

## Addendum #2: Team A Question

The actual question used did not add the restriction the sum  $L + M + N$  is a minimum. Strangely, no appeal was made and there should have been!

Here is the case made by Norm Swanson – Hamilton Wenham

Since no lengths are given, we can choose any convenient lengths. Furthermore, there are many possible locations for point  $Q$  for which  $m\angle PQR$  is a right angle, since any point on a semi-circle with center  $O$  and diameter  $\overline{PR}$  will suffice. By choosing a specific point  $Q_2$ , we can let  $\overline{PS}$  (with length  $h$ ) be an altitude to  $\overline{PR}$ , dividing  $\angle PQR$  into  $30^\circ$  and  $60^\circ$  angles.

Convenient lengths:

$$PQ_2 = 2 \Rightarrow Q_2R = 2\sqrt{3}, PR = 4, a = PS_2 = 1, b = S_2R = 3, h = Q_2S_2 = \sqrt{3}$$

The given formula  $h^2 = \frac{La^2b^2}{Ma^2 + Nb^2}$  becomes  $3 = \frac{9L}{M + 9N}$  or  $1 = \frac{3L}{M + 9N}$  or  $L = \frac{M}{3} + 3N$ .

Since all three constants are positive integers,  $M$  must be a multiple of 3 and we get the minimum sum if we let  $M = 3$ , resulting in  $L = 1 + 3N$ .

Thus, the minimum sum occurs when  $N = 1$  and  $(L, M, N) = \underline{(4, 3, 1)}$ .

However,  $(L, M, N)$  is not unique.

In fact, any triples of the form  $(k + 3j, 3k, j)$ , where  $j$  and  $k$  are positive integers, are solutions.

