[course site]

#DLUPC

Day 1 Lecture 2

The Perceptron

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Acknowledgements

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DCU

Video lecture (DLSL 2017)

Outline

1. Supervised learning: regression/classification
2. Single neuron models (perceptrons)
    a. Linear regression
    b. Logistic regression
    c. Multiple outputs and softmax regression

Types of machine learning

Yann Lecun's Black Forest cake

- "Pure" Reinforcement Learning (cherry)
  - The machine predicts a scalar reward given once in a while.
  - **A few bits for some samples**
- Supervised Learning (icing)
  - The machine predicts a category or a few numbers for each input
  - Predicting human-supplied data
  - **10→10,000 bits per sample**
- Unsupervised/Predictive Learning (cake)
  - The machine predicts any part of its input for any observed part.
  - Predicts future frames in videos
  - **Millions of bits per sample**

- (Yes, I know, this picture is slightly offensive to RL folks. But I'll make it up)

Types of machine learning

We can categorize three types of learning procedures:

1. Supervised Learning:

   $$\mathbf{y} = f(\mathbf{x})$$

2. Unsupervised Learning:

   $$f(\mathbf{x})$$

3. Reinforcement Learning:

   $$\mathbf{y} = f(\mathbf{x})$$

   $$\mathbf{z}$$

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		LECTURES (D5-010)		LAB
Week	Tuesday	15:00	16:00	17:00
1	19/9/2017	Welcome (XG)	Perceptron (XG)	Getting Started (JT)
2	26/9/2017	Multilayer Perceptron (ES)	Visit BSC (JT)	Visit BSC (JT)
3	3/10/2017	Layers (ES)	Frameworks (JT)	Keras & Tensorboard (JT)
4	10/10/2017	Backpropagation (VV)	Software stack (JT)	PyTorch (JT)
5	17/10/2017	Optimizers (VV)	Computational requirements (JT)	Project (XG)
6	24/10/2017	Loss function (JR)	Transfer learning (RM)	TensorFlow (JT)
7	31/10/2017	Methodology (JR)	Incremental (RM)	GPU (JT)
8	7/11/2017	Midterm (ES)	Midterm (ES)	Project (XG)
9	14/11/2017	RNNs (MRC)	Gated RNNs (MRC)	Project (XG)
10	21/11/2017	Attention (MRC)	Reinforcement (XG)	Project (XG)
11	28/11/2017	Unsupervised (XG)	Generative (SP)	GAN (SP)
12	5/12/2017	HOLIDAYS	HOLIDAYS	HOLIDAYS
13	12/12/2017	Project presentations (XG)	Project presentations (MRC)	Project presentations (ES)
14	19/12/2017	Jose M. Álvarez (XG)	Cristian Canton (XG)	Sponsors & Logistics (ES)

Types of machine learning

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Types of machine learning

We can categorize three types of learning procedures:

1. **Supervised Learning:**

$$\mathbf{y} = f(\mathbf{x})$$

We have a labeled dataset with pairs (**x**, **y**), e.g. classify a signal window as containing speech or not:

$\mathbf{x}_1$ = [x(1), x(2), …, x(T)] $\mathbf{y}_1$ = "no"
$\mathbf{x}_2$ = [x(T+1), …, x(2T)] $\mathbf{y}_2$ = "yes"
$\mathbf{x}_3$ = [x(2T+1), …, x(3T)] $\mathbf{y}_3$ = "yes"
...

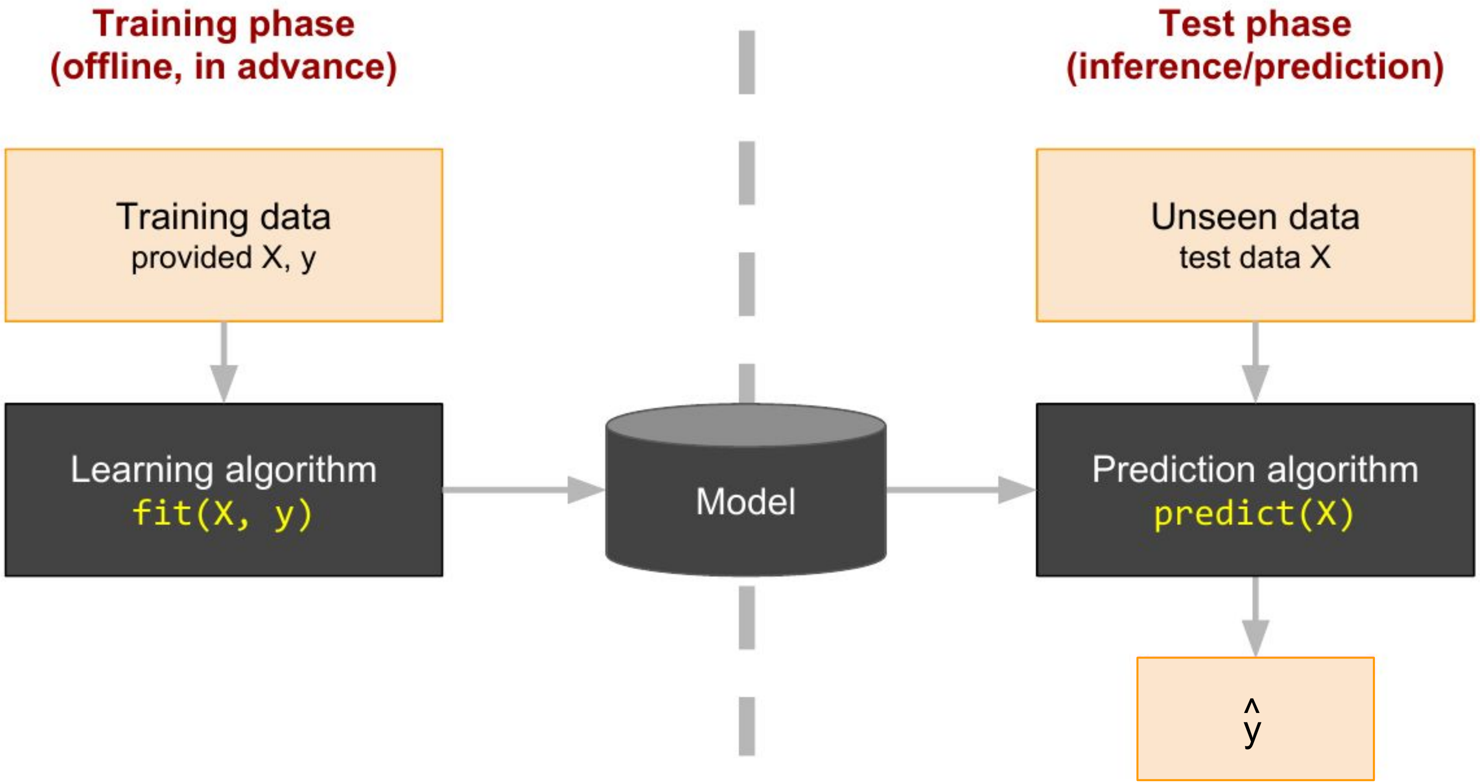
Supervised learning

Fit a function: $\mathbf{y} = f(\mathbf{x}), \quad \mathbf{x} \in \mathbb{R}^m$

Given paired training examples $\{(\mathbf{x}_i, \mathbf{y}_i)\}$

Key point: **generalize well to unseen examples**

Black box abstraction of supervised learning



Regression vs Classification

Depending on the type of target $\mathbf{y}$ we get:

- **Regression:** $\mathbf{y} \in \mathbb{R}^N$ is continuous (e.g. temperatures $\mathbf{y} = \{19^o, 23^o, 22^o\}$)
- **Classification:** $\mathbf{y}$ is discrete (e.g. $\mathbf{y} = \{1, 2, 5, 2, 2\}$).

