MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 – FEBRUARY 2007
ROUND 7 TEAM QUESTIONS

ANSWERS

A) _____ D) _____ feet

B) _____ E) _____ °

C) _____ F) _____

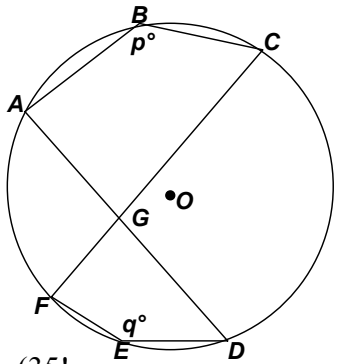
A) Given $f(x) = 3x - 1$ and $g(x) = 5x + 2$
Determine all possible x for which $g^{-1}(f(x)) = f^{-1}(g(x))$

B) Let P = the product of the natural numbers from 1 to 25 inclusive.
Let S = the sum of the natural numbers from 1 to 25 inclusive.
Let $N = P + S$
What is the sum of the rightmost 7 digits of N?

C) In ΔABC, $m\angle A = 30°$, $BC = 7$ and it is known that AB is an integer.
For how many different integer values is ΔABC an acute triangle?

D) Building #1 is <u>twice</u> as tall as building #2. Each building has a lobby with 12 foot ceilings and additional floors with ceilings of uniform height. However, the ceiling heights in building #1 are 6 inches more than in building #2 and building #1 has 3 more floors than building #2. If the ceiling heights in both buildings must be at least 8 feet high, what is the minimum height (in feet) of building #1?

E) Given: $m\angle ABC = p°$ and $m\angle FED = q°$
A, B, C, D, E and F are points on circle O and
G is the point of intersection of chords $\overline{AD}$ and $\overline{CF}$
as indicated in the diagram
Determine the degree measure of $\angle AGC$ in terms of p and q.



F) Given the sequence $t_1 = (29! + 30! + 31!)$, $t_2 = (30! + 31! + 32!)$, …, $t_7 = (35! + 36! + 37!)$
Let p_k be the largest prime factor of t_k.
Determine the value of $\sum_{k=1}^{k=7} p_k$.