

**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2014 SOLUTION KEY**

Round 5

A) Given: $\frac{174-37n}{4n-3}$

$$n = 1 \Rightarrow \frac{137}{1} = 137$$

Note:

As n increases by 1, the numerator will decrease by 37 and the denominator will increase by 4.

$$n = 2 \Rightarrow \frac{100}{5} = 20 \quad n = 3 \Rightarrow \frac{63}{9} = 7 \quad n = 4 \Rightarrow \frac{26}{13} = 2 \quad n = \underline{5} \Rightarrow \frac{-11}{17}$$

B) We are given that $F = k \cdot \frac{a(b+c)}{d^2}$, where k is the proportionality constant.

Substituting, $96 = k \cdot \frac{80 \cdot 12}{16} = 60k \Rightarrow k = \frac{96}{60} = \frac{8}{5}$. Let $(a, b, c) = (x, 2x, 3x)$. Then:

$$50 = \frac{8}{5} \cdot \frac{x(2x+3x)}{12^2} = \frac{8}{5} \cdot \frac{5x^2}{12^2} = \frac{x^2}{18} \Rightarrow x^2 = 18 \cdot 50 = 9 \cdot 100 \Rightarrow x = 30 \Rightarrow c = 90$$

Thus, $(k, c) = \left(\underline{\frac{8}{5}}, \underline{90} \right)$.

C) Suppose I gets h hits in x additional at-bats. To exceed a 0.400 average

$$\frac{56+h}{172+x} > \frac{2}{5} \Rightarrow 280+5h > 344+2x \Rightarrow h > \frac{64+2x}{5} \text{ and } x+172 \geq 400 \Rightarrow x \geq 228.$$

$$x = 228 \Rightarrow h > \frac{64+456}{5} = \frac{520}{5} = 104 \Rightarrow h_{\min} = 105$$

As x increases, so does $\frac{64+2x}{5}$ which forces h to increase as well.

Thus, **105** is the minimum.

Check: $h = 104 \Rightarrow \frac{56+104}{400} = \frac{160}{400} = \frac{2}{5} = 0.400$

$$h = 105 \Rightarrow \frac{56+105}{400} = \frac{161}{400} = 0.4025$$