MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2006 SOLUTION KEY

Round 3

A) Let H and C denote the number of horses and chicks respectively.
Then
$H + C = 360 \rightarrow H = 360 - C$
$4(360 - C) + 2C = 1100 \rightarrow 2C = 1440 - 1100 = 340 \rightarrow C = \underline{\mathbf{170}}$

B) Since the slope of the line is $\frac{1}{4}$, $\frac{b-2006}{2006-a} = \frac{1}{4}$. Inverting both sides and multiplying through by -1, it follows that $\frac{2006-a}{2006-b} = \underline{\mathbf{-4}}$.

C) If she has \$$x$ entering a store, she has \$$\frac{x}{2} - \frac{1}{4}$ when she leaves.
Thus, we half her money and then subtract 0.25. To make life easier, we'll work <u>backwards</u>, by starting with \$0 and first adding 0.25, then doubling!

Upon leaving store #10	9	8	7	6	5	4	3
$0 \rightarrow$	0.5	$\rightarrow$ 1.5	$\rightarrow$ 3.5	$\rightarrow$ 7.5	$\rightarrow$ 15.5 $\rightarrow$	31.5 $\rightarrow$	**<u>63.5</u>**

Round 4

A) $A = 2 - 4/13 = \frac{22}{13}$ and $B = \frac{4+3}{8-3} = \frac{7}{5}$ Computing the difference, $\frac{22(5)-7(13)}{13(5)} = \boxed{\frac{\mathbf{19}}{\mathbf{65}}}$

B) Let $A = \left(\frac{1+x}{1-x}\right)$ Then the equation becomes $A = 4 - 3(1/A) \rightarrow A^2 - 4A + 3 = (A-3)(A-1) = 0$
$\rightarrow A = 1$ or 3. Substituting for A, $1 + x = 1 - x \rightarrow x = \underline{\mathbf{0}}$ or $1 + x = 3(1 - x) \rightarrow 4x = 2 \rightarrow x = \underline{\mathbf{½}}$

C) Method 1:
$\frac{1}{A} + \frac{1}{B} = \frac{17}{25} - \frac{1}{2} = \frac{9}{50} \rightarrow 50A + 50B = 9AB \rightarrow B = \frac{50A}{9A-50}$
B is an integer if and only if $9B$ is an integer. $9B = \frac{450A}{9A-50} = 50 + \frac{2500}{9A-50}$
Thus, $9A$- 50 must be a positive factor of 2500. The smallest possible value of A for which this is true is $A = 6$ and $B = 75 \rightarrow$ **<u>(6, 75)</u>**. In fact, the only ordered pairs are (6, 75) and (75, 6).
Method 2:
$\frac{17}{25} - \frac{1}{2} = \frac{9}{50} = 0.18$
The next largest unit fraction which is less than or equal to 0.18 is $\frac{1}{6} \approx 0.83 \rightarrow A = 6$
$\frac{9}{50} - \frac{1}{6} = \frac{4}{300} = \frac{1}{75} \rightarrow B = 75$ Therefore, $(A, B) =$ **<u>(6, 75)</u>**