

The original problem 5C intended P to be in the interior of the circle, but did not specify that restriction.

Certainly, if $B = P = D$ and $m\angle APC = 90^\circ$, then $\angle APC$ will be

inscribed in a semi-circle and $r = \frac{AC}{2}$

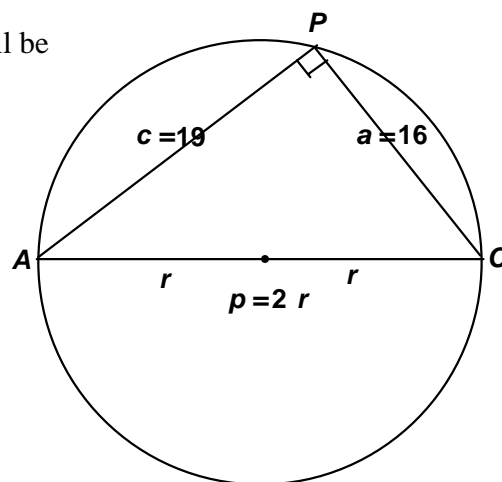
and $AC = \sqrt{16^2 + 19^2} = \sqrt{617}$.

Comparing the squares, $\left(\frac{5\sqrt{17}}{2}\right)^2 = \frac{25 \cdot 17}{4} = \frac{425}{4}$,

$\left(\frac{\sqrt{617}}{2}\right)^2 = \frac{617}{4}$,

and we have an answer larger than the “official” answer.

Both answers were accepted.



Extension: Suppose point P had been specified to be on the circle, but the perpendicular condition were omitted. Could we draw any conclusions about the radius r ?

By making the angle at P acute (or obtuse), what happens to r ?

Consider the radius of a circle circumscribed about a triangle. Let K denote the area of the triangle.

Knowing that $r_{cc} = \frac{abc}{4K}$ and $K = \frac{1}{2}ab \sin \theta$, where θ is the included angle, we have

$r_{cc} = \frac{16 \cdot 19 \cdot p}{4 \left(\frac{1}{2} \cdot 16 \cdot 19 \cdot \sin P \right)} = \frac{p}{2 \sin P} = \left[\frac{p}{2} \cdot \frac{1}{\sin P} \right]$ which reduces to the above answer for $P = 90^\circ$. But,

for any inscribed angle, $\frac{1}{\sin P} \geq 1$. The problem is that P and p are not independent of each other.

Using the Law of Cosines and the fact that $\sin^2 \theta + \cos^2 \theta = 1$, for angle θ , we have

$p^2 = 16^2 + 19^2 - 2 \cdot 16 \cdot 19 \cos P \Rightarrow \cos P = \frac{617 - p^2}{2 \cdot 16 \cdot 19}$ and $\sin^2 P = 1 - \left(\frac{617 - p^2}{2 \cdot 16 \cdot 19} \right)^2$. Consequently,

substituting in the boxed expression above, $r_{cc} = \frac{p}{2 \sqrt{1 - \left(\frac{617 - p^2}{2 \cdot 16 \cdot 19} \right)^2}}$. Constructing a lookup table for

integer values of c from 4 to 34 is enlightening. Mr. TI gives these results:

p	4	5	...	9	10	11	...	24	25
R	13.22	10.97		9.53	9.50	9.51		12.03	12.50
p	26	27	28	29	30	31	32	33	34
R	13.06	13.74	14.56	15.60	16.95	18.80	21.54	26.18	36.74

As P becomes very acute ($P \rightarrow 0 \Rightarrow p \rightarrow 3$) and as P becomes very obtuse ($P \rightarrow 180^\circ \Rightarrow p \rightarrow 35$), as expected from the Triangle Inequality. In either case, r becomes longer and longer ($r \rightarrow \infty$).

Without the perpendicular restriction, there is no maximum value of r .