MASSACHUSETTS MATHEMATICS LEAGUE
MARCH 2004
ROUND 6: PROB & BINOMIAL THEOREM

ANSWERS

A) 105/512

B) 40095

C) 189/8192

A) A fair coin is tossed ten times. What is the probability of getting exactly six heads? Express the answer as a simplified fraction.

$$\frac{{}_{10}C_6}{2^{10}} = \frac{210}{1024} = \frac{105}{512}$$

B) In the expansion of $(x^5 - 3)^{12}$ there is a term of the form kx^{40}. In simplified form, find the value of k

${}_{12}C_r (x^5)^{12-r} (-3)^r \qquad 50 \quad x^{60-5r} = x^{40}$ and $r = 4$

$K = {}_{12}C_4 (-3)^4 = 495 \cdot 81 = 40095$

C) A test has eight multiple choice questions, each with four answer choices. What is the probability of answering exactly five questions correctly by random guessing?

$${}_8C_5 \left(\frac{1}{4}\right)^5 \left(\frac{3}{4}\right)^3 = \frac{56 \cdot 27}{4^8} = \frac{14 \cdot 27}{4^7} = \frac{7 \cdot 27}{2^{13}} = \frac{189}{8192}$$