MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2012 SOLUTION KEY

Team Round

E) - continued

$|x+4|+|x-1|+2\sqrt{|x+4|}\sqrt{|x-1|}=|x-4|$

Case 2: $-4 < x \le 1$

$x+4+1-x+2\sqrt{x+4}\sqrt{1-x}=4-x$

$\Rightarrow\ 2\sqrt{x+4}\sqrt{1-x}=-1-x$

$\Rightarrow\ 4\left(-x^2-3x+4\right)=1+2x+x^2$

$\Rightarrow\ 5x^2+14x-15=0\ \Rightarrow\ \dfrac{-14\pm\sqrt{196+300}}{10}=\dfrac{-14\pm\sqrt{16(31)}}{10}=\underline{\boldsymbol{\dfrac{-7\pm 2\sqrt{31}}{5}}}$ (Both values check!)

[$2\sqrt{31}=\sqrt{124}\approx\sqrt{121}=11\ \Rightarrow\ \approx\dfrac{4}{5},-\dfrac{18}{5}$ Both of these values fall in the required range for case 2.]

Case 3: $1 < x \le 4$

$x+4+x-1+2\sqrt{x+4}\sqrt{x-1}=4-x \Rightarrow 2\sqrt{x+4}\sqrt{x-1}=1-3x$

For any x in this domain, $1-3x<0$. Any solutions would be extraneous, since the left side of the equation <u>must</u> be nonnegative.

Case 4: $x > 4$

$x+4+x-1+2\sqrt{x+4}\sqrt{x-1}=x-4\ \Rightarrow 2\sqrt{x+4}\sqrt{x-1}=-x-7$

For any x in this domain, $-x-7<0$ and any solutions would be extraneous.

F) Note that $\dfrac{n}{(n+1)!}$ can be written as $\dfrac{1}{n!}-\dfrac{1}{(n+1)!}$. $\left[\dfrac{(n+1)!-n!}{n!(n+1)!}=\dfrac{\cancel{n!}\left((n+1)-1\right)}{\cancel{n!}(n+1)!}=\dfrac{n}{(n+1)!}\right]$

Expressing each of the 1999 terms as a difference we have:

$\left(1-\dfrac{1}{2!}\right)+\left(\dfrac{1}{2!}-\dfrac{1}{3!}\right)+\left(\dfrac{1}{3!}-\dfrac{1}{4!}\right)+\ldots+\left(\dfrac{1}{1998!}-\dfrac{1}{1999!}\right)+\left(\dfrac{1}{1999!}-\dfrac{1}{2000!}\right)=1-\dfrac{1}{2000!}$ ➔ <u>**(1, 1, 2000)**</u>

Note that the expression $1-\dfrac{2001}{2001!}$ may be equivalent, but $A+B+C>2002$.