MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 - FEBRUARY 2012 SOLUTION KEY

Team Round

A) $f(x)=2x^4+x^3=x^3(2x+1)$

Thus, $f(h(x))=\left(h(x)\right)^3\left(2h(x)+1\right)=\left(Ax+B\right)^3(2Ax+(2B+1))$.

Expanding $\left(Ax+B\right)^3(2Ax+(2B+1))$, the lead coefficient would be $2A^4$ and the constant term would be $B^3(2B+1)$. Therefore, $2A^4=32\Rightarrow A=\pm 2$ and

$B^3(2B+1)=1\Leftrightarrow 2B^4+B^3-1=0\Rightarrow B=-1$

Consequently, $h(x)=2x-1$ or $-2x+1$.

However, checking the other coefficients of $f(h(x))$, only $A = 2$ produces the correct coefficients for x^3, x^2 and x and $h(x) = 2x - 1$ only.

Thus, $h^{-1}(x)=\frac{x+1}{2}$ and $h^{-1}(3)=\frac{4}{2}=\underline{\mathbf{2}}$.

B) Base 4: $2333_{(4)} = 2(4^3) + (4^3 - 1) = 3(64) - 1 = 191$

Base 5: $344_{(5)} = 3(5^2) + (5^2 - 1) = 99$

Base 6: $155_{(6)} = 36 + 30 + 5 = 71$

Base 7: $56_{(7)} = 35 + 6 = 41$

Base 8: $47_{(8)} = 32 + 7 = 39$

Base 9: $38_{(9)} = 27 + 8 = 35$

Total : **<u>476</u>**

C) $$\frac{2\tan x\left(1-\tan^2 x\right)}{\left(1+\tan^2 x\right)^2}=\frac{2\frac{\sin x}{\cos x}\left(\frac{\cos^2 x-\sin^2 x}{\cos^2 x}\right)}{\sec^4 x}=2\sin x\left(\frac{\cos^2 x-\sin^2 x}{\cos^3 x}\right)\cos^4 x$$

$$=2\sin x\cos x\left(\cos^2 x-\sin^2 x\right)=\sin 2x\cos 2x=\frac{1}{2}\left(2\sin 2x\cos 2x\right)=\frac{1}{2}\sin 4x$$

Thus, (A, B, C) = $\underline{\left(\mathbf{0},\frac{\mathbf{1}}{\mathbf{2}},\mathbf{4}\right)}$.