MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2011 SOLUTION KEY

Team Round - continued

E) $|2x + 1| < x - c$ is equivalent to $-x + c < 2x + 1 < x - c$ which in turn is equivalent to the compound condition $-x + c < 2x + 1$ and $2x + 1 < x - c$

Thus, $x > \frac{c-1}{3}$ and $x < -1 - c \Rightarrow \frac{c-1}{3} < x < -1 - c$.

For this to make any sense at all, we require that $\frac{c-1}{3} < -1 - c \Rightarrow c - 1 < -3 - 3c \Rightarrow c < -\frac{1}{2}$

$c = -1 \Rightarrow$ open interval $\left(-\frac{2}{3}, 0\right)$ - no integer solutions

$-2 \Rightarrow \left(-\frac{3}{3}, 2\right) = (-1, 1) \Rightarrow$ 1 integer solution

$-3 \Rightarrow \left(-\frac{4}{3}, 2\right) \Rightarrow$ 3 integer solutions

$-4 \Rightarrow \left(-\frac{5}{3}, 3\right) \Rightarrow$ 4 integer solutions

$-5 \Rightarrow \left(-\frac{6}{3}, 4\right) \Rightarrow$ 5 integer solutions

$-6 \Rightarrow \left(-\frac{7}{3}, 5\right) \Rightarrow$ 7 integer solutions

Clearly, the solution is unique. Build a table of c-values and n, the corresponding number of solutions for values of c immediately preceding a jump of 2 in the number of solutions.

As c decreases by 3, n increases by 4. $n = 17 \Rightarrow c =$ **<u>−14</u>**.

Check: $n = -14 \Rightarrow (-5, 13) \Rightarrow -4, \ldots, -1, 0, 1, \ldots, 12$

c	***n***
-2	1
-5	5
-8	9
-11	13
-14	17

F) The "Program" searches for twin primes (primes differing by 2) for which the larger is 3 more than a multiple of 4.

Note the twin prime pair (11, 13) is **<u>not</u>** added into the total since the roles of p and q are reversed. The smaller prime is 3 more than a multiple of 4 and the larger prime is 1 more than a multiple of 4.

For $m = 24$, $p = 97$ and $q = 99$ (not prime) and the "program" makes one more pass.

$m = 25 \Rightarrow p = 101$, $q = 103$ (both are primes) and the loop is exited.

T is increased for (5, 7), (17, 19), (29, 31), (41, 43) and (101, 103)

$\Rightarrow T = 12 + 36 + 60 + 84 + 204 =$ **<u>396</u>**.