Team Round

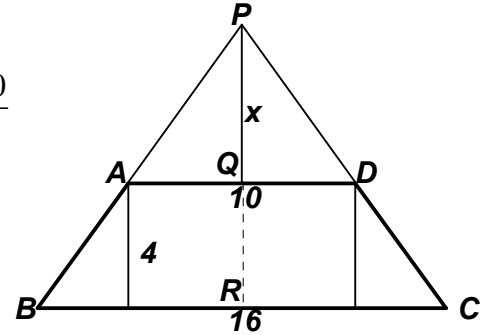A) Extend the non-parallel sides of the trapezoid until they intersect. The triangle formed is a cross-section of the cone from which the frustum was formed. $\Delta PQD \sim \Delta PRC \rightarrow \dfrac{x}{x+4} = \dfrac{5}{8} \rightarrow x = \dfrac{20}{3}$

To avoid computations with fractions, scale the diagram by a factor of 3: See diagram below.
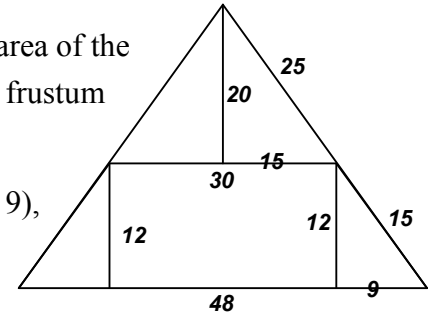
Since the lateral surface area of a cone is given by $\pi r l$, where l is the lateral height, the lateral surface area of the frustum is $\pi \cdot 24 \cdot 40 - \pi \cdot 15 \cdot 25 = \pi(960 - 375) = 585\pi$ and, adding the surface area of the circular bases (225π and 576π), we have the total surface area of the frustum is 1386π.

But we scaled the <u>linear</u> dimensions by 3 (and consequently areas by 9), so the adjusted TSA is $1386\pi/9 = \underline{\textbf{154}\pi}$.

Alternative solution #1: Straightforward - does not scale the numbers, i.e. works with the fractions

Small cone: $l^2 = \dfrac{400}{9} + 25 = \dfrac{400}{9} + \dfrac{225}{9} = \dfrac{625}{9} \rightarrow l = \dfrac{25}{3}$ LA $= \pi(5)\left(\dfrac{25}{3}\right) = \dfrac{125}{3}\pi$

Large cone: $l^2 = \dfrac{1024}{9} + 64 = \dfrac{1024}{9} + \dfrac{576}{9} = \dfrac{1600}{9} \rightarrow l = \dfrac{40}{3}$ LA $= \pi(8)\left(\dfrac{40}{3}\right) = \dfrac{320}{3}\pi$
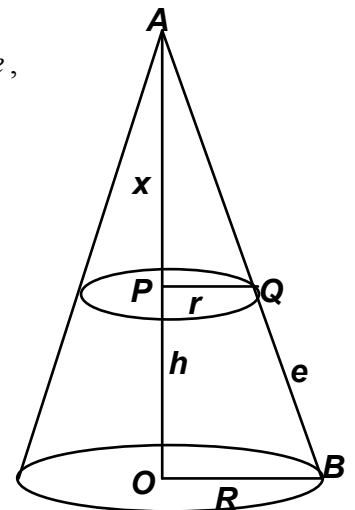
Total surface area of frustum: $25\pi + 64\pi + \left(\dfrac{320}{3}\pi - \dfrac{125}{3}\pi\right) = 89\pi + \dfrac{195}{3}\pi = (89+65)\pi = \underline{\textbf{154}\pi}$

Alternative solution #2 utilizes this formula: LSA(frustum) $= \pi(R+r)e$, where $e^2 = h^2 + (R-r)^2$
$r = 5$, $R = 8$ and $h = 4 \rightarrow e = 5$
Thus, TSA $= \pi(5)^2 + \pi(8+5)(5) + \pi(8)^2 = (25+65+64)\pi = \underline{\textbf{154}\pi}$