

**MASSACHUSETTS MATHEMATICS LEAGUE  
CONTEST 4 - JANUARY 2008 SOLUTION KEY**

**Round 1**

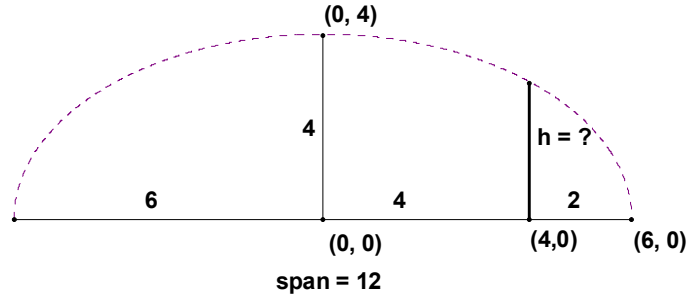
A) Completing the square,  $(x^2 + 4x + \underline{4}) + (y^2 - 2y + \underline{1}) = -F + 4 + 1 = 5 - F = r^2 = 16 \rightarrow F = \underline{-11}$

B) The equation of the ellipse is  $\frac{x^2}{6^2} + \frac{y^2}{4} = 1$

$$\rightarrow x^2 + 9y^2 = 36$$

On the right side, 2 meters from the end of the arch is located at (4, 0). Substituting,

$$y^2 = \frac{36-16}{9} \rightarrow y = \underline{\frac{2\sqrt{5}}{3}}$$



C) The focus of the parabola is located at (2, 3).

The focal width =  $4|a| = 6 \rightarrow a = +3/2$ .

Since the focal chord is vertical, the equation of the parabola has the form  $(y - k)^2 = 4a(x - h)$ , where  $(h, k)$  are the coordinates of the vertex.

$a = +3/2 \rightarrow$  the vertex is at  $(1/2, 3)$ .

Thus, the equation of the parabola is  $(y - 3)^2 = +6(x - \frac{1}{2})$

Substituting  $x = 2.5$ ,  $(y - 3)^2 = 12$ .  $y > 0 \rightarrow y = \underline{3 + 2\sqrt{3}}$