

MASSACHUSETTS MATHEMATICS LEAGUE
JANUARY 2005
ROUND 7: TEAM QUESTIONS

ANSWERS

A) _____ D) _____

B) _____ E) _____

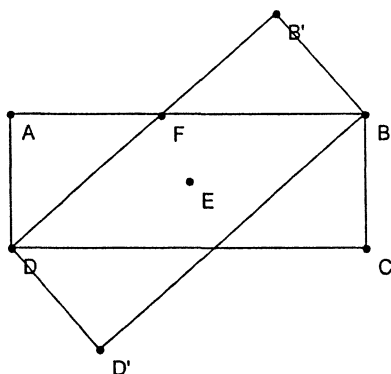
C) _____ F) _____

A) Given $\triangle ABC$ with $A(1, 2)$ $B(4, 6)$ and $C(-8, 11)$ If the angle bisector of $\angle ABC$ intersects \overline{AC} at D find the coordinates of D .

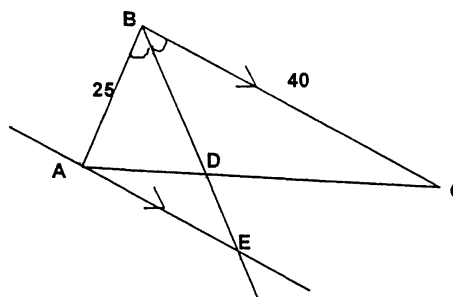
B) Factor : $x^{4n+3} + 4x^{2n+3} + 16x^3$

C) A student solved $\cos(x) + \sin(2x) - \cos(3x) = 0$ by transforming it to the form $(n \cdot \text{foo}(x) + 1) \cdot \text{foo}(nx) = 0$ for some positive integer n and some basic trig function foo . Find the exact value of $n + \text{foo}(\frac{\pi}{3n})$.

D) Rectangle $ABCD$ has $AD=6$ and $AB>AD$. Let E be the point of intersection of its diagonals. Rotate $ABCD$ about E until A lands on D . If the area contained in the intersection of the rotated and original rectangular regions is 45 square units, what was the length of \overline{AB} ?



E) In triangle ABC , $AB = 25$, $BC = 40$ and $AC = 39$. Ray BD bisects angle ABC and line AE is parallel to side BC . Find the exact perimeter of triangle ADE



F) Find all ordered pairs (a,b) which will make the solution to the following equation a set of three consecutive positive integers: $x^3 - 8 = (x - 2)(ax + b)$