MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2015 SOLUTION KEY

Team Round

A) (1, 1) on P ($y = 2x^2 - 8x + C$) $\Rightarrow 1 = 2(1)^2 - 8(1) + C \Rightarrow C = 7$

$y = 2x^2 - 8x + 7 \Leftrightarrow y - 7 = 2(x-2)^2 - 8 \Leftrightarrow (x-2)^2 = \frac{1}{2}(y+1)$ (Parabola opens UP.)

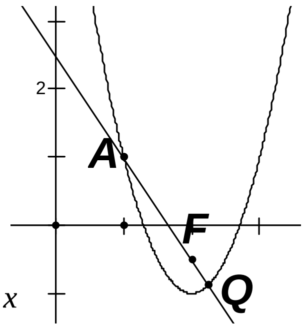
Thus, the vertex of the parabola is at (2, –1) and $4p = \frac{1}{2} \Rightarrow p = \frac{1}{8} \Rightarrow$ focus $F\left(2, -\frac{7}{8}\right)$.

The slope of $\overleftrightarrow{AF}$ is $\dfrac{1+\frac{7}{8}}{1-2} = -\dfrac{15}{8}$ and the equation of $\overleftrightarrow{AF}$ is

$(y-1) = -\frac{15}{8}(x-1)$ or $y = \frac{-15x+23}{8}$. Knowing that $x = 1$ would be a root of the following quadratic equation helps factor the trinomial.

Substituting, $\frac{-15x+23}{8} = 2x^2 - 8x + 7 \Leftrightarrow 16x^2 - 49x + 33 = 0 \Leftrightarrow (16x-33)(x-1) = 0$

$\Rightarrow x_Q = \underline{\mathbf{\frac{33}{16}}}$.



B) Note the original equations $y = \sqrt{x-1}$ and $x = \sqrt{72y+1}$ require that both x and y be nonnegative. Since, in the first equation, $x = 1 \Rightarrow y = 0$ and (1, 0) satisfies the second equation, we have the (trivial) solution **<u>(1, 0)</u>**.

Squaring both sides, we have $\begin{cases} y^2 = x - 1 \Leftrightarrow x = y^2 + 1 \\ x^2 = 72y + 1 \end{cases}$.

Substituting for x in the second equation, $\left(y^2+1\right)^2 = 72y + 1 \Leftrightarrow y^4 + 2y^2 - 72y = 0$

If $y \neq 0$, $y^3 + 2y - 72 = 0$.

By inspection (lucky guess) or synthetic substitution, y = 4 is solution.

Synthetic substitution gives the complete factorization as $(y-4)\left(y^2+4y+18\right)$ and the trinomial factor does not give additional real solutions.

$y = 4 \Rightarrow 16 = x - 1 \Rightarrow x = 17$ and a second solution is the ordered pair $\underline{\mathbf{(17,4)}}$.