

# An Efficient Exact Solution to the ( $l, d$ ) Planted Motif Problem

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# Context of the problem

$(l, d)$  planted motif problem

- ▶ **DNA motif finding**: known as a difficult (NP-complete) problem in computational biology and computer science
- ▶ **motifs**: important sequences that occur repeatedly (but not cleanly, due to mutation) in DNA
- ▶  **$(l, d)$  planted motif problem**: search for a common motif of length  $l$ , allowing for up to  $d$  mismatches due to mutation.

# Context of the problem

$(l, d)$  planted motif problem

*Find a motif of length  $l=8$  across these DNA sequences.  
Each contains the motif with at most  $d=2$  mismatches.*

$S_1$  atcactcgttctcctctaattgtgtaaagacgtactaccgacctta

$S_2$  acgccgaccgggtcccatccttgatatagctcctaacgggcatcagc

$S_3$  tcctgactgcatcgcgatctcggtagtttcctgttcatttttt

$S_4$  ggccctcagcatcgtgcgtcctgctaacacattcccatgcagctt

$S_5$  tgaaaagaatttacggtaaaggatccacatccaatcatttgaaag

*Planted motif:      ? ?*

# Context of the problem

$(l, d)$  planted motif problem

*Find a motif of length  $l=8$  across these DNA sequences.  
Each contains the motif with at most  $d=2$  mismatches.*

$S_1$  at **ca****ctcgtt**ctcctctaattgtgttaaagacgtactaccgacctta

$S_2$  acgccgaccgggtc**ccatc****ctt**gtatagctcctaacgggcatcagc

$S_3$  tcctgactgcatcgcatctcggtagtttcctgt**tcatc****att**ttt

$S_4$  ggccctca**gcatcgt****g**cgctctgctaacacattcccatgcagctt

$S_5$  tgaaaagaattttacggtaaaggatccacatc**caatc****att**tgaaag

*Planted motif:* **ccatcgtt**

# Key concepts

$(l, d)$  planted motif problem

- ▶ an  $l$ -mer is a sequence of length  $l$
- ▶ two  $l$ -mers are  $d$ -neighbors if they have at most  $d$  mismatches

$l=8, d=3,$

$x_1 = \text{cgatc}\text{ctt}$

$x_2 = \text{ccatc}\text{gtt}$

$d_H(x_1, x_2) = 2$

# EMS-GT

## EMS-GT

- ▶ we developed an **exact motif search** (EMS) algorithm that uses the candidate **generate-and-test** (GT) approach
- ▶ **generate** - narrow down the search to a set of candidate motifs
- ▶ **test** - check each candidate to determine if it is a motif

# Demonstration of algorithm

## EMS-GT

$S_1$  atcactcgttctcctctaatagtgtaaagacgtactaccgacctta

$S_2$  acgccgaccggtccgatccttgtatagctcctaacgggcatcagc

$S_3$  tcctgactgcacgcgatctcggtagtttcctgttcacatctttt

$S_4$  ggccctcagcatcgtgcgtcctgctaacacattcccatgcagctt

$S_5$  tgaaaagaattttacggtaaaggatccacatccaatcgtgtgaaag

# Demonstration of algorithm

## EMS-GT



neighborhood of **atcactcg**

$S_1$  atcactcgtttctcctctaattgtgtaaagacgtactaccgacctta

$S_2$  acgccgaccggtccgataccttgtatagctcctaacgggcatcagc

$S_3$  tcctgactgcatcgcgatctcggtagtttcctgttcattcatctttt

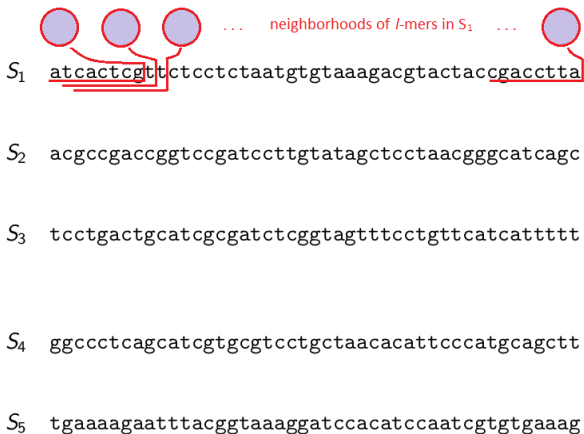
$S_4$  ggccctcagcatcgtgcgtcctgctaacacattcccatgcagctt

