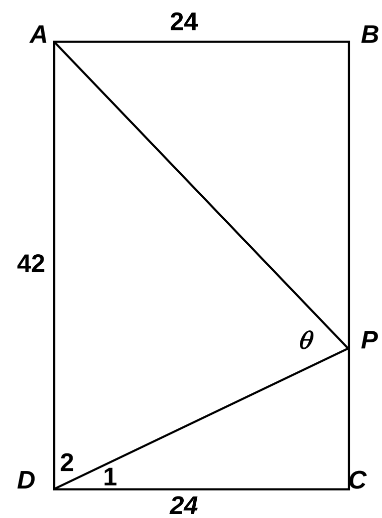
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2013 SOLUTION KEY

Round 1

A) Since 7 must be the length of the hypotenuse, $x = 4\sqrt{3}$.
Since A is the larger acute angle, it must be opposite the longer side.
SOHCAHTOA $\Rightarrow \tan(\angle A) =$ $\underline{\mathbf{4\sqrt{3}}}$.

B) $BP : PC = 16 : 5$ and $BC = 42 \Rightarrow BC = 32$, $CP = 10$.
Using special Pythagorean triples of 3-4-5 and 5-12-13, we have
in ΔPCD, (10, 24, ?) = 2(5, 12, ?) $\Rightarrow PD = 26$
in ΔABP, (24, 32, ?) = 8(3, 4, ?) $\Rightarrow AP = 40$
In ΔPCD, $\cos(\angle 1) = \frac{24}{26} = \frac{12}{13}$
Angles 1 and 2 are complementary, so $\sin(\angle 2) = \cos(\angle 1) = \frac{12}{13}$.
Using the Law of Sines in ΔAPD,
$\frac{\sin\theta}{42} = \frac{\sin\angle 2}{40} \Rightarrow \sin\theta = \frac{42}{40}\cdot\frac{12}{13} = \underline{\mathbf{\frac{63}{65}}}$.



C) Let the three sides have lengths $4k$, $5k$ and $6k$. The smallest angle θ will be opposite the side of length $4k$. Using the law of Cosines, $16k^2 = 25k^2 + 36k^2 - 60k^2\cos\theta°$.
$k \neq 0 \Rightarrow 60\cos\theta = (25+36-16) = 45 \Rightarrow \cos\theta = \frac{3}{4} \Rightarrow \sin\theta = +\frac{\sqrt{7}}{4}$ (since θ must be acute)
The area of any triangle can be computed as $\frac{1}{2}ab\sin C$, where C is the included angle.
Thus, the area of ΔABC is $\frac{1}{2}(5k)(6k)\left(\frac{\sqrt{7}}{4}\right) = 375\sqrt{7} \Rightarrow 15k^2 = 4(375)$.
$\Rightarrow k = 10$ and the perimeter is $\underline{\mathbf{150}}$.