MASSACHUSETTS MATHEMATICS LEAGUE CONTEST 1 - OCTOBER 2011 SOLUTION KEY

Round 6

- A) As a difference of perfect square, $(394387)^2 (394381)^2 = (394387 + 394381) (394387 394381) = (788768)(6) = 4732608$.
- B) Avoiding solving for *x* and *y*, let's try combining the two equations and getting the required expression.

$$A(x + y = 5) + B(2x - 3y = 8) \Rightarrow (A + 2B)x + (A - 3B)y = 5A + 8B$$

If A + 2B = 7 and A - 3B = -8, so we get the required expression 7x - 8y.

Solving for A and B, (A, B) = (1, 3)

The required numerical value is 5(1) + 8(3) = 29.

C)
$$1 - \frac{1}{2 - \frac{1}{3 - \frac{1}{4}}} = 1 - \frac{1}{2 - \frac{1}{\frac{11}{4}}} = 1 - \frac{1}{2 - \frac{4}{11}} = 1 - \frac{1}{\frac{18}{11}} = 1 - \frac{11}{18} = \frac{7}{18}$$

 $(0.2\overline{3} + +0.0\overline{4}) = 0.2\overline{7}$

Converting the repeating decimal,

Let
$$N = 0.2\overline{7}$$
. Then:
$$\begin{cases} 100N = 27.\overline{7} \\ 10N = 2.\overline{7} \end{cases} \Rightarrow 90N = 25 \Rightarrow N = \frac{5}{18}.$$

Thus,
$$\frac{7}{18} + \frac{5}{18} = \frac{2}{3}$$
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