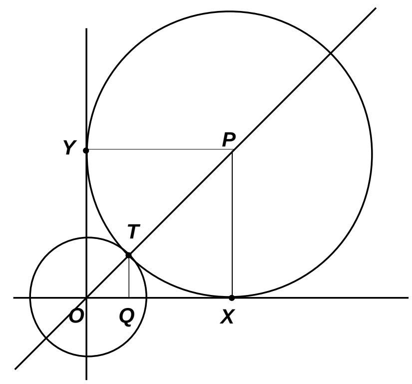
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2012 SOLUTION KEY

Round 3

A) Substituting the coordinates of the point of intersection into both equations,

$$\begin{cases} k = 6m+1 \\ k = \frac{2}{5}\cdot 6 - m \end{cases} \Rightarrow 6m+1 = \frac{12}{5} - m \Rightarrow 7m = \frac{7}{5} \Rightarrow m = \frac{1}{5} \text{ and } k = \underline{\mathbf{\frac{11}{5}}}$$



B) Note that m$\angle POX$ = 45°, so *OXPY* is a square and $a = b$.
$OT = 6 \Rightarrow OQ = 3\sqrt{2}$
Let $PX = PY = r$

$$\Delta TOQ \sim \Delta POX \Leftrightarrow \frac{TO}{PO} = \frac{QO}{XO} \Leftrightarrow$$

$$\frac{6}{r+6} = \frac{3\sqrt{2}}{r} \Rightarrow 2r = r\sqrt{2} + 6\sqrt{2} \Rightarrow$$

$$r = \frac{6\sqrt{2}}{2-\sqrt{2}} = \frac{6\sqrt{2}\left(2+\sqrt{2}\right)}{4-2} = 3\sqrt{2}\left(2+\sqrt{2}\right) = 6\sqrt{2}+6 = \underline{\mathbf{6\left(\sqrt{2}+1\right)}}$$

Alternate solutions:

Consider isosceles right ΔPOX. $OT = 6 \Rightarrow a\sqrt{2} = a + 6 \Rightarrow a = \dfrac{6}{\sqrt{2}-1}$ and the same result follows or, using the P.T. and the quadratic formula, $a^2 + a^2 = (a+6)^2 \Rightarrow a^2 - 12a - 36 = 0$

$$\Rightarrow \frac{12 \pm \sqrt{144 - 4(-36)(-1)}}{2} = \frac{12 \pm \sqrt{144(2)}}{2} = 6 + 6\sqrt{2} \quad \left(6 - 6\sqrt{2} < 0,\ \text{rejected}\right)$$

C) $H = PQ = 4r$. Since *OQ* is the circumference of a base of a cylinder, $OQ = 2\pi r$ and $OP^2 = (2\pi r)^2 + \left(4r\right)^2 = 4r^2\left(\pi^2+4\right)$.

Thus, $OP = 2r\sqrt{\pi^2+4} \Rightarrow \left(A,B\right) = (1,4)$

$$\frac{H^2}{Cr} = 2r \Leftrightarrow \frac{\left(4r\right)^2}{Cr} = 2r \Leftrightarrow 2Cr^2 = 16r^2 \Rightarrow C = 8$$

$(A, B, C) = \underline{\mathbf{(1, 4, 8)}}$.