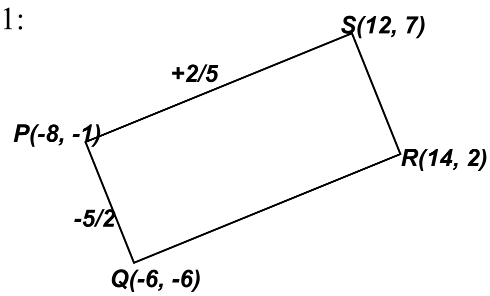
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2010 SOLUTION KEY

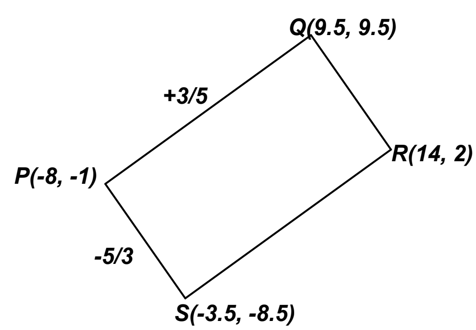
B) You need only consider the rightmost three digits in each case.
$536(536) > 535(537) > 534(538) > 533(539)$
We know this because $535(537) = (536 - 1)(536 + 1) = 536^2 - 1$,
$534(538) = (536 - 2)(536 + 2) = 536^2 - 4$ and $533(539) = (536 - 3)(536 + 3) = 536^2 - 9$
Thus, without multiplying out any of these products, the order from largest to smallest is **CADB**.

C) In rectangle $PQRS$, $\overline{PQ} \perp \overline{QR}$. Thus, the slopes of these segments are negative reciprocals of each other and it follows that product of the slopes will be -1.
$\left(\frac{k+1}{k+8}\right)\left(\frac{k-2}{k-14}\right) = -1 \rightarrow (k + 1)(k - 2) = -(k + 8)(k - 14) \rightarrow k^2 - k - 2 = -k^2 + 6k + 112$
$\rightarrow 2k^2 - 7k - 114 = 0 \rightarrow (2k - 19)(k + 6) = 0 \rightarrow k = 19/2$ or -6
$Q(-6, -6)$ to $P(-8, -1) = 2$ left 5 up
Starting at $R(14, 2)$ and moving 2 left and 5 up, we arrive at S**(12, 7)**
$Q(19/2, 19/2)$ to $P(-8, 1) = 17.5$ left 10.5 down
Starting at $R(14, 2)$ and moving 17.5 left and 10.5 down, we arrive at S**(−3.5, −8.5)**

Case 1:



Case 2:



D) $2y = 5^x - 5^{-x} \rightarrow 2y = 5^x - \frac{1}{5^x} \rightarrow 2y = \frac{(5^x)^2 - 1}{5^x} = \frac{5^{2x} - 1}{5^x} \rightarrow 5^{2x} - (2y)5^x - 1 = 0$
Applying the quadratic formula (treating 5^x as the variable and 1, $(-2y)$ and -1 as the coefficients)
$5^x = \frac{2y \pm \sqrt{4y^2 + 4}}{2} \rightarrow 5^x = y + \sqrt{y^2 + 1}$ $\qquad$ ($y - \sqrt{y^2 + 1} < 0$ and is extraneous)
$x = \underline{\mathbf{\log_5\left(y + \sqrt{y^2 + 1}\right)}}$