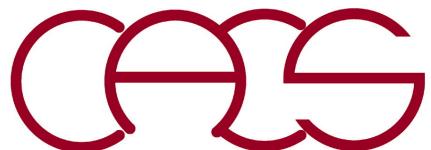


Phys516: Methods of Computational Physics

Aiichiro Nakano

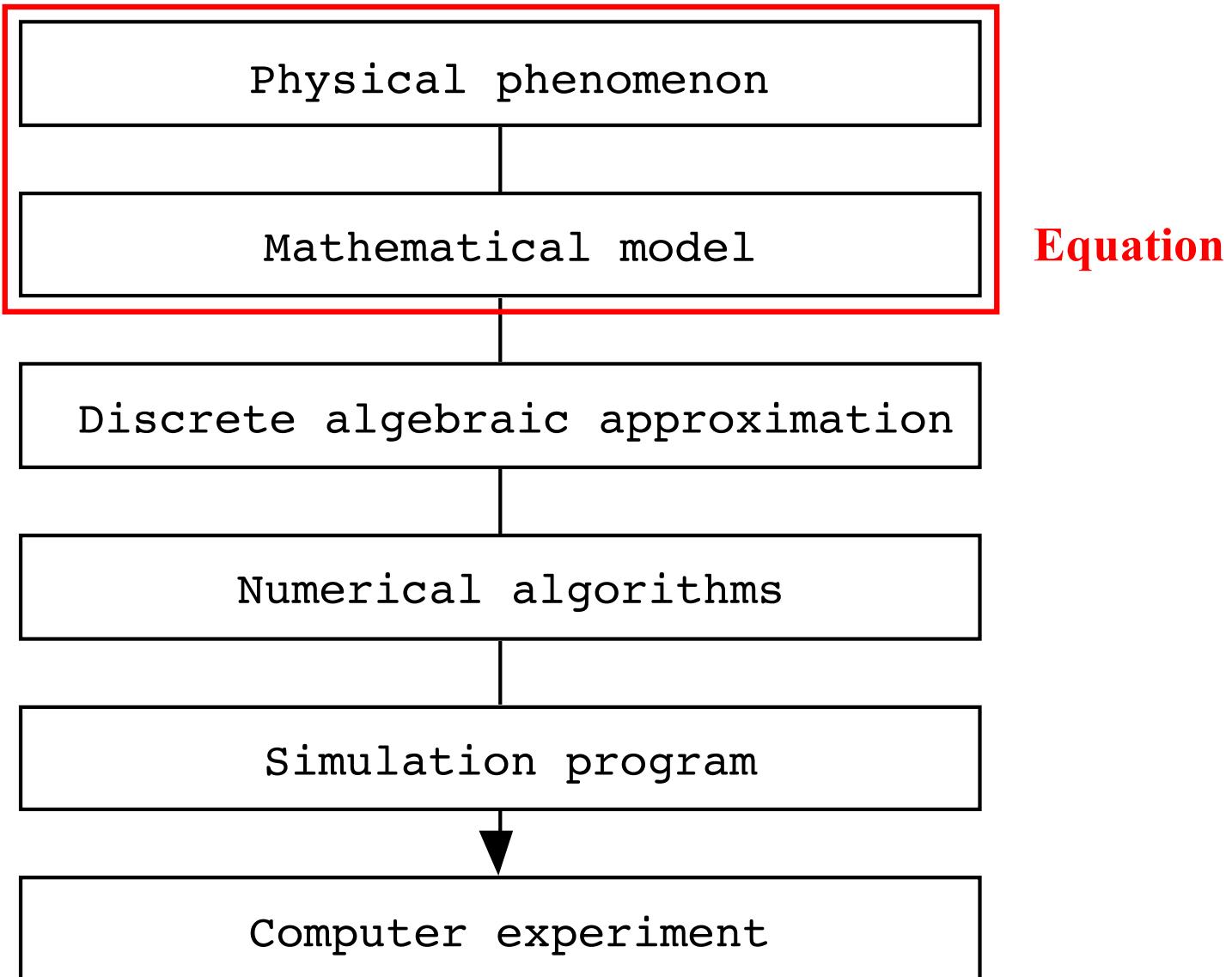
*Collaboratory for Advanced Computing & Simulations
Department of Physics & Astronomy
Department of Computer Science
Department of Quantitative & Computational Biology
University of Southern California*

