hw week 4

	Revenue	Net profit
Acre Computer	138	8
Nadir Soft	87	5
Quixote	111	13

a) false
b) true
c) ~~[illegible]~~ False
d) F → T ⇒ True
e) T ↔ T ⇒ True

2) p → it's below freezing
q → it's snowing

a) $p \wedge q$ ✓
b) $p \wedge \neg q$ ✓
c) $\neg p \wedge \neg q$ ✓
d) $p \vee q$ ✓
e) ~~$p \leftrightarrow q$~~ $p \rightarrow q$ ✓
f) $(p \vee q) \wedge (p \rightarrow \neg q)$ ✓
g) $p \Leftrightarrow q$ ✓

③ a) $p \to \neg p$ 2 rows

b) $(p \vee \neg r) \wedge (q \vee \neg s)$ $2^4 = 16$

c) $q \vee p \vee s \vee \neg r \vee \neg t \vee u)$ $2^6 = 64$

d) $(p \wedge r \wedge t) \leftrightarrow (q \wedge t)$ $2^4 = 16$

④ a) $p \wedge \neg p$

p	¬p	p∧¬p
T	F	F
F	T	F

b) $p \vee \neg p$

p	¬p	p∨¬p
T	F	T
F	T	T

c) $(p \vee \neg q) \to q$

p	q	¬q	p∨¬q	(p∨¬q)→q
T	T	F	T	T
T	F	T	T	F
F	T	F	F	T
F	F	T	T	F

d) $(p \vee q) \to (p \wedge q)$

p	q	p∨q	p∧q	
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

e) $(p \to q) \leftrightarrow (\neg q \to \neg p)$

p	q	¬p	¬q	p→q	¬q→¬p	(p→q)↔(¬q→¬p)
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

f) $(p \to q) \to (q \to p)$

p	q	p→q	q→p	(p→q)→(q→p)
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

(5) first two professors wanted coffee, the last did not want

(6) a) Jon is rich & happy

$p \wedge q \to \neg(p \wedge q) \equiv \neg p \vee \neg q$

Jon is not rich nor happy

b) $p \vee q \to \neg(p \vee q) \equiv \neg p \wedge \neg q$

Carlos will not bicycle and will not run

c) Mei $p \vee q \to \neg(p \vee q) \equiv \neg p \wedge \neg q$

Mei doesn't walk & take bus to class

d) $p \wedge q \to \neg(p \wedge q) \equiv \neg p \vee \neg p$

Ibrahim is not smart nor hardworking

7) a) $(p \wedge q) \rightarrow p$

p	q	$p \wedge q$	$(p \wedge q) \rightarrow p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

→ all Trues → Tautology

b) $p \rightarrow (p \vee q)$

p	q	$p \vee q$	$p \rightarrow (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

→ tautology

c) $\neg p \rightarrow (p \rightarrow q)$ tautology

p	q	$\neg p$	$p \rightarrow q$	$\neg p \rightarrow (p \rightarrow q)$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

d) $(p \wedge q) \rightarrow (p \rightarrow q)$

p	q	$p \wedge q$	$p \rightarrow q$	$(p \wedge q) \rightarrow (p \rightarrow q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

e) $\neg(p \to q) \to p$

p	q	$p \to q$	$\neg(p \to q)$	$\neg(p \to q) \to p$
T	T	T	F	T
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

f) $\neg(p \to q) \to \neg q$

p	q	$p \to q$	$\neg(p \to q)$	$\neg q$	$\neg(p \to q) \to \neg q$
T	T	T	F	F	T
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	F	T	T

(8) if conclusion is True then conditional statement is T.

$p \to q \to$ conclusion

hypoth

a) $(p \wedge q) \to p \equiv \neg(p \wedge q) \vee p \equiv \neg p \vee \neg q \vee p \equiv$

$\Rightarrow \neg q \vee T = T$

b) if p is T, then $p \vee q \to$ True

c) ~~$\neg p \to (p \to q)$~~ if $\neg p$ is T, p - F

then $\to p \to q \to T$, since p - F

$\neg p \to (p \to q) \equiv \neg\neg p \vee (\neg p \vee q) \equiv p \vee (\neg p \vee q) \equiv (p \vee \neg p) \vee q$

$= T \vee q = T$

d) if hypothesis is true then q - T
$p \to q$ - T

e) p - q - F if hypothesis is is true
then $p \to p \not\to T$

f) hypothesis true, $p \to q$ - f
q - false.

