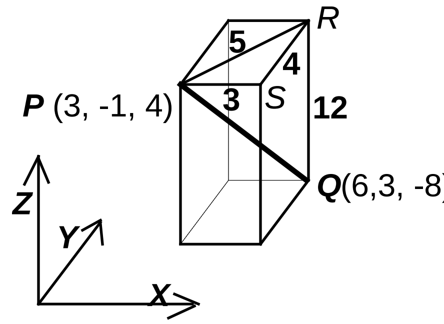
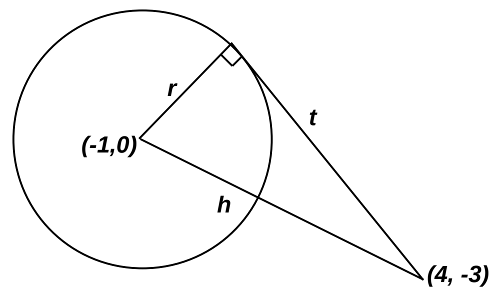
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2011 SOLUTION KEY

Round 3



A) Applying the Pythagorean Theorem, in right ΔPSR ($PS = 3$, $SR = 4$ $\Rightarrow PR = 5$) and in right ΔPRQ ($PR = 5$, $RQ = 12$ $\Rightarrow PQ = 13$).

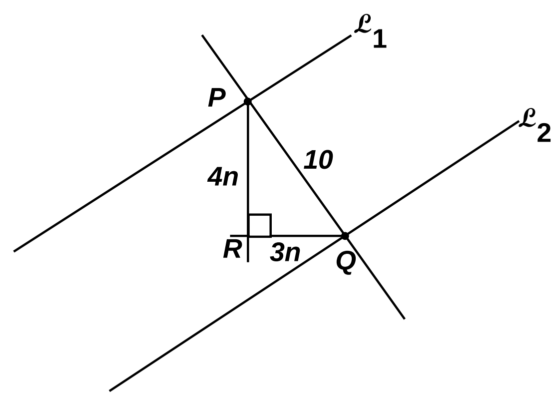
Alternate Solution:
The Pythagorean Theorem extends to 3D as follows:
The distance between points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.
Thus, $PQ = \sqrt{(6-3)^2 + (3-(-1))^2 + (-8-4)^2} = \sqrt{9+16+144} = \sqrt{169} = \underline{\mathbf{13}}$



B) $(x + 1)^2 + y^2 = 6 \Rightarrow$ Center $(-1, 0)$ and $r = \sqrt{6}$
$h = \sqrt{(4+1)^2 + (-3-0)^2} = \sqrt{34} \Rightarrow t^2 = 34 - 6 = 28$
$\Rightarrow t = \underline{\mathbf{2\sqrt{7}}}$
Since translations do not alter distances, translating the circle and given point one unit to the right (so that the circle is centered at the origin) gives the same result.
[$x^2 + y^2 = 6$ and $(5, -3)$]

C) The slope of $\mathscr{L}_2$ is $\frac{3}{4}$ and the slope of $\overleftrightarrow{PQ}$ is $-\frac{4}{3}$.

Therefore, $\frac{PR}{RQ} = \frac{4}{3} \Rightarrow 25n^2 = 100 \Rightarrow n = 2$.



$Q(7 + 6, 11 - 8) = (13, 3)$
The equation of $\mathscr{L}_2$ is
$(y-3) = \frac{3}{4}(x-13) \Leftrightarrow 3x - 4y = 27$

Letting $x, y = 0$, we have X$\underline{\mathbf{(9,\ 0)}}$ and Y$\underline{\left(\mathbf{0,\ -\frac{27}{4}}\right)}$.

Alternate Solution (Norm Swanson)
Since $\mathscr{L}_1 \parallel \mathscr{L}_2$, its equation has the form $3x - 4y + k = 0$.
Any point on $\mathscr{L}_2$ is 10 units from $\mathscr{L}_1$. Applying the point to line distance formula, the distance from (7, 11) on $\mathscr{L}_1$ to $\mathscr{L}_2$ is given by $\frac{|3(7) - 4(11) + k|}{\sqrt{3^2 + 4^2}} = 10$
$\Leftrightarrow |21 - 44 + k| = 50 \Leftrightarrow |k - 23| = 50 \Leftrightarrow k = -27, 53$
$3x - 4y + 53 = 0$ is a line "above" $\mathscr{L}_1$ and is rejected (Q would be to the left of P),
so the equation of $\mathscr{L}_2$ is $3x - 4y - 27 = 0$ and we proceed as above.