MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2008 SOLUTION KEY

Round 3

A) $\frac{1}{\sin 2\theta}+\frac{\cos 2\theta}{\sin 2\theta}=\frac{1+\cos 2\theta}{\sin 2\theta}=\frac{1+2\cos^2\theta-1}{2\sin\theta\cos\theta}=\frac{\cos\theta}{\sin\theta}=\cot(\theta)=1$
→ θ = **<u>45°, 225°</u>**

B) Using the double angle formula, $\sin(x) = 1 - 2\cos^2 40° = -(2\cos^2 40° - 1) = -\cos(80°) = -\sin(10°)$
The related values of 10° in quadrants II, III and IV are 170°, 190° and 350°.
Since $\sin(x)$ is negative only in quadrants III and IV, x = **<u>190° or 350°</u>**.

C) Let $A = Arc\cos(-n/11)$ and $B = Arc\tan\left(-1/\left(2\sqrt{6}\right)\right)$.
Then $\pi/2 < A < \pi$ (quadrant 2) and $-\pi/2 < B < 0$ (quadrant 4)
and $\sin(A + B) = \sin A\cos B + \sin B\cos A = \frac{\sqrt{121-n^2}}{11}\cdot\frac{2\sqrt{6}}{5}+\frac{-1}{5}\cdot\frac{-n}{11}=\frac{2\sqrt{6}\sqrt{121-n^2}+n}{55}$
Thus, $2\sqrt{6}\sqrt{121-n^2}+n=53$ and the radicand $121 - n^2$ must be 6 times a perfect square
Additionally, since $n/11$ is a cosine value, the only possible integer values of n are 1 ... 11.
Only n = **<u>5</u>** satisfies both conditions ($2\sqrt{6}\sqrt{96}+5=2\cdot 6\cdot 4+5=53$).