$3 + \frac{1}{2} = \frac{7}{2} - \frac{8}{2} \cdot \frac{-1}{2}$

Homework W3

① $\log_2\left(\frac{8\sqrt{2}}{16}\right) + \log_2(32) - 2\log_2(4) =$

$= \log_2\left(\frac{2^3 \cdot 2^{\frac{1}{2}}}{2^4}\right) + \log_2 2^5 - 2\log_2 2^2 =$

$= \log_2 2^{\left(-\frac{1}{2}\right)} + \log_2 2^5 - 2\log_2 2^2 =$

$= -\frac{1}{2} + 5 - 4 = \frac{1}{2}$ //

② $\log_3(x-1) + \log_3(x+1) = 2$

$\log_3 (x+1)\cdot(x+1) = 2$

$(x-1)(x+1) = 3^2$

$x^2 - 1 = 3^2$

$x^2 = 10$

$x = \pm\sqrt{10}$

$\sqrt{10} \Rightarrow \approx 3{,}16$

if $x = +\sqrt{10}$

$\log_3 2{,}16 + \log_3 4{,}16) \times 2$ ✓

if $x = -\sqrt{10}$ ✗

$\log_3(-4{,}16) + \log(-2{,}16) \neq 2$

(3) $10000 6% → quarterly

$ min = 20000

$A = P(1 + \frac{r}{n})$

$20000 = 10000 \cdot (1 + \frac{0,06}{4})^{4t}$

$2 = (1 + \frac{0,06}{4})^{4t}$

$2 = (1,015)^{4t}$

$\ln 2 = \ln (1,015)^{4t}$

$\ln 2 = 4t \ln 1,015$

$t = \frac{\ln 2}{4 \ln(1,015)} = 11$ years.

(4) $N(t) = N_0 e^{-kt}$ half life - 5y.

k - ?

$t_{1/2} \rightarrow N(t_{1/2}) = \frac{N_0}{2}$

$\frac{N_0}{2} = N_0 e^{-k \cdot 5}$

$\frac{1}{2} = e^{-k \cdot 5}$

$\ln(\frac{1}{2}) = -5k$ $-0,693 = -5k$

$k = 0,1386$

⑤ 100 gr → 70 gr → 3 hours.

t=? → 20 gr.

$N(t) = N_0 \cdot e^{-k \cdot t}$

$N(3) = 70$

$70 = 100 \cdot e^{-k \cdot 3}$

$0,7 = e^{-3k}$

$\ln(0,7) = -3k$

$-0,35 = -3k$

$k = 0,1189$

$N(t) = 20$ gr.

$20 = 100 \cdot e^{-0,1189 \cdot t}$

$0,2 = e^{-0,1189 \cdot t}$

$\ln(0,2) = -0,1189t$

$-1,6094 = -0,1189t$

$t = 13$ h.

(6) $A(1,2,3)$ $B(4,6,9)$

$\vec{AB} = (4-1,\ 6-2,\ 9-3) = (3,4,6)$

$\|\vec{AB}\| = \sqrt{3^2+4^2+6^2} = \sqrt{61} \approx 7.81$

$u = \dfrac{\vec{AB}}{\|\vec{AB}\|} = \left(\dfrac{3}{\sqrt{61}}, \dfrac{4}{\sqrt{61}}, \dfrac{6}{\sqrt{61}}\right)$

(7) $\vec{v} = 7\hat{i} - 2\hat{j} + 4\hat{k}$

$\vec{v} = \begin{bmatrix} 7 \\ -2 \\ 4 \end{bmatrix}$

$\|\vec{v}\| = \sqrt{7^2+(-2)^2+4^2} = \sqrt{49+4+16} = \sqrt{69}$

(8) $\vec{a} = (2,-1,3)$ $\quad 3\vec{a} - 2\vec{b} =$

$\vec{b} = (-1,4,2)$ $\quad (6,-3,9) - (-2,8,4) =$

$= (8,-11,5)$

(9) $\vec{p} = (1,2,3)$ $\quad \theta = ?$ $\quad \vec{q} = (4,-5,6)$ $\quad \vec{p}\cdot\vec{q} = 4-10+18 = 12$

$\|\vec{p}\| = \sqrt{1+4+9} = \sqrt{14}$ $\quad \|\vec{q}\| = \sqrt{16+25+36} = \sqrt{77}$