$S_5$  tgaaaagaattttacggtaaaggatccacatccaatcgtgtgaaag



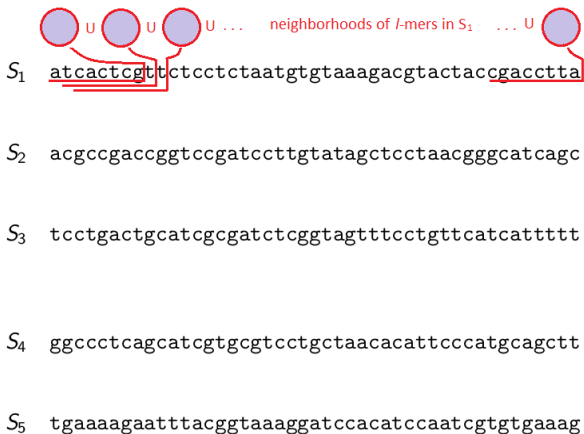
# Demonstration of algorithm

## EMS-GT



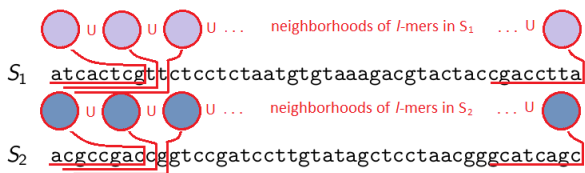
# Demonstration of algorithm

## EMS-GT



# Demonstration of algorithm

EMS-GT



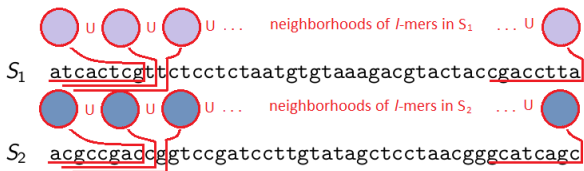
$S_3$  tcctgactgcatcgcgatctcggtagtttcctgttcattcatctttt

$S_4$  ggccctcagcatcgtgcgtcctgctaacacattcccatgcagctt

$S_5$  tgaaaagaattttacggtaaaggatccacatccaatcgtgtgaaaag

# Demonstration of algorithm

EMS-GT



$S_3$  tcctgactgcatcgcgatctcggtagtttcctgttcattcatctttt

$S_4$  ggccctcagcatcgtgcgtcctgctaacacattcccatgcagctt

$S_5$  tgaaaagaattttacggtaaaggatccacatccaatcgtgtgaaaag

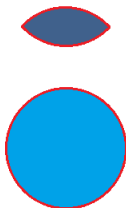
# Demonstration of algorithm

EMS-GT



$S_4$  ggcctcagcatcgtgcgtcctgctaacacattcccatgcagctt

$S_5$  tgaaaagaattttacggtaaaggatccacatccaatcgtgtgaaaag



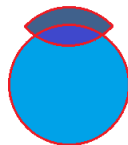
# Demonstration of algorithm

## EMS-GT



$S_4$  ggccctcagcatcgtgcgtcctgctaacacattcccatgcagctt

$S_5$  tgaaaagaatttacggtaaaggatccacatccaatcgtgtgaaag



# Demonstration of algorithm

## EMS-GT



$S_4$  ggccctcagcatcgtgcgtcctgctaacacattcccatgcagctt

$S_5$  tgaaaagaatttacggtaaaggatccacatccaatcgtgtgaaag

# Representing sets with bits

EMS-GT

- ex. the  $d$ -neighborhood of `acgt`, for  $d=2$  (67 neighbors)

`acgt`,

`ccgt`, `gcgt`, `tcgt`, `aagt`, `aggt`, `atgt`,  
`acat`, `acgt`, `actt`, `acga`, `acgc`, `acgg`,

`cagt`, `cggt`, `ctgt`, `ccat`, `ccct`, `cctt`, `ccga`, `ccgc`, `ccgg`,  
`gagt`, `gggt`, `gtgt`, `gcat`, `gcct`, `gctt`, `gcga`, `gcgc`, `gcgg`,  
`tagt`, `tggt`, `ttgt`, `tcat`, `tcct`, `tctt`, `tcga`, `tcgc`, `tcgg`,  
`aaat`, `aact`, `aatt`, `aaga`, `aagc`, `aagg`, `agat`, `agct`, `agtt`,  
`agga`, `aggc`, `aggg`, `atat`, `atct`, `attt`, `atga`, `atgc`, `atgg`,  
`acaa`, `acac`, `acag`, `acca`, `accc`, `accg`, `acta`, `actc`, `actg`.



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EMS-GT

- ▶ We know there are only  $4^l$  possible  $l$ -mers that can be formed with the DNA bases  $\{a, c, g, t\}$ ;

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  - ▶ set to 1 if the corresponding  $l$ -mer is a member of the set,
  - ▶ set to 0 otherwise.

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  - ▶ set to 1 if the corresponding  $l$ -mer is a member of the set,
  - ▶ set to 0 otherwise.
- ▶ For efficiency, EMS-GT stores the  $4^l$  bits as  $\frac{4^l}{32}$  32-bit integers.