Email: anakano@usc.edu



Computational Physics Approach

Study a physical phenomenon using computer—in steps

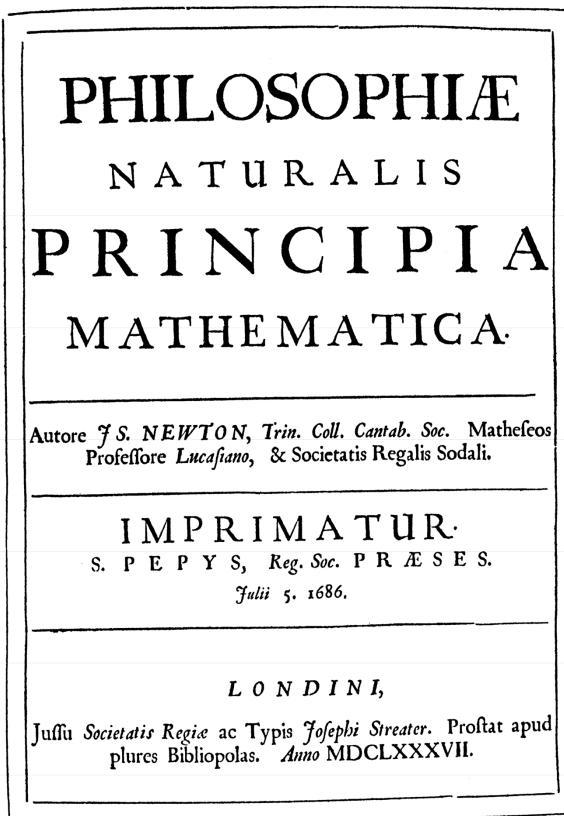


Nature to Math to Computing

The collage includes:

- Griffith Observatory:** A night view of the observatory building and the city lights of Los Angeles.
- ROGER PENROSE - THE ROAD TO REALITY:** The front cover of the book, featuring a blue and green abstract background with the title and author's name.
- Universarium:** An image of a large, circular planetarium screen showing a detailed view of Earth from space.
- La La Land:** A scene from the movie where Ryan Gosling and Emma Stone are dancing on a rooftop at night.
- Big Bang Theory:** A group photo of the main cast members of the TV show.
- Scene from "Centered in the Universe":** A scene from a video or film showing a person looking up at a massive, glowing celestial body.
- NATIONAL BESTSELLER:** A small text at the bottom of the book cover.
- Footnote:** A quote from The New York Times about the book.
- URL:** <https://griffithobservatory.org>
- Scene from "Centered in the Universe":** Another scene from the video, showing a close-up of the Earth against a dark background.

Mathematical Model



TITLE PAGE OF THE FIRST EDITION OF THE PRINCIPIA
(See Appendix, Note 2, page 627)

AXIOMS, OR LAWS OF MOTION¹

LAW I

Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it.

PROJECTILES continue in their motions, so far as they are not retarded by the resistance of the air, or impelled downwards by the force of gravity. A top, whose parts by their cohesion are continually drawn aside from rectilinear motions, does not cease its rotation, otherwise than as it is retarded by the air. The greater bodies of the planets and comets, meeting with less resistance in freer spaces, preserve their motions both progressive and circular for a much longer time.

LAW II²

The change of motion is proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.

If any force generates a motion, a double force will generate double the motion, a triple force triple the motion, whether that force be impressed altogether and at once, or gradually and successively. And this motion (being always directed the same way with the generating force), if the body moved before, is added to or subtracted from the former motion, according as they directly conspire with or are directly contrary to each other; or obliquely joined, when they are oblique, so as to produce a new motion compounded from the determination of both.

LAW III

To every action there is always opposed an equal reaction: or, the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.

Whatever draws or presses another is as much drawn or pressed by that other. If you press a stone with your finger, the finger is also pressed by the

[¹ Appendix, Note 14.]

[² Appendix, Note 15.]

[13]

LAW I

Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it.

LAW II²

The change of motion is proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.

14

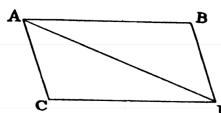
NEWTON'S MATHEMATICAL PRINCIPLES

stone. If a horse draws a stone tied to a rope, the horse (if I may so say) will be equally drawn back towards the stone; for the distended rope, by the same endeavor to relax or unbend itself, will draw the horse as much towards the stone as it does the stone towards the horse, and will obstruct the progress of the one as much as it advances that of the other. If a body impinge upon another, and by its force change the motion of the other, that body also (because of the equality of the mutual pressure) will undergo an equal change, in its own motion, towards the contrary part. The changes made by these actions are equal, not in the velocities but in the motions of bodies; that is to say, if the bodies are not hindered by any other impediments. For, because the motions are equally changed, the changes of the velocities made towards contrary parts are inversely proportional to the bodies. This law takes place also in attractions, as will be proved in the next Scholium.

COROLLARY I

A body, acted on by two forces simultaneously, will describe the diagonal of a parallelogram in the same time as it would describe the sides by those forces separately.

If a body in a given time, by the force M impressed apart in the place A, should with an uniform motion be carried from A to B, and by the force N impressed apart in the same place, should be carried from A to C, let the parallelogram ABCD be completed, and,



by both forces acting together, it will in the same time be carried in the diagonal from A to D. For since the force N acts in the direction of the line AC, parallel to BD, this force (by the second Law) will not at all alter the velocity generated by the other force M, by which the body is carried towards the line BD. The body therefore will arrive at the line BD in the same time, whether the force N be impressed or not; and therefore at the end of that time it will be found somewhere in the line BD. By the same argument, at the end of the same time it will be found somewhere in the line CD. Therefore it will be found in the point D, where both lines meet. But it will move in a right line from A to D, by Law I.

