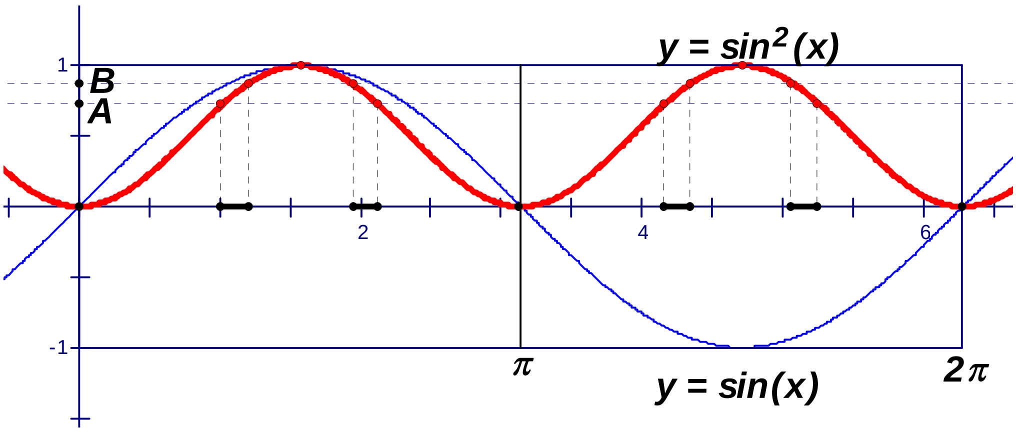
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2015 SOLUTION KEY

Team Round - continued

C) $|8\sin^2 x - 5| < 1 \Leftrightarrow -1 < 8\sin^2 x - 5 < 1 \Leftrightarrow \frac{1}{2} < \sin^2 x < \frac{3}{4} \Rightarrow \frac{\sqrt{2}}{2} < \sin x < \frac{\sqrt{3}}{2}$

As a visual clue, the graphs of $y = \sin(x)$ and $y = \sin^2(x)$ are included below.
The graph of the latter is either on or above the *x*-axis.
The graphs share the same *x*-intercepts and the same maximum value, but notice that between 0 and π, the graph of $y = \sin^2(x)$ is below $y = \sin(x)$, since squaring a value between 0 and 1 produces a smaller value.
Let $A\left(0, \frac{\sqrt{2}}{2}\right)$ and $B\left(0, \frac{\sqrt{3}}{2}\right)$ The horizontal lines through *A* and *B* intersect the graph of $y = \sin^2(x)$ in 8 points over the specified interval $0 \le x < 2\pi$. Since $\frac{\sqrt{2}}{2}$ and $\frac{\sqrt{3}}{2}$ are special values, we recognize the *x*-values (i.e. the coordinates of the endpoints of the solution intervals) as the $\frac{\pi}{4}$ and $\frac{\pi}{3}$ families of related values.



The solution is 4 disjoint intervals, namely
$\underline{\frac{\pi}{4} < x < \frac{\pi}{3}}$, $\underline{\frac{2\pi}{3} < x < \frac{3\pi}{4}}$, $\underline{\frac{5\pi}{4} < x < \frac{4\pi}{3}}$ and $\underline{\frac{5\pi}{3} < x < \frac{7\pi}{4}}$.