# Representing sets with bits

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- ex. the  $d$ -neighborhood of `acgt`, for  $d=2$  (67 neighbors)

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`ccgt`, `gcgt`, `tcgt`, `aagt`, `aggt`, `atgt`,  
`acat`, `acgt`, `actt`, `acga`, `acgc`, `acgg`,

`cagt`, `cggt`, `ctgt`, `ccat`, `ccct`, `cctt`, `ccga`, `ccgc`, `ccgg`,  
`gagt`, `gggt`, `gtgt`, `gcat`, `gcct`, `gctt`, `gcga`, `gcgc`, `gcgg`,  
`tagt`, `tggt`, `ttgt`, `tcac`, `tcct`, `tctt`, `tcga`, `tcgc`, `tcgg`,  
`aaat`, `aact`, `aatt`, `aaga`, `aagc`, `aagg`, `agat`, `agct`, `agtt`,  
`agga`, `aggc`, `aggg`, `atat`, `atct`, `atgt`, `atga`, `atgc`, `atgg`,  
`acaa`, `acac`, `acag`, `acca`, `accc`, `accg`, `acta`, `actc`, `actg`.

## Representing sets with bits

- ▶ ex. the  $d$ -neighborhood of `acgt`, for  $d=2$  (67 neighbors)
- ▶  $l=4$

	0			8				16				24				31													
[0]	0	0	0	1	0	0	0	1	1	1	1	1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
[1]	0	0	0	1	0	0	0	1	1	1	1	1	0	0	0	1	0	0	0	1	1	1	1	1	0	0	0	1	0
[2]	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	1	1	1	1	1	0
[3]	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
[4]	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	1	1	1	1	0	0	0	1	0
[5]	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
[6]	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	1	1	1	1	0	0	0	1	0
[7]	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
	0			8				16				24				31													

# Representing sets with bits

EMS-GT

- ▶ ex. the  $d$ -neighborhood of `acgt`, for  $d=2$  (67 neighbors)
- ▶  $l=4 \rightarrow 4^l = 256$  possible  $l$ -mers

	0	8	16	24	31
[0]	0 0 0 1 0 0 0 1 1 1 1 1 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1				
[1]	0 0 0 1 0 0 0 1 1 1 1 1 1 0 0 0 1 0 0 0 1 0 0 0 1 1 1 1 1 0 0 0 1				
[2]	0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 1 0 0 0 1 1 1 1 1 1 0 0 0 1				
[3]	0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0				
[4]	0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 1 0 0 0 1 1 1 1 1 0 0 0 1				
[5]	0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0				
[6]	0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 1 0 0 0 1 1 1 1 1 0 0 0 1				
[7]	0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0				
	0	8	16	24	31

# Representing sets with bits

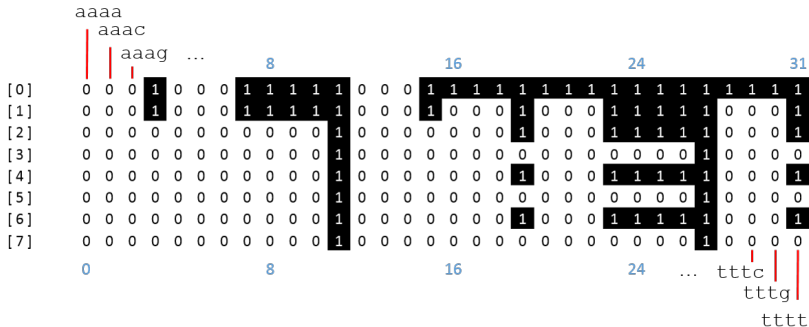
EMS-GT

- ▶ ex. the  $d$ -neighborhood of `acgt`, for  $d=2$  (67 neighbors)
- ▶  $l=4 \rightarrow 4^l = 256$  possible  $l$ -mers  $\rightarrow \frac{4^l}{32} = 8$  32-bit integers

	0	8	16	24	31
[0]	0 0 0 1 0 0 0 1 1 1 1 1 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1				
[1]	0 0 0 1 0 0 0 1 1 1 1 1 1 0 0 0 1 0 0 0 1 0 0 0 1 1 1 1 1 0 0 0 1				
[2]	0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 1 0 0 0 1 1 1 1 1 1 0 0 0 1				
[3]	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0				
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[7]	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0				
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## Representing sets with bits

- ▶ ex. the  $d$ -neighborhood of `acgt`, for  $d=2$  (67 neighbors)
- ▶  $l=4 \rightarrow 4^l = 256$  possible  $l$ -mers  $\rightarrow \frac{4^l}{32} = 8$  32-bit integers

