MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 - FEBRUARY 2017 SOLUTION KEY

Round 6

A) The next rows are: 11 - 60 - 61, 13 - 84 - 85, 15 - 112 - 113
Thus, the required sum is **240**.

Alternately, $15^2 + x^2 = (x+1)^2 = x^2 + 2x + 1 \Rightarrow 2x = 225 - 1 = 224 \Rightarrow x = 112$ and the same result follows or note that the sum of the entries in the 2nd and 3rd columns are odd perfect squares, namely $4+5=9=3^2, 12+13=25=5^2, 24+25=49=7^2, 40+41=81=9^2, \ldots$. Then the sum that goes with 15 is $15^2 = 225$ and 15 + 225 = **240**.

B) $\sum_{n=1}^{n=k} 160(2)^{1-n} = 160(1) + 160\left(\frac{1}{2}\right) + 160\left(\frac{1}{4}\right) + \ldots = (160+80+40+20+10+5) + (2.5+1.25+\ldots)$
The sum of the first 6 terms is 315.
Keep adding terms until the total exceeds 319.
$= (315) + (2.5 + 1.25 + 0.625 + \ldots)$
$315 \Rightarrow 317.5 \Rightarrow 318.75 \Rightarrow 318.75 + 0.625 > 319 \Rightarrow n = \underline{\mathbf{9}}$.

C) The terms alternate positive (odd)/negative (even).
The sum of two consecutive terms (starting with an odd number) is –7.
Since n is odd, we may assume there are k pairs of terms preceding the n^{th} term.
$k = \frac{n-1}{2}$
Therefore, $-7\left(\frac{n-1}{2}\right) + t_n = 87$.
$t_1 = 3$
$t_3 = 17 = 3 + 14 = 3 + 7 \cdot 2$
$t_5 = 31 = 3 + 28 = 3 + 7 \cdot 4$
...
$t_n = 3 + 7(n-1) = 7n - 4$
$\Rightarrow -7\left(\frac{n-1}{2}\right) + (7n-4) = 87 \Leftrightarrow -7n + 7 + 14n - 8 = 174 \Leftrightarrow 7n = 175 \Rightarrow n = 25$
$\therefore t_{25} = 25 \cdot 7 - 4 = \underline{\mathbf{171}}$.