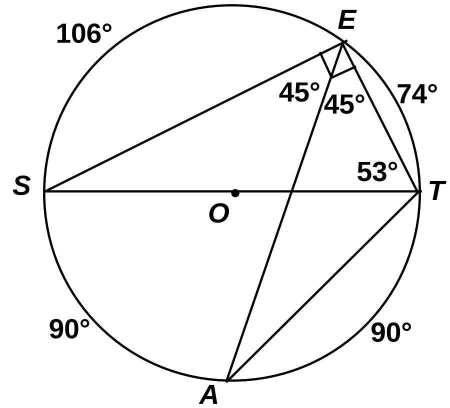
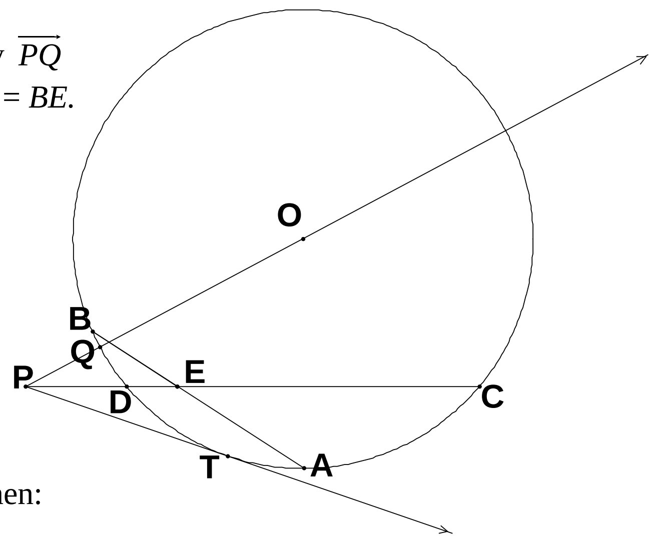
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2013 SOLUTION KEY

Round 5



A) Since $\angle SET$ is inscribed in a semi-circle, it's a right angle. Thus, $m\angle SEA = m\angle TEA = 45°$. As intercepted arcs of $\angle SEA$ and $\angle ETS$, $m\overset{\frown}{SA} = 90°, m\overset{\frown}{SE} = 106° \Rightarrow m\overset{\frown}{ET} = 74°$ $\Rightarrow \overset{\frown}{ETA} = 90 + 74 = \underline{\mathbf{164}}$.



B) Let R denote the second point of intersection of ray $\overrightarrow{PQ}$ and circle O and recall that we were given that $PD = BE$. Applying the product chord theorem,
$AE \cdot BE = CE \cdot DE \Rightarrow 3AE = 1 \cdot 6 \Rightarrow BE = PD = 2$
Applying the tangent-secant rule,
$PD \cdot PC = PT^2 \Rightarrow PT^2 = 2 \cdot (2+7) = 18$ and
$PQ \cdot PR = 18 \Rightarrow 1.5(1.5 + 2r) = 18$
$\Rightarrow 2r + 1.5 = 12 \Rightarrow r = \underline{\mathbf{5.25}}$

C) Let D denote the diameter of one of the circles. Then:

$$3D = \sqrt{162} \Rightarrow 6R = 9\sqrt{2} \Rightarrow R = \frac{3\sqrt{2}}{2}$$

From the diagram at the right, we see that the "star-shaped" region between the circles has an area equal to a square minus a full circle,

i.e. $\left(3\sqrt{2}\right)^2 - \pi\left(\frac{3\sqrt{2}}{2}\right)^2 = 18 - \frac{9\pi}{2}$

Multiplying by 4, the required area is $\underline{\mathbf{72 - 18\pi}}$ or $\underline{\mathbf{18(4-\pi)}}$.