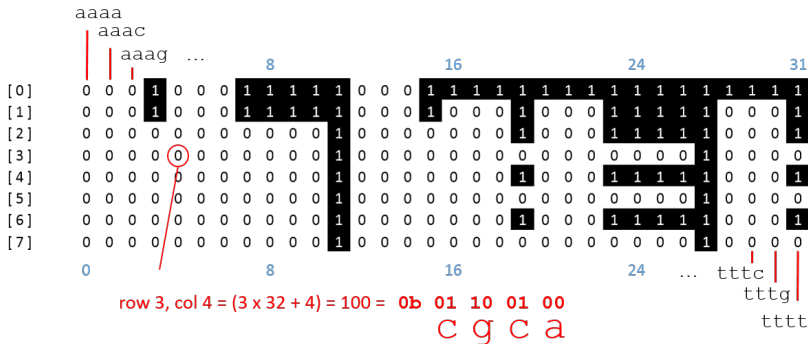




# Representing sets with bits

EMS-GT

- ▶ ex. the  $d$ -neighborhood of **acgt**, for  $d=2$  (67 neighbors)
- ▶  $l=4 \rightarrow 4^l = 256$  possible  $l$ -mers  $\rightarrow \frac{4^l}{32} = 8$  32-bit integers



# Building neighborhoods in blocks

EMS-GT

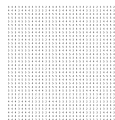
Given the neighborhood bit-array  $N_x$  for  $l$ -mer  $x$ ,  
if we partition  $N_x$  into blocks of  $4^k$  bits each,  $k < l$ ,  
each block conforms to one of  $(k + 2)$  patterns.

# Building neighborhoods in blocks

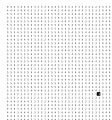
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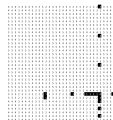
ex. patterns in  $N_{\text{acgtacgtacgt}}$  for  $k=5 \rightarrow 4^5=32 \times 32$  bits per block



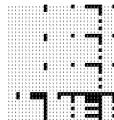
Pattern -1



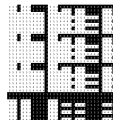
Pattern 0



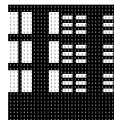
Pattern 1



Pattern 2



Pattern 3



Pattern 4



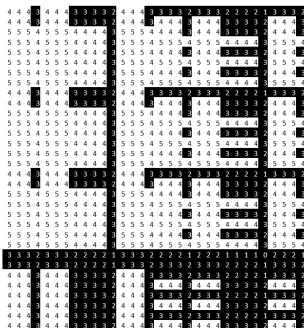
Pattern 5

# Building neighborhoods in blocks

EMS-GT

- **prefix** = first  $(l-k)$  characters,  **$k$ -suffix** = last  $k$  characters

acgtaaaaaaaa →



← acgtaaatTTTT

- due to the alphabetical ordering,  $l$ -mers in the same block all have the same **prefix**, and **differ only in their  $k$ -suffixes**

# Building neighborhoods in blocks

EMS-GT

For the neighborhood  $N_x$  of  $l$ -mer  $x$ ,

- ▶  $x$ 's **prefix** determines which patterns apply to which blocks;
- ▶  $x$ 's *k-suffix* determines the structure of the patterns

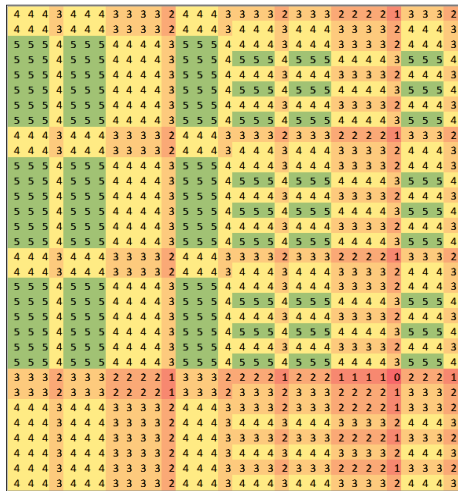
# Building neighborhoods in blocks

EMS-GT

- ▶ ex.  $d=5, k=5$

$x = \text{acgtacgtacgt}$

- ▶ color map: number of mismatches from  $x$ 's suffix  $\text{tacgt}$  of all possible  $k$ -suffixes



# Building neighborhoods in blocks

## EMS-GT

- ▶ ex.  $d=5$ ,  $k=5$

$x = \text{acgtacgtacgt}$

- ▶ if prefix = **cgacatc**  
(6 mismatches from acgtacg)

