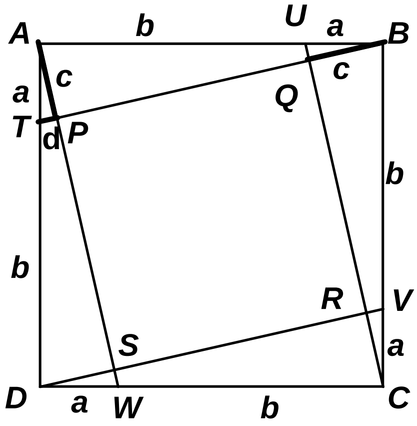
# MASSACHUSETTS MATHEMATICS LEAGUE
# CONTEST 4 - JANUARY 2015 SOLUTION KEY

**Team Round - continued**




E) $\Delta TAP \sim \Delta WAD \Rightarrow \quad \dfrac{short}{hypot} = \dfrac{TP}{DW} = \dfrac{d}{a} = \dfrac{a}{\sqrt{(a+b)^2 + a^2}}$

$$\Rightarrow d = \frac{a^2}{\sqrt{(a+b)^2 + a^2}}$$

Also, $\dfrac{long}{hypot} = \dfrac{AP}{AW} = \dfrac{c}{a} = \dfrac{a+b}{\sqrt{(a+b)^2 + a^2}}$

$$\Rightarrow c = \frac{a(a+b)}{\sqrt{(a+b)^2 + a^2}}$$

Therefore, the side of the square $PS = AW - (c + d)$

$$= \sqrt{(a+b)^2 + a^2} - \left( \frac{a^2}{\sqrt{(a+b)^2 + a^2}} + \frac{a(a+b)}{\sqrt{(a+b)^2 + a^2}} \right)$$

Expressing with a common denominator, this is $\dfrac{\left((a+b)^2 + a^2\right) - a^2 - a(a+b)}{\sqrt{(a+b)^2 + a^2}}$

Expanding and simplifying the numerator, this is $\dfrac{b(a+b)}{\sqrt{(a+b)^2 + a^2}}$

Finally, the required ratio is

$$\frac{area(ABCD)}{area(PQRS)} = \frac{(a+b)^2}{\left( \dfrac{b(a+b)}{\sqrt{(a+b)^2 + a^2}} \right)^2} = \frac{(a+b)^2\left((a+b)^2 + a^2\right)}{b^2(a+b)^2} = \frac{a^2 + (a+b)^2}{b^2} = 1 + \frac{2\left(a^2 + ab\right)}{b^2}$$

$\Rightarrow k = \underline{\boldsymbol{a^2 + ab}}$ or equivalent.