Calculus-Based Science

Calculus has been the principal scientific paradigm for 400 years

Newton, in his efforts to understand the natural laws of the rate of change in motion, used algebra to underpin another new branch of mathematics: calculus (a branch for which von Leibniz is simultaneously and independently credited). Calculus spurred scientists “to go off looking for other laws of nature that could explain natural phenomenon in terms of **rates of change** and found them by the bucketful - heat, sound, light, fluid dynamics, electricity and magnetism” [2].



What's Now? Physics in 100 Years

Computer science will play major roles in physics

- Increasingly, the development of algorithms will become a central focus of theoretical physics. ... Triumphs of creative understanding such as universality (suppression of irrelevant details), symmetry (informed iteration), and topology (emergence of discrete from continuous) are preadapted to **algorithmic thinking**.
- The work of designing algorithms can be considered as a special form of teaching, aimed at extremely clever but literal-minded and inexperienced students—that is, computers—who cannot deal with vagueness. At present those students are poorly motivated and incurious, but those faults are curable. Within 100 years they will become the colleagues and ultimately the successors of their human teachers, with a distinctive style of thought adapted to their talents.
- Two developments will be transformative: **naturalized artificial intelligence** and **expanded sensoria**.

F. Wilczek, Phys. Today 69(4), 32 ('16)

2024 Nobel prizes in physics & chemistry heralded the new age of science

<https://aiichironakano.github.io/phys516/Nobel24.pdf>

This Class: Understanding Simple Math

In your own words



Richard Feynman “On His Father’s Lap”

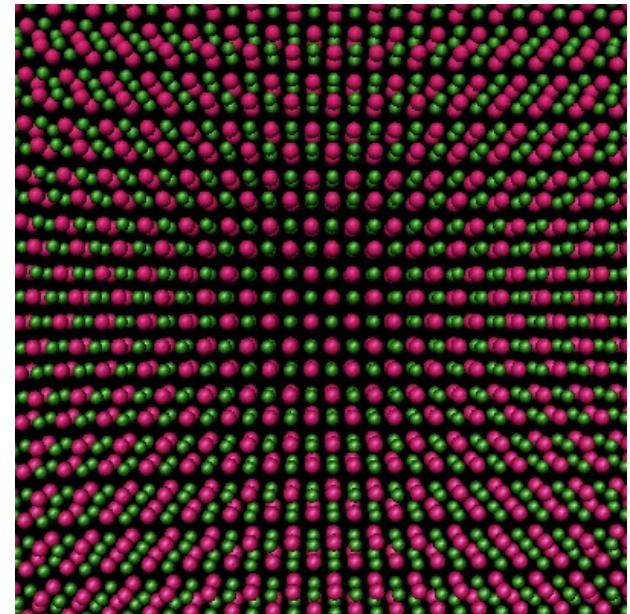
Molecular Dynamics Simulation

- Newton's equation of motion

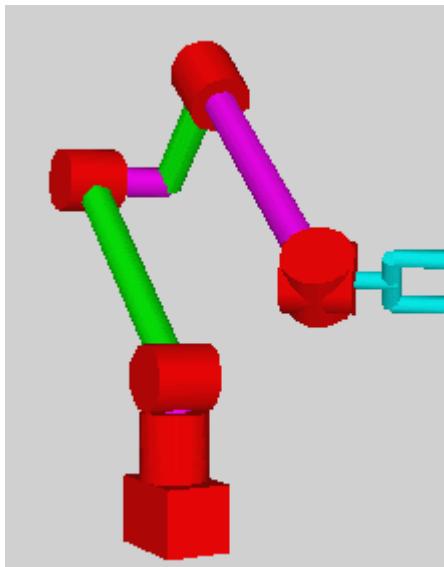
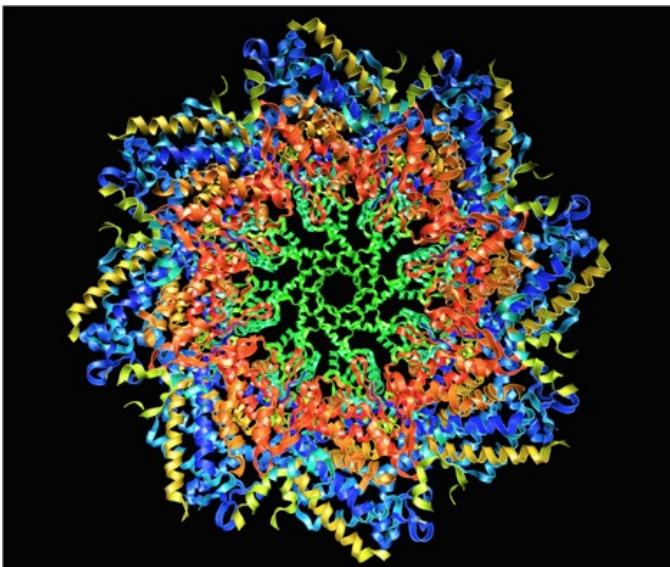
$$m_i \frac{d^2 \mathbf{r}_i}{dt^2} = -\frac{\partial V(\mathbf{r}^N)}{\partial \mathbf{r}_i} \quad (i = 1, \dots, N)$$

