MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2016 SOLUTION KEY

Round 4

A) $R = \dfrac{D_1 + D_2}{T_1 + T_2} = \dfrac{25 + 3020}{3 + .5} = \dfrac{3045}{7/2} = \dfrac{6090}{7} = \underline{\mathbf{870}}$ mph.

B) Assume carpenter can complete the job alone in x days, while the assistant would take $2x$ days. Then:

Their rates are $\dfrac{1}{x}$ and $\dfrac{1}{2x}$ respectively, implying $7\left(\dfrac{1}{x} + \dfrac{1}{2x}\right) = \dfrac{7}{10} \Rightarrow \dfrac{3}{2x} = \dfrac{1}{10} \Rightarrow x = 15$.

If it takes the assistant T days to complete the job, then $\dfrac{1}{30}T = \dfrac{3}{10} \Rightarrow T = \underline{\mathbf{9}}$ days.

C) $x = 1 + \dfrac{2}{3 + \dfrac{4}{x}} \Rightarrow 1 + \dfrac{2}{\dfrac{3x+4}{x}} = 1 + \dfrac{2x}{3x+4} = \dfrac{5x+4}{3x+4}$

$\Rightarrow 3x^2 + 4x = 5x + 4 \Rightarrow 3x^2 - x - 4 = 0 \Leftrightarrow (3x-4)(x+1) = 0 \Rightarrow x = \cancel{-1}, \underline{\mathbf{\dfrac{4}{3}}}$.