

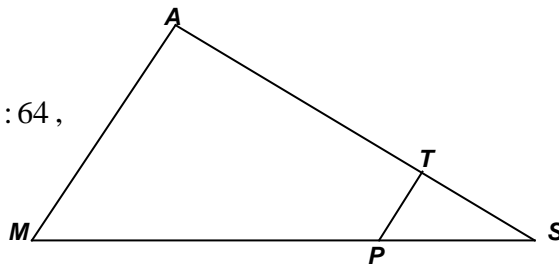
**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 4 - JANUARY 2013 SOLUTION KEY**

Round 5

A) $\overline{TP} \parallel \overline{AM} \Rightarrow \triangle STP \sim \triangle SAM$

Their areas are in a ratio of $18 : (18 + 110) = 18 : 128 = 9 : 64$,
implying their corresponding sides are in a 3 : 8 ratio.

$$\frac{ST}{SA} = \frac{3}{8} \Rightarrow \frac{ST}{TA} = \frac{3}{5}$$



B) Let $ST = x$ and $SA = kx$. Then:

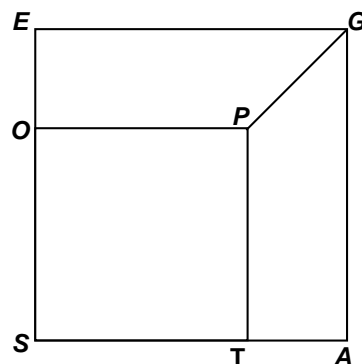
$$PG = SG - SP = kx\sqrt{2} - x\sqrt{2} = 4\sqrt{2} \Rightarrow x(k - 1) = 4$$

$$(xk)^2 - x^2 = 80 \Rightarrow x^2(k^2 - 1) = 80$$

Dividing, $x(k + 1) = 20$

Therefore, $\frac{\cancel{x}(k+1)}{\cancel{x}(k-1)} = \frac{20}{4} = 5 \Rightarrow k + 1 = 5k - 5 \Rightarrow k = \frac{3}{2}, x = 8$

$\Rightarrow \text{area}(SAGE) = 12^2 = \underline{144}$.



C) Regardless of the location of point G on \overline{NE} ,
the area of $\triangle GHI$ is half the area of the rectangle.

Therefore, $\frac{3a}{2} = 9.375 = 9\frac{3}{8} = \frac{75}{8} \Rightarrow a = \frac{25}{4} = 6.25$

Let $GE = x$. Then:

$$\triangle HGE \sim \triangle GIN \Rightarrow \frac{GE}{IN} = \frac{HE}{GN}$$

$$\Rightarrow \frac{x}{3} = \frac{3}{a-x} = \frac{3}{\frac{25}{4}-x} \Rightarrow \frac{25}{4}x - x^2 = 9 \Rightarrow 4x^2 - 25x + 36 = (4x-9)(x-4) = 0 \Rightarrow x = \frac{9}{4} = 2.25 \text{ or } 4$$

Since we were given that $GE > NG$, $GE = \underline{4}$.

