

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 – MARCH 2012
ROUND 1 ALG 2: SIMULTANEOUS EQUATIONS AND DETERMINANTS

ANSWERS

A) (_____ , _____ , _____ , _____)

B) (_____ , _____ , _____)

C) _____

******* NO CALCULATORS IN THIS ROUND *******

A) For a unique ordered quadruple (x, a, b, c) ,
$$\begin{bmatrix} 19-3x & 2x+b \\ x+a & \frac{x}{3}+c \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
, the 2 x 2 identity matrix.
 Compute (x, a, b, c) .

B) There are 3 integers n_1, n_2 and n_3 for which
$$\begin{vmatrix} 1 & x & 2 \\ x & 1 & 2 \\ 1 & 2 & x \end{vmatrix} = 0.$$

 Compute the ordered triple (n_1, n_2, n_3) , where $n_1 < n_2 < n_3$.

C) For positive integers a and b , the system of equations
$$\begin{cases} x + y + 1 = 0 \\ 2x - y + a = 0 \\ 3x + 4y + b = 0 \end{cases}$$
 defines a set of concurrent lines, i.e. they intersect at a common point. Determine (x, y) , the coordinates of the point of concurrency, for which $a + b$ is a maximum.