

MASSACHUSETTS MATHEMATICS LEAGUE
CONTEST 6 - MARCH 2012
ROUND 7 TEAM QUESTIONS
ANSWERS

A) _____ D) _____

B) $K =$ _____ , $J =$ _____ E) _____

C) _____ F) _____

******* NO CALCULATORS IN THIS ROUND *******

- A) Compute the ordered triple (a, b, c) which solves the system $\begin{cases} x + 7y + 5z = 12 \\ 2x + 9y + 4z = 20 \\ 6x + Ay + 3z = 19 \end{cases}$ over the

integers for the smallest possible positive integer value of A .

- B) Suppose $K = a^{2x} + a^{-2x}$ and $J = (a^x + a^{-x}) + (a^{4x} + a^{-4x})$.

a and x are real numbers ($a > 0$), but J and K are both positive integers.

Compute the ordered pair (K, J) , if K is the minimum value for which $J > 2012$.

- C) A circle of radius r ($0 < r < 1$) is centered at $(4, 1)$.

Two particles A and B both starting at $(4 + r, 1)$ rotate around the circle.

A stops at point P after rotating 945° (i.e. counterclockwise).

B stops at point Q after rotating -1140° (i.e. clockwise).

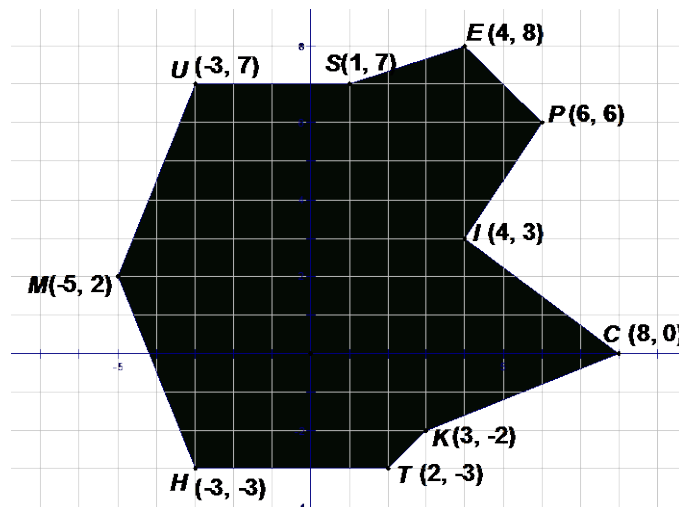
Compute exactly how much closer to the x -axis one point is than the other, in terms of r .

- D) Compute all real values of x for which

$$(x^2 - 2x - 8)^2 = 2(x - 1)^2 + 17.$$

- E) Compute the area of polygon *EPICKTHMUS*.

- F) A license plate consists of 6 distinct nonzero digits. The plate $ABC \cdot DEF$ is considered “memorable” if all six digits are either in increasing or decreasing order, but not necessarily consecutive.
 [Ex: $234 \cdot 567$, $123 \cdot 789$, $875 \cdot 421$ are memorable, $125 \cdot 489$ and $125 \cdot 976$ are not.]



One startled, but alert, eyewitness to a bank

robbery couldn't be specific about the digits, but was sure the plate was “memorable”, while another independent witness reported that the leftmost digit was neither a 1 nor a 2. If both of these witnesses were reliable, how many plates remained for the police to cross check?