- ▶ we cannot use any  $k$ -suffix to form a neighbor of  $x$

```
4 4 4 3 4 4 4 3 3 3 2 4 4 3 3 3 2 3 3 2 2 2 2 1 3 3 3 2
4 4 4 3 4 4 4 3 3 3 2 4 4 3 4 4 4 3 4 4 4 3 3 3 2 4 4 4 3
5 5 5 4 5 5 5 4 4 4 4 3 5 5 5 4 4 4 4 3 4 4 4 3 3 2 4 4 4 3
5 5 5 4 5 5 5 4 4 4 4 3 5 5 5 4 5 5 5 4 5 5 5 4 4 4 3 5 5 4
5 5 5 4 5 5 5 4 4 4 4 3 5 5 5 4 4 4 4 3 4 4 4 3 3 3 2 4 4 4 3
5 5 5 4 5 5 5 4 4 4 4 3 5 5 5 4 5 5 5 4 5 5 5 4 4 4 3 5 5 4
5 5 5 4 5 5 5 4 4 4 4 3 5 5 5 4 4 4 4 3 4 4 4 3 3 3 2 4 4 4 3
4 4 4 3 4 4 4 3 3 3 2 4 4 3 3 3 3 2 3 3 3 2 2 2 2 1 3 3 3 2
4 4 4 3 4 4 4 3 3 3 2 4 4 3 4 4 4 3 4 4 4 3 3 3 2 4 4 4 3
5 5 5 4 5 5 5 4 4 4 4 3 5 5 5 4 5 5 5 4 5 5 5 4 4 4 3 5 5 4
5 5 5 4 5 5 5 4 4 4 4 3 5 5 5 4 4 4 4 3 4 4 4 3 3 3 2 4 4 4 3
5 5 5 4 5 5 5 4 4 4 4 3 5 5 5 4 5 5 5 4 5 5 5 4 4 4 3 5 5 4
5 5 5 4 5 5 5 4 4 4 4 3 5 5 5 4 5 5 5 4 5 5 5 4 4 4 3 5 5 4
5 5 5 4 5 5 5 4 4 4 4 3 5 5 5 4 5 5 5 4 5 5 5 4 4 4 3 5 5 4
5 5 5 4 5 5 5 4 4 4 4 3 5 5 5 4 5 5 5 4 5 5 5 4 4 4 3 5 5 4
5 5 5 4 5 5 5 4 4 4 4 3 5 5 5 4 5 5 5 4 5 5 5 4 4 4 3 5 5 4
5 5 5 4 5 5 5 4 4 4 4 3 5 5 5 4 5 5 5 4 5 5 5 4 4 4 3 5 5 4
3 3 3 2 3 3 3 2 2 2 2 1 3 3 3 2 2 2 2 1 2 2 2 1 1 1 0 2 2 2 1
3 3 3 2 3 3 3 2 2 2 2 1 3 3 3 2 3 3 3 2 3 3 3 2 2 2 2 1 3 3 3 2
4 4 4 3 4 4 4 3 3 3 2 4 4 3 3 3 3 2 3 3 3 2 2 2 2 1 3 3 3 2
4 4 4 3 4 4 4 3 3 3 2 4 4 3 4 4 4 3 4 4 4 3 3 3 2 4 4 4 3
4 4 4 3 4 4 4 3 3 3 2 4 4 3 3 3 3 2 3 3 3 2 2 2 2 1 3 3 3 2
4 4 4 3 4 4 4 3 3 3 2 4 4 3 4 4 4 3 4 4 4 3 3 3 2 4 4 4 3
4 4 4 3 4 4 4 3 3 3 2 4 4 3 3 3 3 2 3 3 3 2 2 2 2 1 3 3 3 2
4 4 4 3 4 4 4 3 3 3 2 4 4 3 4 4 4 3 4 4 4 3 3 3 2 4 4 4 3
```

# Building neighborhoods in blocks

## EMS-GT

- ▶ ex.  $d=5$ ,  $k=5$

$x = \text{acgtacgtacgt}$

- ▶ if prefix = **agacatc**  
(5 mismatches from acgtacg)

