

**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2009 SOLUTION KEY**

Round 3

A) $3(2\cos^2 x - 1) = 2\cos^2 x \rightarrow 4\cos^2 x = 3 \rightarrow \cos x = \pm \frac{\sqrt{3}}{2} \rightarrow x = \underline{\underline{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}}$

B) $(\sqrt{2}\cos\theta - \sqrt{2}\sin\theta)^2 = 3 \rightarrow (\cos\theta - \sin\theta)^2 = \frac{3}{2} \rightarrow 1 - 2\sin\theta\cos\theta = \frac{3}{2}$

$$\rightarrow \sin(2\theta) = -0.5 \rightarrow 2\theta = \begin{cases} 210 + 360n \\ 330 + 360n \end{cases} \rightarrow \theta = \begin{cases} 105 + 180n \\ 165 + 180n \end{cases}$$

$$n = 0 \rightarrow \theta = \underline{\underline{105^\circ, 165^\circ}}$$

$$n = 1 \rightarrow \theta = \underline{\underline{285^\circ, 345^\circ}}$$

C) Since $\tan^2\theta + 1 = \sec^2\theta$, the original equation simplifies to $\sqrt{3}\tan^2\theta + \tan\theta = \sqrt{3}\tan\theta + 1$
 $\rightarrow \sqrt{3}\tan^2\theta - \sqrt{3}\tan\theta + \tan\theta - 1 = 0 \rightarrow \sqrt{3}(\tan\theta - 1) + (\tan\theta + 1) = 0$
 $\rightarrow (\tan\theta - 1)(\sqrt{3}\tan\theta + 1) = 0 \rightarrow \theta = \underline{\underline{45^\circ, 225^\circ, 150^\circ, 330^\circ}}$