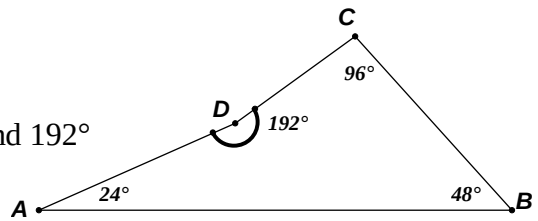
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2016 SOLUTION KEY

Round 6

A) $x+2x+4x+8x=360 \Leftrightarrow 15x=360 \Rightarrow x=24$.
Thus, the 4 angle measures are 24°, 48°, 96° (obtuse) and 192° (reflexive).
The required sum is **<u>288</u>**.

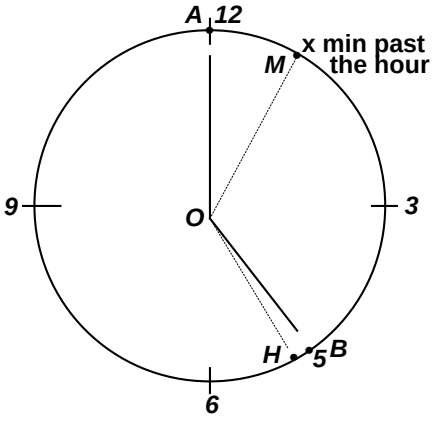


B) $178<\dfrac{180(n-2)}{n}<179 \Rightarrow 178n<180n-360$ and $180n-360<179n$
$\Rightarrow n>180$ and $n<360 \Rightarrow (m,M)=\underline{(\mathbf{181,359})}$.

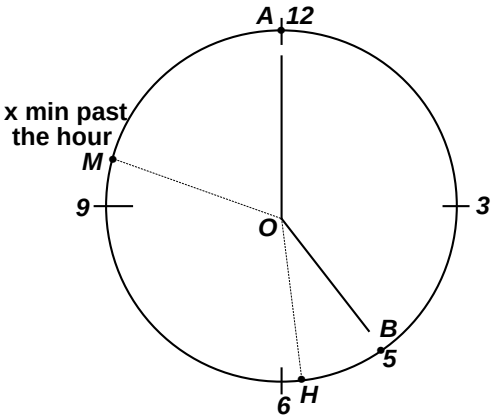
C) The minute hand moves 12 times as fast as the hour hand.
In one hour, the minute hand makes a complete revolution, i.e. turns through an angle of 360° or, equivalently, 6° every minute. The hour hand turns through $\dfrac{360°}{12}=30°$ every hour, or, equivalently, $\dfrac{1}{2}°$ every minute.

<u>In *x* minutes</u>, the minute hand turns through $(6x)°$ and the hour hand turns through $\left(\dfrac{x}{2}\right)°$.

Assume the first 110° angle occurs at *x* minutes past 5:00, i.e. $m\angle MOH=110°$
At 5:00, $m\angle AOB=5(30°)=150°$
At *x* minutes past 5:00, $m\angle AOM=(6x)°$ and $m\angle BOH=\left(\dfrac{x}{2}\right)°$.
Equating two different expressions for $\angle AOH$,
$6x+110=150+\dfrac{x}{2} \Rightarrow x=\dfrac{40}{11}$.



For the second occurrence the diagram looks like the clock below:



Equating two different expressions for $\angle AOM$,
$6x=150+\dfrac{x}{2}+110 \Rightarrow x=\dfrac{520}{11}$
The difference $\dfrac{520-80}{11}=\dfrac{440}{11}=\underline{\mathbf{40}}$ minutes.