- Many-body interatomic potential

$$V = \sum_{i < j} u_{ij}(|\mathbf{r}_{ij}|) + \sum_{i,j < k} v_{jik}(\mathbf{r}_{ij}, \mathbf{r}_{ik})$$



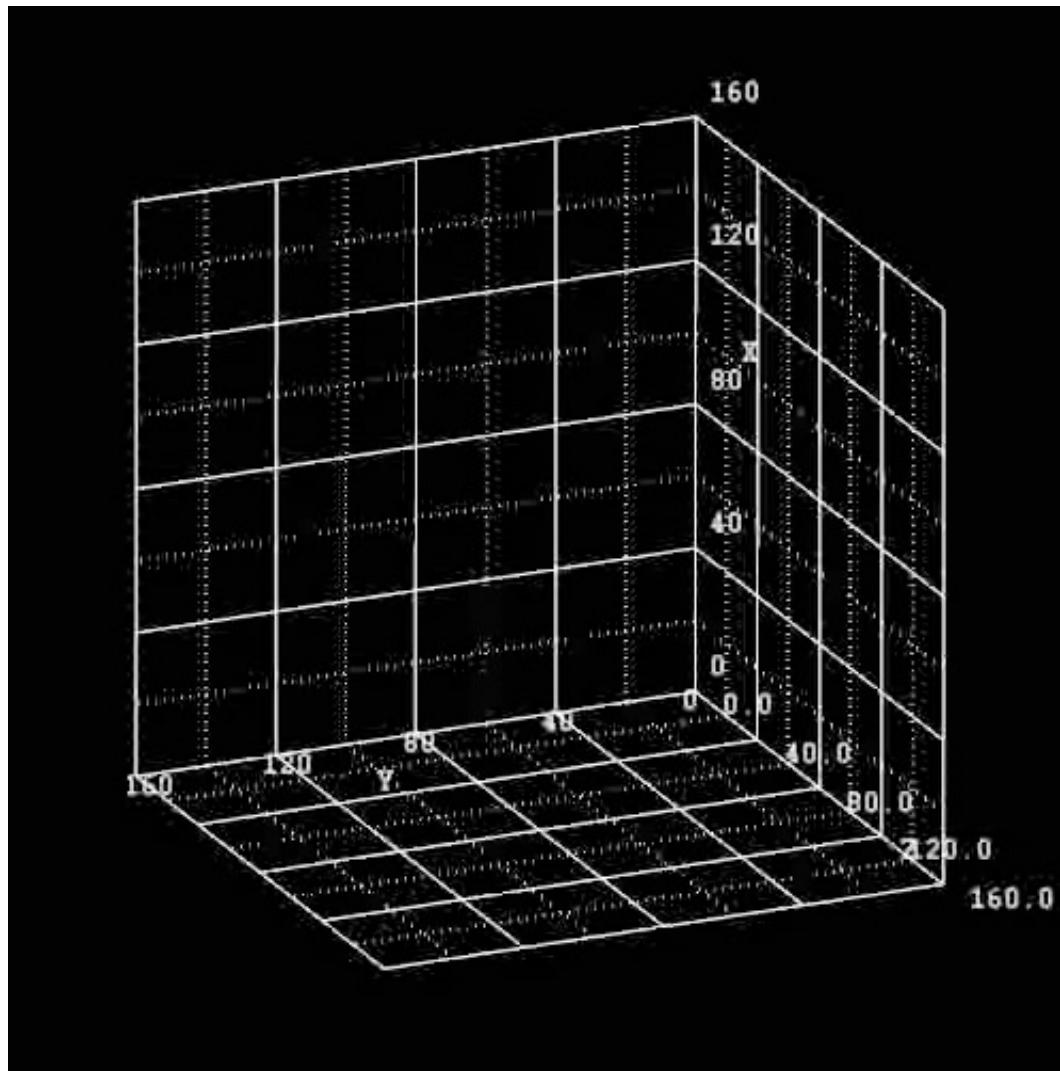
- Application: drug design, robotics, entertainment, etc.



Cancer Modeling

Integrative mathematical oncology

Alexander R. A. Anderson and Vito Quaranta



Pedestrian Crowd Dynamics

Simulating dynamical features of escape panic

Dirk Helbing^{*†}, Illés Farkas[‡] & Tamás Vicsek^{*‡}

Nature 407, 487 ('00)



Buildings 9, 44 (2019)

