MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2013 SOLUTION KEY

Round 6

A) The second term of the geometric progression can be written as $-3i$.

The common ratio r for the geometric progression is $\frac{1}{i} = -i$.

Therefore, the progression is $3,\ -3i,\ -3,\ 3i,\ \boxed{3},\ldots$ consists of a repetition of four terms.

If n is 1 more than a multiple of 4, i.e. $n = 1, 5, 9, \ldots,\ t_n = 3$.

Since 10 is 2 more than a multiple of 4, $t_{10} = \frac{3}{i} = -3i$.

Since 13 is 1 more than a multiple of 4, $t_{13} = 3$

$\Rightarrow\ (-3i+3)^2 = -9-18i+9 = \underline{\boldsymbol{-18i}}$ (Also accept $0-18i$.) Do **not** accept $\frac{18}{i}$.

B) (1) $y^2 = 3x$

(2) $x + 1 - y - 2 = y + 2 - 3 \Rightarrow\ x = 2y$

Thus, $y^2 = 6y \Leftrightarrow y(y-6) = 0$. Since $y \neq 0$, $(x, y) = (12, 6)$

$\Rightarrow$ in the arithmetic progression: $t_{12} = 3 + 11(5) = 58$

$\Rightarrow$ in the geometric progression: $t_5 = 3(2^4) = 48$

The required ratio is **29: 24**

C) $\text{AM} = \text{HM} + 0.1 \Rightarrow \frac{A+2}{2} = \frac{4A}{A+2} + \frac{1}{10}$ Multiplying through by $10(A + 2)$, we have

$5(A+2)^2 = 40A + (A+2) \Rightarrow 5A^2 - 21A + 18 = (5A-6)(A-3) = 0$

Therefore, $A = 3, \frac{6}{5}$.

$A = 3 \Rightarrow GM = \underline{\boldsymbol{\sqrt{6}}}$

$A = 1.2 \Rightarrow GM = \sqrt{2\cdot\frac{6}{5}} = \sqrt{\frac{3\cdot4\cdot5}{25}} = \underline{\boldsymbol{\frac{2\sqrt{15}}{5}}}$ (or $\frac{2}{5}\sqrt{15}$)