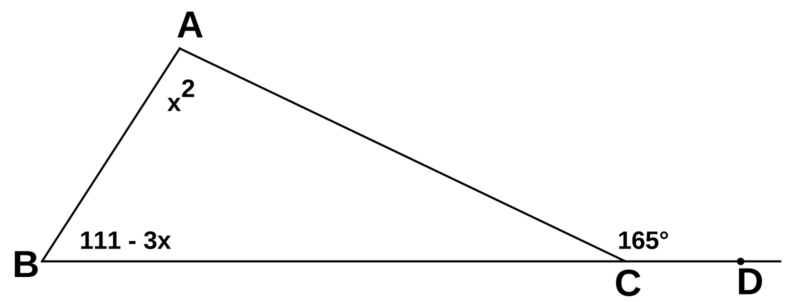
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2015 SOLUTION KEY

Round 6

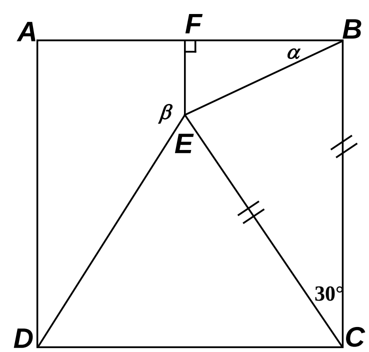
A)



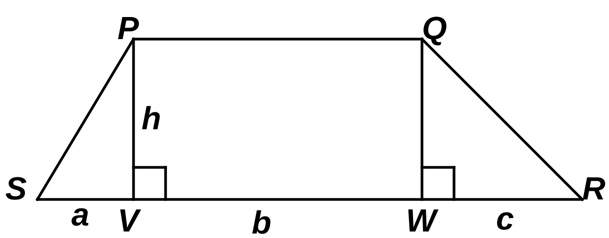
Since the measure of any exterior angle of a triangle is equal to the measure of the sum of the two interior angles, we have $x^2+(111-3x)=165 \Rightarrow x^2-3x-54=(x-9)(x+6)=0 \Rightarrow x=9,-6$

$x=9$ produces angles of 81, 84 and 15, but $x=-6$ produces angles of 36, 129 and 15.
Thus, the largest possible degree-measure is **<u>129</u>**.

B) Since $EC=CD$ and $BC=CD$, by transitivity, $BC=EC$ and ΔBEC is isosceles. $m\angle CBE=m\angle CEB=75° \Rightarrow \alpha=15°$
In trapezoid , $\beta=(360-2\cdot 90°-30°)=150°$.
Therefore, $\alpha+\beta=\underline{\mathbf{165}}°$.



C)



$area(PSV):area(QWR)=\frac{1}{2}ah:\frac{1}{2}ch=2:3 \Rightarrow a:c=2:3$

$\begin{cases} b:c=5:6 \\ a:c=2:3 \end{cases} \Rightarrow a:b:c=4:5:6$

Thus, $a+b+c=4n+5n+6n=60 \Rightarrow n=4 \Rightarrow (a,b,c)=(16,20,24)$

$h:b=9:40 \Rightarrow h=4.5$

Thus, the area of ΔPVR is $\frac{1}{2}4.5(20+24)=4.5(22)=\underline{\mathbf{99}}$.