MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2007
ROUND 7 TEAM QUESTIONS

ANSWERS

A) (_____ , _____) D) _____

B) _____ E) _____

C) _____ F) _____

A) The equation $x^8 - 81 = 0$ has 8 roots which may be expressed the forms:
$\pm A, \quad \pm Ai, \quad \pm(B + Bi), \quad \pm(B - Bi)$
where A and B are positive reals. Compute the ordered pair (A, B).

B) Find all ordered triples (a, b, c) of positive integers that satisfy the following conditions:

- b is prime
- $a + c$ is composite
- $\begin{cases} a + 3b + 5c = 50 \\ 5a + 3b + c = 70 \end{cases}$

C) An equilateral triangle is inscribed in a square whose side has length 1 unit. One vertex of this equilateral triangle coincides with a vertex of the square. Compute the area of this equilateral triangle.

D) Factor completely. $32a^{7x} - 240a^{6x} + 720a^{5x} - 1080a^{4x} + 810a^{3x} - 243a^{2x}$

E) Given: $\sin(4x) = a\sin x\cos x + b\sin^3 x\cos x$
Find the numerical value of $a + 3b$.

F) In regular polygon P, the reduced ratio of the interior angle to the exterior angle is $a : b$.
In regular polygon Q, the reduced ratio of the exterior angle to the interior angle is $c : d$
If a, b, c and d are positive integers, $a + b = 15$ and $c + d = 12$,
compute all possible ratios of the interior angle of P to the exterior angle of Q.