

**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2015 SOLUTION KEY**

Team Round - continued

F) Case 1: 16 is the mean

$$16 = \frac{5a - 4 + a^2}{2} \Rightarrow a^2 + 5a - 36 = 0 \Rightarrow (a + 9)(\cancel{a - 4}) = 0 \Rightarrow \boxed{a = -9}$$

(If $a = 4$, then all three numbers are the same.)

Case 2: $5a - 4$ is the mean

$$5a - 4 = \frac{16 + a^2}{2} \Rightarrow a^2 - 10a + 24 = 0 \Rightarrow (a - 6)(\cancel{a - 4}) = 0 \Rightarrow \boxed{a = 6}$$

Case 3: a^2 is the mean

$$a^2 = \frac{16 + 5a - 4}{2} \Rightarrow 2a^2 - 5a - 12 = 0 \Rightarrow (2a + 3)(\cancel{a - 4}) = 0 \Rightarrow \boxed{a = -\frac{3}{2}}$$

$$\text{The mean is } \frac{-9 + 6 + \left(-\frac{3}{2}\right)}{3} \cdot \frac{2}{2} = \frac{-6 - 3}{6} = \underline{\underline{-\frac{3}{2}}}.$$

Round 1 Question C

Since the midpoint of \overline{PR} is $(-2, 2)$ and the perpendicular is vertical, we have $h = -2$ and the center O is at $(-2, k)$. Since $OR = OQ = r$, we can use the distance formula to find k , and then substitute to find r . The details are left to you.