MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2007 SOLUTION KEY

Round 4

A) The minimum occurs at the vertex which lies on the axis of symmetry of this upward opening parabola. The axis of symmetry occurs at $x = \frac{1+a}{2} = 3 \rightarrow a = 5$

Substituting, $7 = (3 - 1)(3 - 5) + b \rightarrow 7 = -4 + b \rightarrow b = 11$

Thus $(a, b) =$ **<u>(5, 11)</u>**.

B) The sum of the roots is b/a. To maximize the value of this fraction you need to maximize the numerator and minimize the denominator. Thus, b = L, a = S and c = M $\rightarrow$ **<u>b) (S, L, M)</u>**

C) Assume the wire is x feet long. Thus, the weight per foot is $\frac{24}{x}$ and

$\frac{24}{x+1} = \frac{24}{x} - \frac{1}{10} \rightarrow 240x = 240(x + 1) - x(x + 1) \rightarrow 0 = 240 - x^2 - x$

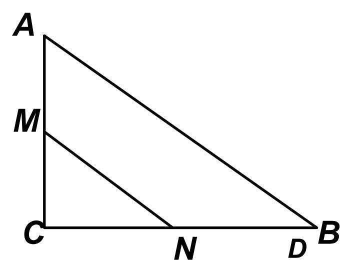
$\rightarrow x^2 + x - 240 = (x + 16)(x - 15) = 0 \rightarrow x = 15$

Therefore, 15 feet of wire weighs 24 ounces $\rightarrow$ 2/15 oz per inch.

Wire needed is $5(4) + 4(5) + 3 = 43$ inches $\rightarrow 86/15 =$ $\underline{\mathbf{5\frac{11}{15}}}$ or **<u>5.73</u>**

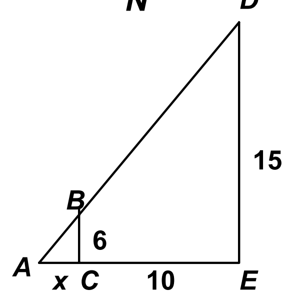
Round 5

A) The area of the 3-4-5 triangle is 6 units^2. The line connecting the midpoints of the legs is parallel to the hypotenuse and cuts off a triangle similar to the original 3-4-5. Since the ratio of their corresponding sides is 1 : 2, their areas are in a ratio of 1 : 4. Since the area of ΔMNC is ¼ the area of ΔABC, the area of the trapezoid is ¾ the area of ΔABC = **<u>4.5</u>**.



B) Since $\Delta ABC \sim \Delta ADE$, $\frac{x}{x+10} = \frac{6}{15} = \frac{2}{5} \rightarrow 5x = 2x + 20$

$\rightarrow 3x = 20 \rightarrow x = \boxed{6\frac{2}{3}}$



C) $\Delta ABC \sim \Delta CAD \rightarrow \frac{AB}{CA} = \frac{AC}{CD} \rightarrow AC^2 = AB(CD) = 12(27) = 18^2 \rightarrow AC = 18$.

Since the ratio of the radii of the inscribed circles is the same as the ratio of the corresponding sides,

$\frac{r_1}{r_2} = \frac{AB}{AC} = \frac{12}{18} = \frac{2}{3} \rightarrow \frac{A_1}{A_2} = \boxed{\frac{4}{9}}$