$\cos\theta = \dfrac{\vec{p}\cdot\vec{q}}{\|\vec{p}\|\cdot\|\vec{q}\|} = \dfrac{12}{\sqrt{14}\cdot\sqrt{77}} = 0,3647$ $\quad \theta = \arccos(0,3647) = 68,61°$

(10) $\vec{u} = (2, -1, 4)$
$\vec{v} = (-8, 4, -16)$

$\vec{u} \cdot \vec{v} = -16 + (-4) + (-64) = -84$

orthogonal → $\vec{v} \cdot \vec{u} = 0$ → since $\vec{u} \cdot \vec{v} \neq 0$ not orthogonal

(11) $A = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 4 & 5 \\ -2 & 1 \end{bmatrix}$

$2A - 3B = \begin{bmatrix} 4 & -2 \\ 0 & 6 \end{bmatrix} - \begin{bmatrix} 12 & 15 \\ -6 & 3 \end{bmatrix} =$

$= \begin{bmatrix} -8 & -17 \\ 6 & 3 \end{bmatrix}$

(12) $C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $D = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$ $E = CD$

$E = C \cdot D = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 5+14 & 6+16 \\ 15+28 & 18+32 \end{bmatrix} =$

$E = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$

(13)
$x + y + 2 = 6$
$2x - y + 3z = 14$
$-3x + 2y - 2z = -10$

$$x + y + z = 6$$
$$2x - y + 3z = 14$$
$$-3x + 2y - 2z = -10$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 2 & -1 & 3 & 14 \\ -3 & 2 & -2 & -10 \end{array}\right] \quad R_2 = R_2 - 2R_1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -3 & 1 & 2 \\ -3 & 2 & -2 & -10 \end{array}\right] \quad R_3 = R_3 + 3R_1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -3 & 1 & 2 \\ 0 & 5 & 1 & 8 \end{array}\right] \quad R_3 = R_3 - \frac{5}{3}R_2$$

$$5 - \frac{5}{3}\cdot(-3) = 0$$

$$1 - \frac{5}{3} = \frac{-2}{3}$$

$$8 - \frac{5}{3}\cdot 2 = \frac{14}{3}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -3 & 1 & 2 \\ 0 & 0 & -\frac{2}{3} & \frac{14}{3} \end{array}\right]$$

$$-\frac{2}{3}z = \frac{14}{3}$$

$$\boxed{z = -7}$$

$$-3y - 7 = 2$$

$$-3y = 9$$

$$\boxed{y = -3}$$

$$x + 1 + \frac{5}{3} = 6 \qquad \frac{3+5}{3} = \frac{14}{3}$$

$$-3y + 1(-7) = 2$$

$$x + (-7) + (-3) = 6$$

$$\boxed{x = 16}$$

14) $B = \left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & -1 \end{array}\right]$ ~~$R_2 = R_2 - (3R_1$~~

$R_1 = R_1 - 2R_2$

$\left[\begin{array}{ccc|c} 1 & 0 & -7 & -10 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & -1 \end{array}\right]$ $R_1 = R_1 + 7R_3$

$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & -17 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & -1 \end{array}\right]$ $R_2 = R_2 - 3R_3$

$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & -17 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & -1 \end{array}\right]$

15) $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ $A^{-1} = ?$

$\left[\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 5 & 3 & 0 & 1 \end{array}\right]$ $R_1 = R_1 : 2$

$\left[\begin{array}{cc|cc} 1 & \frac{1}{2} & \frac{1}{2} & 0 \\ 5 & 3 & 0 & 1 \end{array}\right]$ $R_2 = R_2 - 5R_1$

$\left[\begin{array}{cc|cc} 1 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & -\frac{5}{2} & 1 \end{array}\right]$ $3 - \frac{5}{2} = \frac{1}{2}$

$0 - \frac{5}{2}$ $R_2 = R_2 \cdot 2$

$$\left[\begin{array}{cc|cc} 1 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & -5 & 2 \end{array}\right] \quad R_1 = R_1 - \left(\frac{1}{2} R_2\right)$$

$$\frac{1}{2} - \frac{1}{2}\cdot(-5) = 3$$

~~$\frac{1}{2} - 5 \cdot \frac{1}{2} = \frac{1-5}{2} = -2$~~

$$\left[\begin{array}{cc|cc} 1 & 0 & 3 & -1 \\ 0 & 1 & -5 & 2 \end{array}\right] \quad 0 - \left(\frac{1}{2}\cdot 2\right)$$

$$A^{-1} = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$$