MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2008
ROUND 3 COORDINATE GEOMETRY OF LINES AND CIRCLES

ANSWERS

A) (_____ , _____ , _____)

B) _____

C) (_____ , _____)

A) Given: $2x - 3y - 5 = 0$ and $4x - 6y + 1 = 0$
Let $Ax + By + C = 0$ denote the equation of the line equidistant from the given lines, where A, B and C are integers, $A > 0$ and the GCF(A, B, C) = 1.
Determine the ordered triple (A, B, C).

B) Compute the shortest distance between the circles defined by $\begin{cases} x^2 + y^2 - 4x + 10y + 4 = 0 \\ x^2 + y^2 + 12x - 20y + 55 = 0 \end{cases}$.

Note: Since these two circles do not intersect, the shortest distance between them lies along the segment connecting their centers.

C) There are an infinite number of ordered pairs (x, y) that satisfy the linear equation $6x + 9y = 12$. Let S be set of all ordered pairs (x, y) that satisfy this equation, where both x and y are single digit integers. Compute (A, B), where A is the sum of all the x-coordinates in S and B is the sum of all y-coordinates in S.

Note: The values of x and y may be positive, negative or zero.