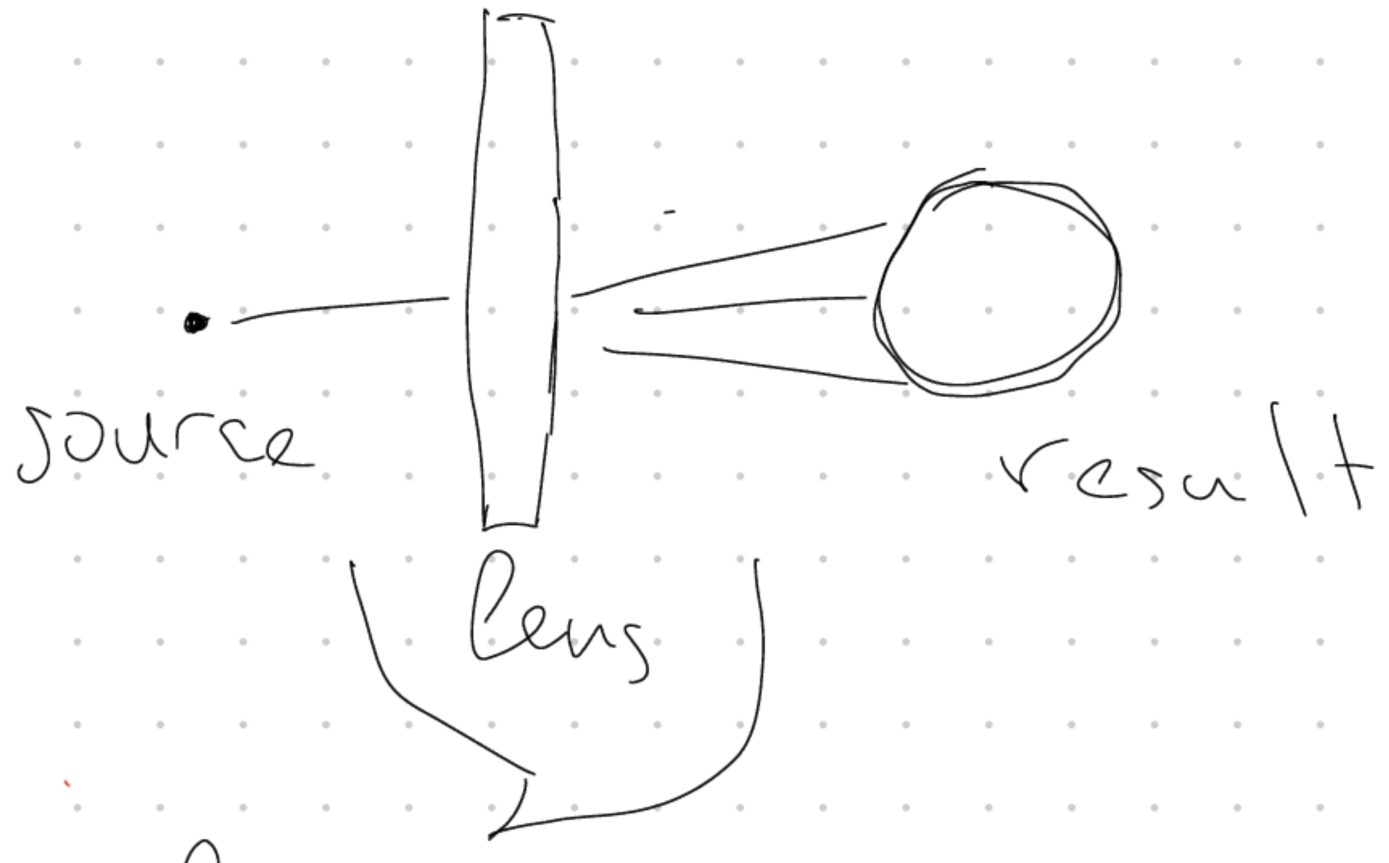