- ▶ adding the  $k$ -suffix **tacgt**  
forms a neighbor of  $x$

```
4 4 4 3 4 4 4 3 3 3 2 4 4 3 3 3 2 3 3 2 2 2 2 1 3 3 3 2
4 4 4 3 4 4 4 3 3 3 2 4 4 3 4 4 4 3 4 4 4 3 3 3 2 4 4 4 3
5 5 5 4 5 5 5 4 4 4 4 3 5 5 5 4 4 4 4 3 4 4 4 3 3 2 4 4 4 3
5 5 5 4 5 5 5 4 4 4 4 3 5 5 5 4 5 5 5 4 5 5 5 4 4 4 3 5 5 4
5 5 5 4 5 5 5 4 4 4 4 3 5 5 5 4 4 4 4 3 4 4 4 3 3 3 2 4 4 4 3
5 5 5 4 5 5 5 4 4 4 4 3 5 5 5 4 5 5 5 4 5 5 5 4 4 4 3 5 5 4
5 5 5 4 5 5 5 4 4 4 4 3 5 5 5 4 4 4 4 3 4 4 4 3 3 3 2 4 4 4 3
4 4 4 3 4 4 4 3 3 3 2 4 4 3 3 3 3 2 3 3 3 2 2 2 2 1 3 3 3 2
4 4 4 3 4 4 4 3 3 3 2 4 4 3 4 4 4 3 4 4 4 3 3 3 2 4 4 4 3
5 5 5 4 5 5 5 4 4 4 4 3 5 5 5 4 4 4 4 3 4 4 4 3 3 3 2 4 4 4 3
5 5 5 4 5 5 5 4 4 4 4 3 5 5 5 4 5 5 5 4 5 5 5 4 4 4 3 5 5 4
5 5 5 4 5 5 5 4 4 4 4 3 5 5 5 4 5 5 5 4 5 5 5 4 4 4 3 5 5 4
5 5 5 4 5 5 5 4 4 4 4 3 5 5 5 4 4 4 4 3 4 4 4 3 3 3 2 4 4 4 3
5 5 5 4 5 5 5 4 4 4 4 3 5 5 5 4 4 4 4 3 4 4 4 3 3 3 2 4 4 4 3
4 4 4 3 4 4 4 3 3 3 2 4 4 3 4 4 4 3 4 4 4 3 3 3 2 4 4 4 3
5 5 5 4 5 5 5 4 4 4 4 3 5 5 5 4 4 4 4 3 4 4 4 3 3 3 2 4 4 4 3
5 5 5 4 5 5 5 4 4 4 4 3 5 5 5 4 5 5 5 4 5 5 5 4 4 4 3 5 5 4
5 5 5 4 5 5 5 4 4 4 4 3 5 5 5 4 4 4 4 3 4 4 4 3 3 3 2 4 4 4 3
5 5 5 4 5 5 5 4 4 4 4 3 5 5 5 4 5 5 5 4 5 5 5 4 4 4 3 5 5 4
5 5 5 4 5 5 5 4 4 4 4 3 5 5 5 4 4 4 4 3 4 4 4 3 3 3 2 4 4 4 3
5 5 5 4 5 5 5 4 4 4 4 3 5 5 5 4 5 5 5 4 5 5 5 4 4 4 3 5 5 4
3 3 3 2 3 3 3 2 2 2 2 1 3 3 3 2 2 2 2 1 2 2 2 1 1 1 1 0 2 2 2 1
3 3 3 2 3 3 3 2 2 2 2 1 3 3 3 2 3 3 3 2 3 3 3 2 2 2 2 1 3 3 3 2
4 4 4 3 4 4 4 3 3 3 2 4 4 3 3 3 3 2 3 3 3 2 2 2 2 1 3 3 3 2
4 4 4 3 4 4 4 3 3 3 2 4 4 3 4 4 4 3 4 4 4 3 3 3 2 4 4 4 3
4 4 4 3 4 4 4 3 3 3 2 4 4 3 3 3 3 2 3 3 3 2 2 2 2 1 3 3 3 2
4 4 4 3 4 4 4 3 3 3 2 4 4 3 4 4 4 3 4 4 4 3 3 3 2 4 4 4 3
4 4 4 3 4 4 4 3 3 3 2 4 4 3 3 3 3 2 3 3 3 2 2 2 2 1 3 3 3 2
4 4 4 3 4 4 4 3 3 3 2 4 4 3 4 4 4 3 4 4 4 3 3 3 2 4 4 4 3
```





# Building neighborhoods in blocks

## EMS-GT

- ▶ ex.  $d=5$ ,  $k=5$

$x = \text{acgtacgtacgt}$

- ▶ if prefix = **agatatg**  
(3 mismatches from acgtacg)

- ▶ any  $k$ -suffix with up to 2 mismatches from **tacgt**  
forms a neighbor of  $x$

