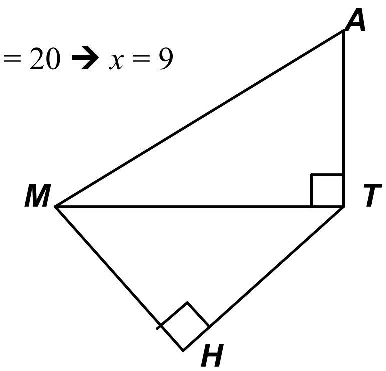
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 3 - DECEMBER 2009 SOLUTION KEY

Round 6

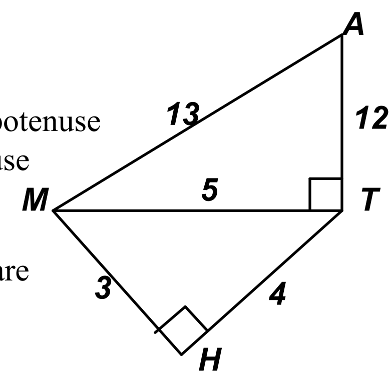


A) $(7x - 3) + (6x + 7) + (95 - 4x) = 180 \rightarrow 9x + 99 = 180 \rightarrow x + 11 = 20 \rightarrow x = 9$
Thus, the angle measures are 60°, 61° and 59°. $m\angle Q =$ **<u>60</u>**

$MA^2 + AT^2 + \left(TH^2 + MH^2\right) =$

B) $MA^2 + \left(AT^2 + MT^2\right) =$

$MA^2 + MA^2 = 2MA^2 = 21^2 + 21^2 = 441 + 441 = \underline{\mathbf{882}}$



Alternate solution (motivated by Pope John XIII mathletes):

Simplify, Simplify, Simplify! Working with 21 as the length of the hypotenuse does not allow both of the other legs to have integer lengths. Instead, let's use $AM = 13$ and look for a pattern. (13 was picked because it allows us to use two common Pythagorean triples, namely 3 – 4 – 5 and 5 – 12 – 13.)
We ignore the fact that the diagram suggests that $MT > AT$, since diagrams are not necessarily drawn to scale.
Note that $MA^2 + AT^2 + TH^2 + MH^2 =$
$3^2 + 4^2 + 12^2 + 13^2 = (9 + 16 + 144) + 169 = 2(169) = 2(13)^2 = 338$
This suggests a pattern: if $AM = x$, then $MA^2 + AT^2 + TH^2 + MH^2 = 2x^2$.
Thus, $21 \rightarrow 2(21)^2 =$ **<u>882</u>**.
The first argument actually proves this contention using the Pythagorean Theorem twice.

C) Let P have n sides and Q have $(n + 8)$ sides. The measure of the exterior angle of Q is 0.5° **<u>less than</u>** the measure of the exterior angle of P. Thus, $\frac{360}{n} - \frac{360}{n+8} = \frac{1}{2} \rightarrow$

$360\left(\frac{8}{n(n+8)}\right) = \frac{1}{2} \rightarrow n(n+8) = 360(16)$

Rather than trying to factor a quadratic trinomial or forcing the quadratic formula, let's look for a factorization of 360(16) where the factors differ by 8. Redistributing factors of 2, 4 and 5 we have $180 \cdot 32 \rightarrow 148$ $\quad 90 \cdot 64 \rightarrow 26$ $\quad 72 \cdot 80 \rightarrow 8$ Bingo!

Thus, Q has 80 sides and $\frac{80(77)}{2} = 40(77) = \underline{\mathbf{3080}}$ diagonals.