MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2015 SOLUTION KEY

Round 4

A) Completing the square on the right side,
$x = y^2 - 10y + 3 \Leftrightarrow x = \left(y^2 - 10y + 25\right) + 3 - 25 \Leftrightarrow (y-5)^2 - 22$
For all values of y, $\left(y-5\right)^2 \geq 0$. Therefore, the minimum value of x (namely m) is –22 and it occurs when $y = 5$. The required ordered pair is $\left(m,k\right) = \underline{\left(-\mathbf{22,5}\right)}$.

B) $2m^2x^2 - 7mx = -3m\left(1+x\right) \Leftrightarrow 2m^2x^2 - 4mx + 3m = 0 \Leftrightarrow m\left(2mx^2 - 4x + 3\right) = 0$
Examining the discriminant of the quadratic factor, $16 - 24m > 0 \Leftrightarrow m < \frac{2}{3}$ (but the quadratic equation disappears for $m = 0$). Thus, $\underline{\boldsymbol{m < \frac{2}{3}, m \neq 0}}$.

C) If $k = -7$, the equation becomes linear and has only one solution.
If $k \neq -7$, the equation is quadratic and, if the discriminant equals zero, there will be exactly one solution. $k^2 - 36(k+7) = 0 \Rightarrow k^2 - 36k - 252 = (k+6)(k-42) = 0$.
Thus, there are three values, $k =$ **<u>42, –6, –7</u>** (in any order).