$$\frac{\frac{1}{b+1}}{1-\frac{1}{b+1}} = \frac{1}{b+1}\left(\frac{1}{\frac{b+1-1}{b+1}}\right) = \frac{1}{b+1}\left(\frac{1}{\frac{b}{b+1}}\right)$$

$$= \frac{1}{\cancel{b+1}} \cdot \frac{\cancel{b+1}}{b} = \frac{1}{b}$$

(1) Prove that e is irrational.

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$

$e \approx 2{,}71828$

$$e = \frac{m}{n} \qquad n \neq 0$$

$$e = \sum_{n=0}^{\infty} \frac{1}{n!}$$

$$e = \cancel{\sum_{n=0}^{\infty}} \frac{1}{n!}$$

$$e = \sum_{n=0}^{\infty}\frac{1}{n!} \qquad e = \frac{a}{b} \qquad b \geq n$$

$$x = b!\left(e - \sum_{n=0}^{b}\frac{1}{n!}\right)$$

$$x = b!\left(\frac{a}{b} - \sum_{n=0}^{b}\frac{1}{n!}\right) = b!\cdot\frac{a}{b} - \sum_{n=0}^{b}\frac{b!}{n!}$$

$$x = \underbrace{a\cdot(b-1)!}_{\text{integer}} - \underbrace{\sum_{n=0}^{b}\frac{b!}{n!}}_{\text{intege}}$$ $\Rightarrow x = \text{integer} - \text{integer}$ [illegible]

$$\frac{\frac{1}{b+1}}{\frac{b+1}{b+1}} = \frac{1}{b+1} \cdot \frac{b+1}{b} = \frac{1}{b}$$

$$x = b!\left(\sum_{n=0}^{\infty} \frac{1}{n!} - \sum_{n=0}^{b} \frac{1}{n!}\right)$$

$$= \cancel{b!}\left(\sum_{n=b+1}^{\infty} \frac{b!}{n!} > 0\right.$$

$$\frac{b!}{n!} = \frac{b(b-1)(b-2)}{n(n-1)(n-2)\ldots(b+1)\cancel{b(b-1)\ldots}}$$

$$= \frac{1}{\underbrace{n(n-1)(n-2)\ \ (b+1)}_{n-b}} < \frac{1}{(b+1)^{n-b}}$$

$\frac{}{b+1}$ smallest term

$a = b+1 \qquad 2 = n$

$n-(b+1)+1$

$n-b$

$$x = \sum_{n=b+1}^{\infty} \frac{b!}{n!} < \sum_{n=b+1}^{\infty} \frac{1}{(b+1)^{n-b}}$$

$k = n-b$

$[a, 2]$; $\boxed{2-a+1}$

$2 = n \qquad a \to b+1$

$n = b+1$

$n-b = 1$

$k = 1$

$$\sum_{k=1}^{\infty} \frac{1}{(b+1)^k}$$

$$\frac{\frac{1}{b+1}}{1-\frac{1}{b+1}} = \frac{1}{b}$$

$\frac{13}{2} = 7 \quad 6$ $\quad 49 - 36 = 13$

$7^2 - 6^2 = 13.$

$$\frac{\frac{1}{b+1}}{1-\frac{1}{b+1}} = \frac{1}{b+1} \cdot \left(\frac{1}{\frac{b+1-1}{b+1}}\right) = \frac{1}{b+1}\left(\frac{1}{\frac{b}{b+1}}\right)$$

