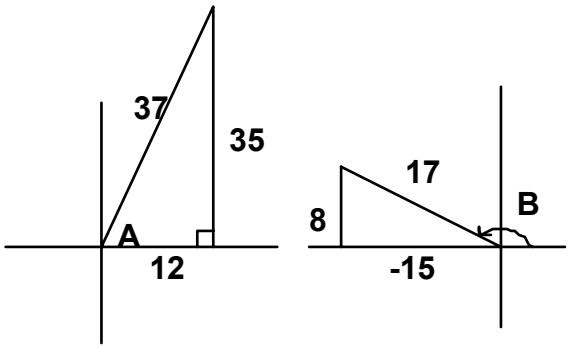
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2007 SOLUTION KEY

Round 3

A) $\sin(A + B) = \sin A\cos B + \cos A\sin B = \frac{35}{37}\cdot\frac{-15}{17} + \frac{12}{37}\cdot\frac{8}{17}$

$= \frac{-525+96}{629} = \boxed{-\frac{\mathbf{429}}{\mathbf{629}}}$



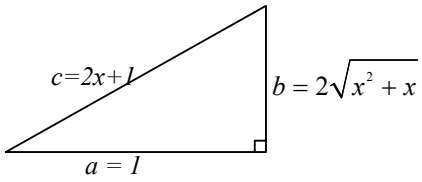
B) $\tan(\theta) = \frac{1}{\sqrt{2}}$ for an <u>acute</u> angle $\theta \rightarrow (\cos(\theta), \sin(\theta)) =$

$(\frac{\sqrt{2}}{\sqrt{3}}, \frac{1}{\sqrt{3}})$

$\cos(\theta + 45°) = \cos(\theta)\cos(45°) - \sin(\theta)\sin(45°) = \frac{\sqrt{2}}{\sqrt{3}}\cdot\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}}\cdot\frac{1}{\sqrt{2}} = \frac{\sqrt{2}-1}{\sqrt{6}}$

$BC = AB\cos(\theta + 45°) = \frac{(\sqrt{2}+1)(\sqrt{2}-1)}{\sqrt{6}} = \frac{1}{\sqrt{6}} = \boxed{\frac{\sqrt{\mathbf{6}}}{\mathbf{6}}}$

C)



Then $\frac{x^2+x}{2x+1} = \frac{b^2/4}{c} = \frac{1}{4}\cdot\frac{b}{1}\cdot\frac{b}{c} = \frac{1}{4}\tan\theta\sin\theta = \frac{1}{4}\frac{\sin^2\theta}{\cos\theta} = \boxed{\frac{\mathbf{1-\cos^2\theta}}{\mathbf{4\cos\theta}}}$