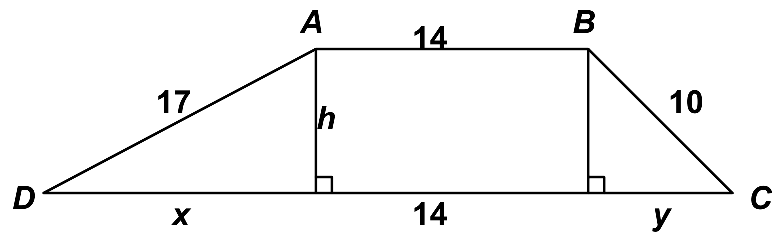
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2008 SOLUTION KEY

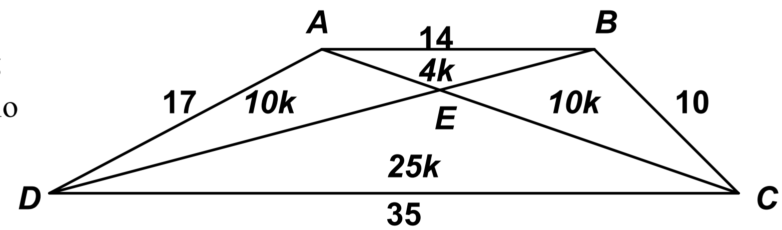
Team Round - continued



E) $\begin{cases} h^2 = 17^2 - x^2 = 10^2 - y^2 \\ x + y + 14 = 35 \end{cases} \rightarrow \begin{cases} x^2 - y^2 = 189 \\ x + y = 21 \end{cases}$

$\rightarrow 21(x - y) = 189 \rightarrow x - y = 9$
Adding, $2x = 30 \rightarrow (x, y) = (15, 6)$
$\rightarrow h = 8$

Thus, Area($ABCD$) = $\frac{1}{2} \cdot 8 \cdot (14 + 35) = 196$



Now $\Delta ABE \sim \Delta CDE$ with sides in a 14 : 35 or 2 : 5 ratio $\rightarrow$ their areas are in a 4 : 25 ratio

Triangles ADE and ABE with the same altitude from A and bases (BE and DE) are in a 2 : 5 ratio must have areas in a 2 : 5 ratio.

A similar argument demonstrates that, although ΔBEC is not congruent to ΔAED, they do have the same area. Thus, the 4 triangles comprising the trapezoid have areas as indicated above.
$49k = 196 \rightarrow k = 4 \rightarrow$ area(ΔBEC) = **<u>40</u>**

F) Let $(a, b) = (x, x + 1)$. Then $x = \frac{20 - c}{2}$ and c must be even (and between 2 and 18 inclusive) to insure that a, b and c are all positive integers.
$\rightarrow (c, a, b) = (2, 9, 10), (4, 8, 9), (6, 7, 8), (8, 6, 7), (10, 5, 6), (12, 4, 5), (14, 3, 4), (16, 2, 3), (18, 1, 2)$
$\rightarrow abc =$ 180, 288, 336, ~~336~~, 300, 240, 168, 96, 36
$\rightarrow$ sum = **<u>1644</u>**