MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2014
ROUND 7 TEAM QUESTIONS

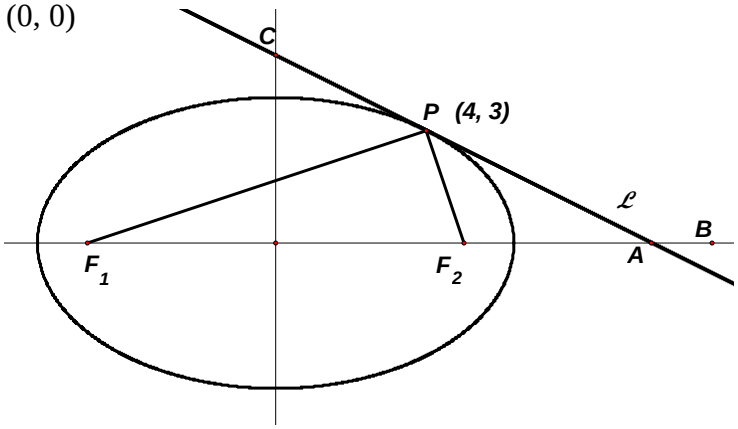
ANSWERS

A) _____ D) _____

B) _____ E) _____

C) _____ F) _____

A) In the figure at the right, the ellipse has center (0, 0) and vertices on the *x*-axis.
Line $\mathcal{L}$ is tangent to the ellipse at *P*(4, 3) and $m\angle F_1PF_2 = 90°$.
Determine the equation of $\mathcal{L}$. Express it in $Ax + By = C$ form, where *A*, *B* and *C* are integers and *A* > 0.



B) Find all real values of *x* for which $\dfrac{4x-1}{4} = \dfrac{1}{\sqrt{4x+1}}$.

C) Solve for *x* over the interval $-2\pi \le x < 0$: $\cos^4 x + 6\cos^2 x + 7\cos x + \cos 3x = -\dfrac{15}{16}$

D) Find all positive integer values of *k* for which $x^2 + kx + k + 11$ can be expressed as the product of two linear binomials with integer coefficients.

E) In ΔPQR, $\overline{ST} \parallel \overline{PQ}$, where *S* lies on $\overline{PR}$ and *T* lies on $\overline{RQ}$.
PS = 3 units and *RS* = *k* units and the area of ΔRST is 100 square units. Compute all possible positive integer values of *k* for which the area of trapezoid *STQP* is also an integer.

F) Let *A* be a positive integer. For a specific value of *A*, the complex fraction $\dfrac{10(x+4)}{\dfrac{5}{x-2} - \dfrac{2}{x-A}}$

is undefined for exactly three different integer values of *x*.
Find the minimum sum of these three values of *x*.