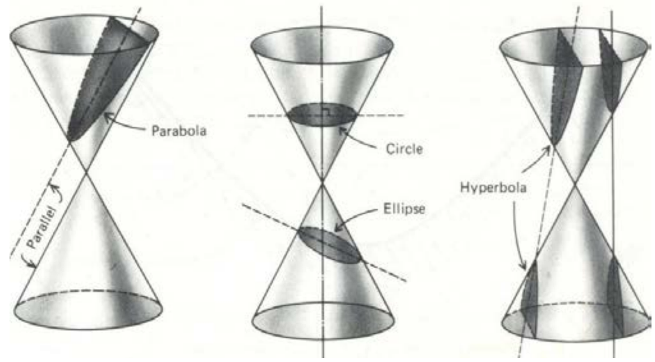
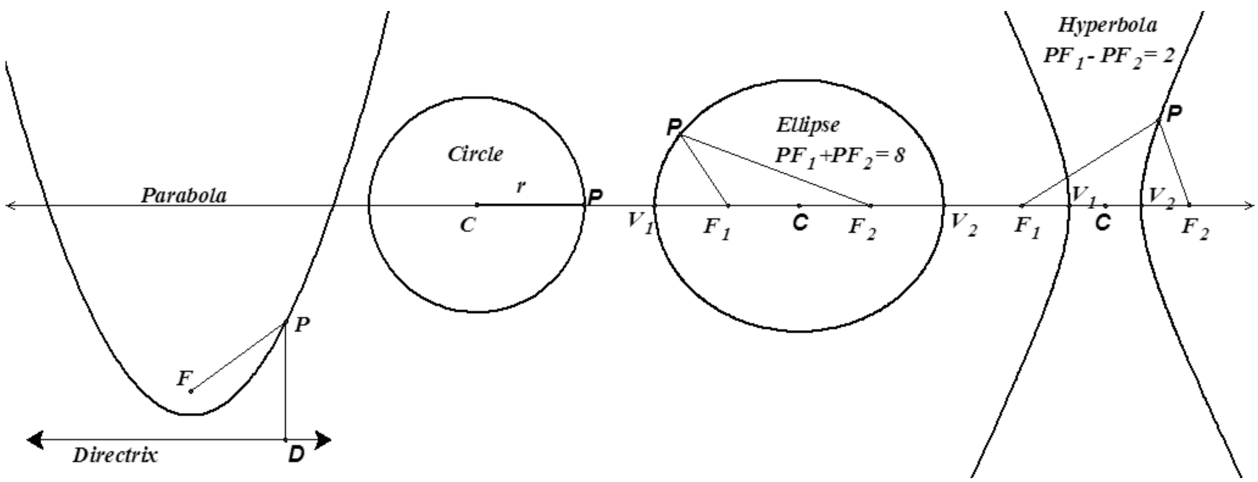
Conic Sections and Eccentricity



The parabola, circle, ellipse and hyperbola are called conic sections because they can all be formed as cross sections of a cone (or pair of cones) as indicated in the diagrams above.

Each is understood to be a set of points in a plane satisfying a certain condition.
The conditions are usually stated as follows:
Parabola - equidistant from a fixed point (the focus) and a fixed line (the directrix)
Circle - at a fixed distance (radius) from a fixed point (center)
Ellipse - the sum of the distances from two fixed points (foci) is constant
Hyperbola – the difference of the distances from two fixed points (foci) is constant.



<u>Eccentricity</u> provides us with a different perspective on these curves and unifies the definitions.
Let F be a fixed point and $\mathscr{L}$ a fixed line. Let $d(P,\mathscr{L})$ denote the distance from point P to line $\mathscr{L}$, measured along a perpendicular drawn from P to $\mathscr{L}$.
Consider the points P in the plane for which $PF = e \cdot d(P,\mathscr{L})$.

$$e=1 \Rightarrow \textit{parabola} \ / \ 0<e<1 \Rightarrow \textit{ellipse} \ / \ e>1 \Rightarrow \textit{hyperbola}$$

The Circle: As e approaches 1, the ellipse becomes more and more elongated; as it approaches zero it becomes more and more circular.
For the ellipse and hyperbola, the distance from the center to a vertex is referred to as <u>a</u> and the distance from the center to a focus as <u>c</u>. For the ellipse and hyperbola, the eccentricity e has a value of $\dfrac{c}{a}$.