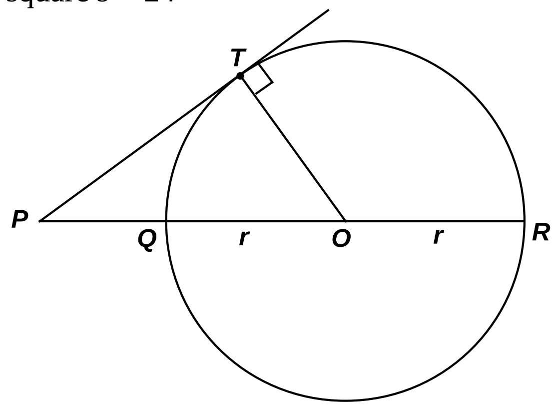
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2012 SOLUTION KEY

Round 5

A) $C = 12\pi \Rightarrow$ radius $r = 6 \Rightarrow$ diameter $d = 12 \Rightarrow$ side of square $s = 24$
$\Rightarrow$ diagonal of the square = $24\sqrt{2}$
$\Rightarrow$ radius of c.c. circle = $12\sqrt{2}$ $\Rightarrow$ area = $\underline{\mathbf{288\pi}}$



B) Draw $\overleftrightarrow{PO}$. This line intersects the circle twice.
The point between P and O is the closet point, Q.
The other point of intersection is point R.
$PR = PQ + QR \Rightarrow 20 = 4 + QR$
Since QR is a diameter, the radius of circle O is 8.
Using the Pythagorean Theorem on ΔTPO,
$PT^2 + 8^2 = 12^2 \Rightarrow PT = \sqrt{80} = \underline{\mathbf{4\sqrt{5}}}$

C) ΔRQP is a 3 – 4 – 5 right triangle (with area 6).
Thus, $\frac{1}{2}(QX)(PR) = 6 \Rightarrow \frac{1}{2} \cdot QX \cdot 5 = 6 \Rightarrow QX = \frac{12}{5}$.
Note since $\Delta RXQ \sim \Delta RQP$, ΔRXQ is a scaled version of a 3 – 4 – 5 triangle.
$\left(\frac{12}{5}, \_\_, 3\right) = \frac{1}{5}(12, \_\_, 15) = \frac{3}{5}(4, \_\_, 5)$
Rather than grinding out the Pythagorean Theorem for ΔRXQ, we see that $XR = \frac{3}{5}(3) = \frac{9}{5} = 1.8$
Therefore, $WR = 3.6$ and $VW = 5 - 1 - 3.6 = \underline{\mathbf{0.4}}$