MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2015 SOLUTION KEY

Team Round - continued

E) All multiples of 2, 3, 5 and 7 are crossed out. Since every composite number less than or equal to 100 is divisible by at least one of these numbers, only the primes remain.
$b-a=2 \Rightarrow (3,5),(5,7),(11,13),(17,19),(29,31),(41,43),(59,61),(71,73)$ - 8 Pairs
$b-a=6 \Rightarrow (23,29),(31,37),(47,53),(53,59),(61,67),(73,79),(83,89)$ - 7 pairs
Therefore, $(k, j)=\underline{(\mathbf{8,7})}$.

F) $\frac{n(n-3)}{2} > 1{,}000{,}000 \Rightarrow n(n-3) > 2(10^6)$.
Since $n > n-3$, $n^2 > n(n-3)$. Therefore, $n > \sqrt{2(10^6)} = 10^3\sqrt{2}$.
If we know that $\sqrt{2} \approx 1.414$, our job is a lot easier.
We start with $n = 1415$.
$1415(1412) = 1{,}997{,}980$ and this product is just a little too small.
$1416(1413) = 2{,}000{,}808$ and we have the minimum, n = **<u>1416</u>.**