Round 1 – continued

Alternate solutions/Generalizations to 1C (Norm Swanson):

Assume ΔABC is isosceles with $AB = AC = \sqrt{7}$ and $BC = 2$. Then: $\cos B = \dfrac{1}{\sqrt{7}} = \sin\left(\dfrac{A}{2}\right)$

Using the double angle formula, $\cos A = 1 - 2\sin^2\left(\dfrac{A}{2}\right) = 1 - 2\left(\dfrac{1}{7}\right) = \dfrac{5}{7} \Rightarrow 7\left(\dfrac{25}{49}\right) - 4\left(\dfrac{1}{7}\right) = \dfrac{21}{7} = \underline{\mathbf{3}}$.

Or

Consider the "collapsed" triangle *ABC* where $A = B = 0°$ and $C = 180°$.
Then: $7\cos^2 A - 4\cos^2 B = 7\cos^2 0° - 4\cos^2 180° = 7(1) - 4(1) = \underline{\mathbf{3}}$

Try proving this generalization:
In any triangle *ABC*, where $\dfrac{BC}{AC} = \dfrac{n}{m}$, $m^2\cos^2 A - n^2\cos^2 B = m^2 - n^2$

This is equivalent to the identity $\dfrac{b\cos A - a\cos B}{b-a} \cdot \dfrac{b\cos A + a\cos B}{b+a} = 1$.