4	4	4	3	4	4	4	3	3	3	2	4	4	4	3	3	3	2	3	3	3	2	2	2	2	1	3	3	3	2
4	4	4	3	4	4	4	3	3	3	2	4	4	4	3	4	4	4	3	4	4	4	3	3	3	2	4	4	4	3
5	5	5	4	5	5	5	4	4	4	3	5	5	5	4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3
5	5	5	4	5	5	5	4	4	4	3	5	5	5	4	5	5	5	4	5	5	5	4	4	4	3	5	5	5	4
5	5	5	4	5	5	5	4	4	4	3	5	5	5	4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3
5	5	5	4	5	5	5	4	4	4	3	5	5	5	4	5	5	5	4	5	5	5	4	4	4	3	5	5	5	4
5	5	5	4	5	5	5	4	4	4	3	5	5	5	4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3
5	5	5	4	5	5	5	4	4	4	3	5	5	5	4	5	5	5	4	5	5	5	4	4	4	3	5	5	5	4
4	4	4	3	4	4	4	3	3	3	2	4	4	4	3	3	3	2	3	3	3	2	2	2	2	1	3	3	3	2
4	4	4	3	4	4	4	3	3	3	2	4	4	4	3	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3
5	5	5	4	5	5	5	4	4	4	3	5	5	5	4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3
5	5	5	4	5	5	5	4	4	4	3	5	5	5	4	5	5	5	4	5	5	5	4	4	4	3	5	5	5	4
5	5	5	4	5	5	5	4	4	4	3	5	5	5	4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3
5	5	5	4	5	5	5	4	4	4	3	5	5	5	4	5	5	5	4	5	5	5	4	4	4	3	5	5	5	4
5	5	5	4	5	5	5	4	4	4	3	5	5	5	4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3
5	5	5	4	5	5	5	4	4	4	3	5	5	5	4	5	5	5	4	5	5	5	4	4	4	3	5	5	5	4
4	4	4	3	4	4	4	3	3	3	2	4	4	4	3	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3
5	5	5	4	5	5	5	4	4	4	3	5	5	5	4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3
5	5	5	4	5	5	5	4	4	4	3	5	5	5	4	5	5	5	4	5	5	5	4	4	4	3	5	5	5	4
5	5	5	4	5	5	5	4	4	4	3	5	5	5	4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3
5	5	5	4	5	5	5	4	4	4	3	5	5	5	4	5	5	5	4	5	5	5	4	4	4	3	5	5	5	4
3	3	3	2	3	3	3	2	2	2	2	1	3	3	3	2	2	2	2	1	1	1	1	1	0	2	2	2	1	
3	3	3	2	3	3	3	2	2	2	2	1	3	3	3	2	3	3	2	2	2	2	2	1	3	3	3	2		
4	4	4	3	4	4	4	3	3	3	2	4	4	4	3	3	3	2	3	3	3	2	2	2	2	1	3	3	3	2
4	4	4	3	4	4	4	3	3	3	2	4	4	4	3	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3
4	4	4	3	4	4	4	3	3	3	2	4	4	4	3	3	3	2	3	3	3	2	2	2	2	1	3	3	3	2
4	4	4	3	4	4	4	3	3	3	2	4	4	4	3	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3
4	4	4	3	4	4	4	3	3	3	2	4	4	4	3	3	3	2	3	3	3	2	2	2	2	1	3	3	3	2
4	4	4	3	4	4	4	3	3	3	2	4	4	4	3	3	3	2	3	3	3	2	2	2	2	1	3	3	3	2
4	4	4	3	4	4	4	3	3	3	2	4	4	4	3	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3

# Building neighborhoods in blocks

## EMS-GT

- ▶ ex.  $d=5$ ,  $k=5$

$x = \text{acgtacgtacgt}$

- ▶ if prefix = **acatatg**  
(2 mismatches from acgtacg)

- ▶ any  $k$ -suffix with up to 3 mismatches from **tacgt**  
forms a neighbor of  $x$

4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3	3	3	3	2	3	3	3	2	2	2	2	1	3	3	3	2	
4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3					
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	4	4	4	3	3	3	3	2	4	4	4	3					
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	5	5	4	4	4	4	3	5	5	5	4						
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	4	4	4	3	4	4	4	3	3	3	2	4	4	4	3		
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	5	5	4	4	4	4	3	5	5	5	4						
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	4	4	4	3	4	4	4	3	3	3	2	4	4	4	3		
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	5	5	4	4	4	4	3	5	5	5	4						
4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3	3	3	3	2	2	2	2	2	1	3	3	3	2				
4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3					
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	4	4	4	3	4	4	4	3	3	3	2	4	4	4	3		
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	5	5	4	4	4	4	3	5	5	5	4						
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	5	5	4	4	4	4	3	5	5	5	4						
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	4	4	4	3	4	4	4	3	3	3	2	4	4	4	3		
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	5	5	4	4	4	4	3	5	5	5	4						
4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3	3	3	3	2	2	2	2	2	1	3	3	3	2				
4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3					
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	4	4	4	3	4	4	4	3	3	3	2	4	4	4	3		
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	5	5	4	4	4	4	3	5	5	5	4						
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	5	5	4	4	4	4	3	5	5	5	4						
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	5	5	4	4	4	4	3	5	5	5	4						
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	5	5	4	4	4	4	3	5	5	5	4						
3	3	3	2	3	3	3	2	2	2	2	1	3	3	3	2	2	2	1	2	2	2	1	1	1	0	2	2	2	1			
3	3	3	2	3	3	3	2	2	2	2	1	3	3	3	2	3	3	3	2	3	3	3	2	2	2	1	3	3	3	2		
4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3	3	3	3	2	2	2	2	2	1	3	3	3	2				
4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3					
4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3	3	3	3	2	2	2	2	2	1	3	3	3	2				
4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3	3	3	3	2	2	2	2	2	1	3	3	3	2				
4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3	3	3	3	2	2	2	2	2	1	3	3	3	2				
4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3	3	3	3	2	2	2	2	2	1	3	3	3	2				

# Building neighborhoods in blocks

EMS-GT

- ▶ ex.  $d=5$ ,  $k=5$

$x = \text{acgtacgtacgt}$

- ▶ if prefix = **ac**atacgt  
(1 mismatch from acgtacg)

