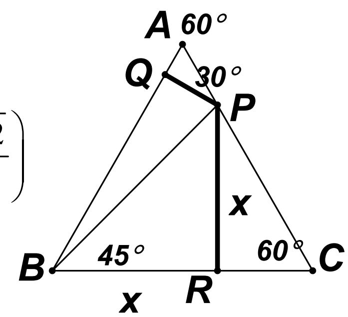
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2006 SOLUTION KEY

Team Round – continued



E) Method 1: Let $BR = PR = x$. Then $RC = \dfrac{x}{\sqrt{3}} \rightarrow PC = \dfrac{2x}{\sqrt{3}} \rightarrow AP = 6 - \dfrac{2x}{\sqrt{3}}$

$\rightarrow PQ = \dfrac{1}{2}\left(6 - \dfrac{2x}{\sqrt{3}}\right)\sqrt{3} = 3\sqrt{3} - x$ Thus, $PQ + PR = 3\sqrt{3} - x + x = \underline{\mathbf{3\sqrt{3}}}$

Method 2: Using the law of sine on ΔBQP, $PQ = x\sqrt{2}\sin 15^\circ = x\sqrt{2}\left(\dfrac{\sqrt{6}-\sqrt{2}}{4}\right)$

$= \left(\dfrac{\sqrt{3}-1}{2}\right)x \rightarrow PQ + PR = \left(\dfrac{\sqrt{3}+1}{2}\right)x$.

$x + \dfrac{x}{\sqrt{3}} = 6 \rightarrow x = 3(3-\sqrt{3}) \quad \rightarrow \left(\dfrac{\sqrt{3}+1}{2}\right) \cdot 3(3-\sqrt{3}) = \dfrac{3}{2}(\sqrt{3}+1)(3-\sqrt{3}) = \underline{\mathbf{3\sqrt{3}}}$

Method 3: Using the theorem

"From any point on or in the interior of an equilateral triangle, **the sum of the altitudes to each side is equal to the length of an altitude of the equilateral triangle**."

$PQ + PR = \dfrac{1}{2} \cdot 6 \cdot \sqrt{3} = \underline{\mathbf{3\sqrt{3}}}$