

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2013
ROUND 6 ALG 2: PROBABILITY AND THE BINOMIAL THEOREM

ANSWERS

A) _____

B) _____

C) (_____ , _____)

******* NO CALCULATORS IN THIS ROUND *******

A) Let $n(X)$ denote the number of integers in set X .

Let $X \cup Y$ denote the union of two sets, i.e. the integers in either X or Y or possibly both.

Consider two sets A and B for which

$n(A) = 11$, $n(B) = 10$ and $n(A \cup B) = 15$

Compute the probability as a percentage that a randomly selected integer from one of the sets is in both of the sets.

B) In the expansion of $(a+b)^n$, where n is an integer, the sum of the coefficients is 8^4 .

For $a = 4$ and $b = -\frac{1}{8}$, if n is even, evaluate the middle term in the expansion; if n is odd,

evaluate the average of the two middle terms. Express your answer as a reduced ratio of integers.

C) A mathlete determines his score by throwing three darts at the board shown at the right. In this league, the minimum possible score is 0 and the maximum score is 9. The innermost circle scores 3 points, while each of the rings scores the points indicated in the diagram at the right. The mathlete hits the target with each dart. The probability of hitting any region is proportional to the area of the region. The probability of getting a score of 4, as a reduced fraction, is $\frac{P}{Q}$. If $AB = BC = CD = DE$, compute

the ordered pair (P, Q) . Assume the dart throws are independent.

