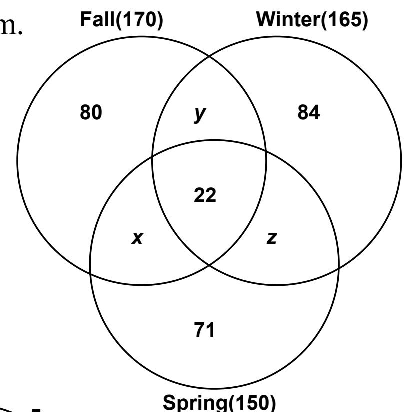
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2007 SOLUTION KEY

Round 4

A) Let B hold x pills. Then A holds $x - 12$ and C holds $2(x - 12)$
$2A + 4B + 3C = 240 \rightarrow 2x - 24 + 4x + 6x - 72 = 240 \rightarrow 12x = 336 \rightarrow x = 28 \rightarrow A = \underline{\mathbf{16}}$

B) Let x = # pages of text in book 2 Then $1^{st} = x + 67$ and $3^{rd} = x - 24$
$3x + 43 = 751 \rightarrow x = 236 \rightarrow 1^{st}$ book has 303 pages. $303 = 3(101)$
Since both these factors are prime, the first book either has 3 chapters of 101 pages each or 101 chapters of 3 pages each. Thus, the maximum number of chapters is **<u>101</u>**.

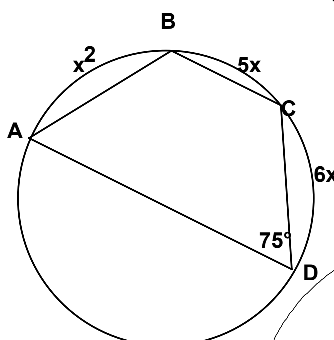
C) The given information translates into the following Venn Diagram.



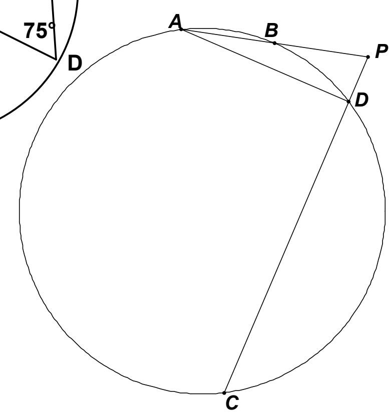
Fall $\rightarrow x + y = 68$
Winter $\rightarrow y + z = 59$
Spring $\rightarrow x + z = 57$
Fall-Winter $\rightarrow x - z = 9$
Thus, $x = 33$, $y = 35$ and $z = 24$ and the total number of girls participating in at least one sport is $170 + 84 + 71 + z = 349$
$\rightarrow 585 - 349 = \underline{\mathbf{236}}$ non-participants

Round 5

A) Appealing to the diagram, as inscribed angles,



$$m\angle D = \frac{1}{2}(x^2 + 5x) = 75$$
$$\rightarrow x^2 + 5x - 150 = (x + 15)(x - 10) = 0$$
$$\rightarrow x = 10$$
$$\rightarrow m\angle A = \frac{1}{2}(5x + 6x) = \frac{1}{2}(110) = \underline{\mathbf{55°}}$$



B) Let $PD = x$. Then $PB(PA) = PD(PC) \rightarrow 4(7) = x(x + 12)$
$\rightarrow x^2 + 12x - 28 = (x + 14)(x - 2) = 0 \rightarrow x = 2$ Applying the Pythagorean theorem, $DA^2 = 49 - 4 = 45 \rightarrow DA = \mathbf{3\sqrt{5}}$

C) $APBQ$ is a kite and $\overline{PQ}$ perpendicularly bisects $\overline{AB}$.
Let N be the point of intersection of $\overline{PQ}$ and $\overline{AB}$. Let $AN = x$, $PN = y$ and $NQ = 8 - y$
Then: $x^2 + y^2 = 25$ and $(8 - y)^2 + x^2 = 49 \leftrightarrow x^2 + y^2 - 16y + 64 = 49$
Substituting, $25 - 16y + 64 = 49 \rightarrow y = \frac{5}{2} \rightarrow x^2 = \frac{75}{4}$

$\rightarrow x = \frac{5\sqrt{3}}{2} \rightarrow AB = \boxed{\mathbf{5\sqrt{3}}}$