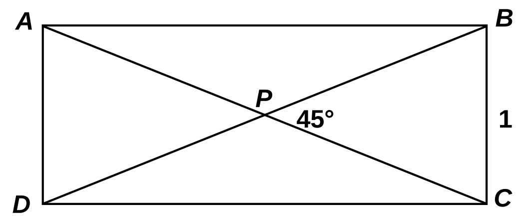
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2016
ROUND 7 TEAM QUESTIONS
ANSWERS

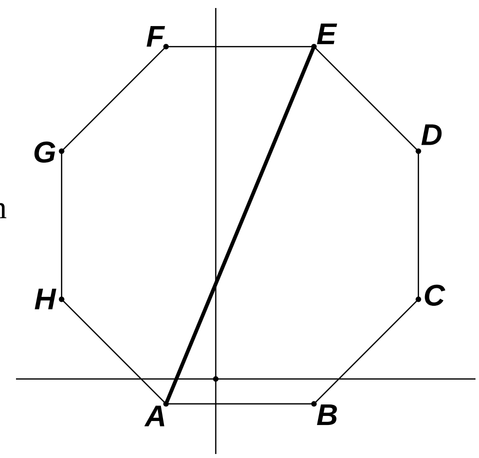
A) (______ , ______ , ______) D) ____________________________

B) ____________________________ E) (___________ , ___________)

C) (___________ , ___________) F) (_______ , _______ , _______)



A) The perimeter of the rectangle $ABCD$, where $BC = 1$, and the diagonals intersect at a 45° angle may be expressed in the form $s + t\sqrt{r}$, where $\sqrt{r}$ is a simplified radical. Compute the ordered triple (s,t,r).



B) Find all integer ordered pairs (n, k) which satisfy the following equation:

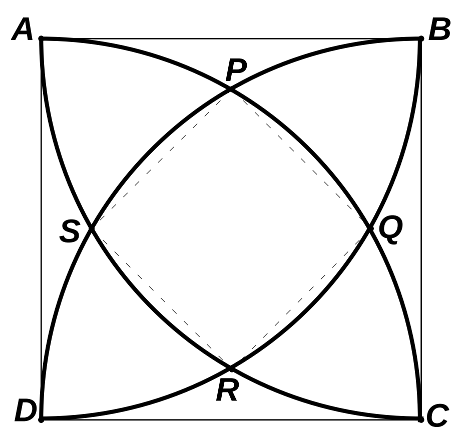
$$n^3 - 3n^2 + 3n = k^3 + 3k^2 + 3k - 17$$

C) A regular octagon is placed on a coordinate system as shown in the diagram. If $A(-2,-1)$ and $B(4,-1)$, compute the coordinates of the x-intercept of $\overline{AE}$.

D) Given: $\log 250 = N$

Give a simplified expression for $\log_5 250$ in terms of N.

E) The golden rectangle is a rectangle with dimensions L_g x W_g, where $L_g > W_g$ which satisfies the proportion $\frac{L_g}{W_g} = \frac{L_g + W_g}{L_g}$. Define a silver rectangle to be a rectangle with dimensions L_s x W_s, where $L_s > W_s$ and diagonal D_s which satisfies the proportion $\frac{L_s}{W_s} = \frac{D_s}{L_s}$. The ratio $\frac{L_g}{W_g} : \frac{L_s}{W_s} = \frac{\sqrt{B}}{A}$, where $A > 1$ is an integer and $A \cdot B$ is a minimum. Compute the ordered pair (A, B).



F) Given square $ABCD$ with side of length 4.
Arcs are drawn from each vertex and the points of intersection form square $PQRS$ as shown. In simplified radical form, the perimeter of $PQRS$ is $a\left(\sqrt{b} - \sqrt{c}\right)$, where a, b, and c are positive integers.
Compute the ordered triple (a,b,c).