MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2013 SOLUTION KEY

Round 2

A) *A* and *B* must be squares of consecutive integers. Since the gap between consecutives squares grows as the squares get larger, we take the two largest 3-digit perfect squares.
$30^2 = 900$, $31^2 = 961$, but $32^2 = 1024$, so the maximum difference is 961 – 900 = **<u>61</u>**.

B) Today is 12/5/2013.
Let DOW denote day of the week. The DOW sequence is MonTueWedThuFriSatSunMon… .
There are 365 days in a year (unless it's a leap year, in which case February 29th makes 366 days). In a 365 day year, there are $\left[\frac{365}{7}\right] = 52$ weeks, plus 1 extra day.
Thus, from one year to the next, a specific date advances one day of the week, unless there is an intervening leapday!
2012 was a leap year and 2016, 2020, 2024 will be as well.
For example, 12/5/2016 falls 2 DOWs after 12/5/2015, because of the extra day 2/29/2016.
The sequence of DOWs for 12/5, starting in 2013 is Thu, Fri(2014), Sat(2015), Mon(2016), Tue (2017), Wed(2018), Thu(**<u>2019</u>**).

C) $\frac{1}{7} = 0.\overline{142857} \Rightarrow d = 3$
Therefore, *N* is an <u>odd</u> multiple of 33. There are 25 primes less than 100:
2,3,5,7, 11,13,17,19, 23,29, 31,37, 41,43,47, 53,59, 61,67, 71,73,79, 83,89, 97
88% of 25 is 22 $\Rightarrow$ *N* is divisible by exactly 3 distinct primes, including 3 and 11.
The smallest digit sum is 6 if *N* can be formed using the digits 0, 1, 2 and 3.
All other sets of possible digits with a digit sum of 6 will have at least one repeated digit.
A 4-digit integer consisting of digits 0, 1, 2 and 3 will always be divisible by 3.
There are 24 arrangements of these *N*-values divisible, only 18 are 4-digit numbers and only 12 of these are odd. Divisibility by 11 narrows the field to 2: 1023 and 2013
The smallest value 1023 = $3 \cdot 11 \cdot 31$ and the next smallest is 2013 = $3 \cdot 11 \cdot 67$. Thus, *N* = **<u>2013</u>**.
(Without the restriction that the digits were distinct, we would have had to consider *N*-values formed from {1, 1, 2, 2}, {1, 1, 1, 3}, {1, 1, 0, 4} and {2, 2, 2, 0}. Only the first set of digits produces a multiple of 11. In this case, the second integer in the list would have been 1221.)

We could also have proceeded by brute force, examining products of the form $3 \cdot 11 \cdot x$, where *x* is a prime such that $x > \left[\frac{1000}{33}\right] = 30$.

31: 1023 (smallest)
37: 1221 (rejected, repeated digits)
41: 1353 (rejected, digit sum = 12)
43: 1419 (rejected, digit sum = 15)
47: 1551 (rejected, digit sum = 12)
53: 1749 (rejected digit sum = 21)
59: 1947 (rejected, digit sum = 21)
61: **<u>2013</u>** Bingo!