MASSACHUSETTS MATHEMATICS LEAGUE CONTEST 4 - JANUARY 2008 SOLUTION KEY

Round 6

Now Then(13yrs ago)

- A) Earl $x x-13 \Rightarrow x-13 = 2(x-18) = 2x-36 \Rightarrow x = 23$ Cousin x-5 x-18
- B) The equation of L_1 is 2x 5y = 10. The point of L_1 closest to P(-1, 15) is the foot of the perpendicular drawn from P to L_1 . Since perpendicular lines have negative reciprocal slopes, the equation of a perpendicular line to L_1 is of the form 5x + 2y = c. Substituting x = -1 and y = 15, we can determine the value of c for which the perpendicular passes through point P.

Thus, c = 25. The solution of the system $\begin{cases} 2x - 5y = 10 \\ 5x + 2y = 25 \end{cases}$ is (5, 0).

C) Alcohol:
$$\frac{3}{4}x + \frac{2}{3}y = \frac{5}{7}(x+y)$$
 and $x+y \ge 24$

Clearing fractions (LCM = 84), $63x + 56y = 60x + 60y \implies 3x = 4y \text{ or } y = \frac{3}{4}x$

$$x + \frac{3}{4}x \ge 24 \rightarrow 7x \ge 96 \rightarrow x > 13 \rightarrow x = 16 \rightarrow (16, 12)$$