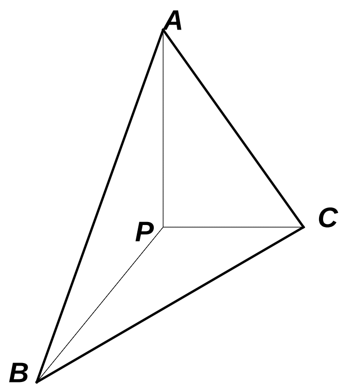
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2013
ROUND 7 TEAM QUESTIONS

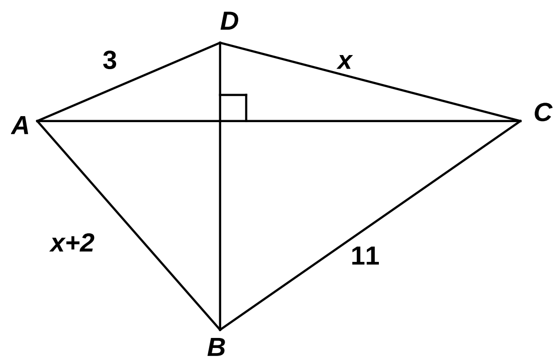
ANSWERS

A) (______ , ______ , ______) D) ____________________________

B) ____________________________ E) ____________________________

C) (__________ , __________) F) (__________ , __________)





A) Each of the angles at vertex P in tetrahedron $PABC$ is a right angle. $PA^2 = 224$, $PB^2 = 560$ and $PC^2 = 65$. The distance from vertex P to the plane ABC, expressed as a simplified radical is $\frac{A}{B}\sqrt{C}$. Determine the ordered triple of integers (A, B, C).

B) Compute the perimeter of quadrilateral $ABCD$ (See diagram above.)

C) If $y = x + 1$, there are unique <u>integer</u> values of x and y for which the points A, B and C are collinear.

$$A(2x+1,\ 3y),\ B(8y-1,\ 9x),\ C(17y+9x,\ 10(x+y)-3)$$

Compute the coordinates of the point closest to the origin.

D) For constants A, B and C, the following equation is an identity, that is, true for all values of x.

$$\frac{3}{(x+1)(x+2)^2} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

Of course, both sides are undefined for $x = -1, -2$. Compute $A^3 + B + C$.

E) Let $x \blacklozenge y = \frac{x+1}{2-y}$ and $S = \{(x, y)\colon\ |x| + |y| \le 4$, where x and y are integers$\}$.

For <u>how many</u> ordered pairs (x, y) does $x \blacklozenge y = y \blacklozenge x$?

F) Consider the <u>closed</u> interval $[6, (2+a)(3+b)]$, where $1 < a \le 10$, $0 < b$ and $ab = 1$.

Let m denote the minimum and M denote the maximum number of integer perfect squares that are included in this interval. Compute the ordered pair (m, M).