

MASSACHUSETTS MATHEMATICS LEAGUE

JANUARY 2004

ROUND 7: TEAM QUESTIONS

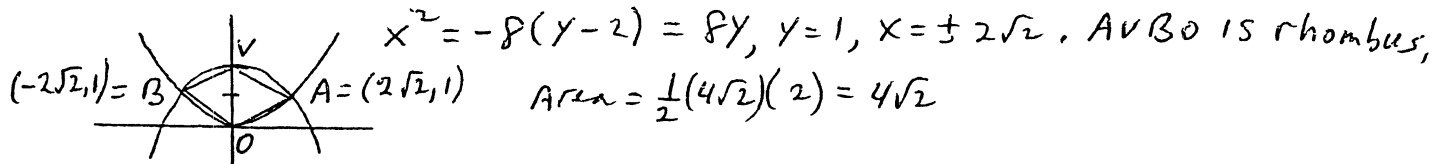
ANSWERS

A) $4\sqrt{2}$ D) ± 2

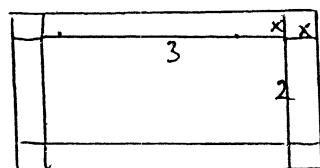
B) 6 E) $1 + \sqrt{5}$

C) $45^\circ, 135^\circ, 225^\circ, 315^\circ$ F) 3

A) A parabola which has vertex V at the focus of the parabola $x^2 = 8y$, and focus at the origin O, intersects $x^2 = 8y$ at points A and B. Calculate the area of quadrilateral AVBO.



B) A two foot by three foot poster is framed with a border of uniform width. If the area of the border is the same as the area of the poster, calculate in inches the width of the border.



$$2(x)(3+2x) + 2(2x) = 6 \quad 2x^2 + 5x - 3 = 0$$

$$6x + 4x^2 + 4x - 6 = 0 \quad (2x-1)(x+3) = 0$$

$$4x^2 + 10x - 6 = 0 \quad x = \frac{1}{2} \text{ ft} = 6 \text{ inches}$$

C) Solve for $0^\circ \leq \theta < 360^\circ$, $\cos 3\theta + \cos \theta = \sqrt{2} \cos 2\theta$

$$2 \cos 2\theta \cos \theta = \sqrt{2} \cos 2\theta$$

$$\cos 2\theta = 0, 2\theta = 90^\circ, 270^\circ, 450^\circ, 630^\circ$$

$$\cos \theta = \sqrt{2}/2 \quad \theta = 45^\circ, 315^\circ$$

Ans $45^\circ, 135^\circ, 225^\circ, 315^\circ$

D) In the equation $ax^2 + bx + c = 0$, a, b, and c are relatively prime integers. If the product of its roots is $-\frac{8}{3}$, and the difference of its roots is $\frac{10}{3}$; calculate two possible values for b.

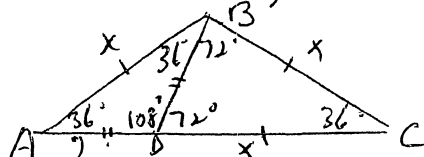
$$rS = -\frac{8}{3} \quad (S + \frac{10}{3})S = -\frac{8}{3}, 3S^2 + 10S + 8 = 0 \quad (3S+4)(S+2) = 0$$

$$r-S = \frac{10}{3} \quad S = -\frac{4}{3}, r = -\frac{4}{3} + \frac{10}{3} = 2; S = -2, r = -2 + \frac{10}{3} = \frac{4}{3}$$

Eq is either $(3x+4)(x-2) = 0$ or $(3x-4)(x+2) = 0$

E) In $\triangle ABC$, $AB = CB$, $\angle B = 108^\circ$, D is on \overline{AC} so that $\angle CBD$ is twice $\angle ABD$. If

DA = $\frac{2}{3}$, calculate CB.



$$\frac{x}{2} = \frac{x+2}{x}, x^2 = 2x + 4$$

$$x^2 - 2x + 1 = 5, (x-1)^2 = 5, x = 1 + \sqrt{5}$$

F) Mary's speed on her bike was 6 mph on the level, 4 mph downhill, and 12 mph uphill. One day it took her one hour to go to Greg's house and return by the same route. How far in miles is it to Greg's house?

$$\frac{d_1}{6} + \frac{d_2}{4} + \frac{d_3}{12} + \frac{d_1}{6} + \frac{d_2}{4} + \frac{d_3}{12} = 1, 3d_1 + 2d_2 + d_3 + d_1 + 2d_2 + d_3 = 12$$