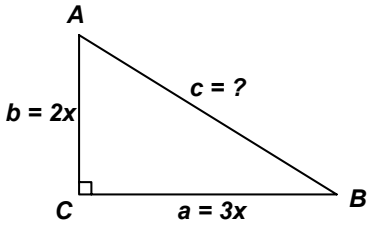
MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 2 - NOVEMBER 2007 SOLUTION KEY

Round 3

A) Area $= \frac{1}{2}bh = \frac{1}{2}\cdot 2x\cdot 3x = 60 \rightarrow x = 2\sqrt{5}$

and $(2x)^2 + (3x)^2 = c^2$

$\rightarrow$ hypotenuse $c = x\sqrt{13} = 2\sqrt{5}\cdot\sqrt{13} = \underline{\mathbf{2\sqrt{65}}}$

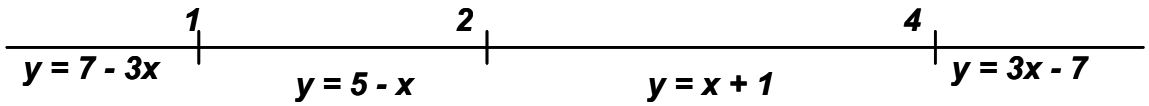


B) Let $PC = QC = x$. Then $PQ = x\sqrt{2}$ and

$\frac{1}{2}x^2 : (36 - \frac{1}{2}x^2) = 1:5 \rightarrow \frac{x^2}{72 - x^2} = \frac{1}{5} \rightarrow 6x^2 = 72$

$\rightarrow x = 2\sqrt{3}$ and $PQ = \underline{\mathbf{2\sqrt{6}}}$

C) The critical points occur at $x = 1$, 2 and 4.
The first equation may be expressed without absolute value over restricted domains as follows:



Thus, the region bounded by this system consists of 4 trapezoids.

$A = \frac{1}{2}\big(1(4+7) + 1(4+3) + 2(3+5) + 4(5+17)\big)$

$= \frac{1}{2}(11 + 7 + 16 + 88) = \frac{122}{2} = \underline{\mathbf{61}}$