Parrondo’s Paradox

Group 4

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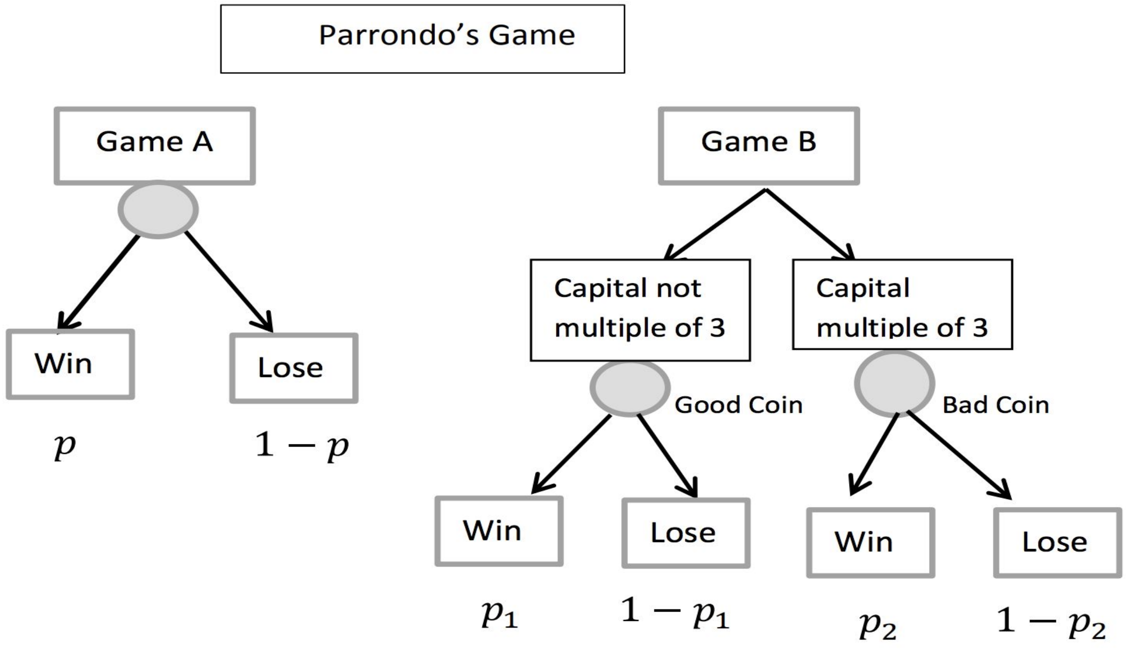
Introduction

- By Juan M. R. Parrondo in 1996
- Losing Games
    - Tendency of losing after some repetitive playing
- Winning Games: Combining 2 losing games will result in a winning game
    - Tendency of winning after some repetitive playing
    - Combination: playing periodically or randomly
- Different types of Parrondo's Paradox
    - Capital Dependent (the most popular)
        - The result of winning/losing only depends on the player's current capital
    - History Dependent
        - Keep track of states of winning/losing and the result of a game depends on the states of game before

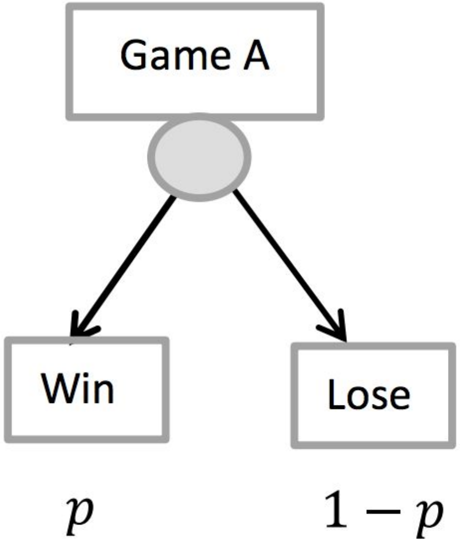
Applications of Parrondo’s Paradox

- Easy to implement - wide range of fields
- Biology
    - Killing virus
        - Low temperature: NO!
        - High temperature: NO!
        - Alternation of extreme hot and cold environments: YES!
- Casino Games
    - Alternation of strategies and games will results in gaining money
    - Which is a trend of winning in Parrondo’s Paradox

