MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2017
ROUND 7 TEAM QUESTIONS

ANSWERS

A) _____ D) S = _____ (x, y) = (____ , ____)

B) _____ E) _____

C) _____ F) _____

A) Compute the ***y*-coordinates** of <u>all</u> points where the vertical line $x = -7$ could intersect a hyperbola which has an asymptote whose equation is $(y-4) = \frac{3}{4}(x+2)$.

B) Factor completely over the integers. $4x^6 - 13x^4 - 13x^2 + 4$

C) Given: $\sin(x) = \frac{1 \pm \sqrt{1-4(N-1)}}{2}$
If $x = (30k)°$, where k is an integer and $0 \leq k < 12$.
Compute all possible <u>rational</u> values of N.

D) Given: $S = x^2 + 4xy + 9y^2 + 4x + 18y + 2017$ for real numbers x and y.
Find m, the <u>minimum</u> value of S, and the ordered pair (x, y) for which it occurs.

E) In right triangle ABC, the bisector of $\angle C$ intersects the hypotenuse $\overline{AB}$ at point D.
Point E is located on $\overline{AC}$ such that $\overline{DE} \perp \overline{AC}$. Compute $\frac{1}{BC} + \frac{1}{AC}$, if $DE = \frac{\sqrt{5}+1}{2}$.

F) Some positive factors of 4000 are multiples of 5 and some are not.
Specifically, k% of the factors of 4000 are multiples of 5.
A is a positive integer less than 4000 with the same number of divisors as 4000, and k% of its divisors are multiples of 5.
B is an integer greater than 4000 with the same number of divisors as 4000, and k% of its divisors are multiples of 5.
Compute the <u>minimum</u> possible value of $B - A$.