MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2008 SOLUTION KEY

Team Round - continued

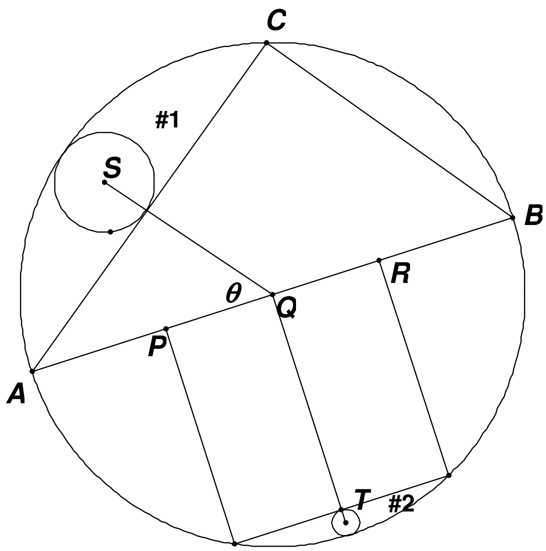
D) The given equation is equivalent to: $12x^2 - 12x = 5 + \sqrt{12x^2 - 12x + 7}$

Substituting $y = \sqrt{12x^2 - 12x + 7}$, we have $y^2 = 12 + y$

$\rightarrow y^2 - y - 12 = (y - 4)(y + 3) = 0 \rightarrow y = 4$ only.

Thus, $12x^2 - 12x + 7 = 16 \rightarrow 12x^2 - 12x - 9 = 3(4x^2 - 4x - 3)$

$= 3(2x + 1)(2x - 3) = 0 \rightarrow x = \underline{\mathbf{-\frac{1}{2}, \frac{3}{2}}}$

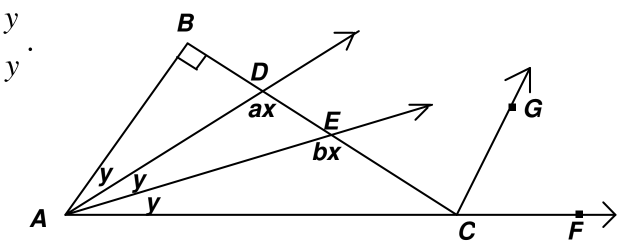


E) Point S lies on the perpendicular bisector of $\overline{AC}$ which will pass through point Q. Point T lies on the perpendicular bisector of the chord parallel to diameter $\overline{AB}$ which will pass through point Q. Let θ denote $\angle SQA$.

Then $\sin(\angle SQT) = \sin(\theta + 90) = \cos(\theta)$.

But $\angle\theta \cong \angle B$ and, therefore, $\cos\theta = \cos B = 6/10 = \underline{\mathbf{3/5}}$.

F) Let $m\angle BAC = 3y, m\angle ADE = 3x$ and $m\angle AEC = 7x$.



Using exterior angles of ΔADE and ΔABD, $\begin{cases} bx = ax + y \\ ax = 90 + y \end{cases}$.

Subtracting, $x = \dfrac{90}{2a - b}$.

Substituting, $y = a\left(\dfrac{90}{2a-b}\right) - 90 = \dfrac{90(b-a)}{2a-b}$

$m\angle BCA = 90 - 3y$ and $m\angle BCF = 90 + 3y$

$\rightarrow m\angle GCF = 45 + \dfrac{3y}{2} = 45 + \dfrac{3}{2}\left(\dfrac{90(b-a)}{2a-b}\right) = 45 + \left(\dfrac{135(b-a)}{2a-b}\right) = 75$

$\rightarrow$ **135*b* – 135*a* = 60*a* – 30*b*** $\rightarrow$ **165*b* = 195*a*** $\rightarrow$ **11*b* = 13*a*** $\rightarrow$ **(*a*, *b*) =** <u>(11, 13)</u>