MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 - FEBRUARY 2017 SOLUTION KEY

Round 2

A) $(1,12) \Rightarrow 112, 121$ (both rejected, $121 = 11^2$)

$(2,11) \Rightarrow 211, \cancel{112}$ $(3,10) \Rightarrow \cancel{310}, 103$ $(4,9) \Rightarrow \cancel{49}, \cancel{94}$

$(5,8) \Rightarrow \cancel{58}, \cancel{85}$ $(6,7) \Rightarrow 67, \cancel{76}$

Additional ordered pairs repeat the same candidates in reverse order.
Thus, the only primes are **67**, **103** and **211** (in any order).
Recall: Testing *N* for primality requires trying prime divisors which when squared are less than *N*.
For *N* = 67, we had to test 2, 3, 5 and 7 only. ($11^2 = 121 > 67$)
For *N* = 103, we had to test these same divisors. $11^2 = 121 > 103$
For *N* = 211, we had to test 2, 3, 5, 7, 11, and 13. ($17^2 = 289 > 211$)

B) Since the side of square *ABCD* is 19, we require pairs $\langle x, y\rangle$ such that $x + y = 19$ and both *x* and *y* are integers, but neither is a multiple of 3. The only possible pairs are $\langle 2,17\rangle$, $\langle 5,14\rangle$ and $\langle 8,11\rangle$ which result in areas of **34**, **70**, and **88**.

C) The ***greatest*** number of coins could be obtained by using the largest number of smaller denominations as possible. Since the number of pennies must be 1 more than a multiple of 5, we use 41 pennies.
$\$17.76 - \$0.41 = \$17.35$
We can use the maximum number of nickels, namely 45.
$\$17.35 - \$2.25 = \$15.10$
The number of dimes must leave a balance that is a multiple of 25.
Thus, 41 dimes leaves $\$15.10 - \$4.10 = \$11.00$ and 44 quarters are required.
The maximum total appears to be $P + N + D + Q = 41 + 45 + 41 + 44 = 171$ coins.
Replacing one quarter with 2 dimes and a nickel, or 5 nickels would increase the total number of coins, but neither of these alternatives would work, since the maximum allowable number of dimes or nickels would be exceeded. **However**, one less quarter, one less nickel and 3 more dimes would maintain the total value and increase the number of coins by 1, without exceeding 45 of any one coin.
The maximum number of coins is $P + N + D + Q = 41 + 44 + 44 + 43 = \underline{172}$.

For the *smallest* number of coins, we must use as many of the larger denominations as possible.
45 quarters $\Rightarrow \$17.76 - \$11.25 = \$6.51$
45 dimes $\Rightarrow \$6.51 - \$4.50 - \$2.01$
40 nickels $\Rightarrow \$2.01 - \$2.00 = \$0.01$
Since the maximum number of quarters is being used, replacing some combination of nickels and dimes with a single quarter is not possible.
Thus, the minimum number of coins is $Q + D + N + P = 45 + 45 + 40 + 1 = \underline{131}$ coins.
The positive difference is **41**.