Round 5 - continued

C) The given information is marked in the diagram below and, taking note of the embedded 30 - 60 - 90 triangle, we know that $EF = \frac{1}{2}$ and $FB = \frac{\sqrt{3}}{2}$.

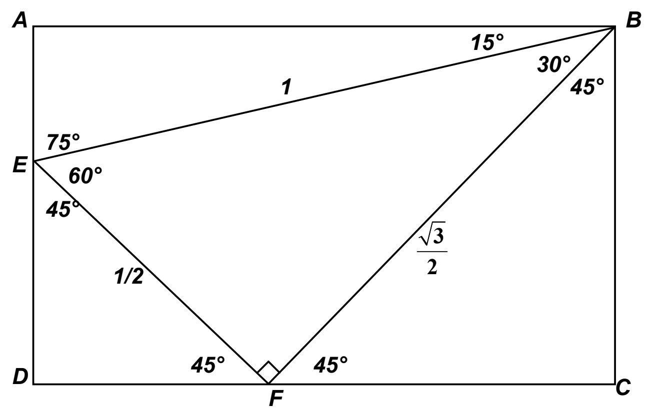
Both ΔEDF and ΔBCF are isosceles right triangles whose sides are in a $1 : 1 : \sqrt{2}$ ratio.

$$FC = BC = AD = \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} = \frac{\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{6}}{4}$$

$$DE = DF = \frac{1}{2} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$$

$$AB = DF + FC = \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$AE = AD - DE = \frac{\sqrt{6}-\sqrt{2}}{4}$$



Thus, $\cos(\angle BED) = \cos(105°) = -\cos(75°) = -\frac{AE}{BE} = \underline{\boldsymbol{\frac{\sqrt{2}-\sqrt{6}}{4}}}$

This approach is an alternative to using the usual expansion of cos(A + B) which would not be known by all students in this round. Thanks again to MaryBeth McGinn for this gem. The question, diagram and solution are adapted directly from the 2008 MML Contest #2 notes and the study guide MML Contest 2 A-Questions + Solutions (05-09).