## MASSACHUSETTS MATHEMATICS LEAGUE
## MARCH 2005 BRIEF SOLUTIONS

**Round One:**

A. c = 5 so b = – 1 so a = –10 so abc = 50.

B. st = 49 = s(17-s) Use quadratic formula to get $s = \dfrac{17 \pm \sqrt{93}}{2} = 8.5 \pm 0.5\sqrt{93}$

C. No solution if determinant of coef. matrix = 0 Solving $\begin{vmatrix} 1 & 2 & 3 \\ 2 & -1 & c \\ c & -2 & 6 \end{vmatrix} = 0$ we have $-6 + 2c^2 - 12 + 2c + 3c - 24 = 0$ so c = -6 or 3.5

**Round Two:**

A. $\sqrt{\dfrac{144 + 64 + 81}{144}} = \dfrac{12 + 8 + 3x}{12}$ so $\sqrt{289} = 17 = 20 = 3x$ so $x = -1$.

B. $1 + 4\sqrt{3} + 12 + \dfrac{2\sqrt{3}}{9} - 3\sqrt{3} + \dfrac{7\sqrt{3}}{9} =$

$13 + \dfrac{36 + 2 - 27 + 7}{9}\sqrt{3} = 13 + 2\sqrt{3}$

C. $\dfrac{3^{n+4}}{3^{n+6}} + \dfrac{3^{n+2}}{3^{n+6}} = \dfrac{1}{3^2} + \dfrac{1}{3^4} = \dfrac{3^2 + 1}{3^4} = \dfrac{10}{81}$

**Round Three:**

A. $p(x) = c\,(x+3)(x-2)^2 = c(x^3 - x^2 - 8x + 12)$ so c=0.5 to have a y-intercept of 6 and sum of coefficients is 0.5(1 – 1 – 8 + 12)=2

B. If the new equation were in y, then $\dfrac{3}{y^4} + \dfrac{5}{y^2} - \dfrac{6}{y} + 2 = 0$ which gives

$3 + 5y^2 - 6y^3 + 2y^4 = 0$

C. Using conjugate pairs of roots to obtain real coefficients:

$x(x - 1 - i)(x - 1 + i)(x - \frac{1}{2} - \sqrt{2})(x - \frac{1}{2} + \sqrt{2}) = x^5 - 3x^4 + \frac{9}{4}x^3 + \frac{3}{2}x^2 - \frac{7}{2}$

then multiply by 4 to obtain integer coefficients OR get two quadratics using sum and product of paired roots and multiply: $x(x^2 - 2x + 2)(x^2 - x + \frac{7}{4})$ etc.