MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2012 SOLUTION KEY

Round 6

A) $|T - 55| \le 50 \Rightarrow -50 \le T - 55 \le +50 \Rightarrow 5 \le T \le 105 \Rightarrow (T_{low}, T_{high}) = (5, 105) \Rightarrow$ **<u>110</u>**

B) The numbers in Carol's list are generated by $1000 - 7k$.
The numbers in Tarah's list are generated by $100 + 13k$.
Let's see if the lists share any common numbers. For the same value of k, does
$1000 - 7k = 100 + 13k$?
Since $900 = 20k$, for $k = 45$, the lists do share a common number.
Multiplying by 7 is easier than by 13, so we substitute in $100 - 7k$.
The common number is $1000 - 7(45) = 1000 - 315 = 685$.
Thus $(C, T) =$ **<u>(685, 685)</u>**.

<u>Note</u>: Since the least common multiple of 7 and 13 is 91, other numbers common to both lists will be of the form $685 + 91n$, for any integer n. For example, $n = \pm 1$ results in 594 and 776.
$594 = 100 + 13(38) = 1000 - 7(58)$ and $776 = 100 + 13(52) = 1000 - 7(32)$
The 39th number in Carol's list and the 59th number in Tarah's list is 594
The 53rd number in Carol's list and the 33rd number in Tarah's list is 776
Between 600 and 700, only 685 is common to both lists and $|C - T| = 0$, the minimum value for the absolute value of a difference.

C) Suppose the walkers rates are x and $2x$.
You may argue later whether the faster walker is Chris or Christine.
When travelling in the same direction, they are separating at $(2x - x) = x$ mph.
$RT = D \Rightarrow x$ (in mph) $\cdot$ T (in hrs) = $\frac{1}{4}$ mi. or T (in hrs) = $\frac{1}{4x}$
$x \cdot \frac{4}{3} + 2x \cdot \frac{4}{3} = 12 \Leftrightarrow 12x = 36 \Leftrightarrow x = 3$. Thus, $T = \frac{1}{12}$ hr. = 5 minutes = **<u>300</u>** seconds.