MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 5 - FEBRUARY 2009
ROUND 7 TEAM QUESTIONS
ANSWERS

A) ____________________________ D) ___________________________

B) ____________________________ E) ____________________________

C) ____________________________ F) ____________________________

A) Consider the following function defined parametrically, i.e. in terms of a third variable (in this case t): $\begin{cases} x = 1 - a^{t} \\ y = 1 + a^{-t} \\ a > 1 \end{cases}$. If $x = \dfrac{2}{3}$, compute y.

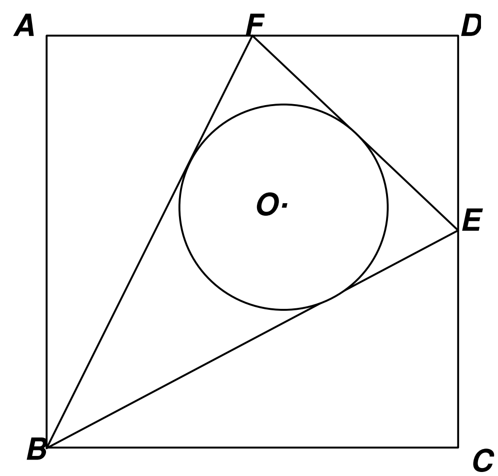
B) For <u>how many</u> positive integers $A < 360$ can the fraction $\dfrac{A}{360}$ be simplified?

C) If $\mathrm{Tan}^{-1}(x) + \mathrm{Sin}^{-1}(x) = \pi/2$ and $x > 0$, compute x^2.

D) The sum of the reciprocals of two integers A and B, not necessarily distinct, equals k times the reciprocal of the sum of A and B.

For certain integer values of k, the ratio $R = \dfrac{A}{B}$ is rational.

Determine <u>all</u> possible ordered pairs (k, R).



E) Given: Square $ABCD$, $AB = 12$, E is the midpoint of $\overline{CD}$, F is the midpoint of $\overline{AD}$ and circle O is inscribed in ΔBEF Compute the length of the radius of circle O.

F) For a finite sequence $A = \left(a_1, a_2, a_3, \ldots, a_n\right)$ of numbers, the Cesaro sum of A is defined to be $\dfrac{S_1 + S_2 + S_3 + \ldots + S_n}{n}$, where $S_k = a_1 + a_2 + a_3 + \ldots + a_k$.

Determine for <u>how many</u> integers $t > 1$, the Cesaro sum of $\left(a_1, a_2, a_3, \ldots, a_t\right)$ is 180 and the Cesaro sum of $\left(1, a_1, a_2, a_3, \ldots, a_t\right)$ is an integer.