MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 – MARCH 2012 SOLUTION KEY

Round 2

A) $A^2 = 6000k = 2(3)\left(10^3\right)k = 2^4 3^1 5^3 k$

If A is an integer, each prime on the right hand side of the equation must be raised to an even power. The smallest value of k which provides this luxury is $k = 3(5) = 15$.

Thus, $A^2 = 2^4 3^2 5^4 \Leftrightarrow A = 2^2 \cdot 3 \cdot 5^2 = 3(100) = 300 \Rightarrow (k, A) = \underline{\mathbf{(15, 300)}}$.

B) $$\left.\begin{array}{l} 2^{-P} = \dfrac{1}{2^P} \\ 3^{-Q} = \dfrac{1}{3^Q} \end{array}\right\} < \frac{0.125}{2} = \frac{2^{-3}}{2} = 2^{-4} = \frac{1}{16} \Rightarrow 2^P > 16 \text{ and } 3^Q > 16$$

$\Rightarrow P \geq 5$ and $Q \geq 3 \Rightarrow (P + Q)_{\min} = \underline{\mathbf{8}}$.

C) $16(27)(128)(1024) = 2^4 3^3 2^7 2^{10} = 2^{21} 3^3$

$\sqrt[12]{16(27)(128)(1024)} = \sqrt[12]{2^{21}3^3} = 2\sqrt[12]{2^9 3^3} = 2\sqrt[4]{2^3 3^1}$

Radicals can be added only if the radicands and the indices are the same.

Thus, $(A, B) = (3,1)$ and the sum would be $5\sqrt[4]{2^3 3^1} = 5\sqrt[4]{24}$ and the required ordered triple is **<u>(5, 4, 24)</u>**.