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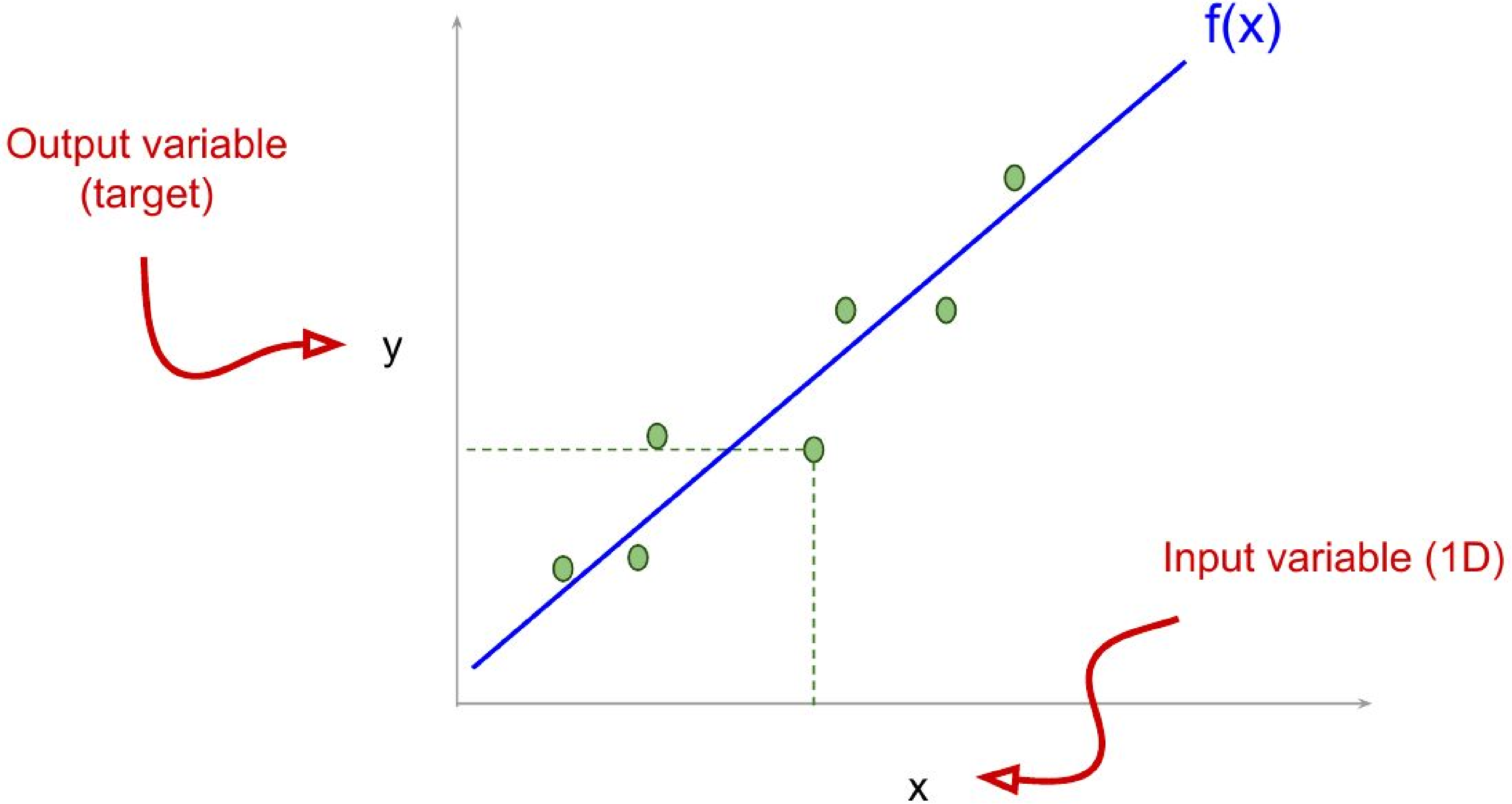
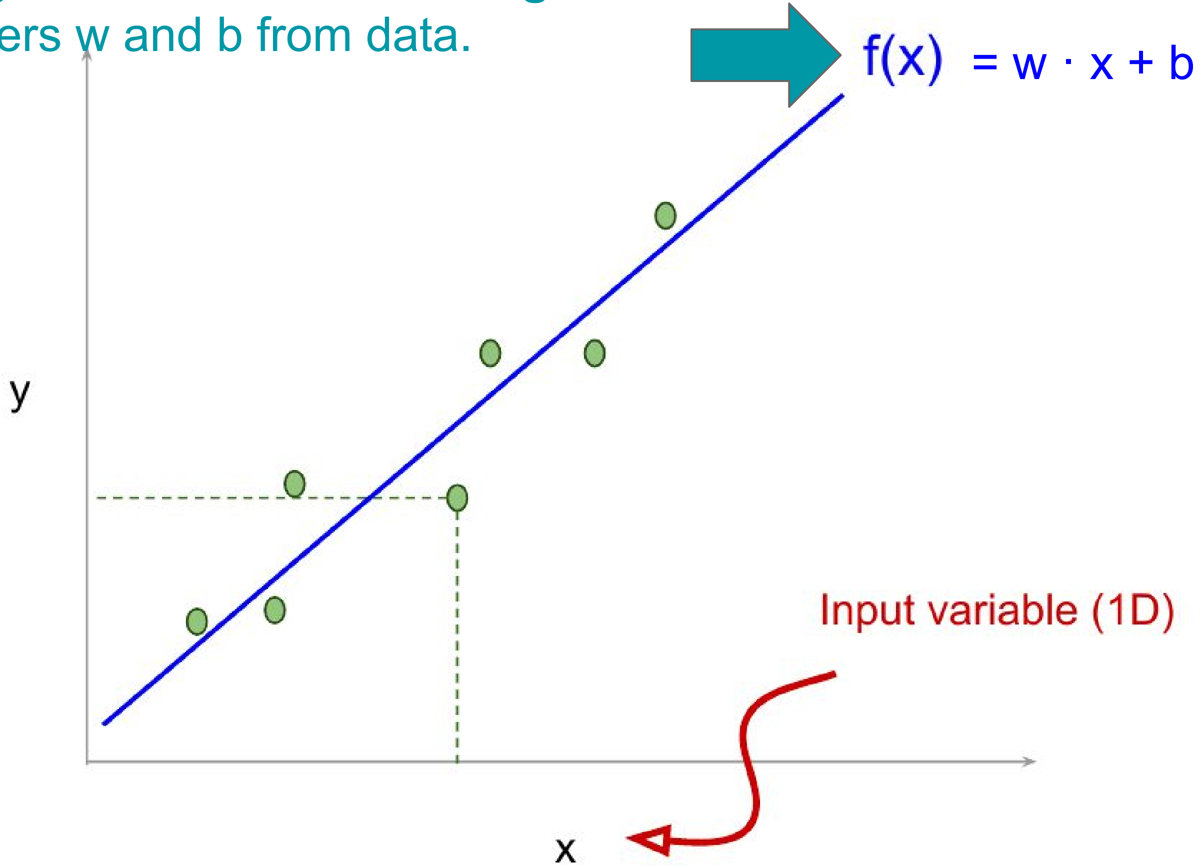
Linear Regression (eg. 1D input - 1D ouput)



Linear Regression (eg. 1D input - 1D ouput)

Training a model means **learning** parameters w and b from data.



Linear Regression (M-D input)

Input data can also be M-dimensional with vector **x**:

$$\boxed{y = \mathbf{w}^T \cdot \mathbf{x} + b} = \; w1 \cdot x1 + w2 \cdot x2 + w3 \cdot x3 + \ldots + wM \cdot xM + b$$

e.g. we want to predict the **price of a house (y)** based on:

x1 = square-meters (sqm)

x2,3 = location (lat, lon)

x4 = year of construction (yoc)

y = price = *w1*·(sqm) + *w2*·(lat) + *w3*·(lon) + *w4*·(yoc) + b

Regression vs Classification

Depending on the type of target $\mathbf{y}$ we get:

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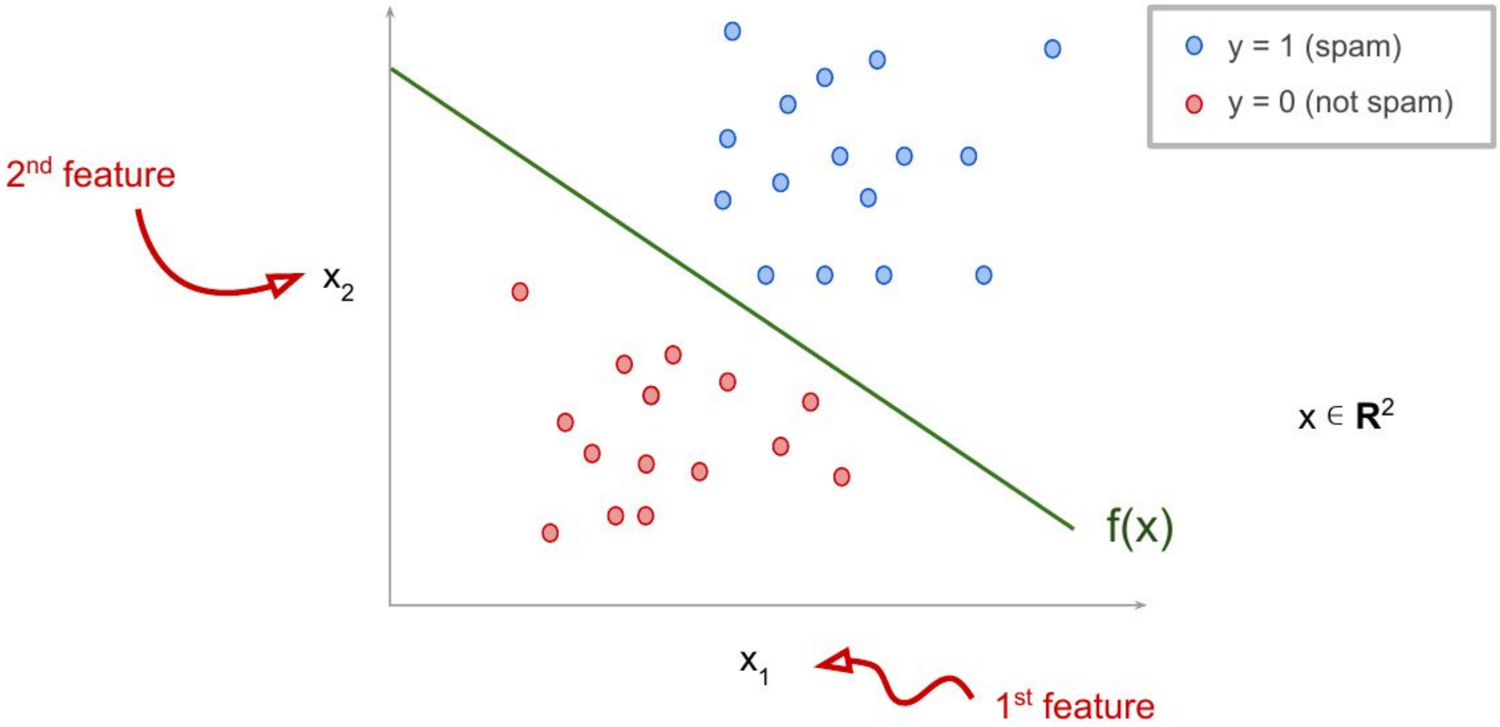
- **Classification:** $\mathbf{y}$ is discrete (e.g. $\mathbf{y}$ = {1, 2, 5, 2, 2}).

