MASSACHUSETTS MATHEMATICS LEAGUE CONTEST 5 - FEBRUARY 2016 ROUND 6 ALG 2: SEQUENCES AND SERIES

ANSWERS

- A) Each term in a sequence (except the first) has a value defined in terms of the previous term. Specifically, the rule is $a_n = a_{n-1}(a_{n-1}+1)$. If $a_1 = 1$, how many terms in this sequence have a value less than 100?
- B) There are two different arithmetic progressions of three numbers a_1 , a_2 , a_3 , each of which has the following properties:
 - $a_1 + a_2 + a_3 = 30$.
 - If 2 is subtracted from the first number, 4 from the second number and 5 from the third number, a geometric progression results.

Compute the ordered triple (a_1, a_2, a_3) , where a_1 is as small as possible.

C) Given: square ABCD

A set of nested squares is drawn inside square ABCD, where AB = 100 cm. The vertices of each nested square are midpoints of the sides of the preceding square, and circle O is inscribed in one of the right triangles formed by the 4^{th} and 5^{th} squares as shown. Compute the radius of the circle inscribed in one of the right triangles formed by the 8^{th} and 9^{th} squares.