Two basic losing games(Coin Tossing)

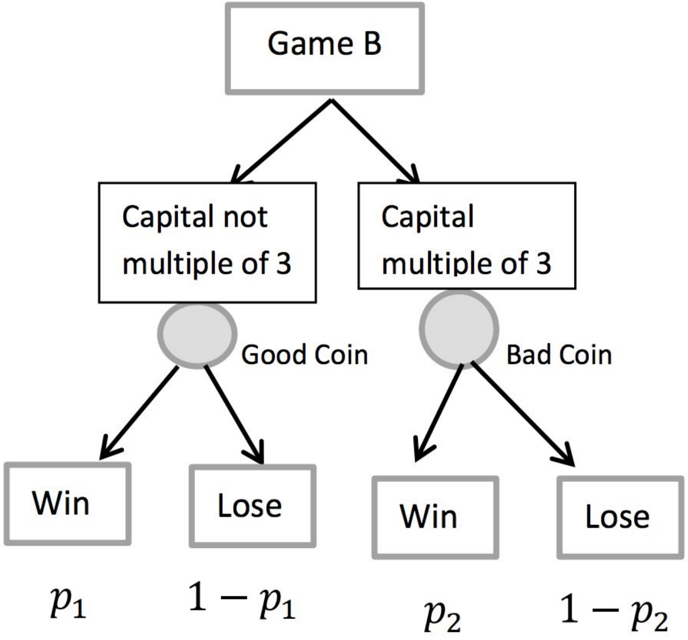


Game A



- If it’s a fair game, p = 0.5
- Increasing p will result in a winning game
- Decreasing p will result in a losing game
- To make it a losing game, we set p = 0.5 - e where e = 0.005

Game B



$$p_1 = 3/4 - \varepsilon$$

$$p_2 = 1/10 - \varepsilon$$

- Basic assumptions: e is same for p1 and p2.
- How Game B to be a losing game?

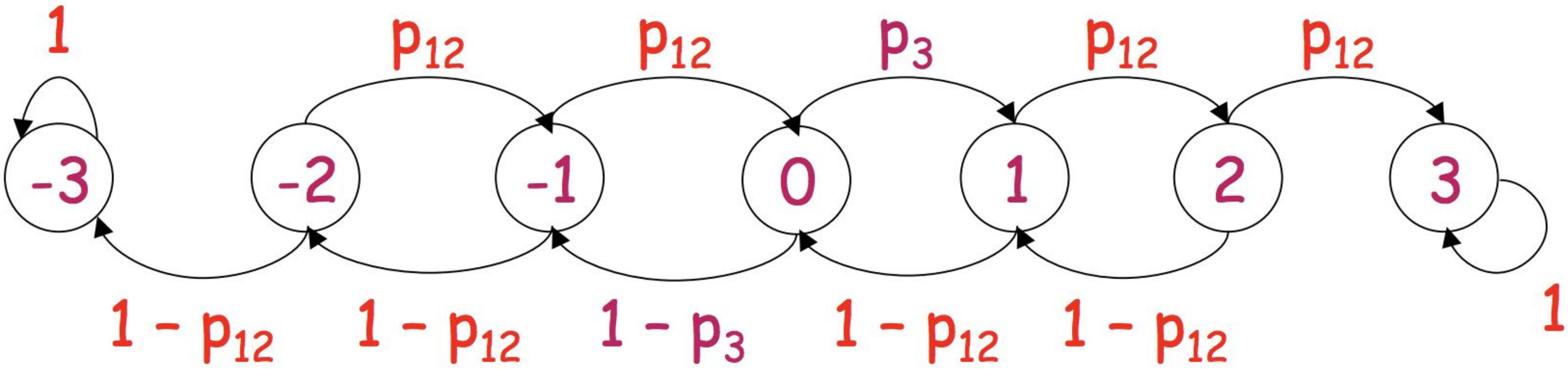
Markov Process

- In game B, you will win \$1 if head comes up, and lose \$1 otherwise. In general, Game B can be stated as: we win with probability p1 if our money is not a multiple of 3, and we win with probability p2 otherwise.
- Assumptions: Let Zj be the probability of winning \$3 before losing \$3 when starting at state j.
- Based on this definition, we have:

