

Note  $\frac{1}{\sqrt{2}+1} = \sqrt{2}-1$

Therefore, the denominator of the continued fraction is equivalent to  $\sqrt{2}+1$ .

Then subtracting 1 from the terms in the sequence for the denominator gives the sequence for  $\sqrt{2}$ , but with a first term of 1 instead of  $3/2$ .

Letting  $r = \sqrt{2}$ , we have:

$$1 < r < \frac{3}{2}, \quad \frac{7}{5} < r < \frac{3}{2}, \quad \frac{7}{5} < r < \frac{17}{12}, \quad \frac{41}{29} < r < \frac{17}{12}$$

Here's another:

The fractions for  $1+\sqrt{2}$  are:  $2/1, 5/2, 12/5, 29/12, 70/29, \dots$

Let  $r = \sqrt{2}$ , the formula  $\frac{\left((1+r)^n - (1-r)^n\right)}{2r}$  gives the values 1, 2, 5, 12, 29, 70, ...

I got this idea from the formula for the Fibonacci numbers.

Suppose we call this sequence  $a(n)$  and we compute  $\frac{a(n+1)}{a(n)} - 1$

What would the limit be?

Could it be  $\sqrt{2}$  ???