MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 – NOVEMBER 2008 SOLUTION KEY

Round 1

A) $\dfrac{1-\dfrac{1}{\sqrt{-6}}}{\sqrt{-2}}-\left(\dfrac{1-\sqrt{-2}}{2}\right)^2=\dfrac{\sqrt{-6}-1}{\sqrt{-2}\cdot\sqrt{-6}}-\dfrac{1-2\sqrt{-2}-2}{4}=\dfrac{i\sqrt{6}-1}{-2\sqrt{3}}-\dfrac{-1-2i\sqrt{2}}{4}$

$=\dfrac{i\sqrt{18}-\sqrt{3}}{-6}+\dfrac{1+2i\sqrt{2}}{4}=\dfrac{-3i\sqrt{2}+\sqrt{3}}{6}+\dfrac{1+2i\sqrt{2}}{4}=\dfrac{-6i\sqrt{2}+2\sqrt{3}}{12}+\dfrac{3+6i\sqrt{2}}{12}=\dfrac{2\sqrt{3}+3}{12}$

→ (A, B) = **<u>(2, 3)</u>**

B) $\left(\sqrt{2}+i\sqrt{3}\right)^{600}\cdot\left(\sqrt{2}-i\sqrt{3}\right)^{600}=\left(\left(\sqrt{2}+i\sqrt{3}\right)\cdot\left(\sqrt{2}-i\sqrt{3}\right)\right)^{600}=\left(2-i^2\cdot 3\right)^{600}=5^{600}$

$=\left(5^2\right)^{300}=\left(5^3\right)^{200}=\left(5^4\right)^{150}=\left(5^5\right)^{120}$ etc → (A, B) = **<u>(625, 150)</u>**

C) Cross Multiplying, $10x+2xi+5yi+yi^2=10+5i$ → $(10x-y)+(2x+5y)i=10+5i$

→ $\begin{cases}10x-y=10\\2x+5y=5\end{cases}$

The usual approach would be to:

multiply the 1st equation by 5 and then add the equations → $\left(x=\dfrac{55}{52}\right)$.

multiply the 2nd equation by −5 and then add the equations → $\left(y=\dfrac{15}{26}\right)$.

Adding these results we have $\underline{\mathbf{\dfrac{85}{52}}}$ Nothing new here!

However, if the system had been $\begin{cases}13x-7y=4\\11x+9y=2.5\end{cases}$, then solving for x and y separately would have been very tedious and unnecessary. The following seems like magic, but it can be a real time saver.

We must find a <u>linear combination</u> of the left hand sides of each equation for which the coefficients of x and y are equal.
Suppose this happens when we multiply $(10x-y)$ by some A and $(2x+y)$ by some B.

Regrouping $A(10x-y)+B(2x+5y)$, we have $(10A+2B)x+(5B-A)y$
Equating the x and y coefficients, $10A+2B=5B-A$ → $11A=3B$. Let's pick $A=3$ and $B=11$.
Multiplying the 1st equation by $A=3$ and the 2nd equation by $B=11$ produces

$\begin{cases}30x-3y=30\\22x+55y=55\end{cases}$ Adding, $52x+52y=85$ → $x+y=\underline{\mathbf{\dfrac{85}{52}}}$

Try the suggested system above the usual way and using the technique outlined above.

$x+y=\dfrac{21}{97}$. How much time did YOU save?