

**MASSACHUSETTS MATHEMATICS LEAGUE**  
**CONTEST 4 – JANUARY 2007**  
**ROUND 7 TEAM QUESTIONS**

**ANSWERS**

- A) \_\_\_\_\_ D) \_\_\_\_\_  
 B) \_\_\_\_\_ E)  $h =$  \_\_\_\_\_  
 C) \_\_\_\_\_ F) \_\_\_\_\_ years

A) Find the equation of the line through  $P(7, 2)$  perpendicular to the segment connecting the points of intersection of the line  $2x - 5y + 3 = 0$  and the parabola  $y = x^2$ . Express your answer as a simplified equation in the form  $ax + by = c$ , where  $a$ ,  $b$  and  $c$  are integers and  $a > 0$ .

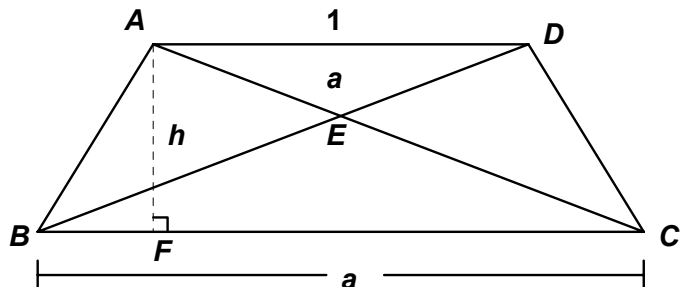
B) Find the sum of all integer values of  $k$  for which the quadratic  $x^2 + kx + k + 11$  can be expressed as the product of two linear binomials with integer coefficients.

C) Find the maximum value of  $k$  for which  $5\sin(x) + 12\cos(x) = k^2 - k + 1$  has a real solution. If necessary, express your answer in terms of a simplified radical.

D) Let  $x = 2 + \frac{1}{a + \frac{1}{2 + \frac{1}{a + \dots}}}$ , where  $a$  is an integer.

Determine the integer value of  $a$  for which  $x$  is a rational number.

E)  $ABCD$  is an isosceles trapezoid. If  $AD = 1$  unit,  $BC = a$  units and the area of  $\triangle ADE$  is  $a$  units<sup>2</sup>, determine the value of  $h$ , the height of the trapezoid, in terms of  $a$ .



F) A man and his grandson have the same birthday. For the grandson's first six birthdays, his grandfather was an integral number of times as old as he was. How old was the grandfather on the grandson's sixth birthday? Assume the grandfather has not yet celebrated his 100<sup>th</sup> birthday.