MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2016 SOLUTION KEY

Round 3

A) Solution #1: (Brute Force)

$n_1+n_2+n_3+n_4+n_5 = x+(x+1)+(x+2)+\boxed{(x+3)}+(x+4) = 5x+10 = 95 \Rightarrow x = 17$

$\Rightarrow n_4 = x+3 = \underline{\mathbf{20}}$.

Solution #2: (a Little Finesse)

The middle integer $n_3 = (x+2)$ is the average of the 5 integers, namely $\frac{95}{5} = 19$.

Therefore, $n_4 = n_3 + 1 = x+3 = \underline{\mathbf{20}}$.

B) 15 minutes = $\frac{1}{4}$ hour $\Rightarrow 40t = 50\left(t-\frac{1}{4}\right) \Leftrightarrow 160t = 200t - 50 \Rightarrow t = \frac{5}{4}$

Thus, my destination is 50 miles away.

40 minutes = $\frac{2}{3}$ hour $\Rightarrow r\left(\frac{2}{3}\right) = 50 \Leftrightarrow r = \underline{\mathbf{75}}$ mph.

C) $\frac{27+(50-k)}{(27+x)+50} = \frac{4}{5} \Leftrightarrow \frac{77-k}{77+x} = \frac{4}{5} \Rightarrow 4x+5k = 77$

$\Rightarrow x = \frac{77-5k}{4} = \frac{76-4k}{4} + \frac{1-k}{4} = 19-k+\frac{1-k}{4}$

Clearly, to insure that x and k are integers, k must increase by 4.
However, this decreases the value of x.
Thus, $(x,k) = \underline{(\mathbf{18,1})}$.