MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2010 SOLUTION KEY

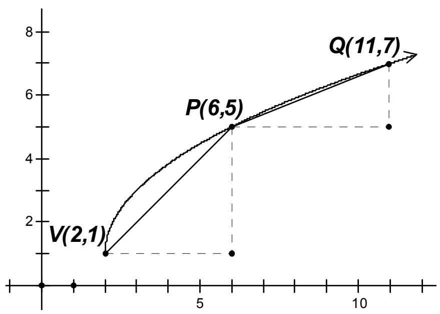
Round 1

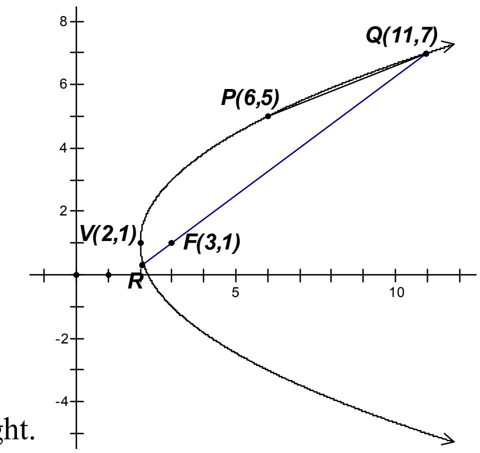
A) Y-intercepts $(x = 0)$: $(y-1)^2 = -1$ ➔ no Y-intercepts
X-intercepts $(y = 0)$: $x^2 - 1 = 1$ ➔ $x = \pm\sqrt{2}$ ➔ $\underline{\mathbf{(\pm\sqrt{2}, 0)}}$

B) Completing the square, $5x^2 + 5y^2 + 15x = 21$ ➔ $5\left(x^2 + 3x + \frac{9}{4}\right) + 5y^2 = 21 + 5\left(\frac{9}{4}\right)$ ➔

$5\left(x + \frac{3}{2}\right)^2 + 5y^2 = 21 + 5\left(\frac{9}{4}\right)$ ➔ Center @ $(-3/2, 0)$

The distance from this point to $2x + 4y + 13 = 0$ can be computed by the point to line distance formula, $r = \dfrac{|2(-3/2) + 4(0) + 13|}{\sqrt{2^2 + 4^2}} = \dfrac{10}{2\sqrt{5}} = \dfrac{5}{\sqrt{5}} = \sqrt{5}$ ➔ $d = \underline{\mathbf{2\sqrt{5}}}$





C) The points $P(6, 5)$ and $Q(11, 7)$ lie on the same side of the axis of symmetry. The parabola must open up <u>or</u> to the right. The slope of $\overline{VP}$ is 1 and the slope of $\overline{PQ}$ is 2/5.

Since the slope is decreasing as we move from left to right, the parabola must open to the right and, therefore has the form $(y-1)^2 = 4p(x-2)$. Substituting $P(6, 5)$, we have $(5-1)^2 = 4p(6-2)$ ➔ $p = 1$. Thus, the focus is at (3, 1) and the slope of $\overleftrightarrow{QR}$ is $\dfrac{7-1}{11-3} = \dfrac{3}{4}$ and the equation of $\overleftrightarrow{QR}$ is $3x - 4y = 5$ or $x = \dfrac{4y+5}{3}$.

Substituting, $(y-1)^2 = 4\left(\dfrac{4y+5}{3} - 2\right)$ ➔ $3(y-1)^2 = 16y - 4$ ➔ $3y^2 - 22y + 7 = 0$

➔ $(3y-1)(y-7) = 0$ ➔ $y = 1/3$ ➔ $(x, y) = \underline{\mathbf{\left(\frac{19}{9}, \frac{1}{3}\right)}}$