Efficient template norm calculation

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These are some notes on the Norm of the signal template present in the likelihood calculations.

$$\ln p = -\frac{1}{2}\langle \delta\mathbf{t}|\delta\mathbf{t}\rangle + \langle \delta\mathbf{t}|\mathbf{h}\rangle - \frac{1}{2}\langle \mathbf{h}|\mathbf{h}\rangle \tag{1}$$

The last term can be precalculated when we are searching for the reduced bases. This is because when we are evaluating how well the RB spans the signal template space, we calculate the following:

$$\left|\left|\mathbf{h} - \sum_i c_i \mathbf{e}_i\right|\right|^2 \tag{2}$$

which could be expanded as:

$$||\mathbf{h}||^2 - 2\left\langle \mathbf{h} \middle| \sum_i c_i \mathbf{e}_i \right\rangle + \left|\left|\sum_i c_i \mathbf{e}_i\right|\right|^2 = ||\mathbf{h}||^2 - 2\sum_i c_i \langle \mathbf{h}|\mathbf{e}_i\rangle + \sum_{i,j} \mathcal{G}_{ij} c_i c_j \tag{3}$$

where $\mathcal{G}$ is the grammian matrix and c_i are projection coeficients.

This is important, because we reduce the order of calculations as the first term can be precomputed before the parameter search, the inner product of the second term is used when calculating the c_i, which means, that we can express this as folows:

$$||\mathbf{h}||^2 - 2\sum_{i,j} \mathcal{G}^{-1}_{ij} \widetilde{c}_i \widetilde{c}_j + \sum_{i,j,k,l} \mathcal{G}_{ij} \mathcal{G}^{-1}_{jk} \mathcal{G}^{-1}_{il} \widetilde{c}_i \widetilde{c}_j = ||\mathbf{h}||^2 - \sum_{i,j} \mathcal{G}^{-1}_{ij} \widetilde{c}_i \widetilde{c}_j \tag{4}$$

where

$$\widetilde{c}_i = \langle \mathbf{h}|\mathbf{e}_i\rangle \tag{5}$$

Also, by precomputing the signal template norm, we do not have to recompute it during MCMC.