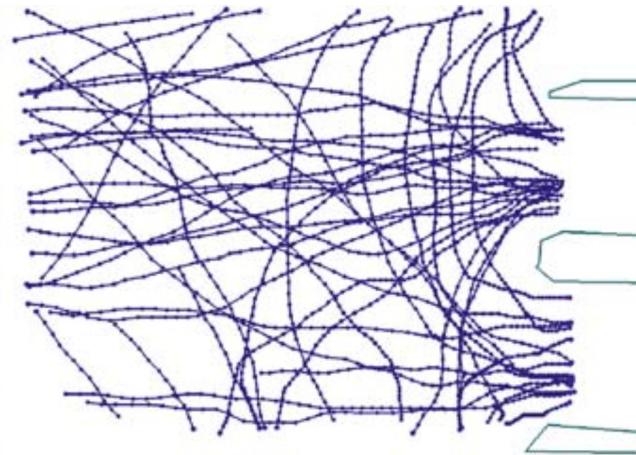
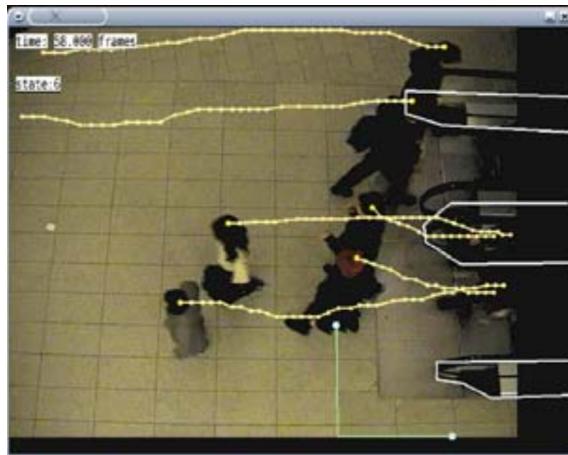


Article

Adaptive Kinetic Architecture and Collective Behavior: A Dynamic Analysis for Emergency Evacuation

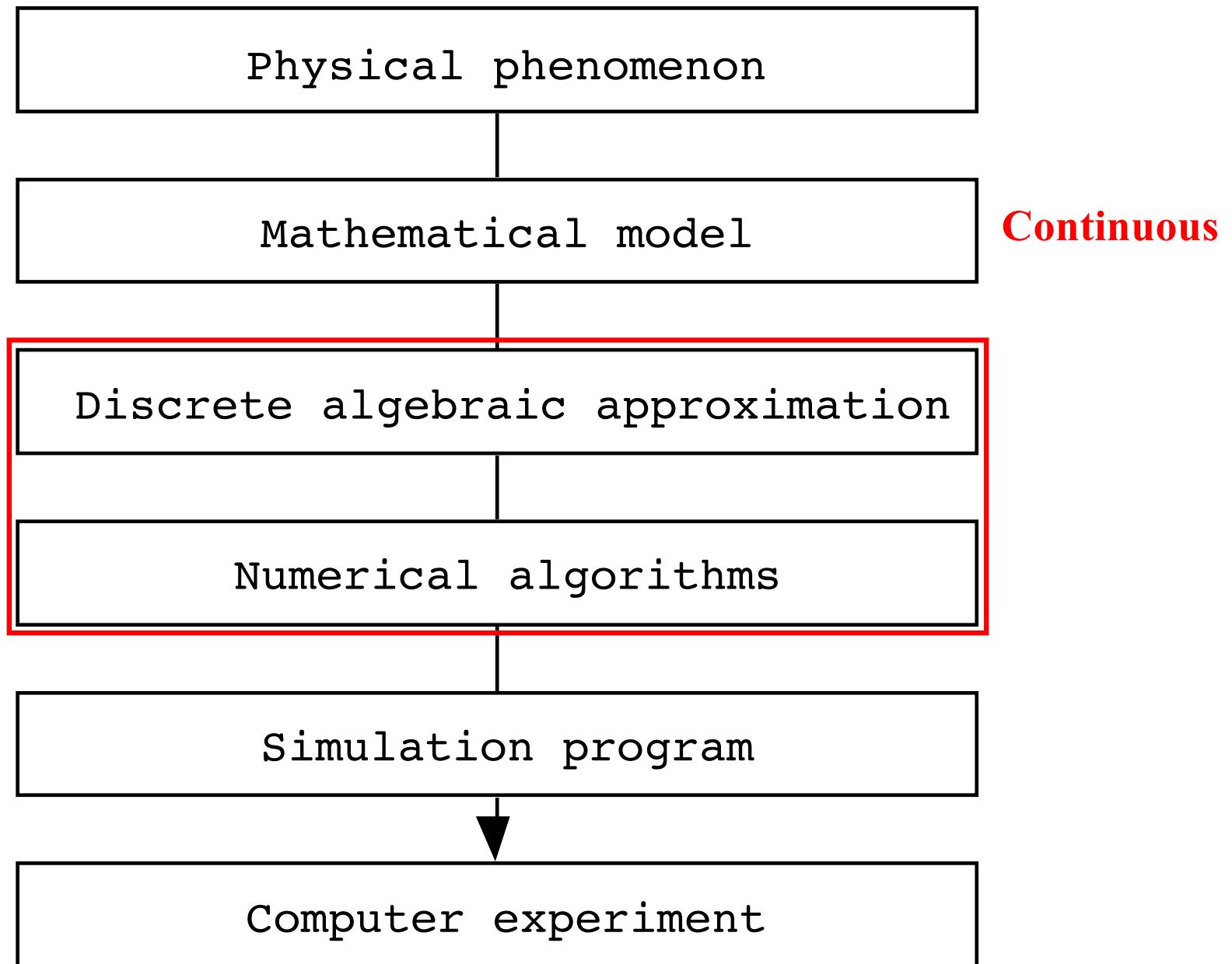
Angella Johnson ^{1,*}, Sizhe Zheng ², Aiichiro Nakano ³ , Goetz Schierle ¹ and Joon-Ho Choi ¹

$$m_i \frac{d\mathbf{v}_i}{dt} = m_i \frac{\nu_i^0(t) \mathbf{e}_i^0(t) - \mathbf{v}_i(t)}{\tau_i} + \sum_{j(\neq i)} \mathbf{f}_{ij} + \sum_W \mathbf{f}_{iW}$$



