MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2014 SOLUTION KEY

Round 1

A) $C = 2\pi r = 8\pi \Rightarrow r = 4 \Rightarrow \text{area}_{\text{base}} = 16\pi$

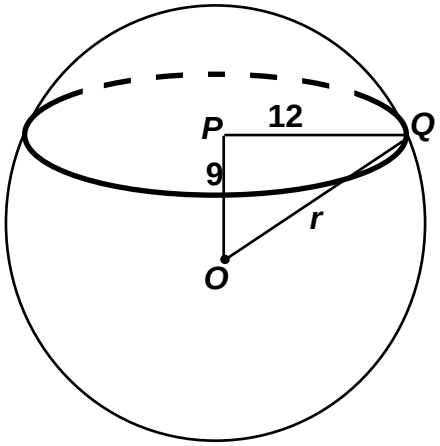
$V = \frac{1}{3}\pi r^2 h \Rightarrow 18\pi = \frac{16}{3}\pi h \Rightarrow h = \frac{54\pi}{16\pi} = \frac{27}{8} \Rightarrow \sqrt[3]{\frac{27}{8}} = \underline{\mathbf{\frac{3}{2}}}$ (or **1.5**)

B) Let r denote the radius of the sphere and O and P be the centers of the sphere and the cross section respectively.

The radius of the cross section is 12. Since $\overline{OP}$ is perpendicular to the cross section, ΔPOQ is a right triangle and $r = OQ = 15$.

The volume of the hemisphere is

$\frac{2}{3}\pi r^3 = \frac{2}{3}\pi(15)^3 = 10(15)^2\pi = \underline{\mathbf{2250\pi}}$



C) The volume of the region "behind" rectangle $PQRS$ is $\frac{1}{2}\cdot 6x\cdot 12 = 36x$. The given ratio is

$\frac{36x}{12^3 - 36x} = \frac{x}{48 - x} = \frac{3}{29} \Rightarrow 29x = 144 - 3x \Rightarrow x = \frac{144}{32} = \frac{9}{2}$

Thus, $\frac{TR}{RV} = \frac{12 - 4.5}{4.5} = \frac{7.5}{4.5} = \frac{5}{3} \Rightarrow (a,b) = \underline{(\mathbf{5,3})}$.