MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 – MARCH 2012 SOLUTION KEY

Round 4

A) Using the Pythagorean Theorem, $\left(\sqrt{n}\right)^2 + \left(\sqrt{n+4}\right)^2 = \left(\sqrt{n+8}\right)^2$

Since $n > 0$, $n + (n + 4) = (n + 8) \Rightarrow n = 4 \Rightarrow$ sides: $2, \sqrt{8}, \sqrt{12} \Rightarrow$ **$2, 2\sqrt{2}, 2\sqrt{3}$**

B) If the line is vertical (or $P = Q$), the slope is undefined (or indeterminate).
Equating the x - coordinates, the line is vertical when $6 - a = 3$, namely for $a = 3$.
For any other value of a, the slope is always the same, so we simply pick another value for a and substitute

$a = 0 \Rightarrow P(6, 4), Q(3, 7) \Rightarrow m = \dfrac{7-4}{3-6} = \dfrac{3}{-3} = -1$

Thus, $(m, k) =$ **$(-1, 3)$**.

FYI: Proof that the slope is always -1 (unless $a = 3$)

$m = \dfrac{4-(7-a)}{(6-a)-3} = \dfrac{-3+a}{3-a} = \dfrac{-\cancel{(3-a)}}{\cancel{3-a}} = -1$

C) $\begin{cases} \dfrac{s-5}{d-5} = \dfrac{5}{2} \\ s+15 = 2(d+7) \end{cases} \Rightarrow s = 2d - 1$. Substituting, $\dfrac{2d-6}{d-5} = \dfrac{5}{2}$

Cross multiplying, $4d - 12 = 5d - 25 \Leftrightarrow d = 13$ and $s = 25$.

If the 10 : 7 ratio occurs in x years, $\dfrac{25+x}{13+x} = \dfrac{10}{7} \Leftrightarrow 175 + 7x = 130 + 10x \Leftrightarrow 3x = 45 \Leftrightarrow x =$ **15**