MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2013 SOLUTION KEY

Team Round - continued

B) The first complex fraction simplifies to $\dfrac{1}{x+\dfrac{1}{x+\dfrac{1}{2}}}=\dfrac{1}{x+\dfrac{2}{2x+1}}=\dfrac{2x+1}{2x^2+x+2}$.

The second complex fraction simplifies to $\dfrac{1}{2+\dfrac{1}{2+\dfrac{1}{x}}}=\dfrac{1}{2+\dfrac{x}{2x+1}}=\dfrac{2x+1}{5x+2}$.

Thus, we require that $\dfrac{5x+2}{2x^2+x+2}=-1$

$\Leftrightarrow 2x^2+x+2=-5x-2$

$\Leftrightarrow 2x^2+6x+4=2(x+1)(x+2)=0$

$\Leftrightarrow x=\underline{\mathbf{-1,-2}}$

C) $\sin^2(t)+\cos^2(t)=1 \Rightarrow 1+\cot^2(t)=\csc^2(t)$

$x^2=25\cot^2(t)$ and $y^2=9\csc^2(t)$ Subtracting, $\dfrac{x^2}{25}-\dfrac{y^2}{9}=\cot^2 t-\csc^2 t=-1$

Thus, $9x^2-25y^2=-225$ or $9x^2-25y^2+225=0 \Rightarrow (A, C, D, E, F)=$ <u>**(9, –25, 0, 0, 225)**</u>

D) $(2x-3)(Bx-1)=5 \Leftrightarrow 2Bx^2-(2+3B)x-2=0$

To insure exactly one root, we set the discriminant equal to zero.

$b^2-4ac=\left(-(2+3B)\right)^2-4(2B)(-2)=0 \Rightarrow 9B^2+28B+4=0$

$\Rightarrow B=\dfrac{-28\pm\sqrt{28^2-4(36)}}{18}=\dfrac{-28\pm\sqrt{4^2\left(7^2-9\right)}}{18}=\dfrac{-28\pm 8\sqrt{10}}{18}=\dfrac{-14\pm 4\sqrt{10}}{9}$

The larger of the two values is $\dfrac{-14+4\sqrt{10}}{9}$. Substituting, 3 for $\sqrt{10}$, we have

$\dfrac{-14+4(3)}{9}=\dfrac{-2}{9} \Rightarrow (P,Q)=\underline{\mathbf{(-2,9)}}$.

The original Team D) question was simply "Approximate the larger of these two values <u>to the nearest hundredth</u>." How could you tackle this question without a calculator?
One possibility is outlined at the end of this solution key and is followed by a discussion of an algorithm for computation of square root without a calculator.