MASSACHUSETTS MATHEMATICS LEAGUE
NOVEMBER 2005 BRIEF SOLUTIONS

Round One:

A. $(a + bi)^2 = a^2 - b^2 + 2abi = 0 + 1i$ so 2ab=1.

B. $(i + 5 - 3 - 7i)^2 = 4 - 24i - 36 = -32 - 24i$

C. $-6 - 2\sqrt{12} - 2 + \dfrac{16i(1 - i\sqrt{3})}{1-(-3)} - (-8) = -4\sqrt{3} + \dfrac{16i + 16\sqrt{3}}{4} = 4i$

Round Two:

A. a+b=30000; 0.09a + 0.08b=350 so a= \$10,000 and a = **\$20,000**

B. C is the average of A and E and also of B and D so sum = 5C so p=14 so Saturday was day 15 and 8 and 1.

C. H+T+U=14; H+U=T+4; subtract. T=10-T so T=5. H+50+U-(H+10U+5)=18 so U=3.

Round Three:

A. Side is 20, diagonal is $20\sqrt{2}$.

B. $\Delta SER \sim \Delta HSR$ ratio 4:5. Area ΔSER is 16/25 of $\Delta HSR = 384$. Subtract from 1200.

C. Area implies other diagonal is 14. Thus $12^2 + x^2 = a^2$ while $12^2 + (14-x)^2 = b^2$, Integer solutions suggest 9-**12**-15 and 5-**12**-13 triangles (14=9+5)

Round Four:

A. n is the difference of factors of 50 so a = 50 – 1 or 25 – 2 or 10 – 5.

B. $3(2x^3 + x^2 - 4x - 2) = 3[\, x^2(2x + 1) - 2(2x + 1)] = \mathbf{3(2x+1)(x^2 - 2)}$

C. Simplify to $x^4 + x^2 + 25 = x^4 + 10x^2 + 25 - 9x^2 = (x^2 + 5)^2 - (3x)^2$ etc.

Round Five:

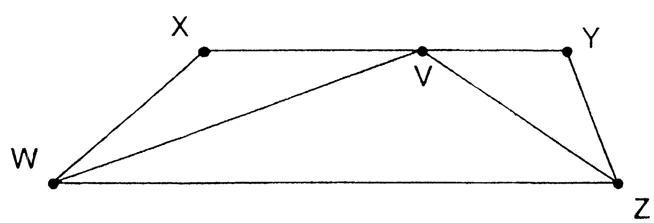
A. $(-2) - 2(\sin^2 \frac{\pi}{12} + \cos^2 \frac{\pi}{12}) + (-\sqrt{3})^2 - 2(1) = -3$

B. 2sinx cos x + sin x = sin x (2cos x +1) so sin x =0 or cos x = -0.5 thus x = 180, 360 or 120, 240

C. ABC equilateral so BC=$10\sqrt{3}$. BCD 30-60-90 so CD=20. CFD isos rt so FD=$10\sqrt{2}$. FDH 30-60-90 so DH = $5\sqrt{6}$

Round Six:

A. ΔWXV and ΔZYV are isosceles so XY=6+8=14 so WZ=(9/5)14.



B. $\angle AEH = 60$, $\angle FGC=45$, $\angle ADF=105$, $\angle AFD=30$.