

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2008
ROUND 7 TEAM QUESTIONS
ANSWERS

- A) () + () i D) _____
 B) _____ E) _____
 C) _____ F) (,)

A) For some integer k , $\|i + 2i^2 + 3i^3 + 4i^4 + 5i^5 + \dots + (4k+1)i^{4k+1}\| = 29$,

where $\|$ denotes absolute value.

Recall: $\|a + bi\| = \sqrt{a^2 + b^2}$

Σ denotes the sum of the indicated terms

\prod denotes the product of the indicated terms.

Now for the value of k determined above, evaluate $\frac{\sum_{n=0}^{n=k} i^{2^n}}{\prod_{n=0}^{n=k} i^{2^n}}$ in simplified $a + bi$ form

B) Solve: $\frac{2}{1 - \frac{1}{1 - \frac{2}{t}}} \geq t^2 - 4$

C) Find the dimensions of a rectangle with maximum area inscribed in an isosceles triangle with sides of length 13, 13 and 10. One side of this rectangle lies on the base of the isosceles triangle.

D) Solve for all real values of x for which $12x(x-1) = 5 + \sqrt{(4x-7)(3x-1)} + 13x$.

E) $\triangle ABC$ is inscribed in circle Q . $AC = 8$ and $BC = 6$.
 Let the circle of maximum area with center S be inscribed in region #1 and the circle of maximum area with center T be inscribed in region #2.
 Compute the $\sin(\angle SQT)$.

F) $\overline{AB} \perp \overline{BC}$, $\angle BAC$ is trisected by \overline{AD} and \overline{AE} ,
 $\angle BCF$ is bisected by \overline{CG} and $m\angle ADE : m\angle AEC = a : b$.
 If the $\text{GCF}(a, b) = 1$ and $m\angle GCF = 75$, compute (a, b) .

