MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2014 SOLUTION KEY

Round 4

A) $x^2 - 13x + 30 = (x-10)(x-3) = 0 \Rightarrow P = 10 - 3 = 7$

$x^2 - 13x - 30 = (x-15)(x+2) = 0 \Rightarrow P = 15 - (-2) = 17$

Thus, $P + Q =$ **<u>24</u>**.

B) The sum of the roots of $2Ax^2 - Bx + C = 0$ is $\dfrac{B}{2A}$. Therefore, we have

$\dfrac{B}{2A} = AC$ or $B = 2A^2C$

To guarantee that B is a perfect square, C must be twice a perfect square.

Thus, the candidates are: $2\left(1^2, 2^2, 3^2, \ldots.\right)$ = 2, 8, 18, … .

Since we are given that $C > 10$, we have $C =$ **<u>18</u>**.

C) Assuming you didn't notice that for $x = 14$, we would have $5 - 4 = 3(3)$ which fails only because of the minus sign on the left side. How else could we determine that $x = 14$ is the extraneous root? Squaring both sides,

$$25 - 10\sqrt{x+2} + x + 2 = 9(x-5)$$
$$72 - 8x = 10\sqrt{x+2}$$
$$36 - 4x = 4(9-x) = 5\sqrt{x+2}$$
$$16(9-x)^2 = 25(x+2)$$
$$16x^2 - 313x + 1246 = 0$$

This looks dreadful, but we were given that there was an extraneous <u>integer</u> solution.

This gives us a partial factorization as $(16x - \boxed{A})(x - \boxed{B}) = 0$, where A and B are integers.

$A + 16B = 313$

$AB = 1246 = 2 \cdot 7 \cdot 89$

Forgetting the numerical values for a moment and considering only the parity of A and B (that is, even or odd), the first equation says A and B cannot both be even, the second equation says that A and B cannot both be odd. Therefore, they have mixed parity and in fact only A odd and B even can satisfy the first equation.

The only even factors of 1246 are 2, 14 and 178.

For $(16x - \boxed{89})(x - \boxed{14})$, we only need check the coefficient of the middle term.

$16 \cdot 14 + 89 = 224 + 89 = 313$ and we have the correct factorization.

The fractional root is $\mathbf{\dfrac{89}{16}}$.

Check: $5 - \sqrt{\dfrac{89}{16} + 2} = 3\sqrt{\dfrac{89}{16} - 5} \Leftrightarrow 5 - \sqrt{\dfrac{121}{16}} = 3\sqrt{\dfrac{9}{16}} \Leftrightarrow 5 - \dfrac{11}{4} = 3 \cdot \dfrac{3}{4} = \dfrac{9}{4}$