MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2014 SOLUTION KEY

Round 4

A) We require that 60 be divisible by 5 consecutive integers.
This is only true for 1 …5 and 2 … 6. The latter gives the smaller sum.
For x = 2, we have $30+20+15+12+10=\underline{\mathbf{87}}$.

B) The 2 hour barrier is equivalent to $2\cdot 60\cdot 60 = 7200$ seconds. Our future marathoner must complete the course in 7199 seconds. At 4:35 per mile, (s)he would take $26.2(275) = 7205$ seconds.
Thus, k = **6**.
Note:
Meb Keflezighi (pronounced Kef-lez-ghee) won the 2014 Boston Marathon in 2:08:37, beating the Kenyan Wilson Chebet by 11 seconds. Meb averaged approximately 4 minutes 54.5 seconds per mile. The marathon is actually 26 miles 385 yards (or 26.21875 miles).

C) $\dfrac{8x^2\left[4+\left(\dfrac{x}{2}-\dfrac{2}{x}\right)^2\right]}{\left(x^2+4\right)^2} \Leftrightarrow \dfrac{8x^2\left[4+\dfrac{x^2}{4}-2+\dfrac{4}{x^2}\right]}{x^4+8x^2+16} \Leftrightarrow \dfrac{16x^2+2x^4+32}{x^4+8x^2+16} \Leftrightarrow \dfrac{2\cancel{\left(x^4+8x^2+16\right)}}{\cancel{x^4+8x^2+16}} = \underline{\mathbf{2}}$