MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2009
ROUND 7 TEAM QUESTIONS
ANSWERS

A) _____ D) _____

B) _____ E) _____

C) _____ F) _____

******* NO CALCULATORS ON THIS ROUND *******

A) Find all possible ordered pairs (x, y) of real numbers that satisfy

$$x^2 + xy + y^2 = 8 \text{ and } x + y = xy + 2$$

given that x and y have the same sign.

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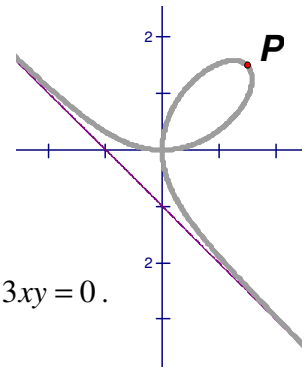
B) Given: N, A and B are positive integers

Determine all values of N for which $\sqrt{N - 24\sqrt{2}} = A - B\sqrt{2}$.



C) The graph at the right is called the Folium of Descartes. Its equation is $x^3 + y^3 - 3xy = 0$.
It is almost indistinguishable from the line $\mathscr{L}$: $x + y + 1 = 0$
as x gets arbitrarily large or arbitrarily small. The line $\mathscr{L}$ is called an asymptote.
If P is the point on the folium farthest from $\mathscr{L}$, compute this distance from P to $\mathscr{L}$.

D) Players in consideration for the baseball Hall of Fame are (O) outfielder Jim Rice, (I) infielder Alan Trammell and (P) pitcher Bert Blyleven. Voters received these two instructions:

"You may vote for the outfielder provided you do not vote for the pitcher."
"You may vote for the pitcher provided you do not vote for the outfielder."

(This is equivalent to "You may vote for the outfielder if and only if you do not vote for the pitcher.")
The results of fan voting were as follows:
500 fans voted (each for at least one these players)
78 voted the infielder and the outfielder (They may or may not have voted for the pitcher.)
80 voted for only the outfielder
320 voted for the infielder or the outfielder, but not the pitcher
90 voted for the infielder and the pitcher (They may or may not have voted for the outfielder.)
96 voted for the outfielder and the pitcher (They may or may not have voted for the infielder.)
30 voted for only the pitcher
How many fans followed the voting instructions?

E) Definition: In a semi-golden rectangle, the ratio of the length of a short side to the length of a long side is the same as the ratio of the length of a long side to the sum of the lengths of a short side and a diagonal.
Find the area of a semi-golden rectangle if the length of a diagonal is $10\sqrt{6}$.

F) Martha makes ¾ of her free throws. The success or failure of making a free throw is independent from one shot to the next. Martha shoots until she hits three free throws or misses two. Find the probability of making three free throws before missing two.