

**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 - FEBRUARY 2013 SOLUTION KEY**

Team Round - continued

- E) The shaded region plus the region bounded by $\triangle SQR$ is a quarter circle with area $\frac{\pi(6)^2}{4} = 9\pi$. Thus, the area of the shaded region is $9\pi - \frac{1}{2} \cdot 6 \cdot 6 = 9(\pi - 2)$.

Draw diagonal \overline{PR} , intersecting \overline{SQ} at point O .

$$PR = 6\sqrt{2} \Rightarrow PO = 3\sqrt{2}.$$

Since points T and M lie on \overline{PR} , $PT + TM + MO = 3\sqrt{2}$ (***)

If r denotes the radius of circle T , $PT = r\sqrt{2}$ (since P, T and the points of tangency on the square form a small square with side r). Therefore, (**) is equivalent to:

$$r\sqrt{2} + r + (6 - 3\sqrt{2}) = 3\sqrt{2} \quad (MO = RM - RO)$$

$$\Leftrightarrow r(\sqrt{2} + 1) = 6(\sqrt{2} - 1)$$

$$\Leftrightarrow r = \frac{6(\sqrt{2} - 1)}{\sqrt{2} + 1} = 6(\sqrt{2} - 1)^2 = 6(3 - 2\sqrt{2})$$

Finally, the required ratio is $\frac{\pi r^2}{9(\pi - 2)} = \frac{\pi \cdot 36 \cdot (3 - 2\sqrt{2})^2}{9(\pi - 2)} = \frac{\pi(68 - 48\sqrt{2})}{\pi - 2}$ and

$$(A, B) = (\underline{68 - 48\sqrt{2}}, \underline{2}) \text{ or } (\underline{4(17 - 12\sqrt{2})}, \underline{2})$$

$4(\sqrt{2} - 1)^4$ is **not** acceptable as the computed value of A .