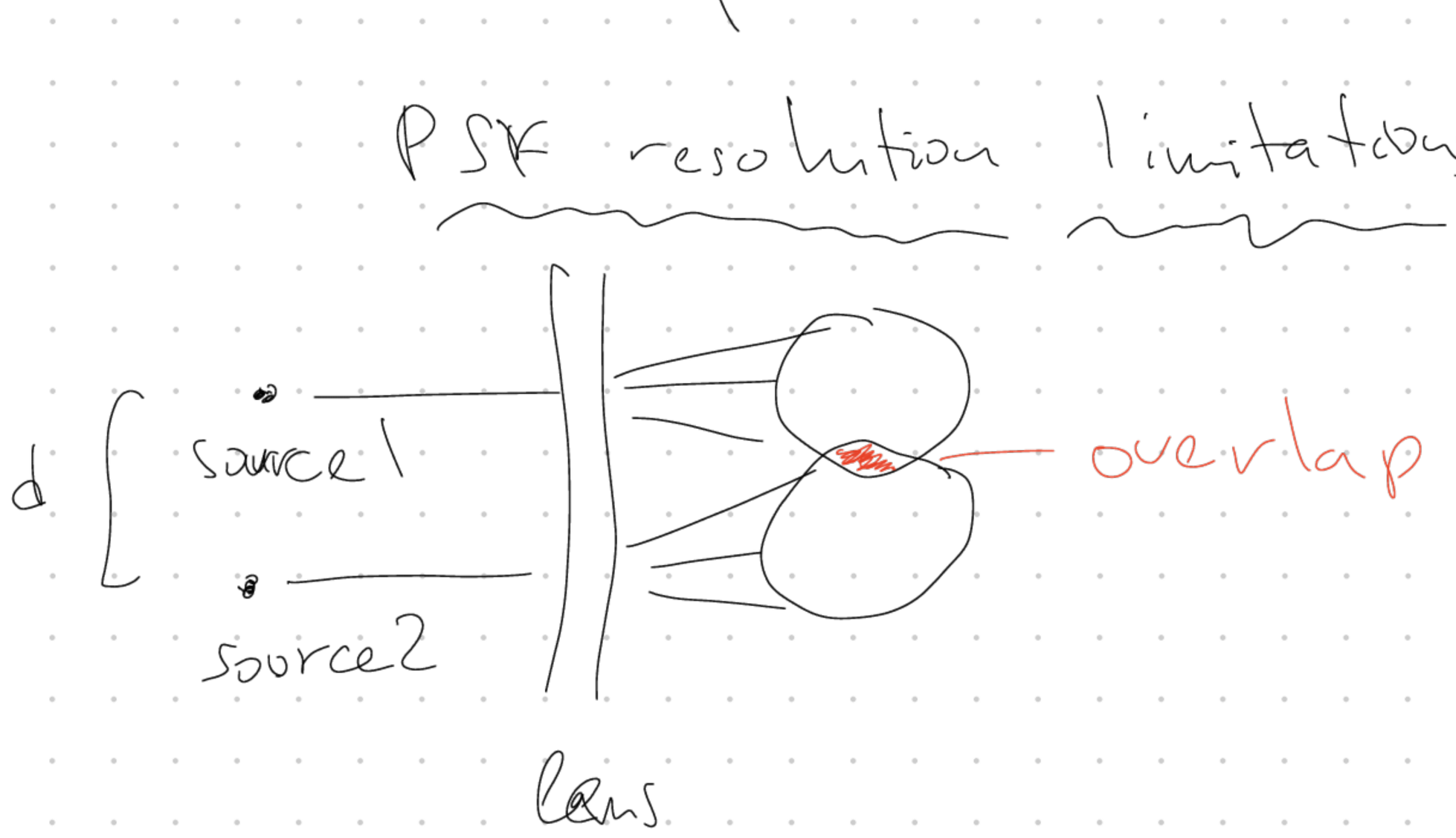
① PSF



