MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 – MARCH 2013 SOLUTION KEY

Round 4

A) Let x denote his average score in the last 4 meets. Then:

$\frac{0+4+4x}{6} = 6 \Leftrightarrow 4x = 32 \Rightarrow x = 8$

He needs 36 points for the year to have an average of 6.
Through 5 meets he has scored 32 points, so he needs only 4 points in meet 6.
Thus, $(k, N) =$ **(4, 8)**.

B) $\left(6x^2 - 8x + 11\right) - \left(-5x^2 + kx + 10\right) = 11x^2 - \left(8 + k\right)x + 1$

This only possible factorizations of this trinomial are $(11x-1)(x-1)$ or $(11x+1)(x+1)$.
Thus, the coefficient of the middle term is ±12.
Equating, $-(8+k) = \pm 12 \Rightarrow 8 + k = \mp 12 \Rightarrow k = \cancel{-20}, \underline{\mathbf{4}}$.

C) At 6 minutes per mile, Paul runs at 10 mph.

At 9 minutes per mile, Ron runs at $\frac{60}{9} = \frac{20}{3}$ or $6\frac{2}{3}$ mph.

Since they meet in 1 hour, $PQ = 10 \cdot 1 + \frac{16}{3} \cdot 1 = 16\frac{2}{3}$ miles.

Reducing his mile time by 1 minute, increases Paul's rate to $\frac{60}{6-1} = 12$ mph

Increasing his mile time by 1 minute decreases Ron's rate to $\frac{60}{9+1} = 6$ mph.

Assuming they meet in T hours at these new rates, $12T + 6T = 16\frac{2}{3} \Rightarrow 54T = 50 \Rightarrow T = \frac{25}{27}$ hour

$\frac{25}{27}$ hr $= \frac{25}{27} \cdot 60 = \frac{500}{9} = 55\frac{5}{9}$ min $= 55$ min $\frac{100}{3}$ sec $= 55$ min $33\frac{1}{3}$ sec

Thus, the elapsed time to cover the distance from P to Q decreased by 4 minutes $26\frac{2}{3}$ seconds, and we have $(A, B) =$ **(4, 27)**.