MASSACHUSETTS MATHEMATICS LEAGUE CONTEST 5 - FEBRUARY 2015 SOLUTION KEY

Round 4

A) The customer must pay 70% of the first discounted price which is 60% of the original price.

$$.7(.6x) = 63 \Rightarrow .42x = 63 \Rightarrow x = \frac{63}{.42} = \frac{6300}{42} = \frac{900}{6} = \underline{150}.$$

B) Applying $R \cdot T = D$ and $R = \frac{D}{T}$, we have $\frac{\text{miles}}{\text{hours}} = \frac{70(2) + 60(3.25)}{2 + 3.25 + \frac{k}{60}} = 50 \Leftrightarrow \frac{335}{5.25 + \frac{k}{60}} = 50$

$$\Leftrightarrow \frac{134}{21 + \frac{k}{15}} = 5 \% \Leftrightarrow 134 = 5 \cdot 21 + \frac{k}{3} \Leftrightarrow 29 = \frac{k}{3} \Leftrightarrow k = \underline{87} \text{ minutes.}$$

C) Let (x, y) denote cups of (41% butterfat, 5% butterfat) required by the recipe. Then:

$$\begin{cases} x + y = 3 \\ .41x + .05y = .19(3) \end{cases} \Rightarrow y = 3 - x \Rightarrow 41x + 5(3 - x) = 57 \Rightarrow 36x = 42 \Rightarrow x = \frac{7}{6} \text{ cups.}$$