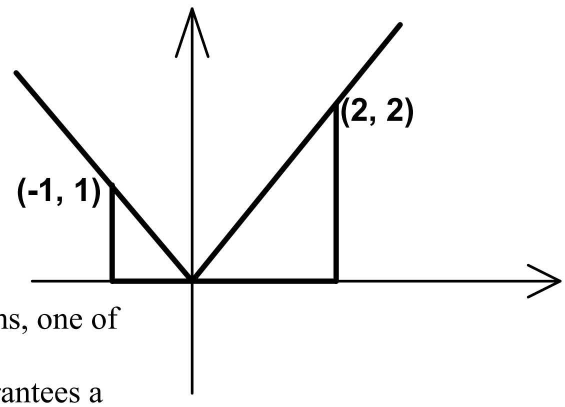
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2007 SOLUTION KEY

Round 5



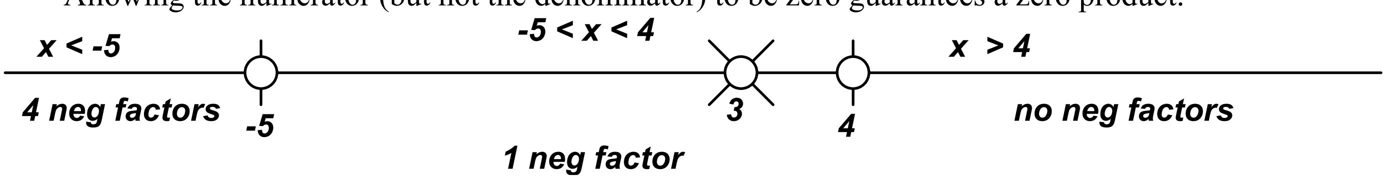
A) The region consists of 2 right triangles

Area = $\frac{1}{2}\cdot 1\cdot 1+\frac{1}{2}\cdot 2\cdot 2$ = $\underline{\mathbf{2.5}}$

B) The quotient on the left-hand side is comprised of 5 terms, one of which is never negative.

Having an <u>even</u> number of factors that are negative guarantees a positive product.

Allowing the numerator (but not the denominator) to be zero guarantees a zero product.

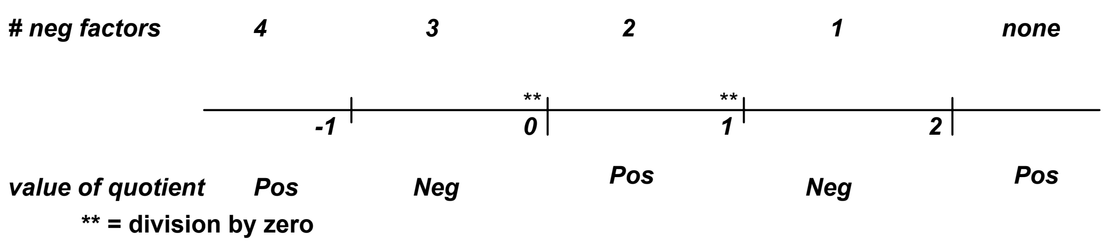


→ **<u>$x < -5$ or $x \geq 4$ or $x = 3$</u>**

C) $\frac{1}{x}\leq\frac{1}{x-1}-\frac{1}{2}$ → $\frac{1}{x-1}-\frac{1}{x}-\frac{1}{2}\geq 0$ → $\frac{2x-2(x-1)-x(x-1)}{2x(x-1)}\geq 0$ → $\frac{2+x-x^2}{2x(x-1)}\geq 0$

→ $\frac{(2-x)(1+x)}{2x(x-1)}\geq 0$ → $\frac{(x-2)(x+1)}{2x(x-1)}\leq 0$

The critical values for this quotient are -1, 0, 1 and 2.



Thus, the solution intervals are: $\underline{-1\leq x<0 \text{ or } 1<x\leq 2}$.