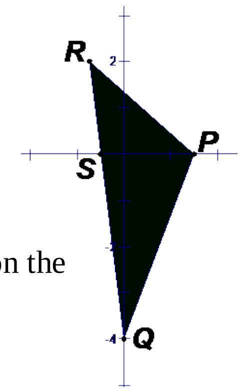
**MASSACHUSETTS MATHEMATICS LEAGUE**
**CONTEST 5 - FEBRUARY 2016 SOLUTION KEY**

## Team Round

A) Since $x=\frac{3}{2}\Rightarrow f(x)=0$, $P\left(\frac{3}{2},0\right)$.

Since $x=0\Rightarrow f\left(x\right)=\frac{-3}{c}$, $Q\left(0,\frac{-3}{c}\right)$.

Since $x=-c$ results in division by 0, $y=f(x)$ has a vertical asymptote at $x=-c$.




Rewriting $f(x)$ as $\dfrac{2-\frac{3}{x}}{1+\frac{c}{x}}$ and letting $x\to\pm\infty$, we see that the horizontal asymptote of $f$ is $y=2$ . Thus, $R\left(-c,2\right)$.

Since $0<c<1$, $-\infty<-\frac{3}{c}<-3$, the graph at the right shows $Q$ as some point on the $y$-axis below $\left(0,-3\right)$ and $R$ as some point between $\left(-1,2\right)$ and $\left(0,2\right)$.

The area of $\Delta PQR$ equals the sum of the areas of $\Delta PSR$ and $\Delta PSQ$.

Thus, we require that $\frac{1}{2}\cdot PS\cdot 2+\frac{1}{2}\cdot PS\cdot\frac{3}{c}=6\Leftrightarrow PS\left(1+\frac{3}{2c}\right)=6$.

Since the slope of line $\overleftrightarrow{RQ}$ is $\dfrac{2+\frac{3}{c}}{-c}=\dfrac{2c+3}{-c^2}$, the equation of $\overleftrightarrow{RQ}$ is $\left(y-2\right)=\dfrac{2c+3}{-c^2}\left(x+c\right)$.

To find $S$, we let $y=0$. $x=\dfrac{2c^2}{2c+3}-c=\dfrac{-3c}{2c+3}$ Thus, $S\left(\dfrac{-3c}{2c+3},0\right)$ and $PS=\dfrac{3}{2}+\dfrac{3c}{2c+3}$

Thus, $\left(\dfrac{3}{2}+\dfrac{3c}{2c+3}\right)\left(1+\dfrac{3}{2c}\right)=6\Leftrightarrow\left(\dfrac{12c+9}{2\cancel{(2c+3)}}\right)\left(\dfrac{\cancel{2c+3}}{2c}\right)=6\Rightarrow 12c+9=24c\Leftrightarrow c=\underline{\boldsymbol{\dfrac{3}{4}}}$ .