Binary Classification (eg. 2D input, 1D ouput)



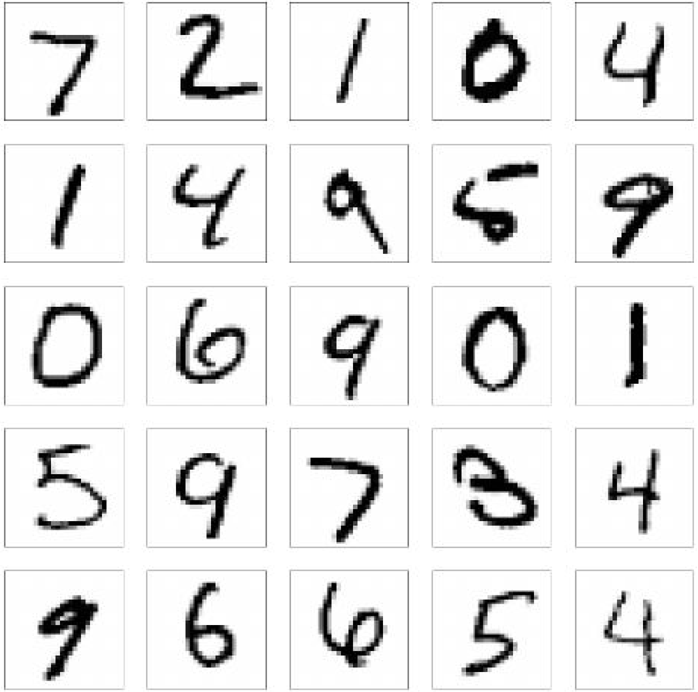
$x \in \mathbf{R}^2$

Multi-class Classification

Produce a classifier to map from pixels to the digit.

- If images are grayscale and 28×28 pixels in size, then $\mathbf{x}_i \in \mathbb{R}^{784}$
- $y_i \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

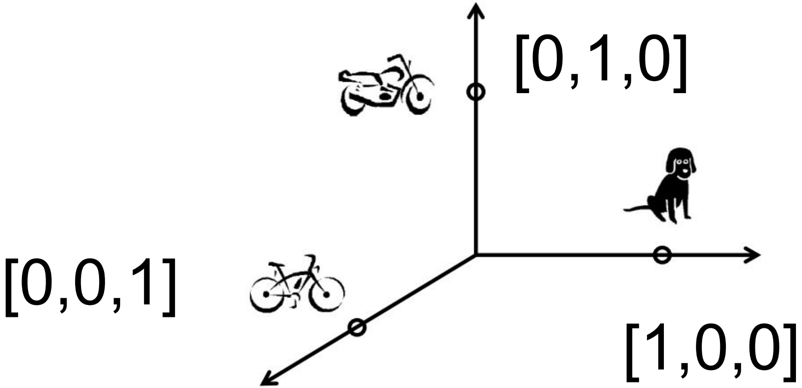
Example of a **multi-class classification** task.



Multi-class Classification

- **<u>Classification</u>:** $\mathbf{y}$ is discrete (e.g. $\mathbf{y} = \{1, 2, 5, 2, 2\}$).
  - Beware! These are unordered categories, not numerically meaningful outputs: e.g. code[1] = “dog”, code[2] = “cat”, code[5] = “ostrich”, ...
  - Classes are often coded as **one-hot vector** (each class corresponds to a different dimension of the output space)

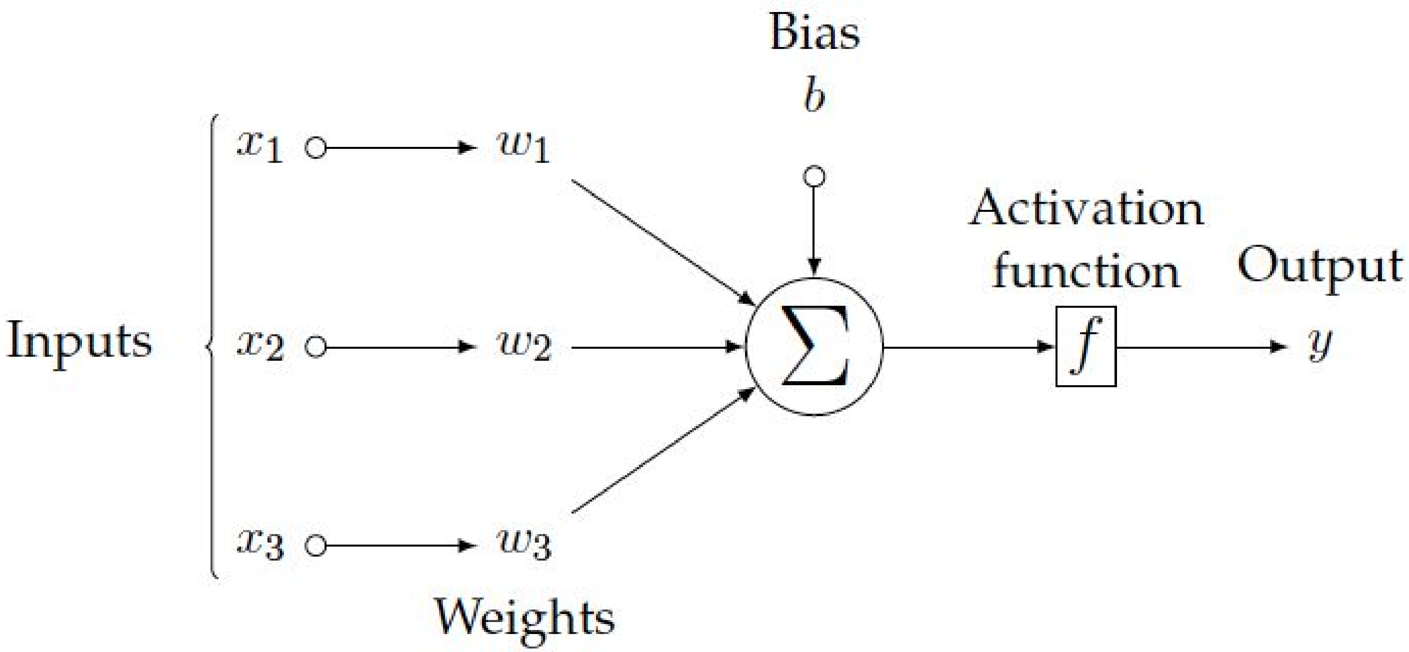
One-hot representation



Perronin, F., CVPR Tutorial on LSVR @ CVPR’14, Output embedding for LSVR

Single Neuron Model (Perceptron)

Both <u>regression</u> and <u>classification</u> problems can be addressed with the perceptron:

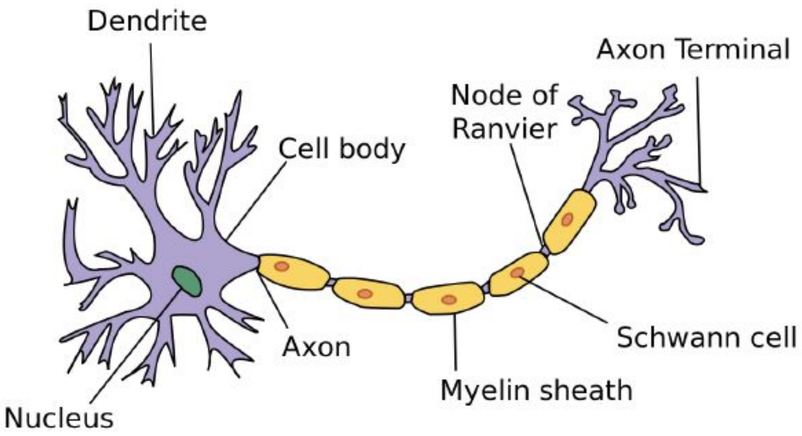


Single neuron model (perceptron)

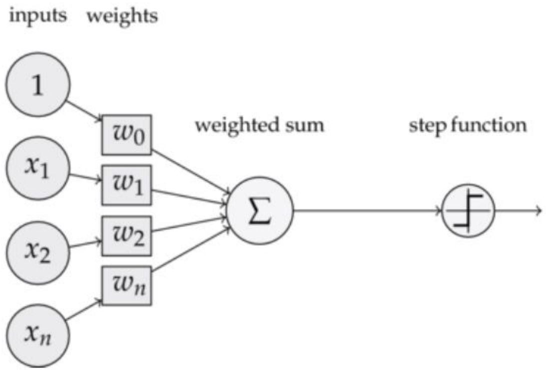
The Perceptron is seen as an **<u>analogy</u>** to a biological neuron.

