MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2016
ROUND 7 TEAM QUESTIONS

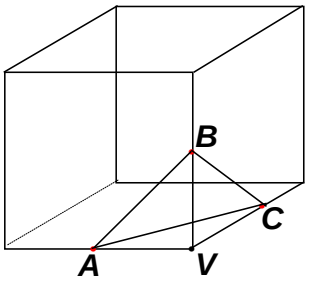
ANSWERS

A) _____ D) _____

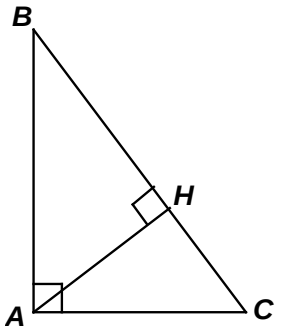
B) _____ E) $\{x \mid$ _____ $\}$

C) (___ , ___ , ___ , ___ , ___) F) (_____ , _____)

A) Points *A*, *B*, and *C* lie on the edges of a cube.
The distance from vertex *V* to the plane containing points *A*, *B*, and *C* is $\frac{m}{n}\sqrt{r}$, in simplified radical form.
If $AB = 13$, $BC = 14$, and $CA = 15$, compute $m+n+r$.



B) Given: ΔABC, with a right angle at *A*. $AB = 9$, $AC = 2\sqrt{10}$
Let *H* be the endpoint of the perpendicular segment drawn from *A* to $\overline{BC}$.
Compute $HB - HC$.



C) Given 5 numbers *v*, *w*, *x*, *y*, and *z*.
If the average of three of them is added to the remaining two, the results are: 21, 23, 25, 27, 25, 27, 29, 29, 31, and 33.
Specify the 5 numbers as $(v_1, v_2, v_3, v_4, v_5)$, where $v_1 \le v_2 \le v_3 \le v_4 \le v_5$.

D) Given: $\frac{2x+3}{(x-2)^3} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{(x-2)^3}$ Compute $A^2 + B^2 + C^2$.

E) Specify the condition which describes those values of *x* (and only those values) which satisfy the following inequality: $2|x^2-1| - |x^2+1| \le 9x+7$
If necessary, the proper use of the connectors "and" / "or" is required.

F) The sequence of Fibonacci numbers $Fib(n)$ is defined by

$$\begin{cases} Fib(n) = Fib(n-1) + Fib(n-2) \text{ for } n \ge 3 \\ Fib(2) = 2 \\ Fib(1) = 1 \end{cases}$$

The first five Fibonacci numbers are 1, 2, 3, 5, and 8. An <u>open</u> interval (a,b) is defined to include all values of *x* satisfying the inequality $a < x < b$.
For some <u>minimum</u> value of $k > 0$, the <u>open</u> interval $(Fib(k), Fib(k+1))$ contains 2 distinct integer perfect cubes j^3 and $(j+1)^3$. Compute the ordered pair (j,k).