- ▶ any  $k$ -suffix with up to 4 mismatches from **tacgt**  
forms a neighbor of  $x$

4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3	3	3	3	2	2	2	2	1	3	3	3	2
4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3	4	4	4	3	3	3	2	4	4	4	3	
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	4	4	4	3	3	3	2	4	4	4	3	
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	5	5	5	4	4	4	3	5	5	5	4	
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	4	4	4	3	3	3	2	4	4	4	3	
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	5	5	5	4	4	4	3	5	5	5	4	
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	4	4	4	3	3	3	2	4	4	4	3	
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	5	5	5	4	4	4	3	5	5	5	4	
4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3	3	3	3	2	2	2	2	1	3	3	3	
4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3	3	4	4	4	3	3	3	2	4	4	4	
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	4	4	4	3	3	3	2	4	4	4	3	
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	5	5	5	4	4	4	3	5	5	5	4	
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	4	4	4	3	3	3	2	4	4	4	3	
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	5	5	5	4	4	4	3	5	5	5	4	
4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3	3	3	3	2	2	2	2	1	3	3	3	
4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3	3	4	4	4	3	3	3	2	4	4	4	
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	4	4	4	3	3	3	2	4	4	4	3	
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	5	5	5	4	4	4	3	5	5	5	4	
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	4	4	4	3	3	3	2	4	4	4	3	
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	5	5	5	4	4	4	3	5	5	5	4	
4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3	3	3	3	2	2	2	2	1	3	3	3	
4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3	3	4	4	4	3	3	3	2	4	4	4	
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	4	4	4	3	3	3	2	4	4	4	3	
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	5	5	5	4	4	4	3	5	5	5	4	
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	4	4	4	3	3	3	2	4	4	4	3	
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	5	5	5	4	4	4	3	5	5	5	4	
3	3	3	2	3	3	3	2	2	2	2	1	3	3	3	2	2	2	2	1	2	2	2	1	1	1	0	
3	3	3	2	3	3	3	2	2	2	2	1	3	3	3	2	2	2	2	1	3	3	2	2	2	1	3	
4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3	3	3	3	2	2	2	2	2	1	3		
4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3	4	4	4	3	3	3	2	4	4	4		
4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3	3	3	3	2	2	2	2	2	1	3		
4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3	4	4	4	3	3	3	2	4	4	4		
4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3	3	3	3	2	2	2	2	2	1	3		
4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3	4	4	4	3	3	3	2	4	4	4		

# Building neighborhoods in blocks

## EMS-GT

- ▶ ex.  $d=5$ ,  $k=5$

$x = \text{acgtacgtacgt}$

- ▶ if prefix = **acgtacg**  
(0 mismatches,  $x$ 's actual prefix)

- ▶ any  $k$ -suffix forms a neighbor of  $x$ , since all  $k$ -suffixes have at most 5 mismatches from **tacgt**

4	4	4	3	4	4	4	3	3	3	3	2	4	4	3	3	3	2	2	2	2	1	3	3	3	2
4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3	4	4	4	3	4	4	3	3	3	2
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	4	4	4	3	4	4	3	3	2	4
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	5	5	5	4	4	4	3	5	5	4
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	4	4	4	3	4	4	3	3	2	4
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	5	5	5	4	4	4	3	5	5	4
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	4	4	4	3	4	4	3	3	2	4
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	5	5	5	4	4	4	3	5	5	4
4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3	3	3	2	3	3	2	2	1	3	3
4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3	4	4	4	3	3	3	2	4	4	3
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	4	4	4	3	3	3	2	4	4	3
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	5	5	5	4	4	4	3	5	5	4
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	5	5	5	4	4	4	3	5	5	4
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	5	5	5	4	4	4	3	5	5	4
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	5	5	5	4	4	4	3	5	5	4
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	5	5	5	4	4	4	3	5	5	4
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	5	5	5	4	4	4	3	5	5	4
5	5	5	4	5	5	5	4	4	4	4	3	5	5	5	4	5	5	5	4	4	4	3	5	5	4
3	3	3	2	3	3	3	2	2	2	1	3	3	3	2	2	2	1	2	2	1	1	1	0	2	2
3	3	3	2	3	3	3	2	2	2	1	3	3	3	2	3	3	2	3	3	2	2	2	1	3	3
4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3	3	3	2	3	3	2	2	1	3	3
4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3	4	4	4	3	3	3	2	4	4	3
4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3	3	3	2	3	3	2	2	1	3	3
4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3	4	4	4	3	4	4	3	3	2	4
4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3	3	3	2	3	3	2	2	1	3	3
4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3	3	3	2	3	3	2	2	1	3	3
4	4	4	3	4	4	4	3	3	3	3	2	4	4	4	3	4	4	4	3	4	4	3	3	2	4

# Performance

EMS-GT