See also <http://www.oasys-software.com/products/engineering/massmotion.html>

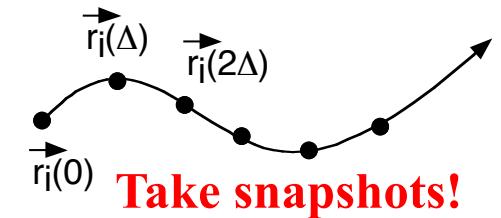
Computational Physics Approach



MD Algorithm

Time discretization: differential → algebraic equation

$$\begin{cases} \vec{r}_i(t + \Delta) = \vec{r}_i(t) + \vec{v}_i(t)\Delta + \frac{1}{2}\vec{a}_i(t)\Delta^2 \\ \vec{v}_i(t + \Delta) = \vec{v}_i(t) + \frac{\vec{a}_i(t) + \vec{a}_i(t + \Delta)}{2}\Delta \end{cases} \quad \vec{a}_i = -\frac{1}{m_i} \frac{\partial V}{\partial \vec{r}_i}$$

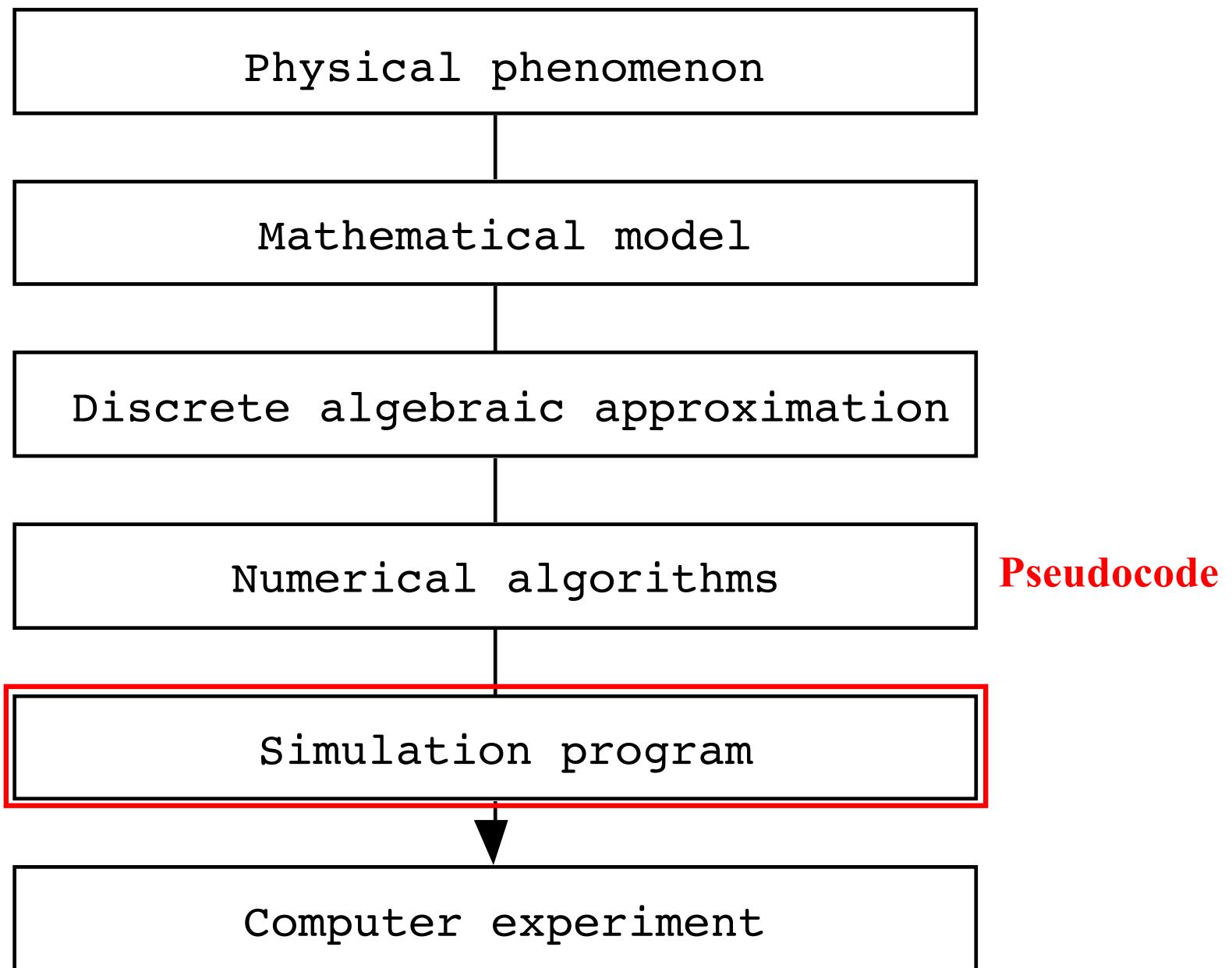


Time stepping: Velocity Verlet algorithm

Given $(\vec{r}_i(t), \vec{v}_i(t))$

1. Compute $\vec{a}_i(t)$ as a function of $\{\vec{r}_i(t)\}$
2. $\vec{v}_i(t + \frac{\Delta}{2}) \leftarrow \vec{v}_i(t) + \frac{\Delta}{2}\vec{a}_i(t)$
3. $\vec{r}_i(t + \Delta) \leftarrow \vec{r}_i(t) + \vec{v}_i(t + \frac{\Delta}{2})\Delta$
4. Compute $\vec{a}_i(t + \Delta)$ as a function of $\{\vec{r}_i(t + \Delta)\}$
5. $\vec{v}_i(t + \Delta) \leftarrow \vec{v}_i(t + \frac{\Delta}{2}) + \frac{\Delta}{2}\vec{a}_i(t + \Delta)$

Computational Physics Approach



MD Program

It's just “how you read” the equation (or algorithm)