Biological neurons fire an impulse once the sum of all inputs is over a threshold.

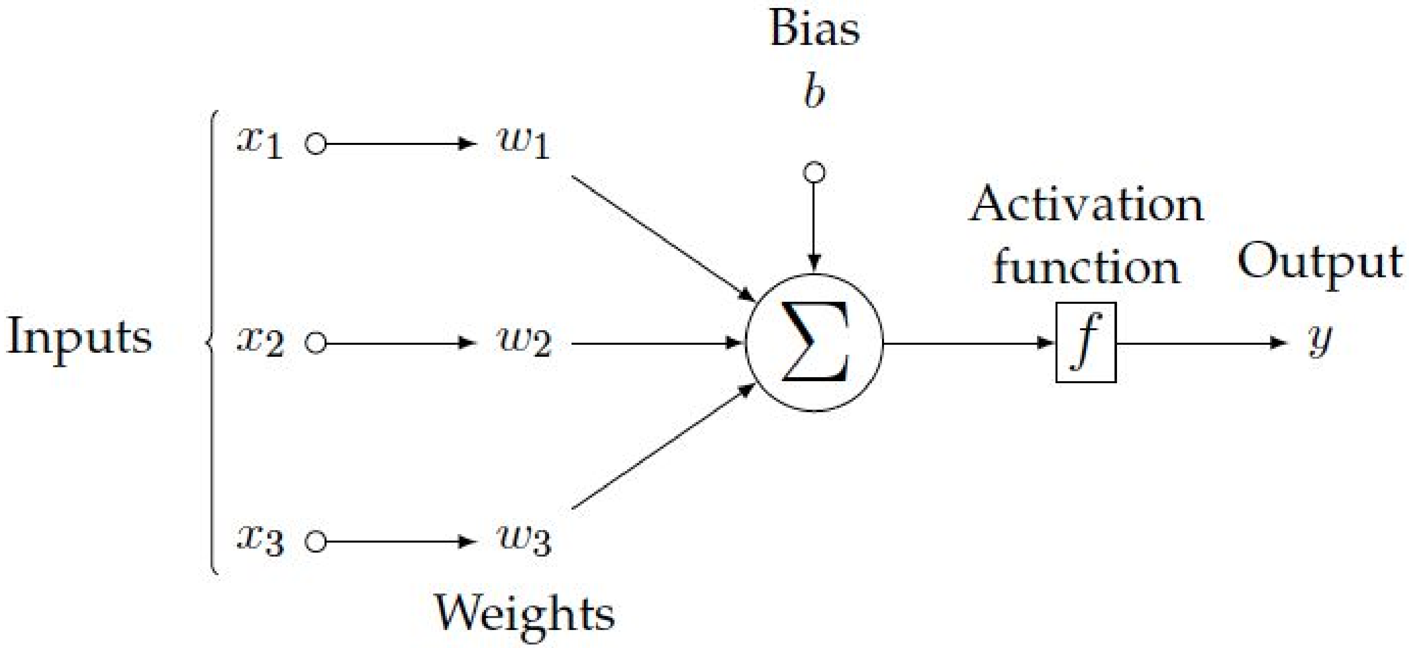
The perceptron acts like a switch (learn how in the next slides...).



Rosenblatt's Perceptron (1958)

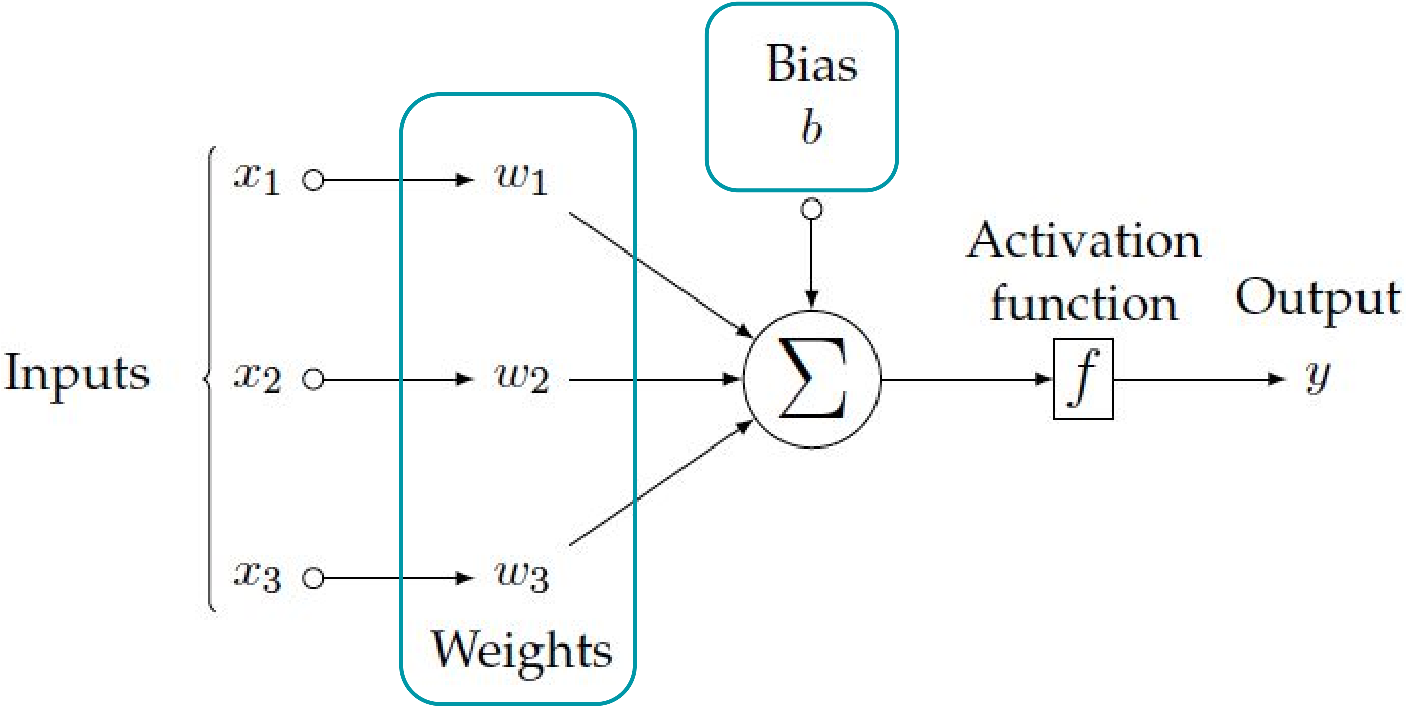


Single neuron model (perceptron)



