

**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2016 SOLUTION KEY**

Round 6

A)

$$\frac{\frac{2+3+4}{36}}{\frac{6+5+4}{36}} = \frac{9}{15} = \frac{3}{5}$$

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

B) The middle term is the 5th term. The 5th term is $\binom{8}{4}(x^{-1})^4(x^3)^4 = \binom{8}{4}x^8 = \frac{8 \cdot 7 \cdot 6 \cdot 5}{1 \cdot 2 \cdot 3 \cdot 4}x^8 = \underline{70x^8}$.

C) Jar #1: 4B and 6W

The two marbles drawn from Jar #1 are either both the same color or one of each.

$$P(BB) = \frac{{}_4C_2}{{}_{10}C_2} = \frac{6}{45} = \frac{2}{15}, \quad P(WW) = \frac{{}_6C_2}{{}_{10}C_2} = \frac{15}{45} = \frac{1}{3} \quad \text{and} \quad P(BW \text{ or } WB) = \frac{{}_4C_1 \cdot {}_6C_1}{{}_{10}C_2} = \frac{24}{45} = \frac{8}{15}$$

Notice that $P(BB) + P(WW) + P(BW \text{ or } WB) = 1$.

$$\text{Jar \#2 initially } (3, 5) \Rightarrow \begin{cases} BB \Rightarrow \text{Jar \#2: } (b, w) = (5, 5) \\ WW \Rightarrow \text{Jar \#2: } (b, w) = (3, 7) \\ BW / WB \Rightarrow \text{Jar \#2: } (b, w) = (4, 6) \end{cases}$$

Thus, the probability of drawing B from Jar #2 is

$$\frac{2}{15} \cdot \frac{1}{2} + \frac{1}{3} \cdot \frac{3}{10} + \frac{8}{15} \cdot \frac{2}{5} = \frac{10+15+32}{150} = \frac{19}{50} \quad (\text{or } \underline{0.38}).$$