Team Round - continued

D) Note: \$1 increase in ticket price produces a 4% decrease in sales; \$1 decrease in ticket price produces a 10% increase in ticket sales.

Solution #1 (strictly arithmetic):
$@7.50$ 160 ticket are sold $\Rightarrow$ \$1200
For cheaper ticket prices, we fill more seats, *but make less money*.
[$6.50(180)=1170$, $5.50(200)=1100$]
For more expensive tickets, we fill fewer seats, *but make more money (up to a point!)*.
$@8.50$ $200(.76)=152$ tickets are sold $\Rightarrow R = \$1292$
$@10.50$ $200(.68)=136$ tickets are sold $\Rightarrow R = \$1428$
$@12.50$ $200(.60)=120$ tickets are sold $\Rightarrow R = \$1500$
Increases are slowing down!!
$@13.50$ $200(.56)=112$ tickets are sold $\Rightarrow R = \$1512$
$@14.50$ $200(.52)=104$ tickets are sold $\Rightarrow R = \$1508$
This <u>*suggests*</u> that T is around \$13.50, but more fine tuning (in 25¢ increments) might produce a larger value of R.
Rather than brute forcing \$13.25 and \$13.75, let's consider an algebraic approach.

Solution #2 (algebraic):
Assuming each \$1 increase produces a 4% decrease in sales.

$$R=(7.5+n)(0.8-0.04n)200=(15+2n)(80-4n)=-8n^2+100n+1200$$

$$\Leftrightarrow R=-8\left(n^2-\frac{25}{2}n+\boxed{\left(\frac{25}{4}\right)^2}\right)+1200+8\boxed{\left(\frac{25}{4}\right)^2}=-8\left(n-\frac{25}{4}\right)^2+1512.50$$

$\Leftrightarrow n=\dfrac{25}{4}$ produces the maximum ticket sales of $R=\$1512.50$

$n=\dfrac{25}{4}\Rightarrow T=\$7.50+\$6.25=\$13.75$.

Thus, $(T,R)=\underline{(\mathbf{13.75,\ 1512.50})}$.