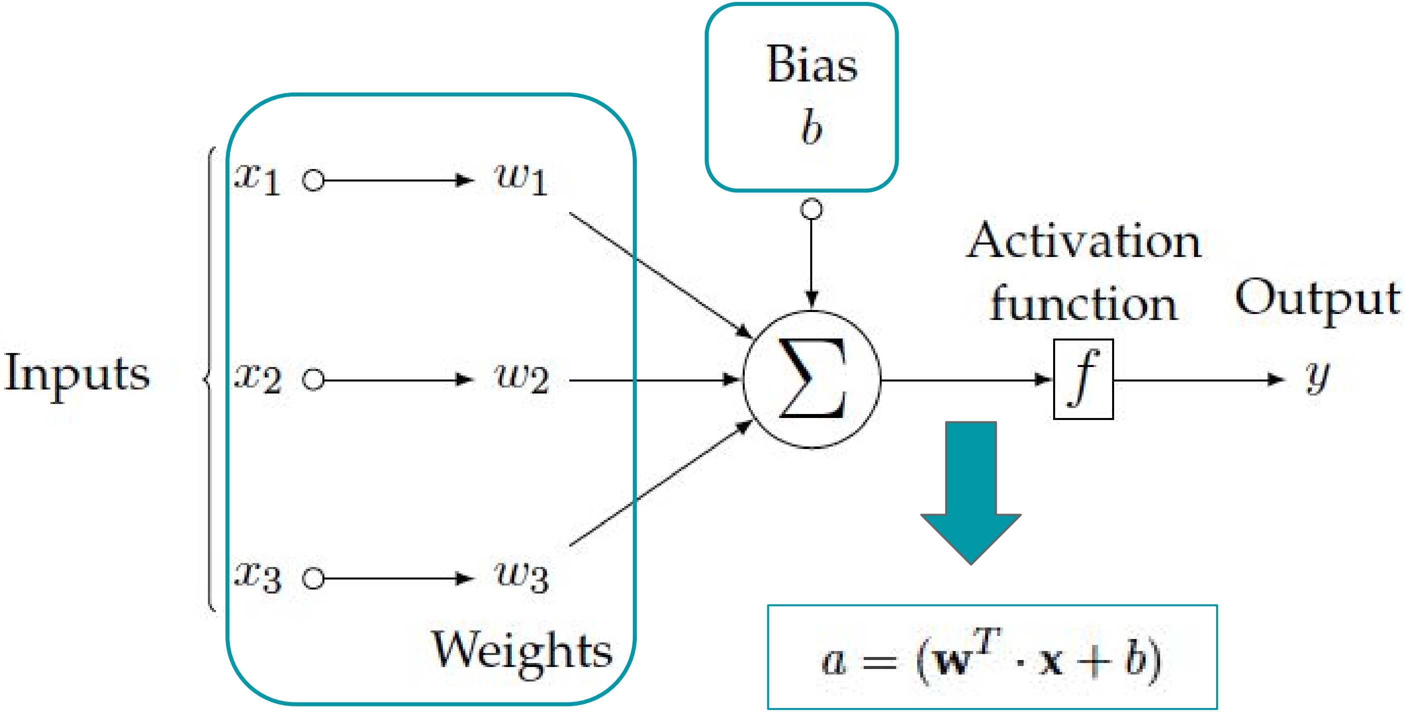
Single neuron model (perceptron)

Weights and bias are the parameters that define the behavior (must be learned).



Single neuron model (perceptron)

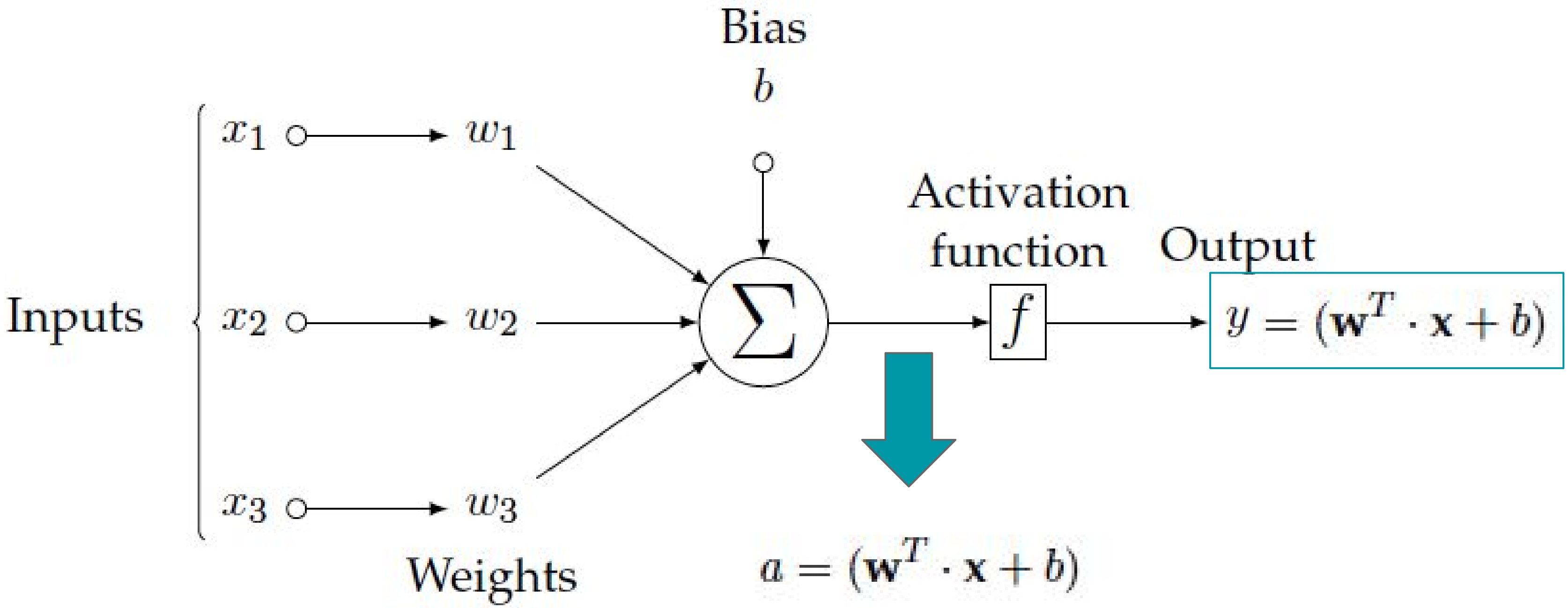
The output y is derived from a sum of the **weighted** inputs plus a **bias** term.



$$a = (\mathbf{w}^T \cdot \mathbf{x} + b)$$

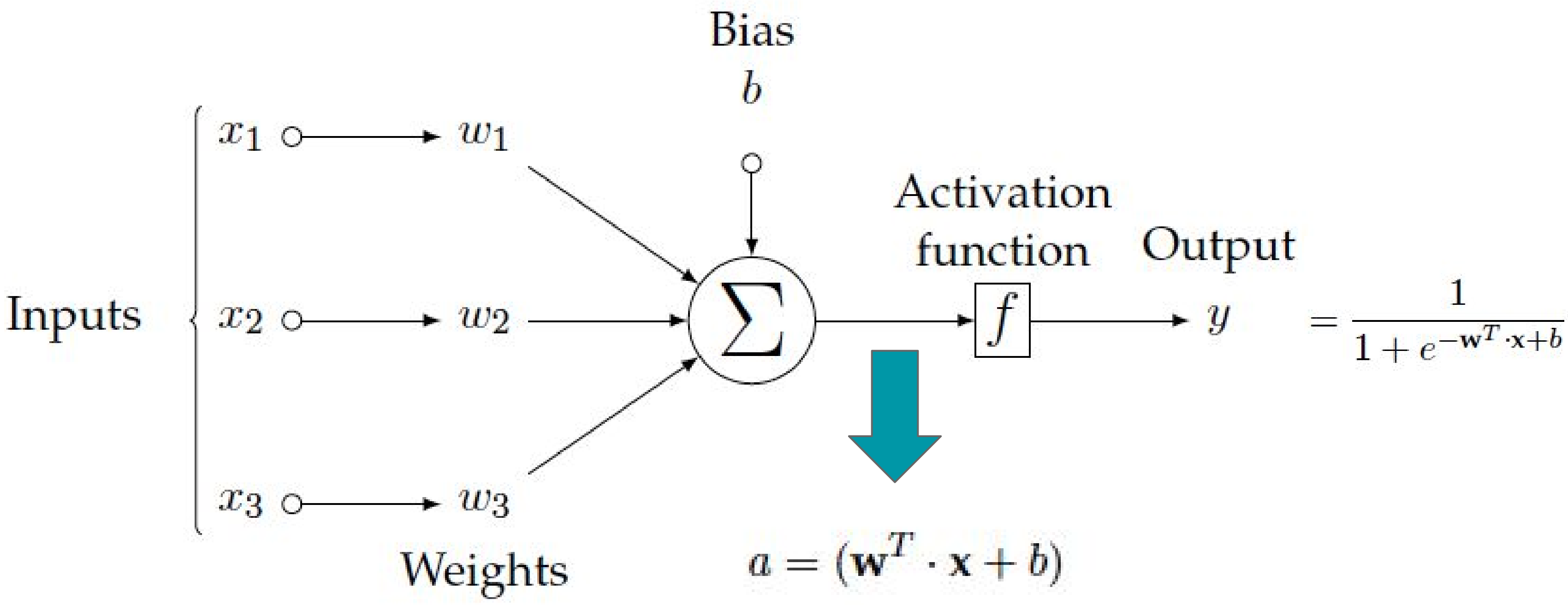
Single neuron model: Regression

The perceptron can solve regression problems when f(a)=a. [identity]



