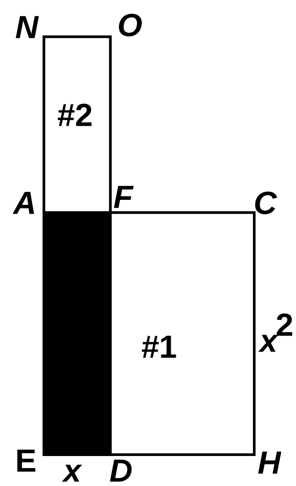
**MASSACHUSETTS MATHEMATICS LEAGUE**
**CONTEST 4 - JANUARY 2014 SOLUTION KEY**

## Round 2

A) One possible factorization is $(x-1)(x-180)=x^2-181x+180$ .
Notice that $k=181$ is the sum of the factors of 180.
Rather than trying all the factor pairs of 180, we notice that to minimize the sum of the factors we want a pair of factors as close together as possible, that is, whose difference is as small as possible. The product $12\cdot 15$ fills the bill and the minimum value of $k$ is **<u>27</u>**.

B) The area of region #1 is $(14-x)x^2$ .
The area of region #2 is $x\left(45-x^2\right)$.
The area of the shaded region is $x^3$.
Therefore, we require that $(14-x)x^2-x\left(45-x^2\right)=x^3$
$\Leftrightarrow 14x^2-x^3-45x+x^3=x^3$
$\Leftrightarrow 14x^2-45x=x^3$
Since $x$ is nonzero, this is equivalent to $14x-45=x^2$
$x^2-14x+45=\cancel{(x-9)}(x-5)=0$ ( $x\neq 9$, since $CH$ can not be 81.)
$x=5\Rightarrow$ the area of $DEAF$ is **<u>125</u>**.




C) $-x(2x-3y-4)=y^2+2y\Leftrightarrow 0=\left(2x^2-3xy+y^2\right)+(-4x+2y)$
Factoring each grouping, we have $(2x-y)(x-y)-2(2x-y)=0$
Factoring out the common factor, $(2x-y)(x-y-2)=0$
Thus, <u>$\boldsymbol{y=2x}$ or $\boldsymbol{y=x-2}$</u>.