

**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2006 SOLUTION KEY**

Round 5

A) $40x \geq 19x + 25000 \rightarrow 21x \geq 25000 \rightarrow x \geq 1190.47+ \rightarrow x_{\min} = \underline{1191}$

B) $|2x - 1| \leq 23 \rightarrow -23 < 2x - 1 < +23 \rightarrow -11 \leq x \leq +12$
 $|x + 1| > 4 \rightarrow x + 1 < -4 \text{ or } x + 1 > +4 \rightarrow x < -5 \text{ or } x > +3$



Thus, the overlap contains integers from -11 to -6 inclusive as well as integers from 4 to 12 inclusive, a total of $6 + 9 = \underline{15}$ integers.

C) The expression under the square root, i.e. the radicand, must be non-negative.

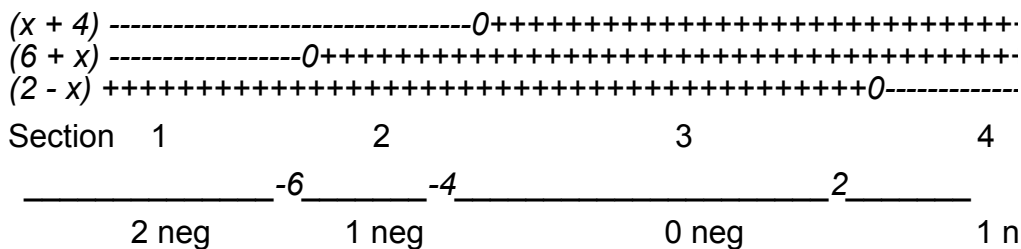
$$\frac{x+4}{12-4x-x^2} = \frac{x+4}{(6+x)(2-x)}$$

The critical values are -4, -6 and +2.

Two factors $(x + 4)$ and $(6 + x)$ are negative for values of x less than the critical value and positive for values of x greater than the critical value.

For $(2 - x)$ the situation is reversed.

The following diagram summarizes this situation:



Thus, in section 1 ($x < -6$) and section 3 ($-4 \leq x < 2$), the quotient is non-negative.

Note: Only -4 is included, since the other critical values would cause division by zero.