

**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2012 SOLUTION KEY**

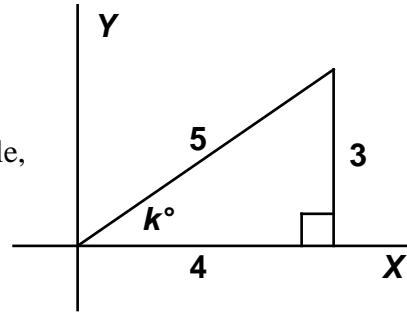
Round 3

A) Think 3 – 4 – 5 right triangle!

$\text{Arc tan}(0.75)$ is the measure of the smaller acute angle,

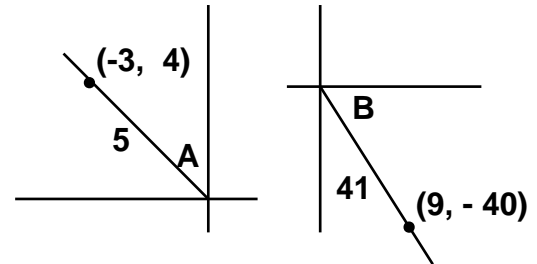
namely k° , since $\tan(k^\circ) = \frac{y}{x} = \frac{OPP.}{ADJ.} = 0.75 = \frac{3}{4}$.

$\sin(k^\circ) = \frac{3}{5}$ (or **0.6**) Recall: SOH-CAH-TOA



B) $\cos(A - B) = \cos A \cos B + \sin A \sin B =$

$$\left(-\frac{3}{5}\right)\left(\frac{9}{41}\right) + \left(\frac{4}{5}\right)\left(-\frac{40}{41}\right) = \frac{-27 - 160}{205} = -\frac{187}{205}$$



C) $\left(2 \sin 7\frac{1}{2}^\circ \cos 7\frac{1}{2}^\circ\right) \left(1 - 2 \sin^2 37\frac{1}{2}^\circ\right) = \sin 15^\circ \cdot \cos 75^\circ =$

$$\begin{aligned} \sin^2 15^\circ &= (\sin(45 - 30))^\circ = (\sin 45 \cos 30 - \sin 30 \cos 45)^\circ = \left(\frac{\sqrt{6} - \sqrt{2}}{4}\right)^2 \\ &= \frac{8 - 2\sqrt{12}}{16} = \frac{8 - 4\sqrt{3}}{16} = \frac{2 - \sqrt{3}}{4} \Rightarrow (A, B, C) = \underline{\underline{(2, 3, 4)}}. \end{aligned}$$

Round 4

A) $\frac{300}{t} + 5 = \frac{320}{t} \Rightarrow 300 + 5t = 320 \Rightarrow t = 4$.

Thus, the faster rate is $\frac{320}{4} = \underline{\underline{80}}$ mph

B) Let D and Q denote the number of dimes and quarters respectively. Then:

$$10D + 25Q = 666 - 156 = 510 \Rightarrow 2D + 5Q = 102$$

$$(D, Q) = (1, 20), (6, 18), \dots, (46, 2), (51, 0)$$

Since there are 3 types of coins in the fountain (51, 0) is rejected and the maximum number of coins in the fountain is $156 + 46 + 2 = \underline{\underline{204}}$.

C) Let X denote the number of days it would take for $(A - C)$ men to complete the job. The time it takes to complete a job is inversely proportional to the size of the works force. The larger

the workforce, the less time it takes to complete the job. Thus, $\frac{A}{A - C} = \frac{X}{B} \Rightarrow X = \frac{AB}{A - C}$

$$\text{Additional days} = x - B = \frac{AB}{A - C} - B = \frac{AB - B(A - C)}{A - C} = \underline{\underline{\frac{BC}{A - C}}}$$