**MASSACHUSETTS MATHEMATICS LEAGUE**
**CONTEST 1 - OCTOBER 2011 SOLUTION KEY**

**Round 2**

A) $x^2 + \left(6\sqrt{3}\right)^2 = \left(13\sqrt{2}\right)^2 \Rightarrow x^2 = 169(2) - 36(3) = 338 - 108 = 230$.
Since $15^2 = 225$ and $16^2 = 256$, we see that 230 is closer to $15^2$ than $16^2$.
Thus, to the nearest integer, $x$ = **<u>15</u>**.

B) $(25+N)^2 + 45^2 = 53^2 \Rightarrow (25+N)^2 = 53^2 - 45^2 = (53+45)(53-45) = 98(8) = 49(16) = 28^2$.
Thus, $25 + N = 28 \Rightarrow N$ = **<u>3</u>**.

**<u>FYI</u>** - Here's how we know that the original $\Delta ABC$ was obtuse:
Let $AB = 53$, then $C$ is the largest angle. Using the Law of Cosines,

$$c^2 = a^2 + b^2 - 2ab\cos C \Rightarrow \cos C = \frac{a^2+b^2-c^2}{2ab} \Rightarrow \cos C = \frac{25^2+45^2-53^2}{2(25)(45)}$$

Since $25^2 + 45^2 - 53^2 = 625 + 2025 - 2809 < 0$, $\cos C < 0$ and $\Delta ABC$ must be obtuse.

C) Let $(a, b, c) = (a, n-1, n)$. Then $a^2 = n^2 - (n-1)^2 = 2n - 1$.
Consider these perfect squares {9, 25, 49, ...}.
Since $a^2$ must be odd, even perfect squares are not considered.
$a = 1$ is rejected , since $a = 1 \Rightarrow n = 1$ and this leaves a leg of length 0.

($a^2 = 2n - 1$)

| ***a*** | ***a*²** | ***N*** | ***a*** | ***b*** | ***c*** | ***Per*** |
|---|---|---|---|---|---|---|
| 3 | 9 | 5 | 3 | 4 | 5 | 12 |
| 5 | 25 | 13 | 5 | 12 | 13 | 30 |
| 7 | 49 | 25 | 7 | 24 | 25 | 56 |
| 9 | 81 | 41 | 9 | 40 | 41 | 90 |
| 11 | 121 | 61 | 11 | 60 | 61 | **<u>132</u>** |

Thus, $(a, b, c)$ = **<u>(11, 60, 61)</u>**.