MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 – JANUARY 2012
ROUND 1 ANALYTIC GEOMETRY: ANYTHING

ANSWERS

A) _____

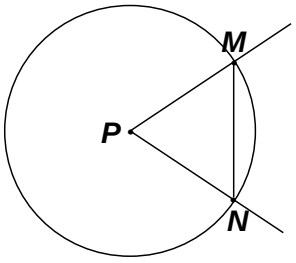
B) (_____ , _____)

C) (_____ , _____)

A) The area of an ellipse is given by the formula πab, where a and b are the lengths of the semi-major and semi-minor axes, respectively.
Compute the absolute value of the difference between the areas of following conic sections:

$$x^2 + y^2 = 36$$
$$4x^2 + 9y^2 = 36$$

B) Given: $C_1 = \left\{(x, y) \mid (x+1)^2 + (y-2)^2 = 64\right\}$
$m\angle P = 60°$, $M(a, b)$ and $N(a, c)$ lie on C_1,
where $a > 0$ and $b > c$.



Determine the ordered pair (a, b).

C) Lines are drawn tangent to the parabola $y = \frac{1}{2}x^2$. If $x = a$, the tangent line has equation $ax - y = \frac{a^2}{2}$. The tangent line through $A(2, 2)$ and the tangent line at B are perpendicular and intersect at point P. Find the coordinates of point P.