$$z_{-3} = 0 \text{ and } z_3 = 1$$

Markov Process

- Z(-2) = (1-p1) Z(-3)+ p1 Z(-1)
- Z(-1) = (1-p1) Z(-2)+ p1 Z0
- Z0 = (1-p2) Z(-1) + p2 Z1
- Z1 = (1-p1) Z0 + p1 z2
- Z2 = (1-p1) Z1 + p1 Z3



Markov Process

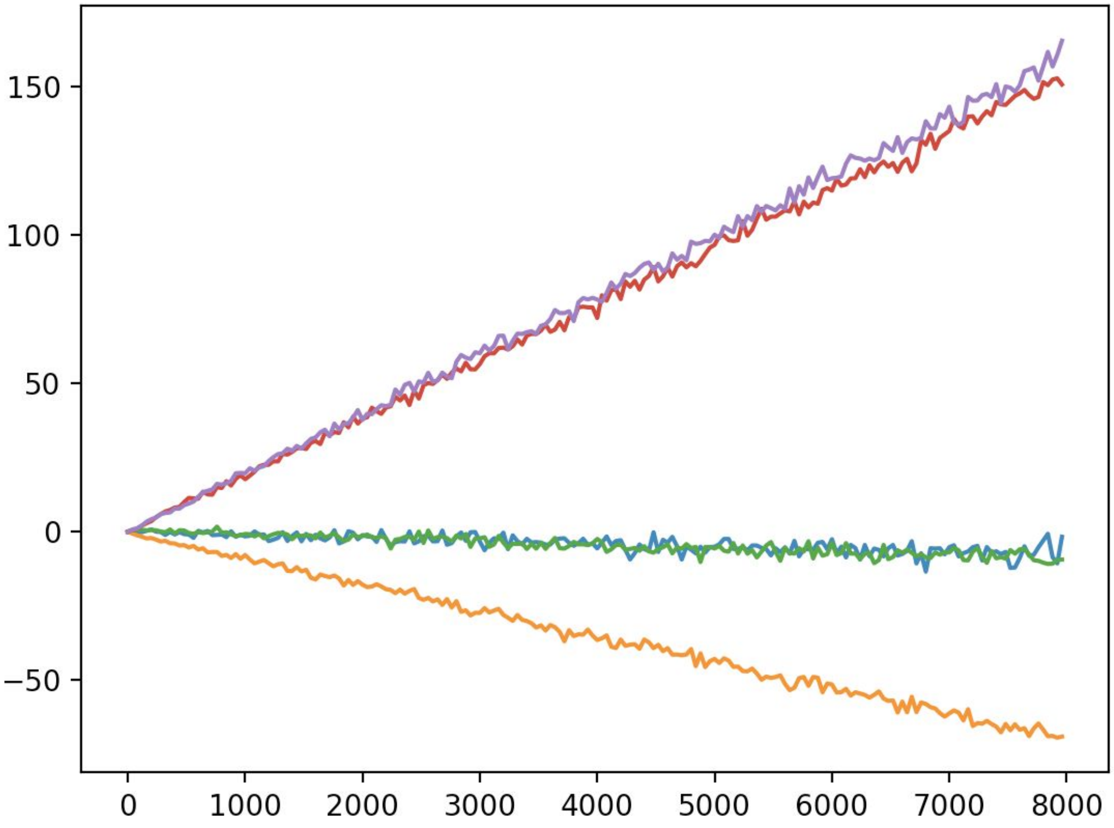
$$z_0 = p_3 p_{12}^2 / ((1-p_3)(1-p_{12})^2 + p_3 p_{12}^2)$$

Z0 = (0.095)x(0.745)x(0.745)/((1-0.095)x(1-0.745)x(1-0.745) + (0.095)x(0.745)x(0.745))
= 0.47 < 0.5

Capital Dependent Games: Random combinations of A and B

- In this combined game, where the sole connection between game A and B is through the capital, the outcome of the game(win or loss) would depend on the relative appearance of game A with respect to game B.
- Interestingly, the combined game is not always winning with any randomness. Too much or too little presence of game A influences the capital in such a way that triggering of the good coin in game B turns less and less.