$$= \frac{1}{\cancel{b+1}} \cdot \frac{\cancel{b+1}}{b} = \frac{1}{b} < 1$$

$0 < x < 1$

$x \Rightarrow$ contradiction

$e = \frac{a}{b} \Rightarrow \quad e \neq \frac{a}{b}$

② HW-2

every odd integer is difference of 2 square

$n = 2k+1 \quad a = k+1 \quad b = k$

$2k+1 \quad n = 11 \quad a = 6 \quad b = 5.$

$2k+1$

$(k+1)^2 - k^2 \quad a^2 - b^2 = (a+b)(a-b) \quad n/2$

$= 2k+1 \quad 3 \quad = \cancel{4^2 - 2^2 = 8 - 4}$

$3 = (2 \cdot 2 + 1)^2 \quad a+2 \quad 3 = 2^2 - 1^2$

$3 = a+b \quad a = 1+b \quad = 3.$

$a - b = 1 \quad 3 = a + 2b \quad (2-1)(2+1)$

$b = 1$

$A = 2k+1$

$a = k+1 \quad b = k$

$\frac{2k+1}{2} = k \quad k+1$

$\frac{15}{2} = 7.6$

$a^2 - b^2 = $ odd.

21 $\quad$ 11 & 10

$11^2 - 10^2 = 121 - 100 = 21$

3

$i + \frac{a}{b} = i \frac{c}{d}$

$i = \frac{c}{d} - \frac{a}{b} = \frac{cb - ad}{db}$

i - rational $\rightarrow$ contradict

$i \cdot i_2 = \frac{a}{b} \rightarrow$ disprove $\quad \sqrt{2} \cdot \sqrt{2} = 2$

4 $\quad \forall i = \quad =$

$\sqrt{2}$ irration. $\quad \sqrt{2} \cdot \sqrt{2} = 2$

p: $x + y \geq 2$
q: $x \geq 1$ or $y \geq 1$

5 $x + y \geq 2$ x, y - real n.
$x \geq 1$ or $y \geq 1$

$p \Rightarrow q \equiv \neg q \to \neg p$

~~$x+y \geq 2$~~

$x \geq 1$ or $y \geq 1$ $x + y \geq 2$

$\neg$ $x \geq 1$ or $\neg y \geq 1$ $x + y \geq 2$

$x < 1$ $y < 1$

$x + y < 1 + 1$

$\boxed{x + y < 2} \to$

6 n - integer $n^3 + 5$ - odd n is even

a) contraposition if n^3+5 odd then n n. even

B) contradiction. p q

$\neg q \to \neg p$

$n = 2k+1$ n is odd $n^3 + 5$ even $8k^3 + 8k^2$

odd $\leftarrow (2k+1)^3 + 5 = 8k^3 + 4k^2 + 2k + 4k^2 + 4k + 1 = + 6k + 1$

n - integer

$\underbrace{n^3+5 \text{ -odd}}_{p}$, $\underbrace{n \text{ is even}}_{q}$,

a) contraposition $p \to q \equiv \neg q \to \neg p$

$\neg q \to \neg p$

n is odd $\quad n^3+5$ - even

$n = (2k+1)$

$n^3+5 = (2k+1)^3+5 = 8k^3+12k^2+6k+6 =$

$= 2(4k^3+6k^2+2k+3) =$ even

$8k^3+12k^2+6k+6$

$= 2(4k^3+6k^2+3k+3) \to$ even

b) n^3+5 is odd $\quad$ n - even

n-odd $\quad \boxed{n = \text{even} \quad n^3+5 - \text{even}}$

$n = 2k \quad \boxed{8k^3+5}$

$n = 2k+1$

$(2k+1)^3+5 = 2(4k^3+6k^2+3k+3) \to$ even

$n^3+5 \to$ odd

ex 8. x & y - integer.

$\underbrace{(x \cdot y \quad x+y \text{ - even})}_{p}$, $\underbrace{(x \text{ \& } y)}_{q}$ are even

$\neg q \to \neg p$

x & y - odd $\qquad x \cdot y \quad x+y$ - odd

$x = 2k+1 \qquad x+y$ - odd

$x \cdot y =$

y ~~even~~

y odd

$x+y = (2k+1) + 2j = 2(k+j)+1$ - odd

$y = 2j+1 \qquad x \cdot y = (2k+1)(2j+1) =$

$= 4kj + 2k + 2j + 1 =$

$= 2(x)+1$ odd

$x = 2n+1$

$y = 2n$

$y = 2n+1$