Single neuron model: Binary Classification

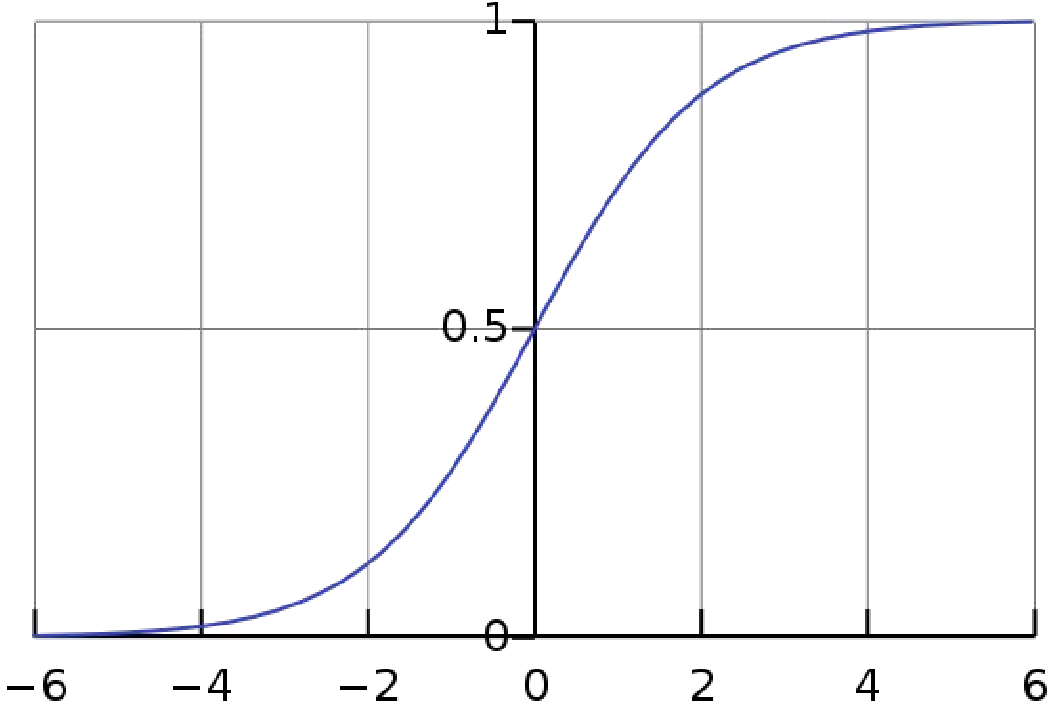
The perceptron can solve classification problems when f(a)=σ(a). [sigmoid]



Single neuron model: Binary Classification

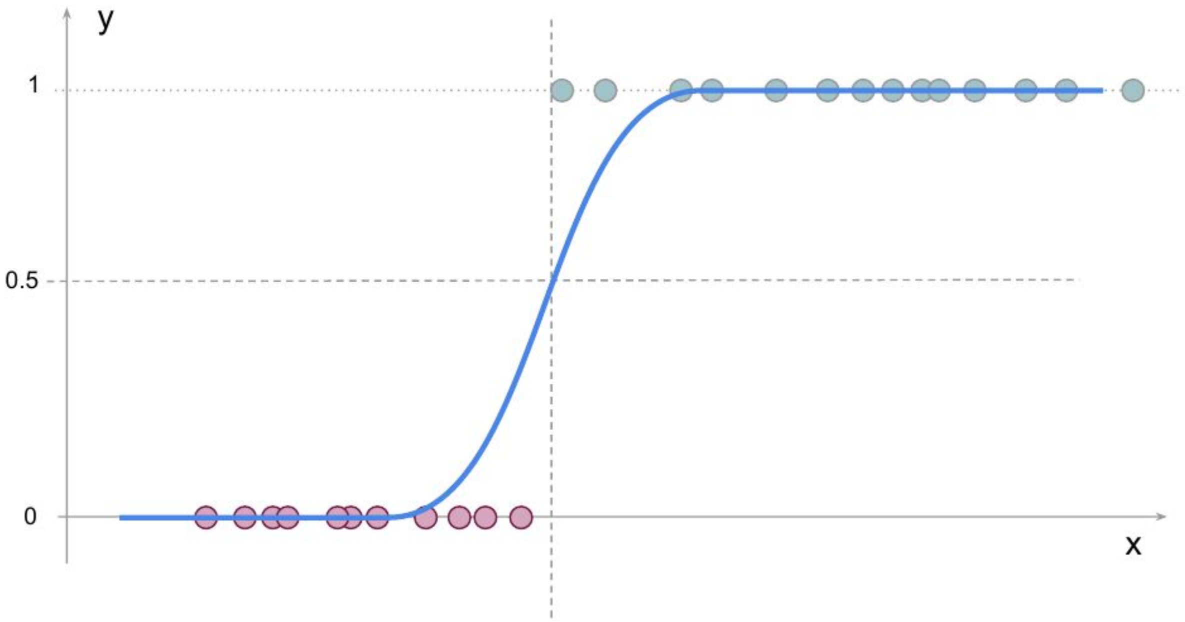
The **sigmoid function σ(x)** or **logistic curve** maps any input x between [0,1]:

$$f(x) = \frac{1}{1+e^{-x}}$$



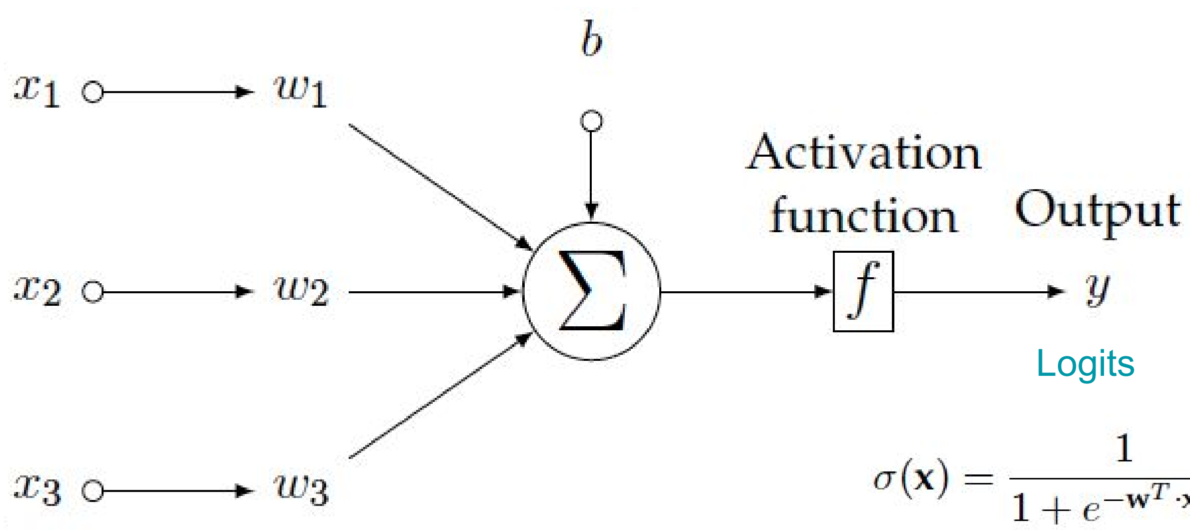
Single neuron model: Binary Classification

For classification, regressed values must be bounded between 0 and 1 to represent probabilities.



Single neuron model: Binary Classification

Setting a **threshold (thr)** at the output of the perceptron allows solving classification problems between two classes (binary) & estimate probabilities:



$$\sigma(\mathbf{x}) = \frac{1}{1 + e^{-\mathbf{w}^T \cdot \mathbf{x} + b}}$$

y > thr → class 1
(eg. green)

y < thr → class 2
(eg. no green)

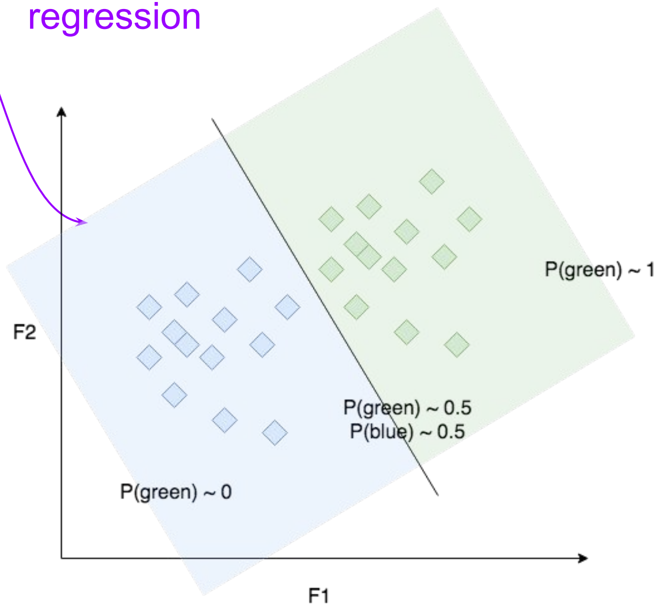
Single neuron model: Binary Classification

Setting a **threshold (thr)** at the output of the perceptron allows solving classification problems between two classes (binary) & estimate probabilities:

Logistic regression

$$\sigma(\mathbf{x}) = \frac{1}{1 + e^{-\mathbf{w}^T \cdot \mathbf{x} + b}}$$

Linear regression



Softmax classifier: Mulitclass

Ideally we would like to predict the *probability* that y takes a particular value given $\mathbf{x}$,