- A
- B
- ABBB
- AABB
- Random(0.5,0.5)



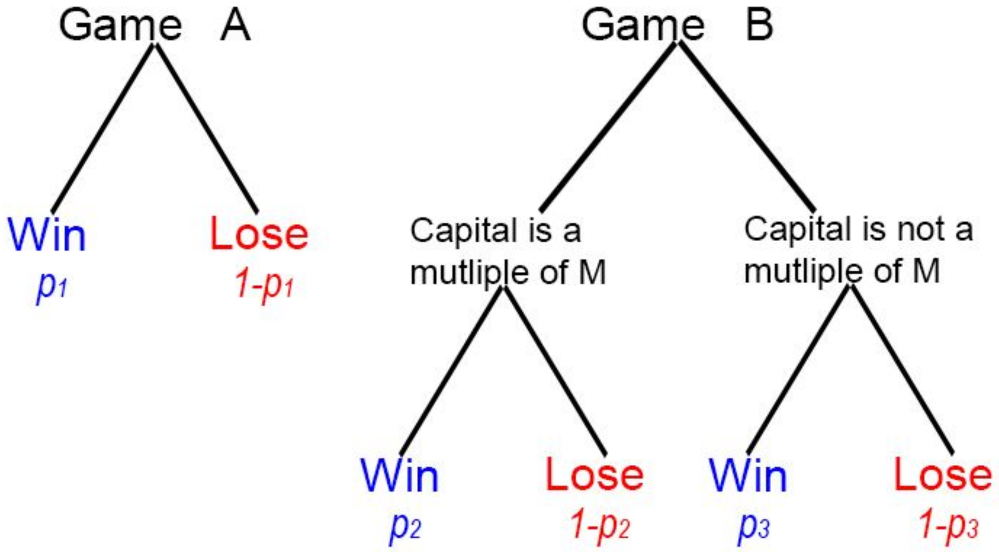
History Dependent Game

One example here

(We didn't do much about this game)

Step $t-2$	Step $t-1$	Coin used in Game B
Lose	Lose	Good coin [Win with $p = 0.9 - \epsilon$]
Lose	Win	Bad Coin [$p = 0.4 - \epsilon$]
Win	Lose	Bad Coin
Win	Win	Bad Coin

Taking a Closer Look

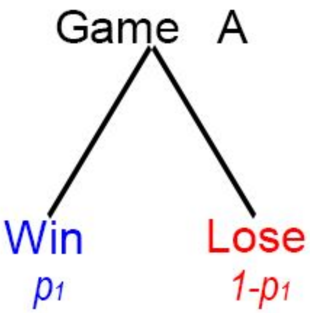


Things to make sure:

- Game A can be fair game
- Game B can be fair game
- Game A with ε is a losing game
- Game B with ε is a losing game

Taking a Closer Look (Game A)

- [x] A fair
- [ ] B fair
- [x] A + ε losing
- [ ] B + ε losing



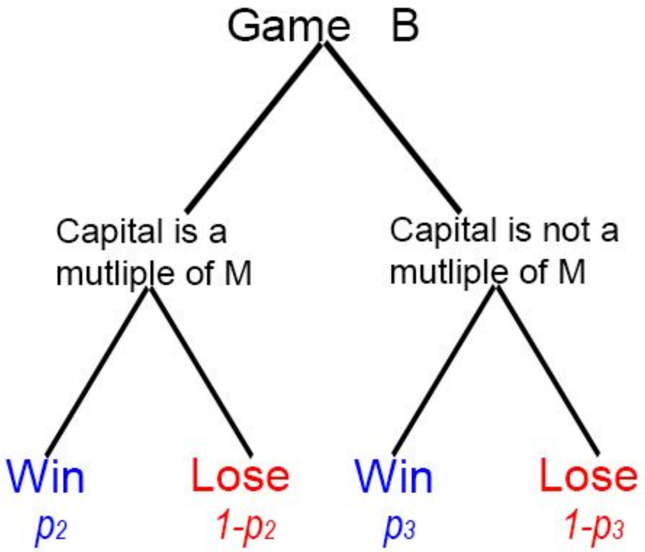
- Like Game A, we can:

$$\begin{aligned} p_1 &= 1 - p_1 \\ 2 * p_1 &= 1 \\ p_1 &= 0.5 \end{aligned}$$

- Increasing p1 will result in a winning game
- Decreasing p1 will result in a losing game

