

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2008
ROUND 1 ALG 2: ALGEBRAIC FUNCTIONS

ANSWERS

A) _____

B) { _____ }

C) _____ feet

A) Given $f: f(x) = \frac{x}{x+1}$, find $f(f(x))$ in simplified form.

B) Definition: $g(x) = \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \end{cases}$

If $f: f(x) = x \left(\frac{1 + g((x+2)(3-x))}{2} \right)$ and the domain of f is $\{x \mid x \leq -2 \text{ or } x \geq 3\}$,
determine the range of f .

C) Initially, a collapsing rectangle R_1 has a length of 40 feet and a width of 30 feet. Its length decreases at a constant rate of 4 ft/sec, while its width decreases at a constant rate of 3 ft/sec. At the same time, an expanding rectangle R_2 has a length of 4 feet and a width of 3 feet. Its length increases at a constant rate of 2 ft/sec and its width increases at a constant rate of 6 ft/sec. Determine the sum of the perimeters of the two rectangles at the time when the area of R_1 is 108 square feet greater than the area of R_2 .