**MASSACHUSETTS MATHEMATICS LEAGUE**
**CONTEST 4 - JANUARY 2011 SOLUTION KEY**

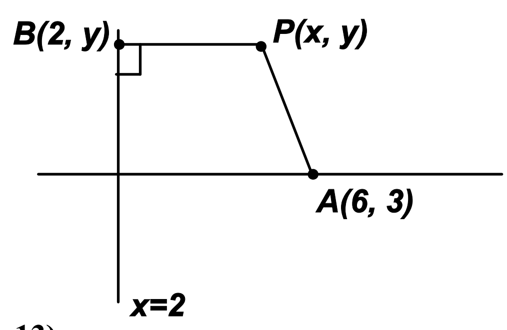
## Round 1

A) The equation $\frac{(x+1)^2}{2}+\frac{(y-3)^2}{4}=1$ defines a vertical ellipse with center at $(-1, 3)$ and $a = 2$.
Thus, the maximum possible value of y is 3 + 2 = **5**.

Alternate solution (I don't know about ellipses.)
Both terms being added are nonnegative and their sum is a constant. Minimizing the value of the $x$-expression will maximize the value of the $y$-expression. Let $x = -1$. Then:
$\frac{(y-3)^2}{4}=1 \rightarrow (y-3)^2 = 4 \rightarrow y - 3 = \pm 2 \rightarrow y = \mathbf{5}$.

B) The equation of the circle $x^2+y^2-8y+3=0$ is equivalent to $x^2+(y-4)^2=13$. The slope of the line from the center (0, 4) to the point of tangency (3, 2) is $\frac{2}{-3}$ and the slope of the tangent line is $+\frac{3}{2}$. Using point-slope, the equation of the tangent line is $(y - 2) = \frac{3}{2}(x - 3)$
$\rightarrow 2y - 4 = 3x - 9 \rightarrow \mathbf{3x - 2y - 5 = 0}$.

C) Equidistant from a point and a line describes a parabola.
Since the line $x = 2$ is vertical the equation must be of the form $(y - k)^2 = 4p(x - h)$.
$(h, k) = (6, 3)$ and since the vertex is at $(4, 3)$, $p = 2$. Thus, the equation of the parabola is $(y - 3)^2 = 8(x - 4)$. Rewriting the equation of the line, we have $x = 3.5 + y$.
Substituting, $(y-3)^2=8\left(y-\frac{1}{2}\right)=8y-4 \rightarrow y^2-14y+13=(y-1)(y-13)=0 \rightarrow y = 1, 13$
$\rightarrow$ **(4.5, 1), (16.5, 13)** or equivalent.




Alternative solution: (I only know the distance formula.)
$PA = PB \rightarrow x-2=\sqrt{(x-6)^2+(y-3)^2}$
$\rightarrow (x-2)^2=(x-6)^2+(y-3)^2$
$\rightarrow -4x+4=-12x+36+(y-3)^2$
$\rightarrow 8x-32=8(x-4)=(y-3)^2$ Substitute for $x$ and solve the resulting equation for $y$, obtaining $(x, y) =$ **(4.5, 1), (16.5, 13)**.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