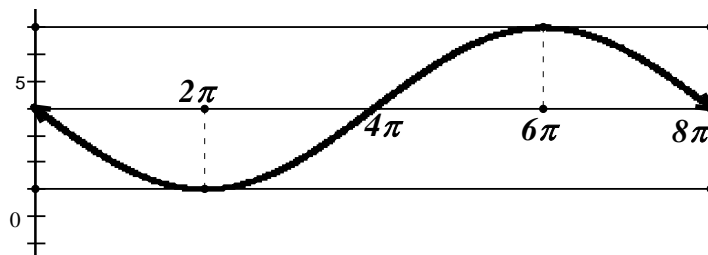


**MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 – MARCH 2014 SOLUTION KEY**

Round 3

A)



The amplitude is 3 and the graph has been flipped. This is solely controlled by the A -value. $A = -3$

The vertical shift is 4. This is solely controlled by the C -value. $C = 4$

The period (which is normally 2π) has been stretched to 8π . This is solely controlled by the B -value.

$$\frac{2\pi}{B} = 8\pi \Rightarrow B = \frac{1}{4}. \text{ Thus, } (A, B, C) = \left(-3, \frac{1}{4}, 4\right) \text{ or } \left(3, -\frac{1}{4}, 4\right).$$

$$\text{B) } \sin 15\alpha = \frac{1}{2} \Leftrightarrow 15\alpha = \begin{cases} 30 + 360n \\ 150 + 360n \end{cases} \Rightarrow \alpha = \begin{cases} 2 + 24n \\ 10 + 24n \end{cases} \Rightarrow \alpha = \begin{cases} 2, 26, 50, 74 \\ 10, 34, 58, \underline{82} \end{cases}$$

$$\begin{aligned} \text{C) } (-1 - \sqrt{3}i)^{100} &= \left(2 \operatorname{cis} \frac{4\pi}{3}\right)^{100} = 2^{100} \operatorname{cis} \frac{400\pi}{3} = 2^{100} \operatorname{cis} \left(\left(132 + \frac{4}{3}\right)\pi\right) = 2^{100} \operatorname{cis} \frac{4\pi}{3} \\ &= 2^{100} \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = \left(-\frac{1}{2}\right) 2^{100} (1 + \sqrt{3}i) = (-1) 2^{99} (1 + \sqrt{3}i) \Rightarrow (A, B) = \underline{(-1, 99)}. \end{aligned}$$