Taking a Closer Look (Game B)

- ☑ A fair
- ☐ B fair
- ☑ A + ε losing
- ☐ B + ε losing



- Like Game A, we can make

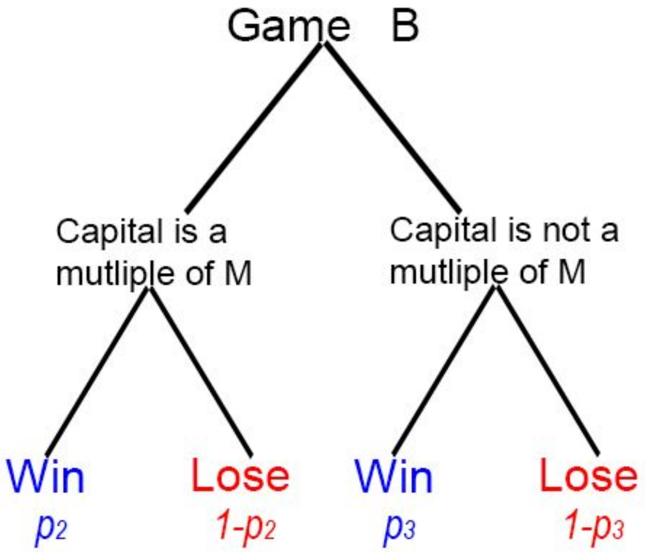
  $$p_2 = 1 - p_2 \text{ and } p_3 = 1 - p_3$$

- But there might be more answers
- We need to use the formula:

  $$p_2 * {p_3}^{M-1} = (1 - p_2) * (1 - p_3)^{M-1}$$

- [x] A fair
- [x] B fair
- [x] A + ε losing
- [ ] B + ε losing

Taking a Closer Look (Game B)



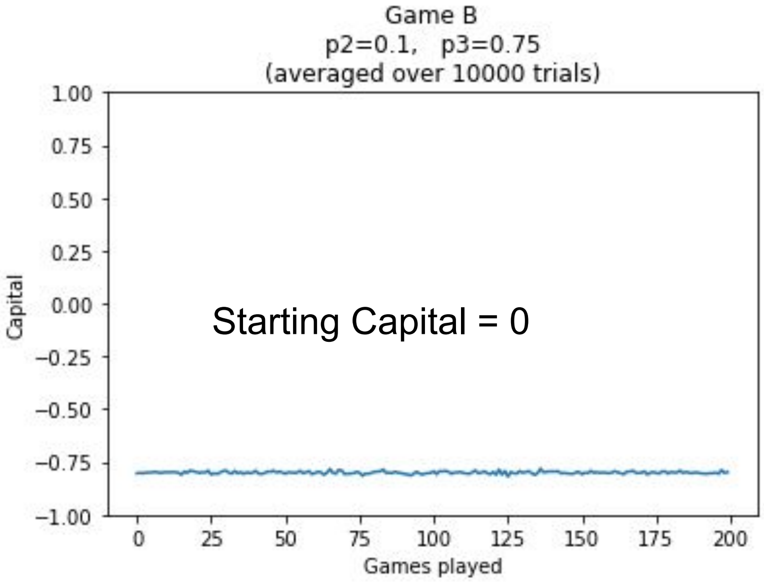
$$p_2 * {p_3}^{M-1} = (1 - p_2) * (1 - p_3)^{M-1}$$

