MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 – MARCH 2009 SOLUTION KEY

Round 3

A) Applying the quadratic formula to $x^2+(1+2i)x+(i-1)=0 \rightarrow x = \dfrac{-1-2i \pm \sqrt{(1+2i)^2-4(i-1)}}{2}$

$= \dfrac{-1-2i \pm \sqrt{(1+4i-4-4i+4)}}{2} = \dfrac{-1-2i \pm 1}{2} \rightarrow$ **$\underline{0-i}$**, **$\underline{-1-i}$**

An alternate solution notes that $x^2+(1+2i)x+(i-1)=0 \Leftrightarrow (x+i)\big(x+(1+i)\big)=0$ and the solutions follow immediately.

B) $f(x)=3^{mx+b}$ and $f(x+2)=27f(x-2) \rightarrow m(x+2)+b=m(x-2)+b+3$

$\rightarrow 2m=-2m+3 \rightarrow m=3/4$

$f(0)=1/3 \rightarrow 3^{m\cdot 0+b}=\frac{1}{3}=3^{-1} \rightarrow b=-1$ Thus, $(m, b) =$ $\underline{\left(\mathbf{\frac{3}{4}, -1}\right)}$

C) The diagram at the right shows region in question.
Maximum occurs when $Ax-2=0 \rightarrow$ Max at $(2/A, 7)$
$y=0 \rightarrow |Ax-2|=7 \rightarrow Ax=2\pm 7 \rightarrow x=-5/A$ or $9/A$

The area is $\frac{1}{2}\left(\frac{14}{A}\right)(7)=2009$

$\rightarrow \frac{49}{A}=2009 \rightarrow A=\frac{49}{2009}=\frac{49}{7^2\cdot 41}=\underline{\mathbf{\frac{1}{41}}}$