(9) $(p \to r) \wedge (q \to r) \stackrel{?}{\equiv} (p \vee q) \to r$

p	q	r	$p \to r$	$q \to r$	$(p \to r) \wedge (q \to r)$	$p \vee q$	$(p \vee q) \to r$
T	T	T	T	T	T	T	T
T	T	F	F	F	F	T	F
T	F	T	T	T	T	T	T
T	F	F	F	T	F	T	F
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	F
F	F	T	T	T	T	F	T
F	F	F	T	T	T	F	T

$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$

in order for $(p \to r) \wedge (q \to r)$ to be F
T F T F

so $r \to F$ & p or q → is T

$p \vee q \to$ true → r - false so $(p \vee q) \to r$ - False

since pvq is true & r is f → in both options
then they are equivalent

⑨ $P(x) \rightarrow x$ can speak Russian

$Q(x) \rightarrow x$ knows computer language C++

D → all students in your school

a) $\exists x (P(x) \wedge Q(x))$ ✓

b) $\exists x (P(x) \wedge \neg Q(x))$ ✓

c) $\forall x (P(x) \vee Q(x))$ ✓

d) $\forall x (\neg (P(x) \vee Q(x)) \Rightarrow$

$\forall x (\neg P(x)) \wedge (\neg (Q(x)))$

⑩ a) $\forall n (n^2 \geq 0)$ - true

b) $\exists n (n^2 = 2)$ - False $n = \pm\sqrt{2}$ → not integer

D → all integers

c) $\forall n (n^2 \geq n)$ → true for all integers

d) $\exists n (n^2 < 0)$ false for n = integer

⑫ D of $P(x) = \{0, 1, 2, 3\}$

a) $\exists x P(x)$ → true for some x

$P(0) \vee (P(1) \vee P(2) \vee (P(3)) \vee P(4)$

b) $\forall x\, P(x) \equiv P(0) \wedge P(1) \wedge P(2) \wedge P(3) \wedge P(4)$

c) $\exists x\, \neg P(x) \equiv \neg P(0) \vee \neg (P(1) \vee \neg P(2) \vee \neg P(3) \vee \neg P(4)$

d) $\forall x \neg P(x) \equiv \neg P(0) \wedge \neg P(1) \wedge \neg P(2) \wedge \neg P(3) \wedge \neg P(4)$

e) $\neg \exists x\, P(x) \equiv \neg (P(0) \vee P(1) \vee P(2) \vee P(3) \vee P(4))$

f) $\neg \forall x\, P(x) \equiv \neg (P(0) \wedge P(1) \wedge P(2) \wedge P(3) \wedge P(4))$

(13) ① - D - students in class ② all people
$C(x) \rightarrow$ "x in your class"

a) ~~$\exists x\, H(x)$~~
~~speak Hindi~~
~~$\exists x\, (C(x) \wedge H(x))$~~

$S(x)$ - student in class
$P(x)$ - person
$H(x)$ - x speak Hindi
$F(x) \rightarrow$ x is friendly
$B(x)$ - Born in California
$M(x) \rightarrow$ x has been in movie
$L(x) \rightarrow$ x → logic programming

a) some one in class can speak Hindi
① $\exists x\, H(x)$ ② $\exists x\, (S(x) \wedge H(x))$

b) ① $\forall x\, F(x)$ ② $\forall x\, (S(x) \rightarrow F(x))$

c) ① $\exists x\, (\neg B(x))$ ② $\exists x\, (S(x) \wedge \neg B(x))$

d) ① $\exists x\, M(x)$ ② $\exists x\, (S(x) \wedge M(x))$

e) ① $\neg \exists x (L(x))$ ② $\forall x (S(x) \rightarrow \neg L(x))$

③ $M(x,y)$ - "x sent y an email message"

$T(x,y)$ - "x has telephoned y"

D - all students

a) $\neg M(Chloe, Koko)$

b) $\neg M(A,S) \wedge \neg T(A,S)$

c) $\neg M(D,J)$

d) $\forall S\, M(S,K)$

e) $\forall C(\neg T(C,N))$

f) $\forall C(T(C,A) \vee M(C,A))$

g) $\exists x \forall y (y \neq x \rightarrow M(x,y))$

h) $\exists x \forall y (y \neq x \rightarrow (M(x,y) \vee T(x,y)))$

i) $\exists x \exists y (x \neq y \wedge M(x,y) \wedge M(y,x))$

j) $\exists x M(x,x)$

k) $\exists x \forall y (x \neq y \rightarrow (\neg M(y,x) \wedge \neg T(y,x)))$

l) $\forall x \exists y (x \neq y \wedge M(y,x) \vee T(y,x))$

m) $\exists x \exists y (x \neq y \wedge M(x,y) \wedge T(y,x))$

n) $\exists x \exists y (x \neq y \wedge \forall z (z \neq x \wedge z \neq y) \rightarrow M(x,z) \vee M(y,z) \vee T(x,z) \vee T(y,z))$