MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2013 SOLUTION KEY

Round 6

A) A regular polygon with n sides (sometimes referred to as an n-gon) has $\frac{n(n-3)}{2}$ diagonals and the exterior angle contains $\frac{360°}{n}$. Thus, we require that

$$\frac{n(n-3)}{2}=n+88 \Leftrightarrow n^2-5n-176=0$$

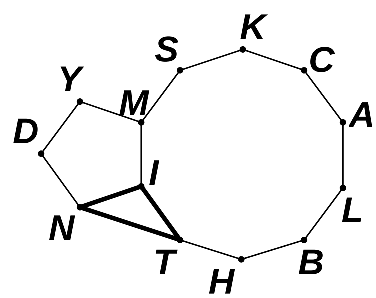
Factoring, we have $(n+11)(n-16)=0 \Rightarrow n=16$ and the exterior angle measure is $\frac{360}{16}=\underline{\mathbf{\frac{45}{2}}°}$ $\left(\underline{\mathbf{22\frac{1}{2}}}°\right.$ or $\left.\underline{\mathbf{22.5°}}\right)$.

B) The angles of a regular pentagon are each $\frac{180(5-2)}{5}=108°$; the angles of a regular decagon are $\frac{180(10-2)}{10}=144°$. Therefore,

$m\measuredangle TIN=360-(m\measuredangle MIN+m\measuredangle MIT)=360-(108+144)=108$.

Since ΔTIN is isosceles, its base angles each measure $\frac{180-108}{2}=\frac{72}{2}=36$.

Thus, the required sequence is **<u>36, 36, 108</u>**.



C) Since $EM=\frac{x}{2}\sqrt{3}$, $AR=2x$, we have

$$\left(x\left(1+\frac{\sqrt{3}}{2}\right)\right)^2+(2x)^2=d^2 \Rightarrow \frac{d^2}{x^2}=4+\left(1+\frac{\sqrt{3}}{2}\right)^2$$

$$=4+1+\sqrt{3}+\frac{3}{4}=\frac{23}{4}+\sqrt{3}=\underline{\mathbf{\frac{23+4\sqrt{3}}{4}}}$$