=N(A)$
4) With respect to the given definitions, no ticket is both lucky and unlucky.
5) With respect to the given definitions, at least one ticket is both lucky and unlucky.



C) In square *ABCD*, the squares in opposite corners have sides 1 and 2, respectively. A square with side *PQ* has an area equal to sum of the areas of the small squares.
Compute the area of the shaded region.

D) Given: $a^2+b^2+ab=12,\ b^2+c^2+bc=13,\ a^2+c^2+ac=19$
If $a>0$, compute (a,b,c) over the rational numbers.

E) The value of the expression $N=\sin\left(\frac{11\pi}{3}\right)+\cos(600°)+\sin^2\left(\frac{9\pi}{4}\right)-\tan^2(495°)-3\tan(540°)$
satisfies the inequality $a < N < b$, where a and b are integers and $b - a = 1$.
Compute the ordered pair (a, b).

F) ΔABC is known to be isosceles, but it is not known which angle is the vertex angle.
$\overrightarrow{BP}$ is a <u>trisector</u> of $\angle B$, so that $m\angle PBC < m\angle ABP$ (P is on $\overline{AC}$).
$\overrightarrow{CQ}$ is a <u>bisector</u> of $\angle C$ (Q is on $\overline{AB}$).
$\overrightarrow{BP}\cap\overrightarrow{CQ}=\{D\}$. $m\angle BDC=140°$. Compute <u>all</u> possible $m\angle A$.