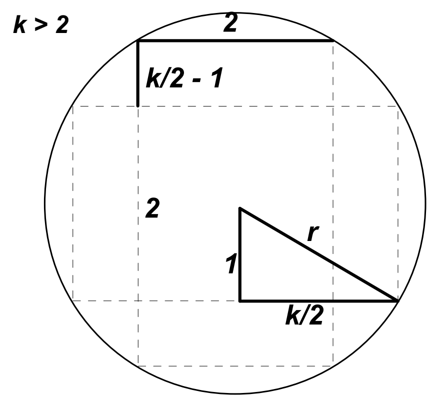
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2010 SOLUTION KEY

Team Round – continued

D) Let (A, 24, P) denote the current ages of Al, Sue and Pam respectively.
8 years ago, their ages were (A – 8, 16, P – 8) and (A – 8) + (P – 8) = 24 or $A + P = 40$.
Clearly, if A – 8 denotes a positive integer, the minimum value of A is 9.
Since Al is a younger brother, $A < 24$, but the second condition that Pam is an older sister puts a more restrictive condition on the maximum value of A.
Substituting $P = 40 - A$, we have $40 - A > 24$ or $A < 16$ and the maximum value of A is 15.
Thus, there are **7** possible values of A (15 – 9 + 1).

E) The area bounded by the overlapping rectangles: $4\left(2\left(\frac{k}{2}-1\right)\right) + 4 = 4k-4 = 4(k-1)$



The area of the circle: $\pi r^2 = \pi\left(1+\frac{k^2}{4}\right)$

$4(k-1) = \frac{\pi r^2}{2}$ ➔ $8(k-1) = \pi\left(1+\frac{k^2}{4}\right)$

➔ $\pi k^2 - 32k + 4(\pi+8) = 0$ ➔ $k = \frac{32 \pm \sqrt{32^2 - 16\pi(\pi+8)}}{2\pi}$

$= \frac{32 \pm 4\sqrt{64-\pi(\pi+8)}}{2\pi} = \frac{16 \pm 2\sqrt{64-\pi(\pi+8)}}{\pi}$

The "+" sign gives the maximum value of k and we have
(A, B, C, D) = **(16, 2, 64, 8)**

For curiosity sake, $k \approx 8.521121922\cdots$ produces an area that is approximately half that of the circle. Check it out!
Using the "–" sign, $k \approx 1.664794436\cdots$ and must be rejected.
(You may want to verify that if $k < 2$, then $k = \frac{16 - 2\sqrt{64-\pi(\pi+8)}}{\pi+8} \approx 0.4694217542\cdots$.)