MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 – NOVEMBER 2012 SOLUTION KEY

Round 2

A) My ideal weight W must be 195 + 10 – 5 = 200 lbs.
$195 - x = 200 - 9 \Rightarrow x = 4$
Thus, my new weight tomorrow is 195 + 4 = **<u>199</u>**.

B) $\begin{cases} x+y=17 \\ xy>42 \end{cases} \Rightarrow x(17-x)>42 \Rightarrow x^2-17x+42=(x-3)(x-14)<0 \Rightarrow 3<x<14.$
The largest value occurs when x is as large as possible, namely 16(13) + 4 = 208 + 4 = **<u>212</u>**.

An equivalent problem in the language of bases would have been:

Let N = <u>x y</u> be a two-digit <u>base 16 integer</u>.
(x is the <u>most</u> significant digit and y is the <u>least</u> significant digit.)
In base 10, $x + y = 17$ and $xy > 42$.
What is the largest possible value of N in base 10?

The base 16 number would be represented as $16x + y$ in base 10.
Recall in base 10 there are 10 digits, the largest being 9 (1 less than the base).
Similarly, in base 16 there are 16 digits and the largest is equivalent to 15 in base 10.
They are usually represented 0, 1, 2, … ,9 , A = 10, B = 11, … F = 15.
So D = 13 is a legal digit in base 16.
Base 16 (commonly called hexadecimal) is frequently used in computer science to represent computer addresses. On a 64-bit operating system, a legal address might be 0000 4A73 FFB1 8660. Spaces added for legibility only. Who knows, the first letter you type in your next homework assignment might be stored there!
N's largest value is $D4_{(16)}$ = 16(13) + 4 = **<u>$212_{(10)}$</u>**.

C) The possible sums for the 6 sheets are: 3, 7, 11, 15, 19 and 23
All of these are odd numbers.
Adding any three of these numbers will result in an odd total; therefore 1) is false.
The minimum sum of lost pages numbers is (3 + 7 + 11) = 21; the maximum (15 + 19 + 23) = 57; therefore 3) is true.
If I lost 7, 19 and 23, my total would be 49; therefore 2) is true.
The total of the 6 numbers is 78.
If lost and found totals are equal, then they must both be 39.
The units digits of my 6 numbers include all the odd digits and 3 occurs twice.
Since I desire a sum ending in 9, I have 2 possibilities: numbers ending in 3,7 and 9 or 3, 5 and 1.
Thus, the only possibilities are: 3, 7, 19 7, 19, 23 3, 11, 15 or 11, 15, 23
None of these total 39, so 4) is false.
Therefore, the only true statements are **<u>2 and 3</u>**.