MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2015 SOLUTION KEY

Round 6

A) $2x^2 + A(x-B) = (2x-3)(x+6) \Leftrightarrow 2x^2 + Ax - AB = 2x^2 + 9x - 18$
If these two trinomials are to give the same values for every value of x, then
$A = 9$ and $AB = 18 \Rightarrow (A,B) = \underline{\mathbf{(9,2)}}$.

B) Using as many of the larger denominations as possible first will minimize the number of coins needed.
$\left\lfloor \frac{100}{31} \right\rfloor = 3 \Rightarrow 100 - 31 \cdot 3 = 7$ which can be returned as one $2¢$ coin and one $5¢$ coin.
Therefore, only **5** coins are needed.

C) Let Linda and Sam be x and y years old today. Then:
$x - 10 = 2(y-10) - 1 \Rightarrow x = 2y - 11$ and
$x + 13 = \frac{5}{6}(y + 29) \Rightarrow 6x - 5y = 67$
Substituting for x, $6(2y-11) - 5y = 67 \Rightarrow 7y = 133 \Rightarrow (x,y) = (27,19)$
Suppose the required 6 : 5 ratio occurs in T years. Then:
$\frac{27+T}{19+T} = \frac{6}{5} \Leftarrow 135 + 5T = 114 + 6T \Rightarrow T = \underline{\mathbf{21}}$