MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2017 SOLUTION KEY

Round 4

A) $720 = 6! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 = 2^4 3^2 5^1$
The first few factorial numbers are 1, 2, 6, 24, 120, 720.
$720 = 1 \cdot 720 = 6 \cdot 120 = 1! \cdot 6! = 3! \cdot 5! \Rightarrow (x, y) = \underline{\mathbf{(1,6), (3,5)}}$.

B) Squaring both sides, we have $N^2 = k^2 + 2017 \Rightarrow N^2 - k^2 = (N+k)(N-k) = 2017$.
However, since 2017 is prime, its only factors are 1 and 2017.
Therefore, $\begin{cases} N+k = 2017 \\ N-k = 1 \end{cases} \Rightarrow 2N = 2018 \Rightarrow (k, N) = \underline{\mathbf{(1008,1009)}}$

C) $\boxed{\begin{array}{r} 1 \\ 2n-5 \overline{)3n+25} \\ \underline{2n-5} \\ n+30 \end{array}} \Rightarrow \dfrac{3n+25}{2n-5} = 1 + \dfrac{n+30}{2n-5}$

$\dfrac{n+30}{2n-5}$ is the fractional part of the mixed number equivalent of $\dfrac{3n+25}{2n-5}$.

Evaluating the fractional part, $n = 1,2,3,4,5,6 \Rightarrow \dfrac{31}{-3}, \boxed{\dfrac{32}{-1}} = -32, \boxed{\dfrac{33}{1}} = 33, \dfrac{34}{3}, \boxed{\dfrac{35}{5}} = 7, \dfrac{36}{7}$

*** As the numerator of these fractions increases by 1, the denominator increases by 2.
The positive quotients will continue to get smaller. Eventually, the denominator will exceed the numerator and the search must stop.
We will get an integer quotient when the numerator is a multiple of the denominator.

Continuing the pattern, $n = 7,8,9,...,15 \Rightarrow \dfrac{37}{9}, \dfrac{38}{11}, \boxed{\dfrac{39}{13}}, \dfrac{40}{15}, \dfrac{41}{17}, \dfrac{42}{19}, \dfrac{43}{21}, \dfrac{44}{23}, \dfrac{45}{25}$

So a 3:1 ratio was a possibility, but a 2:1 ratio was not.

Is a 1 : 1 ratio possible? $45 + k = (25 + 2k) \Leftrightarrow k = 20 \Rightarrow \dfrac{65}{65}$ or $n + 30 = 2n - 5 \Rightarrow n = 35$.

The sum of the *n*-values is 2 + 3 + 5 + 9 + 35 = **<u>54</u>**.