MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 1 - OCTOBER 2016 SOLUTION KEY

Round 6

A) Invoking the PEMDAS rule, $2+3\cdot 4-x\div 6=7$ is equivalent to $2+(3\cdot 4)-\dfrac{x}{6}=7$

$\Rightarrow 14-\dfrac{x}{6}=7 \Rightarrow \dfrac{x}{6}=7 \Rightarrow x=\underline{\mathbf{42}}$

B) Sums triggering the paintbrush are 3, 5, 6, 9, 10, 12 and 15, since the smallest sum is 2 and the largest 16. $(r=2)+(c=1...8)\Rightarrow 3,5,6,9,10$ $(r=4)+(c=1...8)\Rightarrow 5,6,9,10,12$ $(r=6)+(c=1...8)\Rightarrow 9,10,12$

All other rows generate 4 triggering sums.
Thus, there will be $5\cdot 4+2\cdot 5+3=\underline{\mathbf{33}}$.

	1	2	3	4	5	6	7	8	
1		x		x	x			x	4
2	x		x	x			x	x	5
3		x	x			x	x		4
4	x	x			x	x		x	5
5	x			x	x		x		4
6			x	x		x			3
7		x	x		x			x	4
8	x	x		x			x		4

C) Let x denote the length of the third side.

According to the triangle inequality, $\begin{cases} x+17>38 \\ 17+38>x \end{cases} \Rightarrow 22\le x\le 54$

The perimeter is 55 + x. The minimum value of x producing a multiple of 4 is 25
The values of x are of the form 25 + 4k. k = 7 produces the largest possible value of x, namely 53.
Thus, there are **<u>8</u>** possible lengths for the third side.