Homework № 1

Chapter 1.10 (Probability)

№ 5 Suppose we toss a fair coin until we get exactly two heads. Describe the sample space S. What is the probability that exactly k tosses are required?

$\Omega = \{(\omega_1, \omega_2, \omega_3, \dots, H) : \omega_i \in \{H, T\}\}$

I have described sample space in a such way because we need exactly 2 Heads and the last one is Head.

If $k<2$, the $P(X=k)=0$.

Probability of requring H is $p=\frac{1}{2}$ and $q=1-p=\frac{1}{2}$

Prob of finding H in $k-1$ tosses is

$$\binom{k-1}{1}\cdot\left(\frac{1}{2}\right)\cdot\left(\frac{1}{2}\right)^{k-2}=(k-1)\cdot\frac{1}{2}\cdot\frac{1}{2^{k-2}}=$$

$$=\frac{k-1}{2^{k-1}}$$

And plus probability of getting H in last toss, because they're independent.

$$\frac{k-1}{2^{k-1}}\cdot\frac{1}{2}=\frac{k-1}{2^k}.$$

$\boxed{\text{Answer: } P(X=k)=\frac{k-1}{2^k}}$

$\sqrt{\underline{o}}\ 13$ Suppose that a fair coin is tossed repeatedly until both a head and tail have appeared at least once.

(a) Describe the sample space Ω.

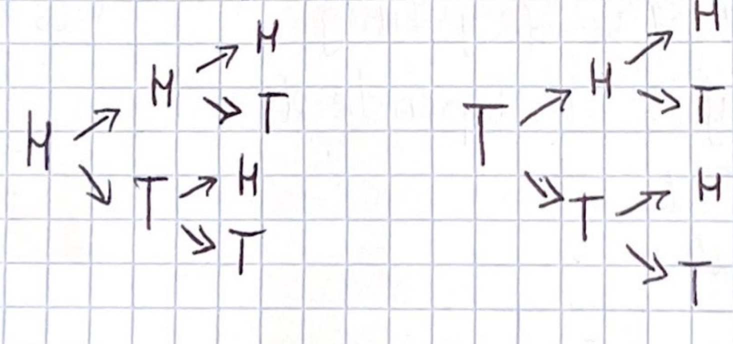
$$\Omega = \{(\omega_1, \omega_2, \omega_3, \dots, \omega_k, \omega_{k+1}) : \omega_i \in \{H, T\}\}$$

$\omega_k \neq \omega_{k+1}$

$\omega_1, \omega_2, \dots = \omega_k$.

(b) What is the probability that three tosses will be required?

When 3 tosses



$\Omega = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

If we think about probability that three tosses will be required is

HHT or TTH

So, $P(X=3) = \frac{2}{8} = \frac{1}{4}$

$\boxed{\text{Answer: } \frac{1}{4}}$

Chapter 2.14 (Random variables)

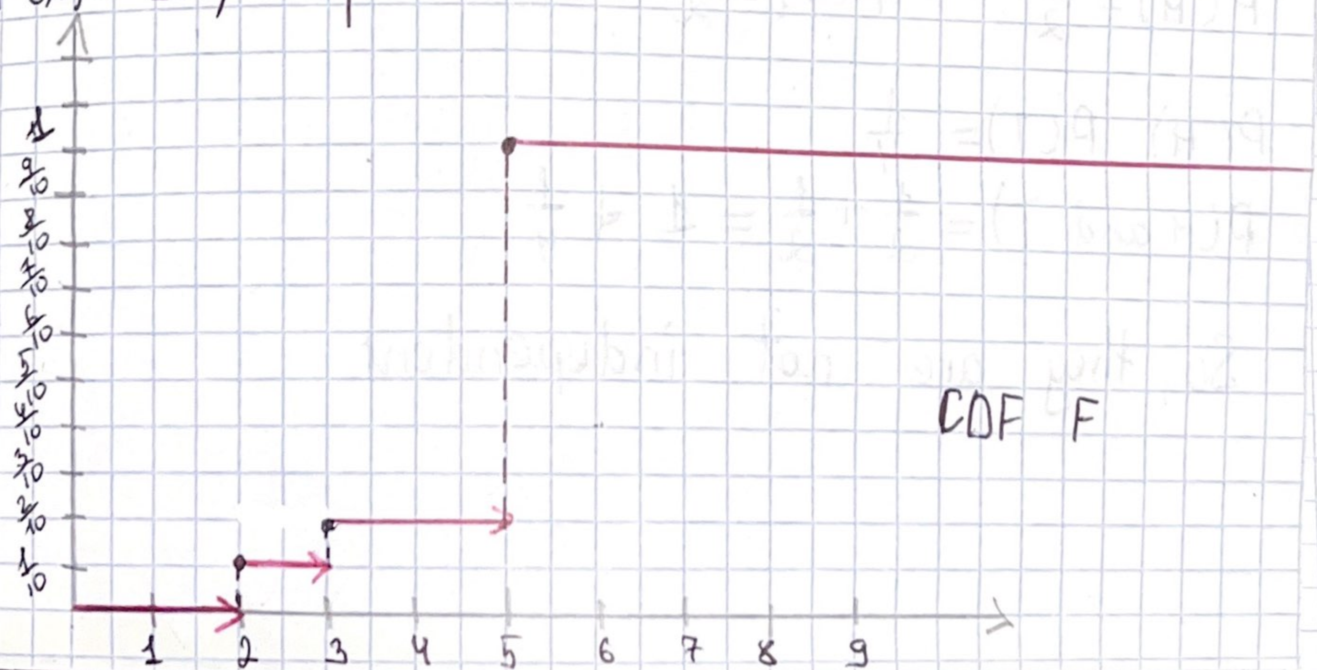
№ 2 Let X be such that $P(X=2) = P(X=3) = \frac{1}{10}$ and $P(X=5) = \frac{8}{10}$. Plot the CDF F. Use F to find $P(2 < X \leq 4{,}8)$ and $P(2 \leq X \leq 4{,}8)$.

