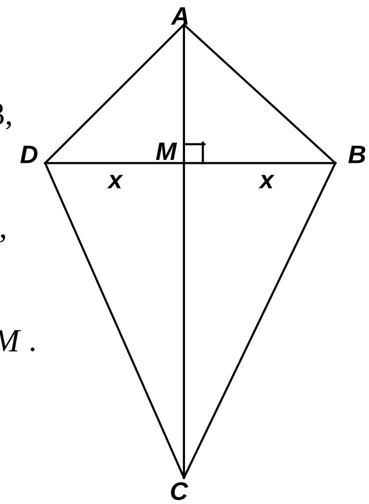
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2015 SOLUTION KEY

Round 2

A) The hypotenuse of ΔABC has length 17 $\left(8^2+15^2=289=17^2\right)$, producing a perimeter of 40.
Since the sides of similar triangles (and hence the perimeters) are proportional, we have $40k>2^{10}=1024 \Rightarrow k \geq 26$.
Thus, the minimum length of the hypotenuse of ΔDEF is $26 \cdot 17 = \underline{\mathbf{442}}$.

B) Since either 2*x* = 18 or 2*x* = 52, a leg of a right triangle *BAM* must be either 9 or 26. Listing the common right triangles, we have 3-4-5, 5-12-13, 7-24-25, 9-40-41, 11-60-61, $\boxed{\text{13-84-85}}$, …
If 26 were a <u>leg</u> of one of the right triangles, the lengths of two sides of the kite would be 170 and our perimeter would already exceed 112. Therefore, we are sure that the legs on the short diagonal must both be 9.
We try 3(3-4-5) = 9-12-$\boxed{15}$ as the dimensions of ΔBAM and ΔDAM.
112 – 30 = 82 This results in dimensions of 9-40-41 for ΔCDM and ΔCBM.
Thus, the long diagonal is divided into segments of lengths 12 and 40.
$\frac{40}{12}=\frac{10}{3} \Rightarrow \underline{\mathbf{13}}$.



C) Given: Right triangle *ABC*, with sides of length $AB=2\sqrt{k}$, $AC=\sqrt{30}$, and $BC=k\sqrt{2}$.
Case 1: $\overline{AB}$ is the hypotenuse.
$4k=30+2k^2 \Leftrightarrow k^2-2k+15=0$ which has no solutions over the real numbers.

Case 2: $\overline{AC}$ is the hypotenuse
$30=4k+2k^2 \Leftrightarrow k^2+2k-15=(k-3)(k+5)=0 \Rightarrow k=3$, producing an area of
$\frac{1}{2}\cdot 2\sqrt{3}\cdot 3\sqrt{2}=\underline{\mathbf{3\sqrt{6}}}$.
Case 3: $\overline{BC}$ is the hypotenuse
$4k+30=2k^2 \Leftrightarrow k^2-2k-15=(k-5)(k+3)=0 \Rightarrow k=5$, producing an area of
$\frac{1}{2}\cdot 2\sqrt{5}\cdot \sqrt{30}=\underline{\mathbf{5\sqrt{6}}}$.