MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 – NOVEMBER 2013 SOLUTION KEY

Round 4

A) $x^2+2x+1=10000 \Leftrightarrow (x+1)^2=10^4 \Leftrightarrow x+1=\pm 100 \Leftrightarrow x=$ **99**, **−101**

Alternately, $x^2+2x+1=10000 \Leftrightarrow x^2+2x-9999=0$
Since 9999 is divisible by 99 [99·101], we have factors which differ by 2.
$(x-99)(x+101)=0 \Rightarrow x=$ **99, −101**

B) $x(6x-5y)+4y(x-4y)+y^2 \Leftrightarrow 6x^2-xy-15y^2=$ $\mathbf{(3x-5y)(2x+3y)}$

C) $\dfrac{4-7x-2x^2}{5-11x+2x^2}=3+\dfrac{38}{x-12} \Leftrightarrow \dfrac{(4+x)(1-2x)}{(5-x)(1-2x)}=\dfrac{3x+2}{x-12}$ Note: $x \neq 5, \dfrac{1}{2}, 12$

Cancelling the common binomial factors on the left side and cross multiplying, we have
$(4+x)(x-12)=(5-x)(3x+2)$
$4x-48+x^2-12x=15x+10-3x^2-2x$
$4x^2-21x-58=0$
$(4x-29)(x+2)=0$
$x=$ **−2, $\dfrac{29}{4}$** (or **7.25**)

FYI:

Check: $x=-2 \Rightarrow \dfrac{4+14-8}{5+22+8}=\dfrac{10}{35}=\boxed{\dfrac{2}{7}}$, $3+\dfrac{38}{-14}=3-\dfrac{19}{7}=\boxed{\dfrac{2}{7}}$

$$x=\frac{29}{4} \Rightarrow \frac{4-\frac{203}{4}-\frac{841}{8}}{5-\frac{19}{4}+\frac{841}{8}}=\frac{32-406-841}{40-638+841}=\frac{-1215}{243}=\boxed{-5},$$

$$3+\frac{38}{\frac{29}{4}-12}=3+\frac{38(4)}{29-48}=3+\frac{38(4)}{-19}=3-8=\boxed{-5}$$