

**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2011 SOLUTION KEY**

Round 2

- A) Since primes are odd (except for 2), the difference between two primes is odd if and only if one of the two primes is 2. Therefore, the two primes must be 2 and 47, resulting in a sum of **49**.
- B) There are $400(365)$ days plus 100 leap days minus leap days in 1700, 1800 and 1900.
 $[400(365) + 100 - 3] = 146097$ days or, dividing by 7, **20,871** weeks

FYI: The Julian calendar (aka Julius Caesar) was instituted in 45BC, specifying a year to be exactly 365.25 days long. A pretty amazing result for the time! The currently accepted year length is 365.2425 days, i.e. 365 days 5 hours 49 minutes and 12 seconds – a discrepancy of 10 minutes and 48 seconds per year. Not much of an error, but over the centuries it added up. The Gregorian calendar we use today was adopted in most countries in 1582. According to a Papal Bull issued by Pope Gregory XIII on 2/24/1582, the day after Thursday 10/4/1582 was to be Friday 10/15/1582. The accumulated error had grown to 10 days. Imagine the reaction from the general population!! People who accept as truth:

“There are three types of people: those who do math and those who don’t.”

The last country to make the transition was Greece on Thursday, March 1st **1923**. The day before had been Wednesday, February 15th (on the old Julian calendar). The gap had grown to 13 days. Great stuff! Check it out for yourself.

- C) $10101_{(-2)} = 1(16) + 1(4) + 1 = 21$
 $11010_{(-2)} = 1(16) + 1(-8) + 1(-2) = 6$
 Thus, the sum in base 10 is 27.
 Converting to base (-2) , the place values are 1, -2 , 4, -8 , 16, -32 , 64, ...
 Since 27 is odd and all place values are even except 1, we must have a 1 in the unit’s position.
 $(64)1 + (-32)1 = 32$ too large
 $32 + (-8)1 = 24$ too small
 $24 + (4)1 = 28$ too large
 $28 + (-2)(1) = 26$ too small
 $26 + (1)1 = 27$
 \Rightarrow **1101111** _{(-2)}

Alternate solution: (do the arithmetic in the given base!!)

10101

11010 What is $1 + 1$ in base (-2) ? $2_{(10)} = 110_{(-2)}$ Why?

?1111

$$[1(-2)^2 + 1(-2)^1 + 0(-2)^0 = 1(4) + 1(-2) + 0(1) = 4 - 2 + 0 = 2]$$