Week 2 HW

(1) $S = 5 + 9 + 13 + \dots 89$

$a_1 = 5$, $d = 4$, $a_n = 89$

$89 = 5 + (n-1) \cdot 4$

$\frac{84}{4} = \frac{(n-1)4}{4}$

$21 = n - 1$

$n = 22$ terms

$$\boxed{\sum_{k=1}^{22} (5 + (k-1) \cdot 4)}$$

(2) $\sum_{K=3}^{15} (2k+1) \Rightarrow$ start at $k=1$

$$\sum_{K=1}^{13} \left(2(k+2)+1\right) = \sum_{k=1}^{13} 2k+5$$

(3) $a_1 = 12$ $a_n = a_{n-1} + d$ $a_{10} = 57$ $d = ?$ $a_{25} = ?$

$d = \frac{a_y - a_x}{y - x}$ $y \geq x$ $\boxed{d = 5}$

$d = \frac{57-12}{10-1} = \frac{45}{9} = 5$ $5 = \frac{a_{25} - 12}{25-1} =$

$\Rightarrow 120 = a_{25} - 12$ $\boxed{a_{25} = 132}$

2, 2, 5·5 53

4) find ~~all~~ sum of all multiples of 7 btw 100 & 1000

$a_1 = 7 \cdot 15 = 105$

$a_n = 7 \cdot 142 = 994$ $\quad n = \frac{a_n - a_1}{d} + 1 = 128$

$d = 7$

$$S_n = \frac{2 \cdot 105 + (128-1) \cdot 7}{\cancel{2}} \cdot \overset{64}{\cancel{128}} = 70336$$

5) $\sum_{k=1}^{n} (3k+2)$ $\quad n$ - ? where $S = 2650$.

$\boxed{\text{check for } 41}$

$2650 = \frac{10 + 40 \cdot 3}{2} \cdot 41$

$2650 \neq \frac{130}{2} \cdot 41 = 2665$

$\boxed{\text{check for } 40}$

$2650 \neq \frac{10 + 39 \cdot 3}{2} \cdot 40 = 2540$

$a_1 = 5$ $\quad d = 3$

$a_2 = 8$

$2650 = \frac{2 \cdot 5 + (n-1) \cdot 3}{2} \cdot n \Rightarrow$

$5300 = (10 + 3n - 3)n$

$5300 = 7n + 3n^2$

$3n^2 + 7n - 5300 = 0$

$n \approx 40.8$

$\boxed{\text{there's no } n \text{ for } S = 2650}$

⑥ $a_5 = 20$

$a_{15} = 60$

$d = \frac{60-20}{15-5} = \frac{40}{10} = 4.$

$a_{10} = a_5 + (10-5) \cdot 4 = 20 + 20 = 40.$

$a_{10} = 40 = \frac{a_{15} + a_5}{2} = \frac{60+20}{2} = 80$ //

⑦ 20 step $\quad a_1 = 5\text{ cm} \quad d = 0{,}5 \quad a_{20} = a_1 + (n-1) \cdot d =$

$= 5 + 19 \cdot 0{,}5 = 14{,}5$

$S = \frac{a_1 + a_n}{2} \cdot n = \frac{5+14{,}5}{2} \cdot 20 =$

$= 195\text{ cm}$

⑧ $a_1 = 11$

$d = 3 \quad n$ where $S_n > 1000.$

$S_n = \frac{2a_1 + (n-1) \cdot d}{2} \cdot n = \frac{22 + (n-1) \cdot 3}{2} \cdot n =$

$= \frac{22n + 3n^2 - 3n}{2} = \frac{3n^2 + 19n}{2}$

random $n = 20 \quad \frac{1200 + 380}{2} = 790$

$n = 25 \quad \frac{1875 + 475}{2} = 1175.$

$\boxed{n = 23 \quad \frac{1587 + 437}{2} = 1012}$ //

9) $\sum_{k=3}^{12} \left(4\left(\frac{1}{2}\right)^k\right)$ $k=0$.

$j = k-3$

$\sum_{k=3}^{12} 4\left(\frac{1}{2}\right)^k \Rightarrow \sum_{j=0}^{9} 4\left(\frac{1}{2}\right)^{j+3} = \sum_{j=0}^{9} \frac{1}{2}\cdot\left(\frac{1}{2}\right)^j = \sum_{j=0}^{9} \left(\frac{1}{2}\right)^{j} \cdot \frac{1}{2}$

$\sum_{k=0}^{9} \left(\frac{1}{2}\right)^k \cdot \frac{1}{2}$

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10) $a_{10} - ?$ if $a_2 = -6$ $a_5 = 48$.

$a_n = a_1 \cdot r^{n-1}$

$48 = (-6) \cdot r^{5-2} = -8 = r^3$

$r = -2$.

