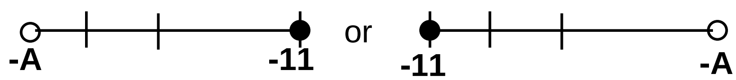
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2016 SOLUTION KEY

Round 4

A) $\left(3x-2\right)\left(3x+1\right)=4 \Leftrightarrow 9x^2-3x-6=0 \Leftrightarrow 3\left(3x^2-x-2\right)=3\left(3x+2\right)\left(x-1\right)=0 \Rightarrow x=\underline{\mathbf{-\frac{2}{3}, 1}}$

B) Examining the factorization of $180=2^2\cdot 3^2\cdot 5^1$, we see 180 has 18 positive factors which will form 9 ordered pairs (A, B) where $A\cdot B=180$ and $A>B>0$.
GCF = 1: (180, 1), (45, 4), (36, 5), (20, 9)
GCF = 2: (90, 2), (18, 10)
GCF = 3: (60, 3), (15, 12)
GCF = 6: (30, 6)
Thus, $\left(j,k\right)=\underline{\left(\mathbf{4,5}\right)}$.

C) $\dfrac{(x+2)^2-81}{(7-x)(A+x)}\ge 0 \Leftrightarrow \dfrac{x^2+4x-77}{(7-x)(A+x)}\ge 0 \Leftrightarrow \dfrac{(x+11)(x-7)}{(7-x)(A+x)}\ge 0 \Leftrightarrow \dfrac{-x-11}{A+x}\ge 0$, provided $x\ne 7$.
or, equivalently, $\dfrac{x+11}{A+x}\le 0$. The solution set is between -11 and -*A*.
However, we must consider two cases: -*A* to the left of -11 and -*A* to the right of -11



To guarantee exactly 3 integer solutions, *A* = **<u>8</u>** (-11, -10, -9) or **<u>14</u>** (-13, -12, -11).