Round 5 - continued

C) Alternate solution:

$$\frac{z}{z+y}=2 \rightarrow \frac{\frac{z}{x}}{1+\frac{y}{x}}=2 \quad \text{and} \quad \frac{y}{x+z}=3 \rightarrow \frac{\frac{y}{x}}{1+\frac{z}{x}}=3$$

Rearranging, we get: $\begin{cases} \frac{y}{x}-3\frac{z}{x}=3 \\ 2\frac{y}{x}-\frac{z}{x}=-2 \end{cases}$ Solving we get $\frac{z}{x}=-\frac{8}{5}$ and $\frac{y}{x}=-\frac{9}{5} \rightarrow \frac{y}{z}+1=\frac{17}{8}$

Thus, $\frac{z}{y+z}=\frac{1}{\frac{y}{z}+1}=\underline{\boldsymbol{\frac{8}{17}}}$

Generalization
(contributed by Shing S. So Dept. Math and Computer Science – University of Central Missouri)

Given: $\frac{z}{x+y}=a$ and $\frac{y}{x+z}=b$, find $\frac{z}{y+z}$ in terms of *a* and *b*.

$\frac{z}{y+z}=\frac{1}{\frac{y}{z}+1}$ so we solve for $\frac{y}{z}$.

From $\frac{z}{x+y}=a$ we have $z = ax + ay$

From $\frac{y}{x+z}=b$ we have $y = bx + bz$

Substituting we have $z = ax + a(bx + bz) = ax + abx + abz$, so $\frac{z}{x}=\frac{a+ab}{1-ab}$.

Substituting again we have $y = bx + b(ax + ay) = bx + abx + aby$, so $\frac{y}{x}=\frac{b+ab}{1-ab}$.

Now $\frac{y}{z}=\frac{\frac{y}{x}}{\frac{z}{x}}=\frac{b+ab}{a+ab}$ and $\frac{y}{z}+1=\frac{b+ab+a+ab}{a+ab}=\frac{2ab+a+b}{a+ab}$

Inverting we have the required ratio, namely $\frac{z}{y+z}=\underline{\boldsymbol{\frac{a+ab}{2ab+a+b}}}$

Note if the values of the two given ratios are reversed we have $\frac{z}{y+z}=\underline{\boldsymbol{\frac{b+ab}{2ab+a+b}}}$,

and these two values sum to 1.