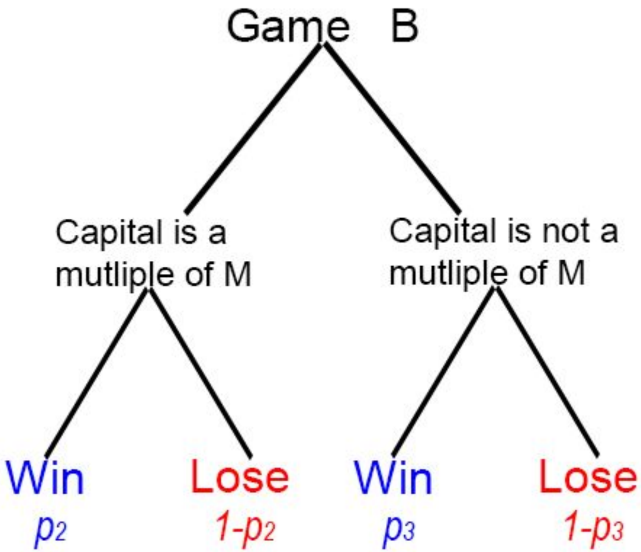
$$p_2 = 0.1, p_3 = 0.75, M = 3$$

$$0.1 * 0.75^{3-1} = (1 - 0.1) * (1 - 0.75)^{3-1}$$
$$0.1 * 0.75^2 = 0.9 * 0.25^2$$
$$0.1 * 0.5625 = 0.9 * 0.0625$$
$$0.05625 = 0.05625$$

Taking a Closer Look (Game B Graph)

- ☑ A fair
- ☑ B fair
- ☑ A + ε losing
- ☐ B + ε losing





Taking a Closer Look (Game B Graph)

- ☑ A fair
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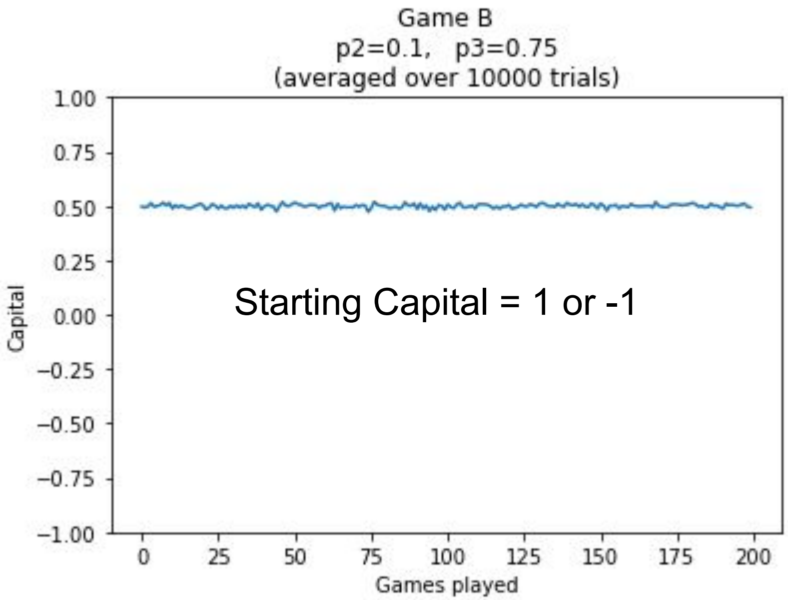
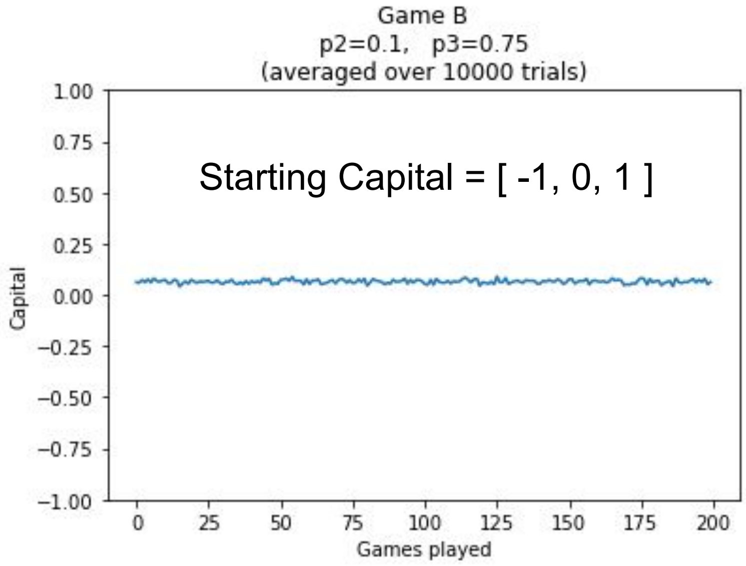




