MASSACHUSETTS MATHEMATICS LEAGUE CONTEST 1 - OCTOBER 2010 SOLUTION KEY

Team Round – continued

F) Alternate solution (Tuan Le) - 3 cases Case 1: 2Ps are next to each other, but O and I are not PP, L and N create 4 spaces to place the remaining letters. { PP L N}
The PP, L and N can be arranged in $3! = 6$ ways and there are $4 \cdot 3 = 12$ ways to place the O and I
→ $6 \cdot 12 = 72$
Case 2: OI are next to each other, but the Ps are not Again OI, L and N create 4 spaces to place the remaining letters. { OI L N} Arrange OI next to each other – 2 ways Arrange OI, L and N – 6 ways
Use 2 of the 4 spaces for the Ps - $\binom{4}{2} = \frac{4!}{2!2!} = 6$ $\Rightarrow 2(6)(6) = 72$
2 2(0)(0) 12

Case 3: Both OI and PP in adjacent positions Arrange the 4 items. Only OI can be flipped. $\{OI, PP, L, N\} \rightarrow 4! \cdot 2! = 48$

Since the total number of anagrams is 6! - 1 = 359, the number of anagrams without adjacent Ps and without adjacent vowels is $359 - (2(72) + 48) = \underline{167}$.