MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2016 SOLUTION KEY

Round 4

A) $A=\dfrac{995+(-95)}{2}=450$. Solving for x, $7x-6=1800 \Rightarrow x=\dfrac{1806}{7}=\underline{\mathbf{258}}$.

B) The distance is irrelevant. Call the run-leg $8x$, walk-leg x, and the jog-leg $3x$, implying the total distance is $12x$. Then:

$$D=RT \Leftrightarrow R=\frac{D}{T} \Rightarrow R=\frac{12x}{\frac{8x}{12}+\frac{x}{3}+\frac{3x}{6}}=\frac{12}{\frac{2}{3}+\frac{1}{3}+\frac{1}{2}}=\frac{12}{\frac{3}{2}}=\underline{\mathbf{8}}.$$

C) The path taken will travel up, right, down and left and repeat until all 100 cells are numbered. The chart on the left keeps track of the number of cells in each "leg" of the spiral, the cell number and location of the last cell in each leg. For a relatively small grid, the table on the right is sufficient for numbering the cells in a spiraling pattern. Determining (R,C) of the terminal cell is a matter of reading the row at the left and the column at the top. For a larger grid, where numbering would be *beyond* tedious, the chart on the left hints at the pattern which would lead to a generalization, namely a 4-cycle, with *pairs* of decreasing leg lengths, after the initial leg length which equals the size of the square grid. If the last leg is only partially traversed before the terminal cell is reached, the pattern of doubles will be broken, as is the case in this example**.

Leg Length	Cell #	Row	Column	Direction
10	10	10	1	Right
9	19	10	10	**Down**
9	28	1	10	**Left**
8	36	1	2	**Up**
8	44	9	2	**Right**
7	51	9	9	Down
7	58	2	9	Left
6	64	2	3	Up
3**	67	**5**	**3**	

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
10	10	11	12	13	14	15	16	17	18	19
9	9	44	45	46	47	48	49	50	51	20
8	8	43	70	71	72	73	74	75	52	21
7	7	42	69	88	89	90	91	76	53	22
6	6	41	68	87	98	99	92	77	54	23
5	5	40	67	86	97	100	93	78	55	24
4	4	39	66	85	96	95	94	79	56	25
3	3	38	65	84	83	82	81	80	57	26
2	2	37	64	63	62	61	60	59	58	27
1	1	36	35	34	33	32	31	30	29	28

Thus, $(R,C)=\underline{(\mathbf{5,3})}$.