MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2010 SOLUTION KEY

Round 2

A) Since the digit sum of 7659, namely 27, is divisible by 9, 9 must be a factor of 7659.
Thus, 7659 = 9(851) and we need to determine whether 851 is prime or factors as a product of smaller primes. If any prime factor exists, there must be one smaller than $\sqrt{851}$. Since $30^2 = 900 > 851$, we restrict our search to primes less than or equal to 29.
2 and 5 obviously fail. By long division, 7, 11, 13, 17 and 19 fail, but 851 = 23(37).
Thus, the largest prime factor of 7659 is **<u>37</u>**.

B) Converting from base 7: $2144_{(7)} = 2\left(7^3\right)+1\left(7^2\right)+4\left(7^1\right)+4\left(7^0\right) = 686+49+28+4 = 767_{(10)}$

The **<u>shortcut</u>** looks like synthetic substitution:

$$\begin{array}{r|rrrr} & 2 & 1 & 4 & 4 \\ & & 14 & 105 & 763 \\ \hline 7 & 2 & 15 & 109 & \mathbf{767} \end{array}$$

Converting to base 4: The digit values in base 4 are $\underset{1024}{\underline{4^5}}\ \underset{256}{\underline{4^4}}\ \underset{64}{\underline{4^3}}\ \underset{16}{\underline{4^2}}\ \underset{4}{\underline{4^1}}\ \underset{1}{\underline{4^0}}$

Since 767 < 1024, only the five rightmost positions will be filled with a digit.

$\dfrac{767}{256} \to \mathbf{2}\ r255$ or 767 – 2(256) = 767 – 512 = 255. The leftmost digit must be 2.

Continuing, $\dfrac{255}{64} \to \mathbf{3}\ r63\ \ \dfrac{63}{16} \to \mathbf{3}\ r15\ \ \dfrac{15}{4} \to \mathbf{3}\ r\mathbf{3}$, we see the remaining digits are 3333.

The **<u>shortcut</u>** looks like long division, recording quotients and *remainders*.
Eventually, the divisor (in this case 4) will be larger than the dividend, the quotient will be zero and we will have the last remainder.
The remainders are the digits and they are read from the bottom up.

$$\begin{array}{l} 4\underline{|767} \to r_1 = 3 \\ 4\underline{|191} \to r_2 = 3 \\ 4\underline{|47} \to r_3 = 3 \\ 4\underline{|11} \to r_4 = 3 \\ 4\underline{|\ 2} \to r_5 = 2 \\ \quad 0 \end{array}$$

Reading up, we have **<u>$23333_{(4)}$</u>**.

C) $42875 = 25(1715) = 125(343) = 5^3 \cdot 7^3$ or $35^3 \cdot 1^3$
Thus, $n = 3$ and $(a + b, a - b) = (7, 5)$ or $(35, 1)$
$(7, 5) \to (a, b) = (6, 1)$
$(35, 1) \to (a, b) = (18, 17)$
The maximum value of $abn = (18)(17)(3) = 18(51) =$ **<u>918</u>**

$$\begin{array}{l} 5\underline{|42875} \\ 5\underline{|8574} \\ 5\underline{|1715} \\ 7\underline{|343} \Rightarrow 5^3 \cdot 7^3 \\ 7\underline{|49} \\ \quad 7 \end{array}$$

Created with nitroPDF professional
download the free trial online at nitropdf.com/professional