MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2010 SOLUTION KEY

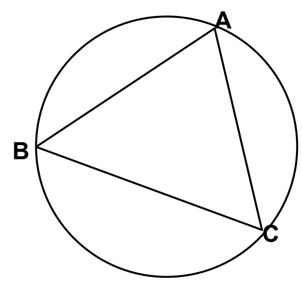
Round 4 - continued

C) Let (Tom, Sherry) = $(T, S) = (x + 18, x)$ denote their current ages.
87 years ago, $(T, S) = (x – 69, x – 87)$.
Let A and B be the digits used to denote their ages 87 years ago.

Now we have: $\begin{cases} x-69=10A+B \\ x-87=10B-A \end{cases} \rightarrow 9A – 9B = 18 \rightarrow A – B = 2$

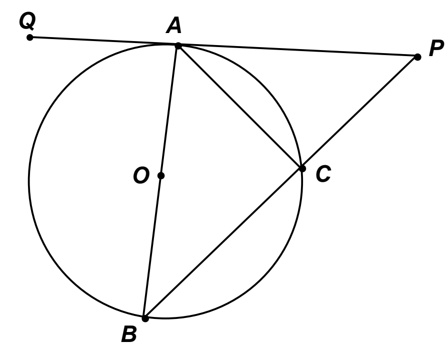
$(A, B) = (3, 1), (4, 2), (5, 3)$ – all rejected, but $(6, 4) \rightarrow 64 + 46 = 110$
Therefore, $x – 69 = 64 \rightarrow x = 133 \rightarrow T =$ **151**

Alternate (faster if not better) solution:
Examining two-digit integers with digits reversed, we quickly determine that Tom and Sherry must be 64 and 46 years old respectively (64 + 46 = 110). Adding 87 to Tom's age (the older sibling), we get his current age of **151**.



Round 5

A) Since $\angle B$ is an inscribed angle,
$m\angle B = 4x + 2 = ½(9x – 3) \rightarrow x = 7$
and $m\angle C = 180 – (m\angle A + m\angle B) = 180 – 54 –30 =$ **96**



B) Since $BP : BC = 9 : 5$, let $PC = 4x$ and $BC = 5x$.
Using the tangent-secant relationship, $(5x)(9x) = 12^2$.
$36x^2 = 144 \rightarrow x = 2$. A chord through the center of a circle must be a diameter; therefore ΔBAP is a right triangle.
$BP = 18$, $AP = 12 \rightarrow AB^2 = 324 – 144 = 180$
Since $\angle ACB$ is inscribed in a semicircle, it must be a right angle.
Thus, $AC^2 = AB^2 – BC^2 = 180 – 100 = 80 \rightarrow AC =$ $\mathbf{4\sqrt{5}}$
[Easier, use the Pythagorean Theorem directly on ΔPCA.
$AC^2 = 12^2 – 8^2 = 80 \rightarrow AC =$ $\mathbf{4\sqrt{5}}$.]

C) The region accessible to the goat is:
I: ¾ of a circle w/radius 20' (center at H)
II: ¼ of a circle w/radius 10' (center at E)
III: ½ of a circle w/radius 4' (center at R)
IV: ¼ of a circle w/radius 5' (center at S)

$$\frac{3}{4}400\pi+\frac{1}{4}100\pi+\frac{1}{2}16\pi+\frac{1}{4}25\pi$$
$= \pi(300 + 25 + 8 + 6.25)$
$=$ $\mathbf{339.25\pi}$ or $\mathbf{\dfrac{1357\pi}{4}}$