MASSACHUSETTS MATHEMATICS LEAGUE
FEBRUARY 2005
ROUND 7: TEAM QUESTIONS

ANSWERS

A)______________ **D)**______________

B)______________ **E)**______________

C)______________ **F)**______________

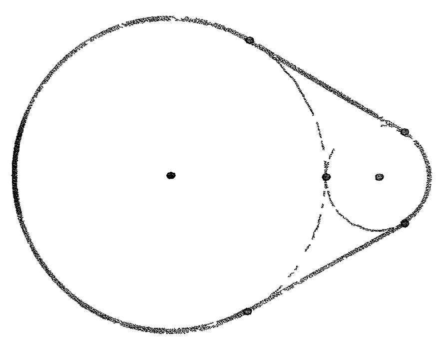
A) Suppose $f(x) = \dfrac{ax+7}{x-1}$ for some constant a, and $g(x) = \dfrac{x+1}{7}$. If one solution for $f^{-1}(x) = g^{-1}(x)$ is $x = 3$ find the exact value of the other solution.

B) Let T = 1 + 3 + 5 + . . + (2k – 1) for some k between 1 and 99 inclusive
Let S = 2 + 4 + 6 + … + (2j) for some j between 1 and 99 inclusive
If k and j are distinct integers find the maximum value of k+j for which S–T is a multiple of 25.

C) If $Tan^{-1}(A) + Tan^{-1}(\frac{4}{A}) = Tan(\frac{-5}{3}) = n\pi$ for the function Tan^{-1}, real A, and integer n. find all possible ordered pairs (A, n).

D) On her road trip Sue drove part at 60 mph, part at 45 mph, and the rest at 36 mph. She spent twice as much time at 45 mph as at 60 mph but drove twice as far at 36 mph as she did at 60 mph. Her average speed for the trip was A/B mph where A and B are relatively prime. Find A+B.

E) A tight band is wrapped around two externally tangent circles as shown below. If the smaller circle's radius is one third that of the larger circle and the length of the band is exactly $36 + 14\pi\sqrt{3}$ find the area of the larger circle.



F) A sequence S has the unusual property that any four consecutive terms of the form $s_{2n}, s_{2n+1}, s_{2n+2}, s_{2n+3}$ form an arithmetic progression when n is even but form a geometric progression when n is odd. Although $s_2 = s_4 = -2$, S is <u>not</u> a constant sequence. Find in simplest form $10\,s_0 + s_{14}$