$F(x) = 0$, if $X < 2$

$F(x) = \frac{1}{10}$, if $2 \leq X < 3$

$F(x) = \frac{2}{10}$, if $3 \leq X < 5$

$F(x) = 1$, if $X \geq 5$



Using F (our graphic)

$$P(2 < X \leq 4{,}8) = \frac{2}{10} - \frac{1}{10} = \frac{1}{10}$$

$$P(2 \leq X \leq 4{,}8) = \frac{2}{10}.$$

№ 11a Suppose we toss a coin once and let p the probability of heads. Let X denote the number of heads and let Y denote the number of tails.

(a) Prove that X and Y are dependent.

$\Omega = \{H, T\}$

$P(H) = \frac{1}{2}$ $\quad$ $P(T) = \frac{1}{2}$

$P(H) \cdot P(T) = \frac{1}{4}$

$P(H \text{ and } T) = \frac{1}{2} + \frac{1}{2} = 1 \neq \frac{1}{4}$

So, they are not independent.

Chapter 1.10 (Probability)

Ex.22

22. (Computer Experiment.) Suppose we flip a coin n times and let p denote the probability of heads. Let X be the number of heads. We call X a binomial random variable, which is discussed in the next chapter. Intuition suggests that X will be close to n p. To see if this is true, we can repeat this experiment many times and average the X values. Carry 1.10 Exercises 17 out a simulation and compare the average of the X's to n p. Try this for p = .3 and n = 10, n = 100, and n = 1, 000.

My solution:

```
from statistics import mean
import numpy as np

p = 0.3

X1 = []
X2 = []
X3 = []
for i in range(100000):
    a = np.where(np.random.uniform(low = 0, high = 1, size = 10) < p, 1, 0)
    b = np.where(np.random.uniform(low = 0, high = 1, size = 100) < p, 1, 0)
    c = np.where(np.random.uniform(low = 0, high = 1, size = 1000) < p, 1, 0)
    X1.append(a)
    X2.append(b)
    X3.append(c)

print("For n = 10:")
print(mean(sum(X1) / len(X1)) * 10)
print(10 * p)
print()
print("For n = 100:")
print(mean(sum(X2) / len(X2)) * 100)
print(100 * p)
print()
print("For n = 1000:")
print(mean(sum(X3) / len(X3)) * 1000)
print(1000 * p)
```

Answer to the code:

```
C:\Users\Айжан\Desktop\Advanced Statistics>C:/Users/Айжан/AppData/Local/Microsoft/WindowsApps/python3.9.exe "c:/Users/Айжан/Desktop/Advanced Statistics/homewo
rk 1/chapter1(22).py"
For n = 10:
3.00157
3.0

For n = 100:
30.00551
30.0

For n = 1000:
300.01511
300.0
```

First value is the mean, and the second value is np. And they are close to each other.

Chapter 2.14(Random Variables)

Ex.13b

Let X ∼ N(0, 1) and let Y = eX. (a) Find the pdf for Y . Plot it. (b) (Computer Experiment.) Generate a vector x = (x1,...,x10,000) con sisting of 10,000 random standard Normals. Let y = (y1,...,y10,000) where yi = exi . Draw a histogram of y and compare it to the pdf you found in part (a).