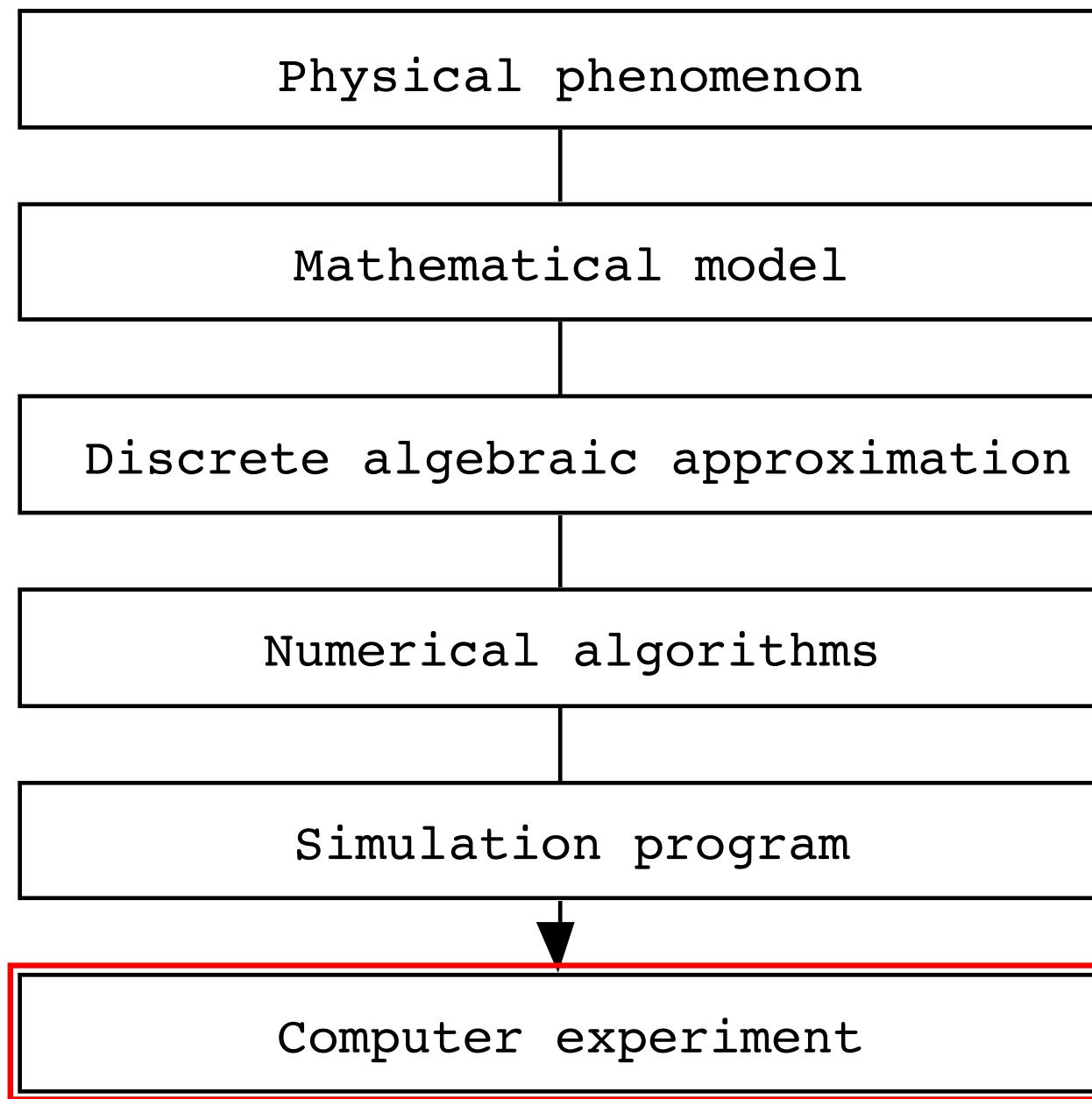
```
for (i=0; i<nAtom; i++)            $\vec{v}_i(t + \frac{\Delta}{2}) \leftarrow \vec{v}_i(t) + \frac{\Delta}{2} \vec{a}_i(t)$ 
    for (k=0; k<3; k++)
        rv[i][k] = rv[i][k] + DeltaT/2*ra[n][k];

for (i=0; i<nAtom; i++)            $\vec{r}_i(t + \Delta) \leftarrow \vec{r}_i(t) + \vec{v}_i(t + \frac{\Delta}{2})\Delta$ 
