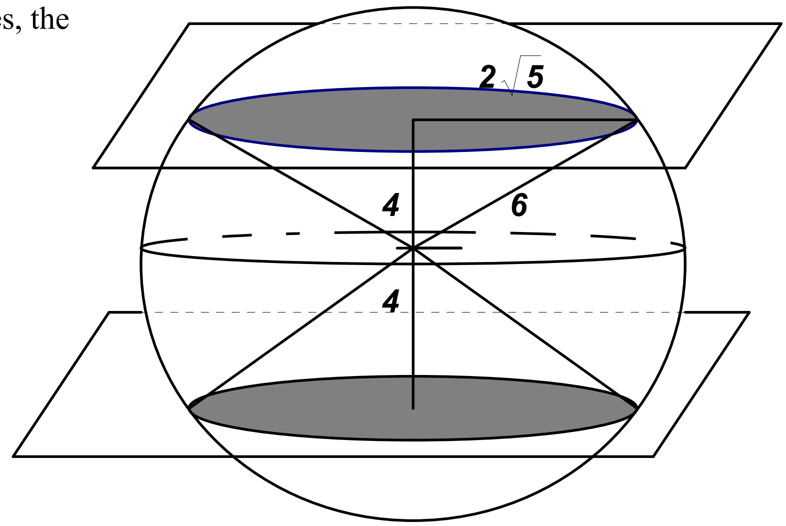
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2007 SOLUTION KEY

Round 1

A) Let x denote the increase in radius or height. Then $\pi(10+x)^2(4) = \pi(10)^2(4+x)$
$\rightarrow 4(100+20x+x^2) = 400+100x \rightarrow 4x^2 - 20x = 4x(x-5) = 0 \rightarrow x = \underline{\mathbf{5}}$

B) The regions consist of two congruent circles, the intersection of a sphere and two parallel planes.

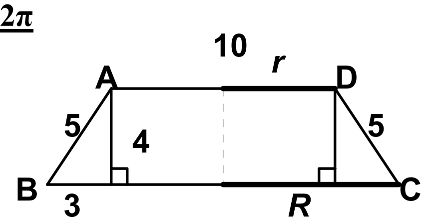
Area = $2\pi(2\sqrt{5})^2 = \underline{\mathbf{40\pi}}$

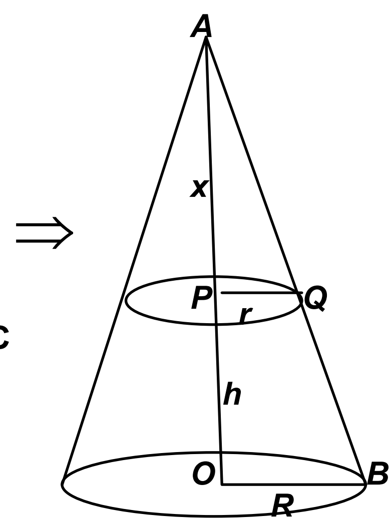


C) Method #1 [using V(frustum) = $\frac{\pi h}{3}\left(R^2 + Rr + r^2\right)$]

$r = 5$, $R = 8$ and $h = 4$

Thus, V = $\frac{1}{3}\pi(4)[25+40+64]$ = $\underline{\mathbf{172\pi}}$





Method #2 [V(cone$_1$) – V(cone$_2$)]

(Note: $\Delta APQ \sim \Delta AOB \rightarrow \frac{x}{x+h} = \frac{r}{R} \rightarrow x = \frac{hr}{R-r}$)

$\frac{x}{x+4} = \frac{5}{8} \rightarrow x = \frac{20}{3}$

V(cone$_1$) = $\frac{1}{3}\pi(8)^2\left(4+\frac{20}{3}\right) = \frac{2048\pi}{9}$ V(cone$_2$) = $\frac{1}{3}\pi(5)^2\left(\frac{20}{3}\right) = \frac{500\pi}{9}$

Therefore, V(frustum) = $\frac{1548\pi}{9}$ = $\underline{\mathbf{172\pi}}$