MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2012
ROUND 7 TEAM QUESTIONS

ANSWERS

A) _____ D) _____

B) _____ E) _____

C) _____ F) $ _____ . __ __

A) Compute the slopes of both tangent lines to $x^2 + y^2 = 25$ from $P(7, -1)$.

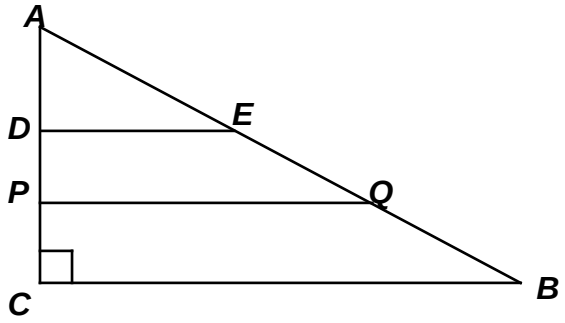
B) $\dfrac{x^4 + 3x^2 + k}{x^2 + 3}$ is an integer for exactly five positive integer values of x.
Determine the smallest possible positive integer value of k.

C) Solve for x over $0 \le x < 360°$: $\sin 4x - \cos 2x = 4 \sin x \cos x - 1$

D) For positive integer constants k, A and B, $x^2 - (k-4)x + 6k - 2 = (x - A)^2 + B^2$ is an identity in x.
Find all possible ordered triples (k, A, B).

E) Given: $\overline{AC} \perp \overline{BC}$, $\overline{PQ}$ is a median of trapezoid $BCDE$
$AC = 5$, $BC = 12$ and $AD = k$

Determine the range of values of k for which
the perimeter of $PQBC$ is less than or equal to 25.



F) In Maine, at one time, a customer could buy a Frisbee for the selling price of $\$X$, plus a 5% sales tax. However, due to inflation, the vendor raised his selling price by 25% and, adding insult to injury, Maine raised the sales tax to 8%. As a result, the cost to the customer for a Frisbee, including the tax, was an additional 48¢. Compute X (to the nearest cent).