MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2015 SOLUTION KEY

Round 5

A) The area of circle 1 is $\pi\left(\sqrt{3}\right)^2 = 3\pi$

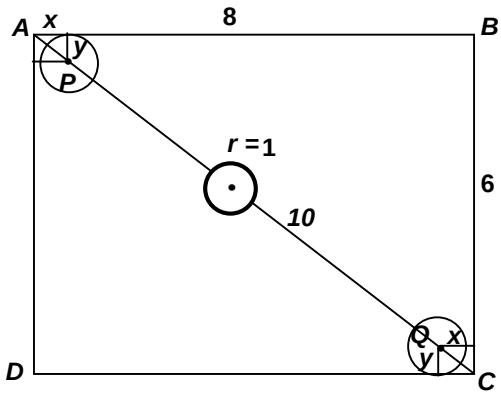
The area of circle 2 is $\pi\left(\frac{\sqrt{15}}{2}\right)^2 = \frac{15}{4}\pi = 3.75\pi$

Thus, the required ratio is $\frac{15/4}{3} = \underline{\mathbf{\frac{5}{4}}}$.

B) By similar triangles, $\frac{y}{x} = \frac{6}{8}$ and $r = 1 \Rightarrow y = 1, x = \frac{4}{3}$,

(diag) $d = AP = CQ = \frac{5}{3}$

Therefore, $PQ = 10 - 2\left(\frac{5}{3}\right) = \underline{\mathbf{\frac{20}{3}}}$.



C) Let x denote $OA = OP = OC$, radii of circle O. Then:

In ΔBOC, $x^2 + 10^2 = (20 - x)^2 \Rightarrow$

$100 = 400 - 40x \Rightarrow x = 7.5$

$AB = 20 \Rightarrow PB = 5$

$\Delta ROP \sim \Delta COB \Rightarrow \frac{7.5}{7.5+5} = \frac{y}{10} \Rightarrow \frac{15}{25} = \frac{3}{5} = \frac{y}{10}$

$\Rightarrow y = 6 \Rightarrow PQ = \underline{\mathbf{12}}$