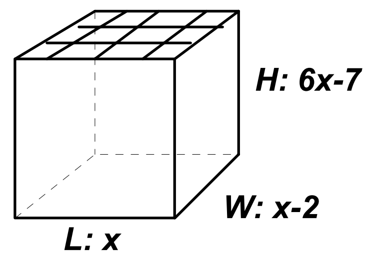
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 – MARCH 2007 SOLUTION KEY

Round 3

A) Using synthetic substitution, $p(x)$ factors to $(x-1)(x^2+x-1)$ Using the quadratic formula on the second factor, the remaining roots are $\frac{-1\pm\sqrt{5}}{2}$ and the larger is $\underline{\mathbf{\frac{\sqrt{5}-1}{2}}}$.

B) $p(x) = a(x+1)(x-2)(x-½)$ and $p(0) = a(1)(-2)(-1/2) = 2 \rightarrow a = 2$
Thus, $p(x) = (x+1)(x-2)(2x-1)$.
Substituting $x = 1$ gives us the remainder of division by $(x-1)$: 2(-1)(1) = **-2**

C) $V = x(x-2)(6x-7) = 6x-5$
$\rightarrow 6x^3 - 19x^2 + 14x = 6x - 5$
$\rightarrow 6x^3 - 19x^2 + 8x + 5 = 0$



Synthetic substitution shows that $x = 1$ is a root and the quotient is $6x^2 - 13x - 5 = (3x+1)(2x-5)$. Checking the roots of 1, 5/2 and -1/3, only $x = 5/2$ gives three positive dimensions.
The dimensions of the box are 5/2 x 1/2 x 8 and

the surface area of the open box is

top		left/right faces		front/back faces		
1(1/2 · 5/2)	+	2(1/2 · 8)	+	2(5/2 · 8)	=	5/4 + 8 + 40 = **49.25**.

Round 4

A) Since 1 c = 8 oz, we need 24 oz of 1% milk.
We must determine the ratio of water and 4% milk that produces 1% milk.
To convert the entire half gallon (64 oz) of 4% milk to 1% milk by adding x oz of water:
$.04(64) + 0.00x = .01(64+x) \rightarrow 256 = 64 + x \rightarrow x = 192$
4% : water = 64 : 192 → 1 : 3 ratio
Let n denote the # ounces of 4% milk. Then $n + 3n = 24 \rightarrow n = 6$
→ milk: **6 oz** water: **18 oz**

B) The x- and y-intercepts of $\frac{x}{3a}+\frac{y}{4a}=1$ are (3a, 0) and (0, 4a).
The distance between these points is $\sqrt{25a^2} = 5|a| = 35 \rightarrow a = \underline{\pm 7}$

C) $(4^x + 4^x) = 2(4^x) = 2(2^{2x}) = 2^{2x+1}$
$(3^x + 3^x + 3^x) = 3(3^x) = 3^{x+1}$
2^{2x+1} has $(2x+2)$ factors, namely $1 = 2^0, 2 = 2^1, 4 = 2^2, 8 = 2^3, \ldots\ 2^{2x+1}$
Similarly, 3^{x+1} has $(x+2)$ factors, namely $1, 3, 9, 27, \ldots\ 3^{x+1}$
The factors of N are found by multiplying any factor in the first list by any factor in the second list. Since any pair of numbers selected from these lists are relatively prime, there will be $(2x+2)(x+2)$ products with no duplicates. Thus, N has $\underline{\mathbf{2(x+1)(x+2) = 2x^2+6x+4}}$ factors (or equivalent)