

MASSACHUSETTS MATHEMATICS LEAGUE  
OCTOBER 2003  
ROUND 1 - VOLUMES & SURFACES

ANSWERS

A)  $132\pi$

B)  $30\pi$

C)  $2064\pi$

A) A right circular cone of radius 4 and height 8 is cut by a plane parallel to its base and 4 units out of its center. Find the total surface area of solid remaining leaving the result in terms of  $\pi$ .



$$2\pi \cdot 4 \cdot 4 = 32\pi \quad \text{Surface area of top cone}$$

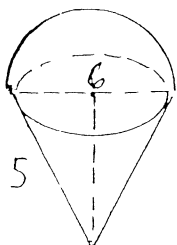
$$2\pi \cdot 4 \cdot 4 = 32\pi \quad \text{Surface area of bottom cone}$$

$$10\pi + 12\pi = 22\pi$$

$$2\pi R^2 = 2\pi \cdot 4^2 = 32\pi \quad \text{Total surface area} = 22\pi + 32\pi = 54\pi$$

Ans:  $108\pi + 24\pi = 132\pi$

B) A right circular cone (apex down) as shown has a slant height of 5 cm and a base diameter of 6 cm. A hemisphere is sitting on top of the cone. Find the volume in terms of  $\pi$  of the solid formed by the cone and the hemisphere.



$$V_S = \frac{4}{3}\pi r^3 \quad V_C = \frac{1}{3}\pi r^2 h$$

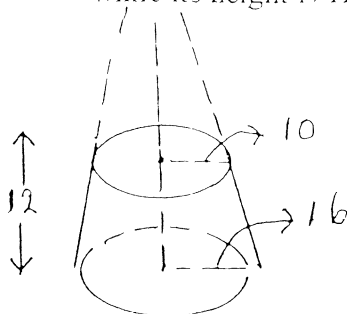
$$V_H = \frac{1}{3}\pi \cdot 3^2 \cdot 4 = 12\pi$$

$$r = 3, h = \sqrt{5^2 - 3^2} = 4$$

$$V_S = \frac{4}{3}\pi \cdot 3^3 = 36\pi$$

Ans:  $30\pi$

C) The truncated cone shown was formed by cutting off the top of a right circular cone with a plane parallel to its base. The radii of the bases of the truncated cone are 10 cm and 16 cm while its height is 12 cm. Calculate the volume of truncated cone in terms of  $\pi$ .



$$V = \frac{1}{3}\pi h (r_1^2 + r_1 r_2 + r_2^2)$$

$$= \frac{1}{3} 12\pi (100 + 160 + 256) = 4\pi \cdot 516$$

$$= 2064\pi$$