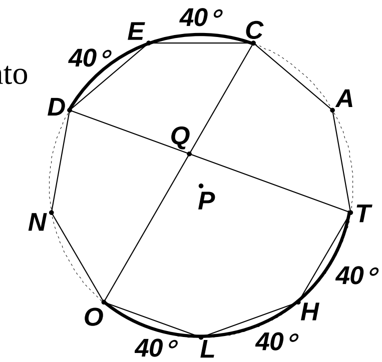
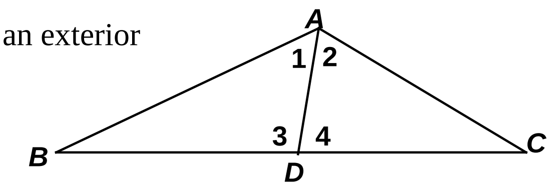
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 – MARCH 2013 SOLUTION KEY

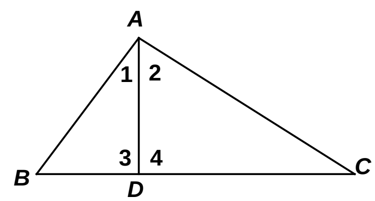
Round 5

A) The nine vertices of regular polygon DECATHLON divide circle P into $\dfrac{360°}{9}=40°$ arcs. Since the measure of an angle formed by intersecting chords of a circle equals the average of the intercepted arcs, we have $m\angle DQC = m\angle OQT = \dfrac{80+120}{2} = \underline{\mathbf{100}}°$.



B) $\angle 1$ in ΔBAD is congruent to $\angle 2$, 4 or C in ΔADC . As an exterior angle of ΔBAD , $\angle 4$ can not be congruent to $\angle 1$ (or $\angle B$).
If $\angle 1$ were congruent to $\angle 2$, then $\angle C$ would have to be congruent to $\angle B$ and $\angle 3$ would have to be congruent to $\angle 4$, forcing $\Delta BAD \cong \Delta CAD$ and $AB = AC$ which is not the case.
Thus, $m\angle 1 \neq m\angle 2$.
The only alternative left is
$m\angle 1 = m\angle C \Rightarrow m\angle 2 = m\angle B = 37°,\ m\angle 3 = m\angle 4 = 90°$ and $m\angle 1 = 53°$
Thus, the required sum is $\underline{\mathbf{143}}$°.





Alternate Solution: $\Delta BAD \sim \Delta ACD$ (Other correspondences lead to contradictions as argued above.) Thus, $m\angle C = m\angle 1$. In ΔBAD , since $m\angle B = 37°$, $m\angle BDA = 143 - m\angle 1 = 143 - m\angle C$,
Transposing, $m\angle BDA + m\angle C = \underline{\mathbf{143}}$. A more accurate diagram is shown above.

C) Let (BD, BC, AB) = (x, y, z) as indicated in the diagram.
Drop a perpendicular (h) from D to $\overline{AB}$. Then: $x = \dfrac{5}{2}z,\ x = \dfrac{5}{4}y$

Since $DE = 6\sqrt{3}$, the required area is $2\cdot\dfrac{1}{2}\cdot 6\sqrt{3}\cdot y + \dfrac{1}{2}zh = 6\sqrt{3}\cdot y + \dfrac{zh}{2}$

$$\begin{cases} y^2 + 108 = x^2 \\ x = \dfrac{5}{4}y \end{cases} \Rightarrow \frac{9}{16}y^2 = 108 \Rightarrow y^2 = 16(12) \Rightarrow y = 8\sqrt{3}\ ,\ x = 10\sqrt{3}$$

and $z = 4\sqrt{3}$ Thus, $h^2 = 300 - 12 = 288 = 144(2) \Rightarrow h = 12\sqrt{2}$.
The required area is $48(3) + 24\sqrt{6} = 24\left(6+\sqrt{6}\right) \Rightarrow$ **(24, 6, 6)**.