

**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2013 SOLUTION KEY**

Team Round

A) Your first line of attack might be to try the Pythagorean Theorem.

$$\left(\left[\frac{x}{8}\right]-1\right)^2 + (x-1)^2 = x^2 \Rightarrow \left(\left[\frac{x}{8}\right]-1\right)^2 = 2x-1 \Rightarrow \left[\frac{x}{8}\right]^2 - 2\left[\frac{x}{8}\right] + 1$$

$$\Rightarrow 2(x-1) = \left[\frac{x}{8}\right]\left(\left[\frac{x}{8}\right]-2\right)$$

But then what??

A quicker solution would be to list the Pythagorean triples where the length of the hypotenuse is 1 more than the long leg and note a pattern.

3	4	5
5	12	13
7	24	25
9	40	41

The gap between the long legs is growing by 4 as the gap between the short legs remains

constant at 2. Since $\left[\frac{5}{8}\right]-1 \neq 3$, $\left[\frac{13}{8}\right]-1 \neq 5$, $\left[\frac{25}{8}\right]-1 \neq 7$, and $\left[\frac{41}{8}\right]-1 \neq 9$, we must continue the pattern.

11	60	61	$\left[\frac{61}{8}\right]-1 = 7-1 = 6 \neq 11$
13	84	85	$\left[\frac{85}{8}\right]-1 = 10-1 = 9 \neq 13$
15	112	113	$\left[\frac{113}{8}\right]-1 = 14-1 = 13 \neq 15$
17	144	145	$\left[\frac{145}{8}\right]-1 = 18-1 = 17$ Bingo!

Therefore, the perimeter of $\triangle ABC$ is $17 + 144 + 145 = \underline{\mathbf{306}}$.