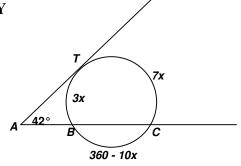
MASSACHUSETTS MATHEMATICS LEAGUE CONTEST 5 – FEBRUARY 2009 SOLUTION KEY

Round 5

A)
$$42 = \frac{1}{2}(7x - 3x) \rightarrow x = 21 \rightarrow BC = 360 - 10(21) = 150^{\circ}$$

B) Let
$$(x, y) = (CE, DE)$$
. Thus, $CD = x + y$.

Applying the product chord theorem. 2(10-2) = 36 = xyAs integer factors of 36, the possible ordered pairs are: (x, y) = (1, 36), (2, 18), (3, 12), (4, 9) and (6, 6) - or vice versa. Since CD < 20, the last three ordered pairs give us CD = 15, 13 or 12.



C) Let BD = DE = EC = CP = x. Using the tangent-secant relationship $AP^2 = CP \cdot BP$. Substituting, $108 = x(4x) \Rightarrow x^2 = 27 \Rightarrow x = 3\sqrt{3}$

Thus, the required area is $\pi \left(\frac{9\sqrt{3}}{2}\right)^2 - \pi \left(\frac{3\sqrt{3}}{2}\right)^2$

$$=\pi\left(\frac{81\cdot 3-9\cdot 3}{4}\right)=\pi\left(\frac{3\cdot 72}{4}\right)=54\pi$$