Probability Space

$$p_2 * {p_3}^{M-1} = (1 - p_2) * (1 - p_3)^{M-1}$$

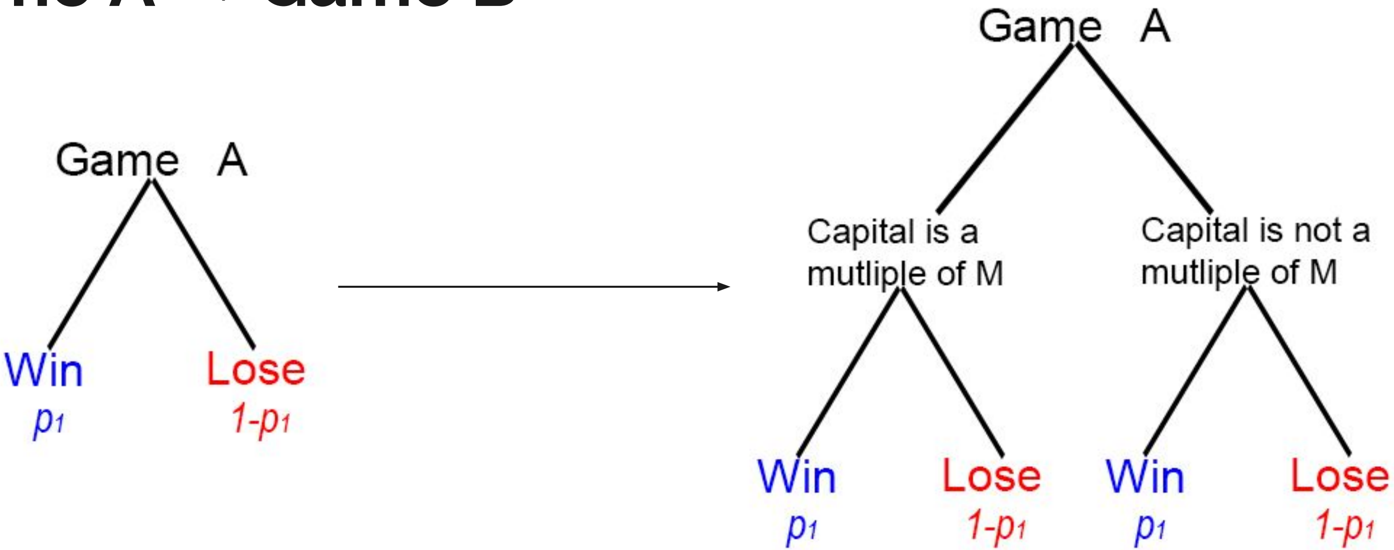
$$\frac{p_2 * {p_3}^{M-1}}{(1 - p_2) * (1 - p_3)^{M-1}} = 1$$

M = 3

$$p_2 = \frac{(1 - p_3)^2}{2{p_3}^2 - 2p_3 + 1}$$

☑ A fair
☑ B fair
☑ A + ε losing
☑ B + ε losing

Game A → Game B


Game A
Win
p_1
Lose
$1-p_1$
Game A
Capital is a mutliple of M
Capital is not a mutliple of M
Win
p_1
Lose
$1-p_1$
Win
p_1
Lose
$1-p_1$

Parrondo's Paradox

- Game A and Game B are have negative capital returns
- they were in the losing game section.
- However, if we combine them by connecting them ...

Parrondo's Paradox

- The resulting game is more in the winning section than the losing section

More Parrondo's Paradox

More Parrondo's Paradox

M = 5

Thank you