light bends and transforms

PSF = f(light transform/source)

PSF resolution limitations



PSF limits the distance (d) between sources, i.e. sets lower bound.

PSF= Invariance

- PSF → Linear Shift-Invariant (LSI)
  - Image = Obj * PSF
  - Restoration: $F(Obj) = \frac{F(Image)}{F(PSF)}$

(2a) $$F(\omega) = \int f(x)\, e^{-i\omega x}\, dx$$

$$f(x) = \Pi(x): \quad F(\omega) = \int_{-x_0}^{x_0} A\, e^{-i\omega x}\, dx =$$

$$= A \int_{-x_0}^{x_0} e^{-i\omega x}\, dx = \frac{A}{-i\omega}\, e^{-i\omega x}\Big|_{-x_0}^{x_0}$$

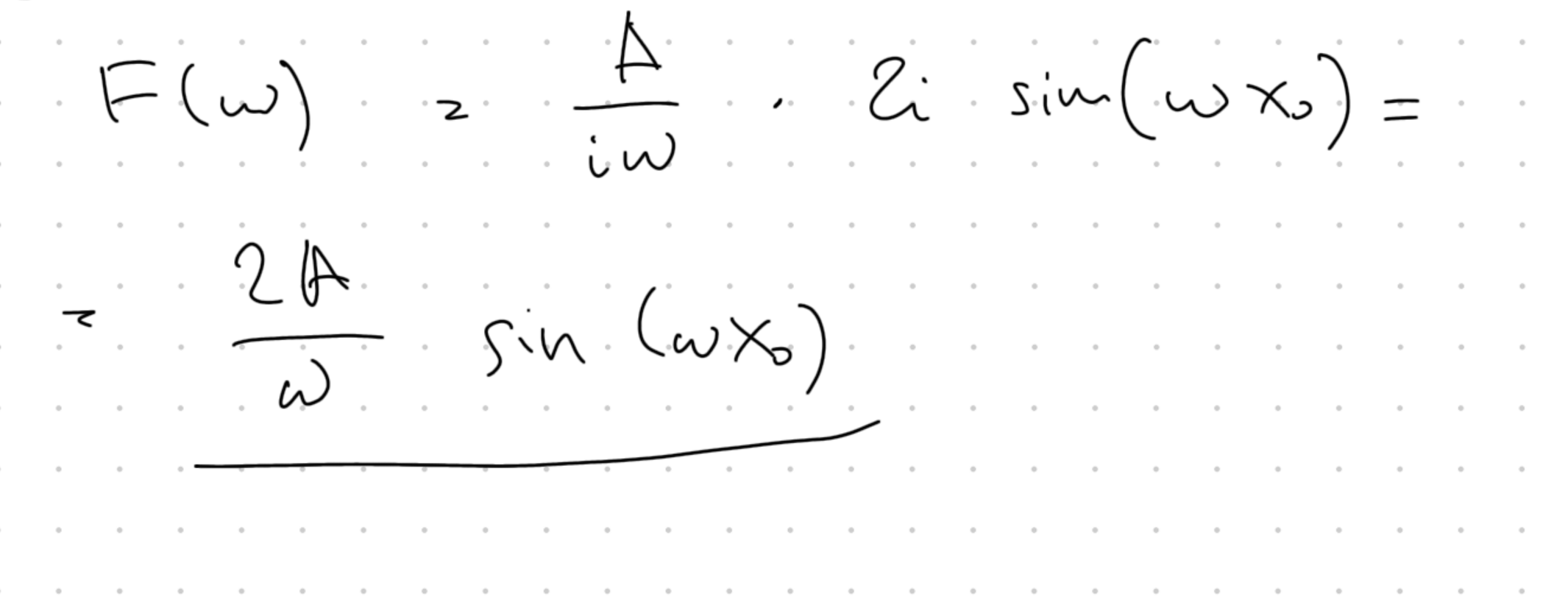
$$= \frac{A}{-i\omega}\left(e^{-i\omega x_0} - e^{i\omega x_0}\right)$$

By Euler's formula:

$$F(\omega) = \frac{A}{i\omega} \cdot 2i \sin(\omega x_0) =$$

$$= \frac{2A}{\omega} \sin(\omega x_0)$$

$A = 1$, $x_0 = 1$:



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Theorem:

$$\int_{-\infty}^{\infty} |f(x)|^2 dx = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(\omega)|^2 d\omega$$

$$\int_{-\infty}^{\infty} \Pi(x) dx = \int_{-x_0}^{x_0} |A|^2 dx = 2x_0 A^2$$

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} |F(\omega)|^2 d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left| 2A \frac{\sin(\omega x_0)}{\omega} \right|^2 d\omega$$

$$= \frac{4A^2}{2\pi} \int_{-\infty}^{\infty} \frac{\sin^2(\omega x_0)}{\omega^2} d\omega$$

$$= \frac{4A^2 x_0}{2\pi} \int_{-\infty}^{\infty} \frac{\sin^2(u)}{u^2} du =$$

$$= \frac{4A^2 x_0}{2\pi} \cdot \pi = 2x_0 A^2$$

$$\rightarrow \int_{-\infty}^{\infty} \Pi(x) dx = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(\omega)|^2 d\omega$$