Team Round - continued

E) Let x denote the speed of the truck in feet/sec.

The length of each stripe is $x(1)$ feet.

The lengths of the gaps are $x(.3), x(.4), x(.5)$.

Thus, $AB = 4x + 1.2x = 104 \Rightarrow 5.2x = 104 \Rightarrow x = \underline{\mathbf{20}}$ feet/sec.

The stripes are 20 feet long and between x stripes there are $(x - 1)$ intervals.

If the first interval is 3/10 sec., the second 4/10 sec., then the $(x-1)^{\text{st}}$ interval is $(x+1)$ sec.

We require that $20x + 20 \cdot \dfrac{1}{10}(3 + 4 + ... + (x+1)) \le 5280$

Applying the summation formula for the arithmetic series $\langle 1 + 2 \rangle + 3 + 4 + ... + (x+1)$, we

have $20x + 2\left(\dfrac{(x+1)(x+2)}{2} - (1+2) \right) \le 5280$

$\Leftrightarrow 20x + \left(x^2 + 3x + 2 - 6 \right) \le 5280 \Leftrightarrow x^2 + 23x = x(x + 23) \le 5284$

By trial and error, let's start with $x = 60$ to get close to 5284 $(60 \cdot 83 = 4980, \ 61 \cdot 84 = 5124,$
$62 \cdot 85 = 5270)$. Thus, $\underline{\mathbf{62}}$ stripes and the intervening 61 intervals totals 5270 feet and the

spraying mechanism is now in the off cycle for the next $\dfrac{64}{10} = 6.4$ seconds, resulting in a gap

of $20(6.4) = 128$ feet and the 63rd stripe does not start until 5398 feet.