MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 - FEBRUARY 2011
ROUND 7 TEAM QUESTIONS

ANSWERS

A) _____ D) _____

B) _____ E) _____

C) _____ F) _____

******* NO CALCULATORS ON THIS ROUND *******

A) Given: $f(x) = \dfrac{3x+1}{2(x-1)}$, $g(t) = \dfrac{1}{3t-2}$

Determine all values of m for which $f^{-1}(m) \le g^{-1}(m)$.

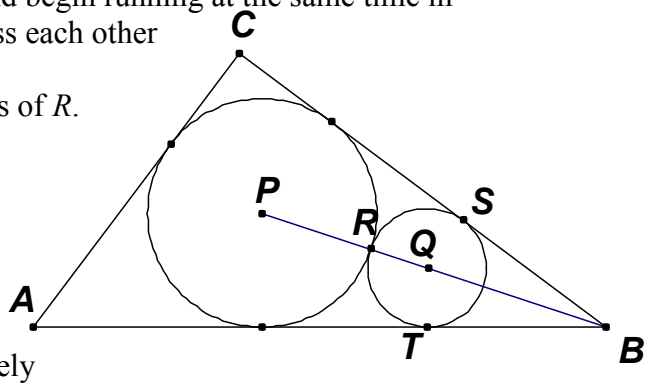
B) The sum of the digits of a two-digit positive <u>even</u> integer N is 9.
Determine the <u>sum</u> of all possible values of N which have 12 divisors.

C) Given: $a > 0$, $b > 0$ and $Tan^{-1}\left(\dfrac{a}{b}\right) + Sin^{-1}\left(\dfrac{a}{b}\right) = 90°$. Compute $\dfrac{a^2}{b^2}$.

D) Faster runner #1 (running at R feet/sec) completes A laps around a quarter-mile track in the time that runner #2 completes B laps, where A and B are relatively prime positive integers. If the two runners start at the same point on the track and begin running at the same time in <u>opposite</u> directions at the rates specified above, they pass each other for the first time in 45 seconds. If A and B are integers, where $0 < A - B < 3$, compute <u>all</u> possible integer values of R.

Note: 1 mile = 5280 feet

E) $AC = 6$, $BC = 8$ and $AB = 10$
Circle P is inscribed in ΔABC.
Circle Q is tangent to $\overline{BC}$ and $\overline{AB}$ at S and T respectively and to circle P at R.
Compute BS.



F) The sum of the first three terms in an infinite geometric progression of rational numbers is 1792. The sum of the first 11 terms is 2047. If the 56th term in an arithmetic progression is equal to the sum of the terms in this infinite geometric progression and the first term a and the common difference d are positive integers (with $a < 50$), compute the 55th term of this arithmetic progression.