MASSACHUSETTS MATHEMATICS LEAGUE CONTEST 3 - DECEMBER 2007 SOLUTION KEY

Round 5

A) Let x denote the sales tax (in cents).

$$\frac{20}{1} = \frac{399 - x}{x} \Rightarrow 20x = 399 - x \Rightarrow 21x = 399 \Rightarrow x = 19 \Rightarrow \text{price} = \$3.80$$

B) Let A, B denote the 1st and 2nd number respectively. Then $\begin{cases} A = B + x \\ A + B = y \end{cases}$

$$\Rightarrow$$
 2B + x = y \Rightarrow B = $\frac{y-x}{2}$ and $A = \frac{y-x}{2} + x = \frac{y+x}{2}$

Thus,
$$A : B = (y + x) : (y - x)$$

C) We were given that the pressure varies according to the formula $P = kAv^2$, where k is a proportionality constant to be determined.

Substituting for the first set of conditions,
$$3 = kA18^2 \rightarrow k = \frac{3}{A \cdot 18^2}$$

Substituting for the second set of conditions,
$$P = \frac{3}{A \cdot 18^2} (2A) (30)^2 = \frac{6 \cdot 30^2}{18^2} = \frac{6 \cdot 5^2}{3^2} = \frac{50}{3}$$