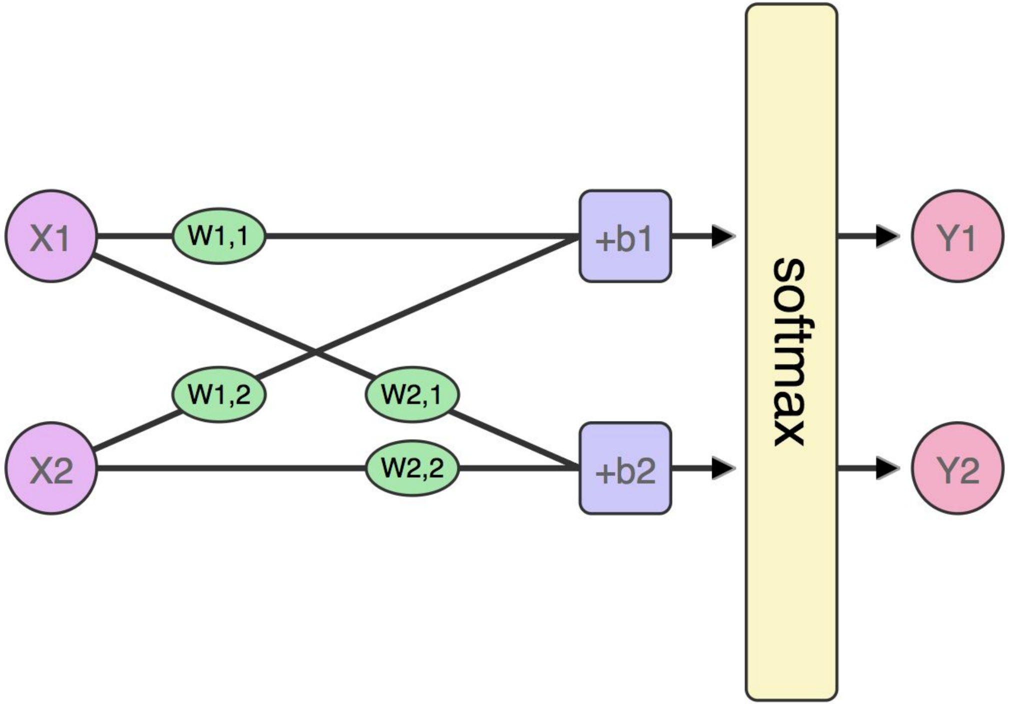
$$P(y = j|\mathbf{x}), \quad j \in \{1, \dots, K\}$$

with:

$$\sum_{j=1}^{K} P(y = j|x) = 1$$

The logistic regression classifier does this for the binary case. The **softmax classifier** extends it to the multiclass case.

Softmax classifier: Multiclass



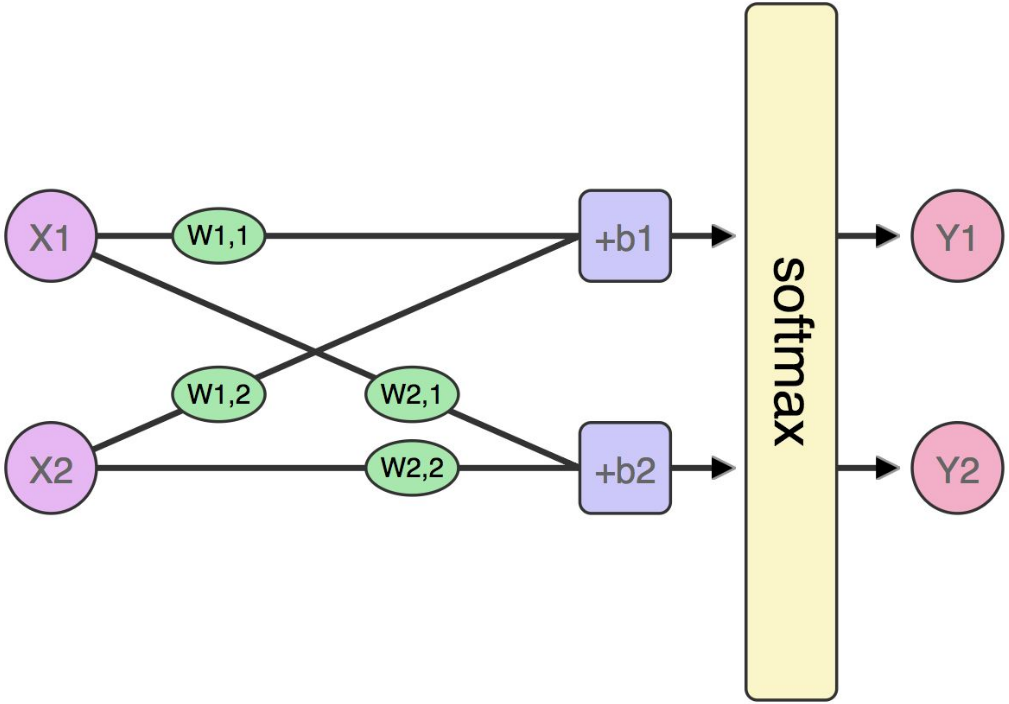
Probability estimations for each class can also be obtained by **softmax normalization** on the output of two neurons, one specialised for each class.

Softmax regression

$$P(y = k|\mathbf{x}) = \frac{\exp \mathbf{x}^T \mathbf{w}_k}{\sum_{n=1}^{N} \exp \mathbf{x}^T \mathbf{w}_n}$$

J. Alammar, "A visual and interactive guide to the Basics of Neural Networks" (2016)

Softmax classifier: Multiclass



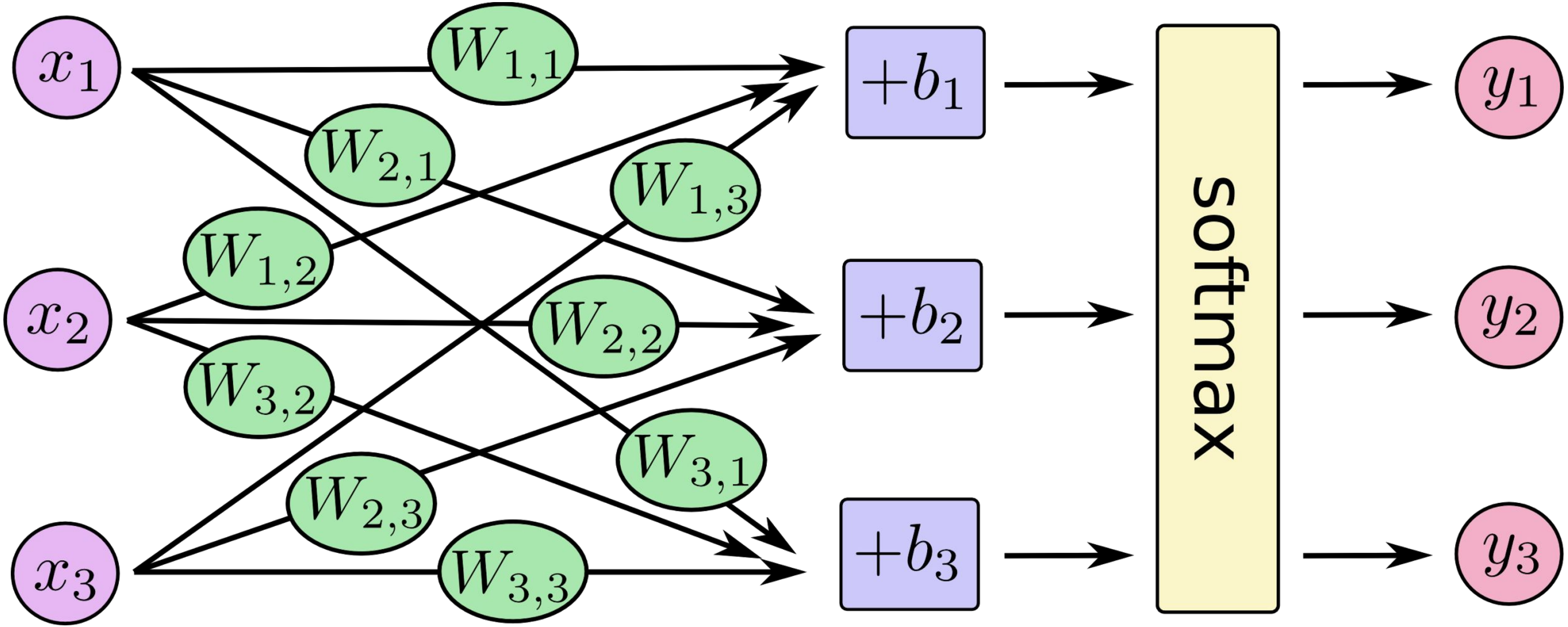
Softmax regression

$$P(y = k|\mathbf{x}) = \frac{\exp \mathbf{x}^T \mathbf{w}_k}{\boxed{\sum_{n=1}^{N} \exp \mathbf{x}^T \mathbf{w}_n}}$$

Normalization factor so that the sum of probabilities sum up to 1.

