MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 - FEBRUARY 2014 SOLUTION KEY

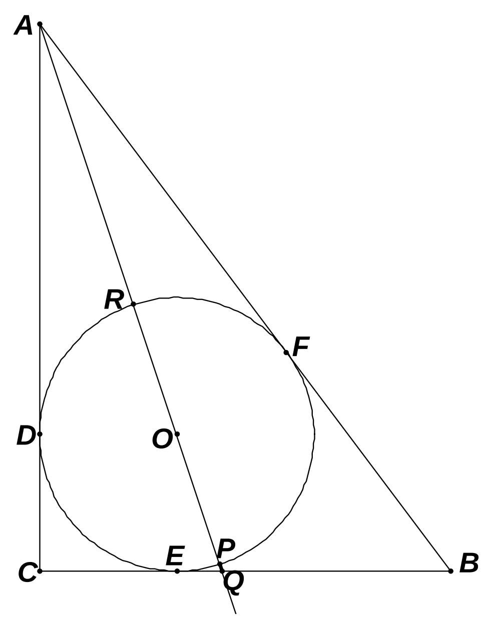
Team Round

D) Let the 5 notes be designated Q, H, W, D and E.

Then: $$\begin{cases} Q+H+W+D+E=48 \\ Q+2H+4W+1.5D+.5E=16\cdot 4=64 \\ D=E=x \\ Q=7E=7x \\ Q=2H+4W \end{cases}$$

$$Q+(2H+4W)+(1.5D+.5E)=64 \Leftrightarrow 2Q+2E=64 \Leftrightarrow 14x+2x=64 \Rightarrow x=4,\ Q=28$$
$$Q+H+W+D+E=48 \Leftrightarrow 28+H+W+2(4)=48 \Rightarrow \underline{H+W=12}$$
$$Q+(2H+4W)+(1.5D+.5E)=64 \Leftrightarrow 28+2H+4W+8=64 \Leftrightarrow \underline{H+2W=14}$$
Subtracting, $W=2$ and $H=\underline{\mathbf{10}}$.

E) $\overrightarrow{AO}$ is an angle bisector $\Rightarrow \dfrac{QC}{4}=\dfrac{3-QC}{5} \Rightarrow QC=\dfrac{4}{3}$.



The radius of the inscribed circle is given by

$r=\dfrac{A}{s}=\dfrac{\frac{1}{2}\cdot 3\cdot 4}{\frac{1}{2}(3+4+5)}=\dfrac{6}{6}=1$. Since $OECD$ must be a square,

$EQ=\dfrac{1}{3} \Rightarrow OQ^2=1^2+\left(\dfrac{1}{3}\right)^2 \Rightarrow OQ=\dfrac{\sqrt{10}}{3}$. Applying the Pythagorean Theorem to ΔADO, $AO=\sqrt{10}$.

The required ratio $\dfrac{AO}{PQ}$ is

$$\frac{\sqrt{10}}{\frac{\sqrt{10}}{3}-1}=\frac{3\sqrt{10}}{\sqrt{10}-3}\cdot\frac{\sqrt{10}+3}{\sqrt{10}+3}=\frac{30+9\sqrt{10}}{10-9}=\frac{30+9\sqrt{10}}{1}$$

$\Rightarrow k=\underline{\mathbf{30+9\sqrt{10}}}$ or $\underline{\mathbf{3\left(10+3\sqrt{10}\right)}}$.