My solution:

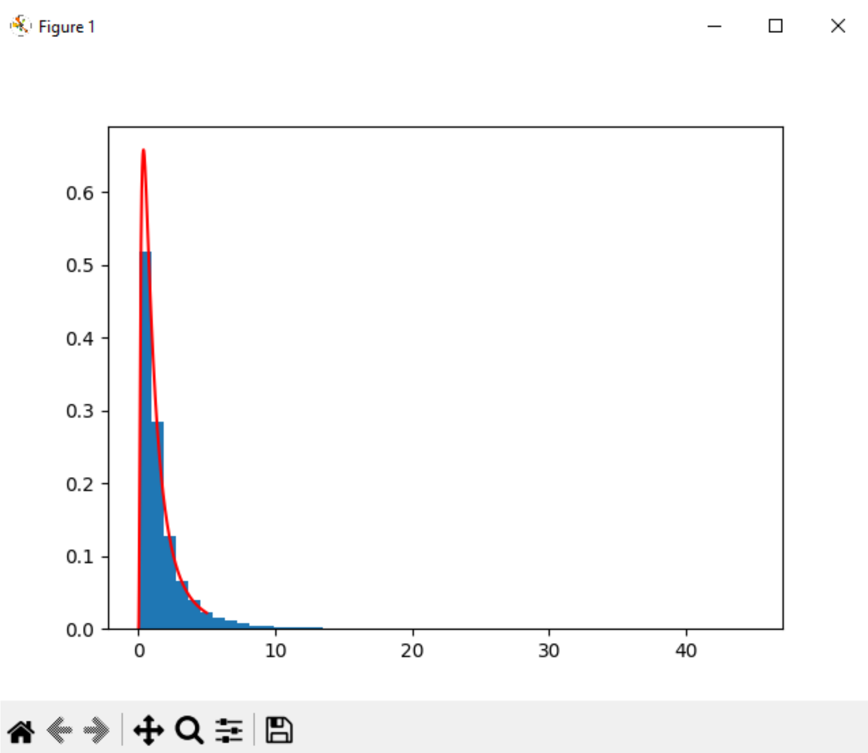
```
from cmath import exp
import numpy as np
from scipy.stats import norm
import matplotlib.pyplot as plt

n = 10000
x = norm.rvs(size=n)
y = np.exp(x)

b = np.arange(0.01, 5, step = 0.01)
a = norm.pdf(np.log(b)) / b

plt.hist(y, bins=50, density=True)
plt.plot(b, a, label='true density', color = 'r')
plt.show()
```

Answer to the code:



Ex.18

Let X ~ N(3, 16). Solve the following using the Normal table and using a computer package. (a) Find P(X < 7). (b) Find P(X > −2). (c) Find x such that P(X>x) = .05. (d) Find P(0 ≤ X < 4). (e) Find x such that P(|X| > |x|) = .05.

My solution:

```
import numpy as np
from scipy.stats import norm

#X ~ N(3, 16)

x = "P(X < 7): "
#cdf(x, loc=0, scale=1) = Cumulative distribution function.
y = norm.cdf(7, loc=3, scale=4)
print(x + str(y))

x = "P(X > -2): "
y = 1 - norm.cdf(-2, loc=3, scale=4)
print(x + str(y))

x = "Find x such that P(X > x) = 0.05: "
#ppf(q, loc=0, scale=1) = Percent point function (inverse of cdf — percentiles).
y = norm.ppf(0.95, loc=3, scale=4)
print(x + str(y))

x = "Find P(0 ≤ X < 4): "
y = norm.cdf(4, loc=3, scale=4) - norm.cdf(0, loc=3, scale=4)
print(x + str(y))

x = "Find x such that P(|X| > |x|) = 0.05: "
y = norm.ppf(0.975, loc=3, scale=4)
print(x + str(y))
```

Answer to the code:

```
C:\Users\Айжан\Desktop\Advanced Statistics>C:/Users/Айжан/AppData/Local/Microsoft/WindowsApps/python3.9.exe "c:/Users/Айжан/Desk
rk 1/chapter2(18).py"
P(X < 7): 0.8413447460685429
P(X > -2): 0.8943502263331446
Find x such that P(X > x) = 0.05: 9.57941450780589
Find P(0 ≤ X < 4): 0.3720789733060555
Find x such that P(|X| > |x|) = 0.05: 10.839855938160216
```