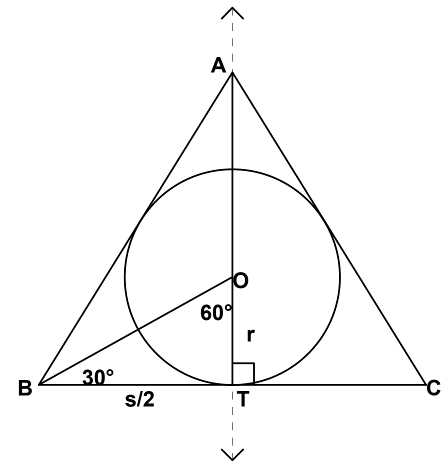
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2007 SOLUTION KEY

Team Round



A) Consider the diagram at the right.

The volume of the cone is given by $\frac{1}{3}\pi\left(\frac{s}{2}\right)^2\left(\frac{s}{2}\sqrt{3}\right) = \frac{\sqrt{3}\pi s^3}{24}$

The volume of the sphere is given by $\frac{4}{3}\pi\left(\frac{s\sqrt{3}}{6}\right)^3 = \frac{\sqrt{3}\pi s^3}{54}$

The difference is $5\pi\sqrt{3}s^3\left(\frac{1}{24}-\frac{1}{54}\right) = 5\pi\sqrt{3}s^3\left(\frac{54-24}{24(54)}\right) = \underline{\boldsymbol{\frac{5\pi\sqrt{3}}{216}s^3}}$

B) Examining right triangles in which the lengths of the hypotenuse and long leg differ by 1:

a	***b***	***c***	**Per**	**Factors**
3	4	5	12	3(4)
5	12	13	30	5(6)
7	24	25	56	7(8)
9	40	41	90	9(10)

Apparently, the perimeter is given by $a(a + 1)$
We want a as large as possible with $a(a + 1) \leq 1000$ $a = 31 \rightarrow 992$, $a = 32 \rightarrow 1056$
Thus, $a = 31$ and $31^2 + b^2 = (b + 1)^2 \rightarrow 961 = 2b + 1 \rightarrow b = 480$ and the sides are **<u>(31, 480, 481)</u>**

C) Let n denote the number of pencils originally bought for 30¢.
Let p denote the cost of a single pencil(in cents) .
Then $np = 30$ and $(n + 3)(p - 27/12) = 30$.
$(n + 3)(p - 2.25) = np - 2.25n + 3p - 6.75 = 30$
Cancelling, $-2.25n + 3p - 6.75 = 0 \rightarrow -9n + 12p - 27 = 0 \rightarrow -3n + 4p - 9 = 0 \rightarrow p = \frac{9+3n}{4}$

Substituting, $np = n\cdot\frac{9+3n}{4} = 30 \rightarrow 3n^2 + 9n - 120 = 0 \rightarrow (3n + 24)(n - 5) = 0$
$\rightarrow n = 5$ and $p = 6$
which means 8 pencils could be bought for the lower price of 6 – 9/4 = 3.75 cents→ **<u>(8, 3.75)</u>**

D) $\frac{1}{1+\frac{2}{x+3}} = 4-0.\overline{5} \rightarrow \frac{x+3}{x+5} = 4-\frac{5}{9} = \frac{31}{9} \rightarrow 9x + 27 = 31x + 155 \rightarrow x = \frac{-128}{22} = \frac{-64}{11}$

Note: $\frac{64}{-11}$ is disallowed since $A < B$. If $\frac{-64+n}{11}$ must be an integer and $n > 0$, the minimum possible value of n is **<u>9</u>**.