MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2010 SOLUTION KEY

Round 1

A) Draw a diagram and label the edges of the solid x, $2x$ and $3x$. Then the total surface area of the solid is given by $2\left(2x^2+3x^2+6x^2\right)=198 \rightarrow x=3$.

Thus, the volume is $6x^3 = 6(3)^3 =$ **162** units^3.

B) $$\frac{4\left(\frac{1}{2}ls\right)}{4\left(\frac{1}{2}ls\right)+s^2}=\frac{5}{8} \rightarrow \frac{2ls}{2ls+s^2}=\frac{2l}{2l+s}=\frac{5}{8} \rightarrow 16l = 10l + 5s \rightarrow 6l = 5s \rightarrow s : l = \underline{\mathbf{6 : 5}}$$

C) If h denotes the height of the cylinder, then $r^2=100-h^2$.

Additionally, $\dfrac{\pi r^2 h}{\frac{2}{3}\pi(10)^3}=\dfrac{3r^2h}{2(10)^3}=\dfrac{9}{16} \rightarrow r^2h=3(5)^3=375$ (***)

Substituting for r^2, $r^2h=\left(100-h^2\right)h=375 \rightarrow h^3-100h+375=0$.

Since $h < 10$ and an integer, we note that $h = 1, 2, 3$ and 4 must be rejected (incorrect units digits) and, trying $h = 5$, we immediately see that it works. [125 – 500 + 375 = 0]

Therefore, substituting in (***), $r^2 = 75 \rightarrow r = \underline{\mathbf{5\sqrt{3}}}$

[Aside: $h^3-100h+375=(h-5)(h^2+5h-375)=0$

The quadratic factor gives additional values of $\dfrac{5\left(1\pm\sqrt{13}\right)}{2}$, but neither is an integer.]