MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2008 SOLUTION KEY

Round 3

A) $0.1\overline{x} = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

Multiplying both sides by 10, $1.\overline{x} = \frac{10}{6} = \frac{5}{3}$

Subtracting 1 from both sides, $0.\overline{x} = \frac{2}{3} = 0.666666...$ → $x =$ **6**

B) Clearly, fewer than half the members were absentees, so 60 is too high as a guesstimate.
40 is a good place to start, but it's not divisible by 3, so let's start with 39.
Two-thirds of 39 is 26 so there would be 65 members present, but 65 + 39 = 104 < 120.
(2/3)(42) → 28 → 70 present → 112 members – still too small
(2/3)(45) → 30 → 75 present → 120 members!
Since a quorum is 80 members, we need **5** more members.

An alternative solution (Let algebra do the heavy lifting.):
Let p denote the number of members present. Then # members absent = $a = (120 - p)$

$p = (120 - p) + \frac{2}{3}(120 - p) = \frac{5}{3}(120 - p)$ → $3p = 600 - 5p$ → $8p = 600$ → $p = 75$

$\frac{2}{3}(120) = 80$ members required for a quorum. Thus, **5** more members were needed.

Note: The problem does not say that the difference between the percentage of those present and the percentage of those absent is 66 2/3%, so $(a, p) = \left(\frac{1}{6}, \frac{5}{6}\right)$ or $\left(16\frac{2}{3}\%, 83\frac{1}{3}\%\right)$, where the difference between the fractions is 2/3 (or the percentages 66 2/3%) is a misinterpretation of the problem.

C) From $\begin{cases} x = 1 - 2t \\ y = \frac{t}{2} + 1 \end{cases}$, we have $y = \frac{t}{2} + 1$ → $t = 2(y - 1)$ → $x = 1 - 4(y - 1)$ → $x + 4y = 5$
→ $(n, c) =$ **(4, 5)**

Alternate solution: Let $t = 0$ → $(x, y) = (1, 1)$. Let $t = 2$ → $(-3, 2)$ Substituting, $\begin{cases} 1 + n = c \\ -3 + 2n = c \end{cases}$
→ $-3 + 2n = 1 + n$ → $(n, c) =$ **(4, 5)**