MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2011
ROUND 2 ALG1: EXPONENTS AND RADICALS

ANSWERS

A) _____

B) (_____ , _____), (_____ , _____)

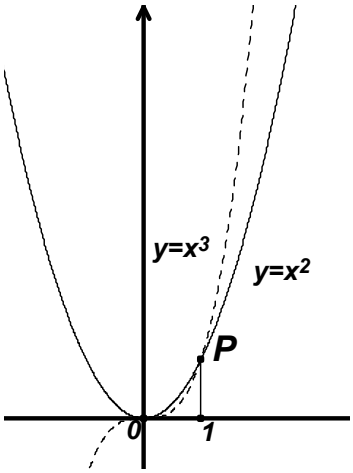
C) (_____ , _____)

******* NO CALCULATORS ON THIS ROUND *******

A) Clearly, the cube of a positive integer is <u>always</u> greater than or equal to the square of a positive integer. However, this is not true <u>for all positive real numbers</u> as you can see from the graph at the right. The value of x^2 exceeds the value of x^3 over the interval $0 < x < 1$, since the solid line is above the dotted line. The <u>difference</u> D between x^2 and x^3 is largest for one of the following values:

$$\left(x=\frac{1}{3}\right), \left(x=\frac{1}{2}\right) \text{ or } \left(x=\frac{2}{3}\right).$$

Compute the <u>largest</u> possible value of D.



B) Find <u>both</u> ordered pairs of integers (A, B) that satisfy the system $\begin{cases}(A+B)^3=-8\\(A-B)^2=2^{2^3}\end{cases}$.

Recall: $x^{y^z}=x^{\left(y^z\right)}$

C) In simplified form, $\sqrt{37-20\sqrt{3}}=a+b\sqrt{3}$, where a and b are integers.
Determine the quadrant in which point $P(a, b)$ lies and compute its distance from the origin.
Express your answer as an ordered pair (quadrant, distance).