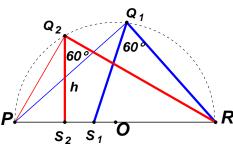
Addendum #2: Team A Question

The actual question used did not add the restriction the sum L + M + N is a minimum. Strangely, no appeal was made and there should have been!

Here is the case made by Norm Swanson – Hamilton Wenham

Since no lengths are given, we can choose any convenient lengths. Furthermore, there are many possible locations for point Q for which $m \angle PQR$ is a right angle, since any point on a semi-circle with center O and diameter \overline{PR} will suffice. By choosing a specific point Q_2 , we can let \overline{PS} (with length h) be an altitude to \overline{PR} , dividing $\angle PQR$ into 30° and 60° angles.



Convenient lengths:

$$PQ_2 = 2 \implies Q_2R = 2\sqrt{3}, PR = 4, a = PS_2 = 1, b = S_2R = 3, h = Q_2S_2 = \sqrt{3}$$

$$PQ_2 = 2 \Rightarrow Q_2 R = 2\sqrt{3}, PR = 4, a = PS_2 = 1, b = S_2 R = 3, h = Q_2 S_2 = \sqrt{3}$$
The given formula $h^2 = \frac{La^2b^2}{Ma^2 + Nb^2}$ becomes $3 = \frac{9L}{M + 9N}$ or $1 = \frac{3L}{M + 9N}$ or $L = \frac{M}{3} + 3N$.

Since all three constants are positive integers, M must be a multiple of 3 and we get the minimum sum if we let M = 3, resulting in L = 1 + 3N.

Thus, the minimum sum occurs when N = 1 and (L, M, N) = (4, 3, 1).

However, (L, M, N) is <u>not</u> unique.

In fact, any triples of the form (k+3j,3k,j), where j and k are positive integers, are solutions.