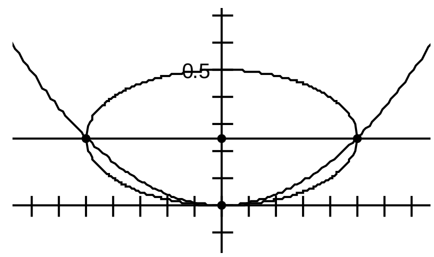
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2014 SOLUTION KEY

Round 1

A) Substituting, $b = 3^2 \Rightarrow A(3, 9)$; $4 = a^2 \Rightarrow B_1(+2, 4)$ or $B_2(-2, 4)$
The longer distance is $AB_2 \Rightarrow \sqrt{(3-{}^{-}2)^2+(9-4)^2} = \sqrt{2(5)^2} = \underline{\mathbf{5\sqrt{2}}}$

B) $y = 4px^2 = 1x^2 \Rightarrow p = \frac{1}{4}$



The focus of the parabola is at $\left(0, \frac{1}{4}\right)$.
Its vertex is at (0, 0) and the endpoints of its focal chord are located $\frac{1}{2}$ unit to the right and left. Thus, for the ellipse,
$a = \frac{1}{2}$ (the semi-major axis), $b = \frac{1}{4}$ (the semi-minor axis) and
for all ellipses, $a^2 = b^2 + c^2 \Rightarrow c^2 = \frac{3}{16} \Rightarrow c = \frac{\sqrt{3}}{4}$
The eccentricity (defined as $\frac{c}{a}$) is $\underline{\mathbf{\frac{\sqrt{3}}{2}}}$.

C) Completing the square,
$$3y^2 - x^2 + 24y + 14x = 49 \Leftrightarrow 3\left(y^2 + 8y + 16\right) - \left(x^2 - 14x + 49\right) = 49 + 48 - 49 \Leftrightarrow$$
$$3(y+4)^2 - (x-7)^2 = 48 \Leftrightarrow$$
$$\frac{(y+4)^2}{16} - \frac{(x-7)^2}{48} = 1$$
Thus, the conic is a hyperbola with center at (7, −4) and a vertical major axis.
Since $a^2 = 16$, $b^2 = 48$ and $c^2 = a^2 + b^2$ for the hyperbola, $c = 8$ and
the coordinates of the foci are $\left(7, -4 \pm 8\right) \Rightarrow \underline{\mathbf{(7,4),\ (7,-12)}}$.

<u>FYI</u>:
At the end of this solution key, you will find additional comments about eccentricity and the conic sections - circle, ellipse, parabola and hyperbola.