MASSACHUSETTS MATHEMATICS LEAGUE
FEBRUARY 2006
ROUND 7: TEAM QUESTIONS
ANSWERS

A)_____   D)_____

B)_____   E)_____

C)_____°_ F)_____

A)   Suppose $f(x) = \dfrac{2x-1}{x-3}$ For what values of x does $2 \cdot f(x) \cdot f^{-1}(x) = 2$?

B)   Suppose m and n are twin primes (i.e. primes differing by 2) and suppose the digits used to form m and n are distinct. If the smallest such pair of numbers and the largest such pair of number are all added together, what is the sum?

C)   In $\triangle ABC$, $\angle B = \csc^{-1}(\dfrac{\sqrt{34}}{3})$ and $\angle A = \tan^{-1}(0.25)$. Find m$\angle C$ in degrees.

D)   I am half as old as my mother was when my brother was twelve years younger than I am now. My brother was born when my mother was 26. If the sum of my brother's and my own current ages is 36, how old was my mother when I was born?

E)   In the diagram at the right, m$\angle DCB = 30°$,
    $AC = 4$, $IC = 6$ and $BC = 18$. The exact <u>positive</u> difference
    between the distances of the two chords from the center of the
    circle is $a - b\sqrt{c}$ for integers a, b, and c.
    Evaluate $b^2 c - a$.
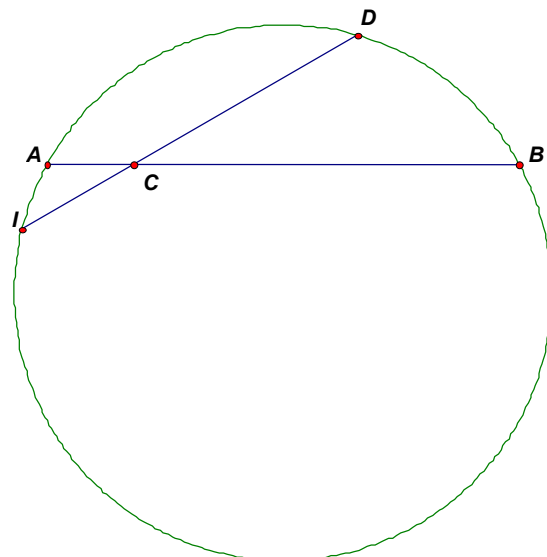
F) $T_n = 3n + 2$. For some integers j and k, $j > k > 6$,
    T_k will be the geometric mean between T_6 and T_j.
    Find the smallest possible value of the sum $j + k$.