$a_{10} = (-6) \cdot (-2)^{10-2} = (-6) \cdot (-2) =$

$= 6 \cdot 2^9 \quad 3072$

$-6 = a_1 (-2)$

$a_1 = 3$

$a_{10} = 3 \cdot (-2)^9 = -1536$ //

11) $a_4 = 54 \quad a_7 = 1458 \quad r - ?$

$$a_7 = a_4 \cdot r^{7-4} \Rightarrow \frac{1458}{54} = \frac{54 \cdot r^3}{54}$$

$$27 = r^3$$

$$\boxed{r = 3}$$

12) $S_{15} - ? \quad a_1 = 8 \quad r = \frac{3}{4}$

$$a_{15} = 8 \cdot \left(\frac{3}{4}\right)^{14}$$

$$\cancel{S_n = \frac{8\left(\left(\frac{3}{4}\right)^{15} - 1\right)}{\frac{3}{4} - 1} = \frac{8 \cdot \left(\frac{3^{15} - 4^{15}}{4^{15}}\right)}{-\frac{1}{4}} =}$$

$$\cancel{= -32 \cdot \frac{3^{15} - 4^{15}}{4^{15}} = -32 \cdot \frac{14348907 - 1073741824}{1073741824}}$$

$$\cancel{= +32 \cdot 1059392917}$$

$$\cancel{S_n = \frac{8\left(\left(\frac{3}{4}\right)^{15} - 1\right)}{\left(\frac{3}{4}\right) - 1} =}$$

$$S_n = a_1 \frac{1 - r^n}{1 - r}$$

$$S_n = 8 \cdot \frac{1 - \left(\frac{3}{4}\right)^{15}}{1 - \frac{3}{4}} = 8 \cdot \left(\frac{1 - r^{15}}{\frac{1}{4}}\right) =$$

$$= 32 \cdot \left(1 - r^{15}\right) \approx 32$$

r^{15} - small number

13) $P(x) = x^5 - 4x^3 + x^2 + 7$
Degree: 5 4 terms.

14) $(2x^4 - 3x^2 + x - 5) + (x^3 - 2x^2 + 4x + 7) =$

$= 2x^4 - 5x^2 + x^3 + 5x + 2 =$

$= 2x^4 + x^3 - 5x^2 + 5x + 2$

15) $(x^2 - x + 2)(x^2 + x + 1) = \cancel{x^4 + x^3 + x^2 - x^3 - x^2} - x$
$\cancel{+ 2x^2 + 2x + 2} =$

$= \cancel{x^4 + 2x^2 + 2x + 2}$

$x^4 + \cancel{x^3} + \cancel{x^2} - \cancel{x^3} - \cancel{x^2} - x + 2x^2 + 2x + 2 =$

$= x^4 + 2x^2 + x + 2$

16) GCD & LCM

$24x^3y^3z^5$ $36x^5y^3z^2$

$2 \cdot 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot z \cdot z \cdot z \cdot z \cdot z$ $2 \cdot 2 \cdot 3 \cdot 3 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot z \cdot z$

$GCD = 12x^3y^3z^2$

$LCM = 72 \cdot x^5 \cdot y^3 \cdot z^5$

17) $x^4 - 13x^2 + 36 = (x^2 - 9)(x^2 - 4) = (x+2)(x-2)(x+3)(x-3)$

x -9 $+3$ 2
x^2 -4 -3 2

8 · 9 · 10

18) $(2x+3y)^5 =$

$n=5$ 1 5 10 10 5 1

$1(2x)^5\cdot(3y)^0 + 5\cdot(2x)^4\cdot(3y)^1 + 10\cdot(2x)^3\cdot(3y)^2 + 10(2x)^2\cdot(3y)^3 +$
$+ 5\cdot(2x)^1(3y)^4 + 1(2x)^0\cdot(3y)^5 =$
$= 32x^5 + 240x^4y + 720x^3y^2 + 1080x^2y^3 + 810xy^4 + 243y^5$

19) $6x^3 + 11x^2 - 31x + 15 \mid 3x-2$

$-/6x^3 - 4x^2$ $\qquad 2x^2+5x-7$

$15x^2 - 31x$

$-/15x^2 - 10x$

$-21x + 15$

$+/-21x + 14$

1

$(2x^2+5x-7)(3x-2)+1 = 6x^3 + 11x^2 - 31x + 15$

20) $f(x) = 2x^4 - 5x^3 + x^2 - 4$

$p = -4$ $\qquad p = \pm 1, \pm 2, \pm 4$
$q = 2$ $\qquad q = \pm 1, \pm 2$

$\frac{p}{q} = \pm\frac{1}{1}; \pm\frac{1}{2}$ | $\frac{p}{q} = \pm\frac{2}{1}; \pm\frac{2}{2}$ | $\frac{p}{q} = \pm\frac{4}{1}; \pm\frac{4}{2}$

$\boxed{\frac{p}{q} = \pm 1, \pm 2, \pm 4, \pm\frac{1}{2}}$