MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2013 SOLUTION KEY

Round 3

A) The slope of $\overleftrightarrow{PQ}$ is $\frac{0+6}{8-0}=\frac{3}{4}$. Thus, the equation of the parallel line is $(y-15)=\frac{3}{4}(x-2)$.
$\Leftrightarrow 4y-60=3x-6 \Leftrightarrow$ **$\underline{3x-4y=-54}$**

B) $x=2t+1 \Rightarrow t=\frac{x-1}{2}$ Substituting, $y=6t-5=6\left(\frac{x-1}{2}\right)-5=3x-8$
Thus, $3k-8=5k \Rightarrow k=$ **$\underline{-4}$**

Alternately, we require that $y = 5x$ for $x = k$. Therefore, $6t-5=5(2t+1) \Rightarrow 4t=-10 \Rightarrow t=-\frac{5}{2}$
$\Rightarrow x=k=2\left(-\frac{5}{2}\right)+1=$ **$\underline{-4}$**

C) If there are S short rows with $x - 3$ stars each, then there are $S + 1$ long rows with x stars each.
$S(x-3)+(S+1)(x)=40 \Leftrightarrow (2S+1)x=40+3S \Rightarrow x=\frac{40+3S}{2S+1}$
There must be at least one short row.
$(S, x) = \left(1,\frac{43}{3}\right),\left(2,\frac{46}{5}\right),\left(3,\frac{49}{7}\right),\left(4,\frac{52}{9}\right),\left(5,\frac{55}{11}\right)$
According to the chart above,
one possibility is 3 short rows of $7 – 3 = 4$ stars each and 4 long rows of 7 stars each or 5 short rows of $5 – 3 = 2$ stars each and 6 rows of 5 stars each, but the latter exceeds the maximum number of rows. Thus, two consecutive rows contain $4 + 7 =$ **$\underline{11}$** stars.