**MASSACHUSETTS MATHEMATICS LEAGUE**
**CONTEST 1 - OCTOBER 2012 SOLUTION KEY**

## Round 4

A) Let $A$ and $B$ denote the number of jelly beans in first and second jars respectively.

$$\begin{cases} .4A + 1.2B = 1200 \\ A + B = 1200 \end{cases}$$

The first equation simplifies to $A + 3B = 3000$. Subtracting, $B =$ **<u>900</u>** and $A =$ **<u>300</u>**.

B) For $x =$ **<u>0, ±1</u>**, one of the three fractions in either the numerator or denominator is undefined, but there is an additional value as well.

$$\frac{\frac{3}{x}}{\frac{1}{x+1}+\frac{2}{x-1}} = \frac{3}{x} \div \frac{x-1+2(x+1)}{(x+1)(x-1)} = \frac{3}{x} \cdot \frac{(x+1)(x-1)}{3x+1}$$

Thus, $\underline{\mathbf{-\frac{1}{3}}}$ also causes division by zero.

C) $2\frac{A}{B} + 4\frac{A+1}{B} = 6 + \frac{2A+1}{B} \Rightarrow \frac{2A+1}{B} = 1.375 = \frac{11}{8} \Rightarrow (A,B) = \underline{\mathbf{(5,8)}}$

Other possibilities exist, but $A + B > 13$. For example, cross multiplying, $8(2A+1) = 11B$.
$11B$ is a multiple of 8 and, since 8 does not divide into 11, $B$ must be a multiple of 8.
Therefore, $8(2A+1) = 11(8k)$, where $k$ is an integer.
It follows that $2A+1 = 11k$. For all integer values of $A$, the left side denotes an odd integer, so $k$ must be odd.
$k = 3 \Rightarrow A = 16,\ B = 24$, but $k = 5 \Rightarrow A = 27,\ B = 40$
Thus, another relatively prime ordered pair ($A$, $B$) exists, but $A + B = 67$.