

**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 - FEBRUARY 2012
ROUND 7 TEAM QUESTIONS
ANSWERS**

- A) _____ D) _____
B) _____ E) _____ °
C) (_____ , _____ , _____) F) _____

******* NO CALCULATORS ON THIS ROUND *******

- A) Given: $f(x) = 2x^4 + x^3$, $f(h(x)) = 32x^4 - 56x^3 + 36x^2 - 10x + 1$
If $h(x) = Ax + B$, where A and B are integer constants, compute $h^{-1}(3)$.
- B) Let $M(b)$ be the base 10 representation of the minimum natural number in base b that has a digit sum greater than 10. For example, $M(10) = 29$ and $M(3) = 122222_{(3)} = 845_{(10)}$.
Compute $\sum_{b=4}^{b=9} M(b)$. Recall: \sum is the summation symbol. (Ex: $\sum_{x=1}^{x=4} x^2 = 1 + 4 + 9 + 16 = 30$)

- C) Compute the ordered triple (A, B, C) for which the following equation is an identity, for all values of x for which both sides of the equation are defined.

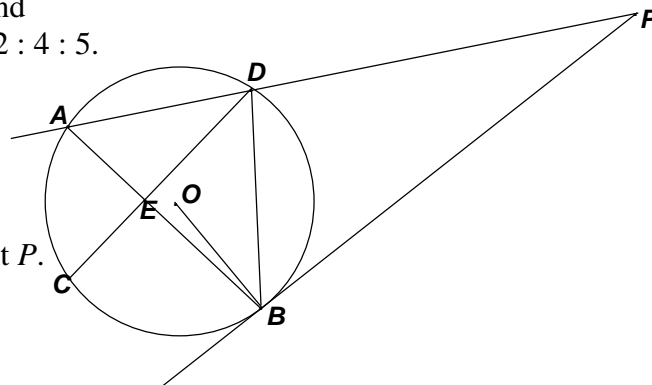
$$\frac{2 \tan x (1 - \tan^2 x)}{(1 + \tan^2 x)^2} = A + B \sin(Cx)$$

- D) In a special summer session of Hogwarts School of Witchcraft and Wizardry, three courses were offered to new students: Charms (C), Potions (P) and Flying (F).
Every student chose to take at least one course and some chose to take multiple courses.
Let XY denote taking both course X and course Y .
Let $X + Y$ denote taking either course X or course Y (or both).
Let $n(X)$ denote the number of students signed up for course X .

Given: $n(C) = 30$, $n(C + P + F) = 116$, $n(CPF) = 6$ and
 $n(CP) : n(CF) : n(PF) = n(C) : n(P) : n(F) = 2 : 4 : 5$.

Compute the largest possible number of students who could have signed up just for flying.

- E) \overline{AB} and \overline{CD} are chords in circle O that intersect at point E . A secant line through points A and D and a line tangent to the circle at point B intersect at point P .
If $m\angle DBA = m\angle ADC + 10^\circ$, $m\angle P = 5^\circ$ and
 $m\angle AED : m\angle BED = 4 : 5$, compute $m\angle EBO$.



- F) The sum of an infinite geometric progression with first term a and common multiplier r , is one more than the sum of its first two terms. If $2 \leq a \leq 6$, compute all possible values of r .