MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2009 SOLUTION KEY

Round 1

A) $f(f(3)) = f(-7) = -(-7)^3 = 343$
$f(-1) = -(-1)^3 = 1$
$f(f(1/4)) = f(5/4) = -7$
$343 + 1 - 7 =$ **<u>337</u>**

B) Inputting 5/6 to f^{-1} is equivalent to outputting 5/6 from f. Therefore, there is no need to determine the f^{-1} function rule. Simply set the given function equal to 5/6,
i.e. solve $x - \frac{1}{x} = \frac{5}{6}$.
Thus, $6x^2 - 5x - 6 = (3x + 2)(2x - 3) = 0 \rightarrow x =$ **<u>$\frac{3}{2}$</u>**

C) $x = 2 \rightarrow 4a + 2b + 3 = 3 \rightarrow 2a + b = 0$
$x = -1 \rightarrow a - b + 3 = 9 \rightarrow a - b = 6$ Thus, $(a, b) = (2, -4)$
$f(x) = 2x^2 - 4x + 3$ and $f(4) = 2(16) - 4(4) + 3 = 32 - 16 + 3 = 19$
$2c^2 - 4c - 16 = = \rightarrow c^2 - 2c - 8 = (c + 2)(c - 4) = 0 \rightarrow c =$ **<u>-2</u>**