J. Alammar, “A visual and interactive guide to the Basics of Neural Networks” (2016)

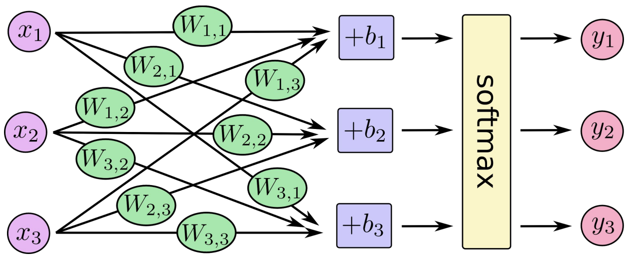
Softmax classifier: Multiclass (3 classes)



TensorFlow, [“MNIST for ML beginners”]

Softmax classifier: Multiclass (3 classes)

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \text{softmax}\begin{pmatrix} W_{1,1}x_1 + W_{1,2}x_2 + W_{1,3}x_3 + b_1 \\ W_{2,1}x_1 + W_{2,2}x_2 + W_{2,3}x_3 + b_2 \\ W_{3,1}x_1 + W_{3,2}x_2 + W_{3,3}x_3 + b_3 \end{pmatrix}$$

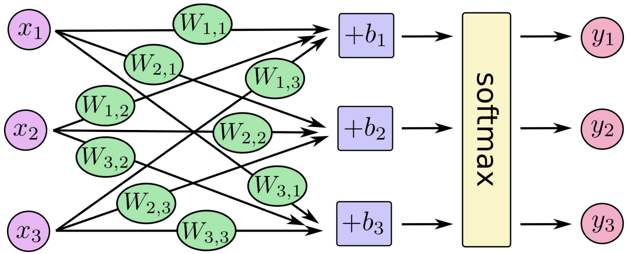


TensorFlow, [“MNIST for ML beginners”]

Softmax classifier: Multiclass (3 classes)

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \text{softmax}\left(\begin{bmatrix} W_{1,1} & W_{1,2} & W_{1,3} \\ W_{2,1} & W_{2,2} & W_{2,3} \\ W_{3,1} & W_{3,2} & W_{3,3} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \right)$$

$$\boxed{y = \text{softmax}(Wx + b)}$$

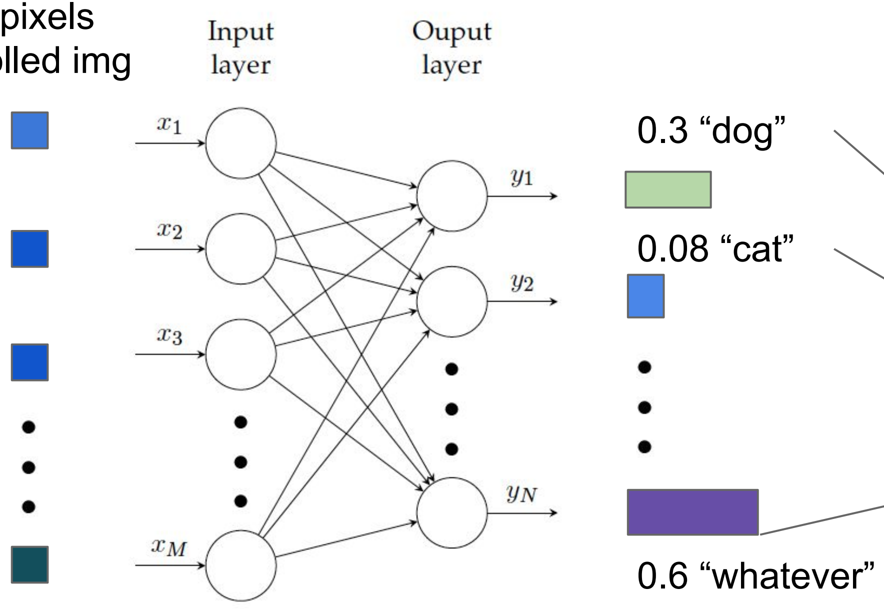


TensorFlow, [“MNIST for ML beginners”]

Softmax classifier: Multiclass (3 classes)

Multiple classes can be predicted by putting many neurons in parallel, each processing its binary output out of N possible classes.

raw pixels
unrolled img



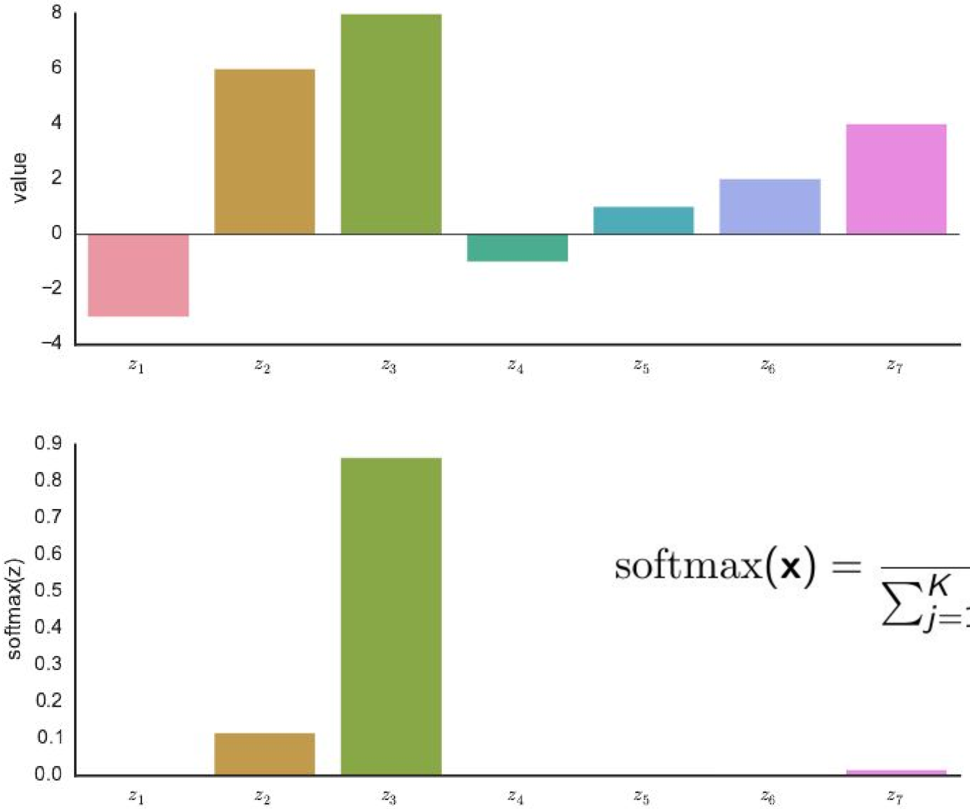
Softmax function

$$P(y=k|\mathbf{x}) = \frac{\exp \mathbf{x}^T \mathbf{w}_k}{\sum_{n=1}^{N} \exp \mathbf{x}^T \mathbf{w}_n}$$

Normalization factor, remember: we want a pdf at the output! → all output P’s sum up to 1.

38

Effect of the softmax



$$\mathrm{softmax}(\mathbf{x}) = \frac{1}{\sum_{j=1}^{K} \exp(x_j)} \begin{bmatrix} \exp(x_1) \\ \exp(x_2) \\ \vdots \\ \exp(x_K) \end{bmatrix}$$

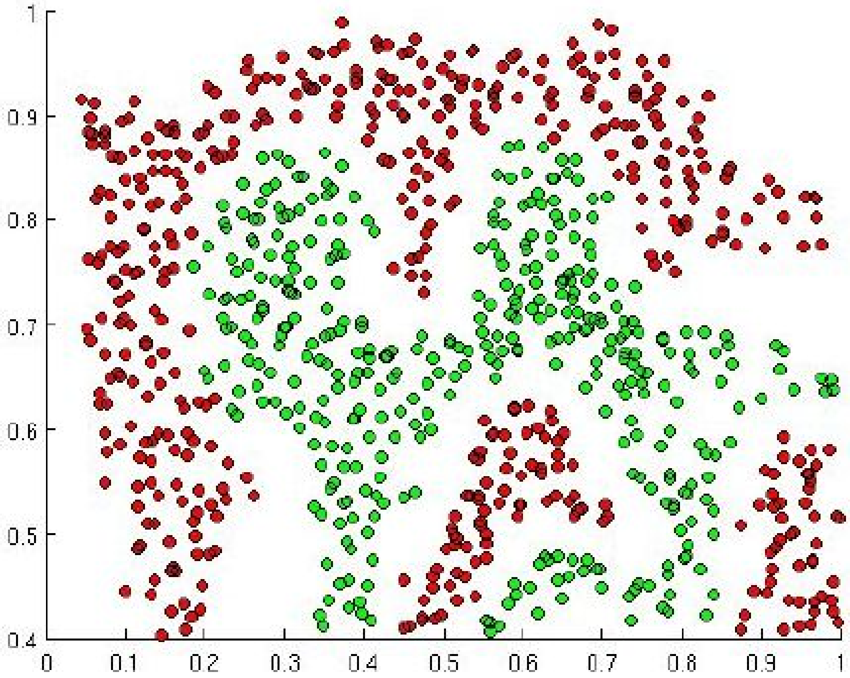
Next lecture...

Perceptrons can only produce linear decision boundaries.

Many interesting problems are not linearly separable.

Real world problems often need non-linear boundaries

- Images
- Audio
- Text



Questions?