    for (k=0; k<3; k++)
        r[i][k] = r[i][k] + DeltaT*rv[i][k];

ComputeAccel();      // r[ ][ ] → ra[ ][ ]

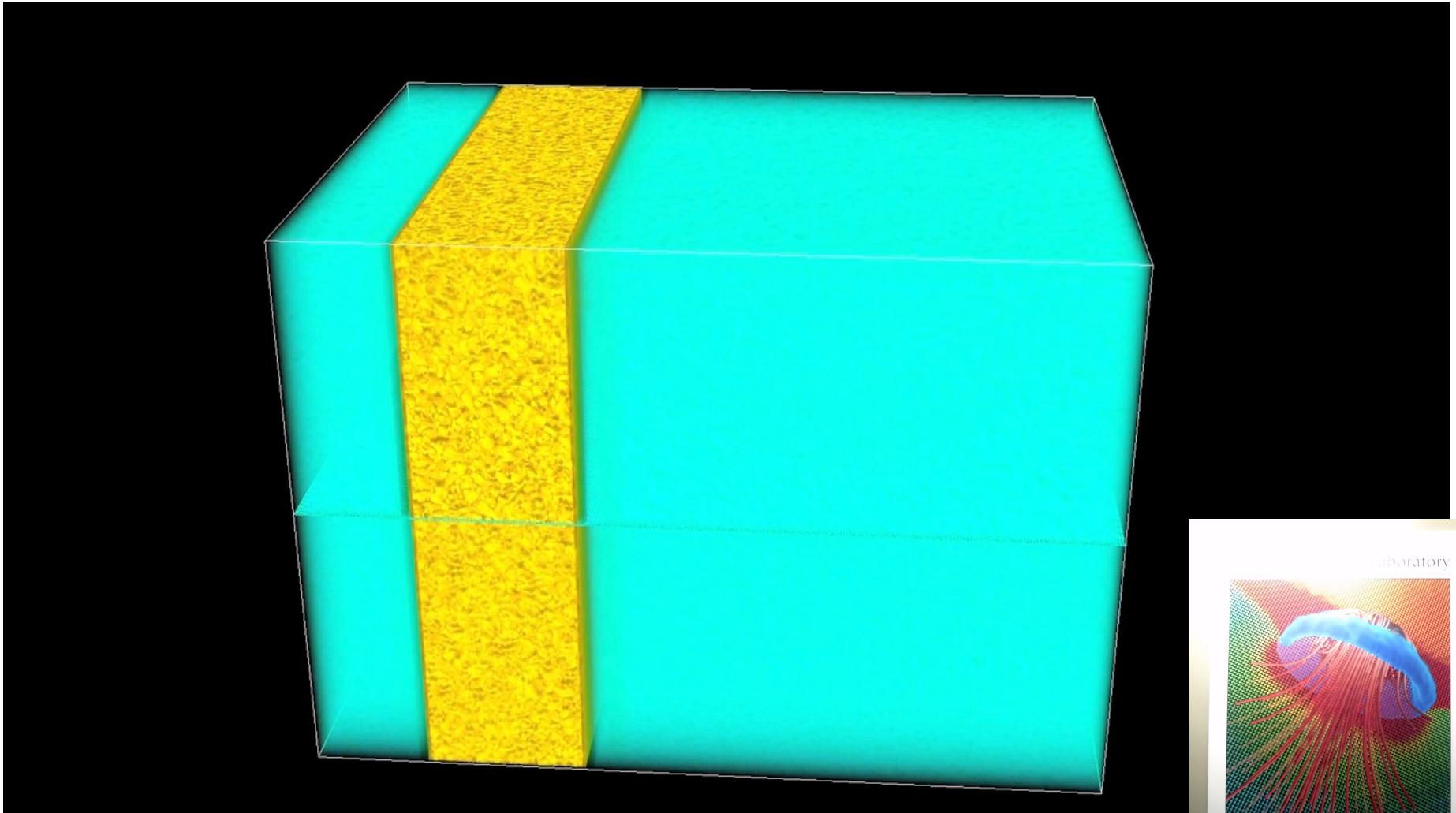
for (i=0; i<nAtom; i++)            $\vec{v}_i(t + \Delta) \leftarrow \vec{v}_i(t + \frac{\Delta}{2}) + \frac{\Delta}{2} \vec{a}_i(t + \Delta)$ 
    for (k=0; k<3; k++)
        rv[i][k] = rv[i][k] + DeltaT/2*ra[i][k];
```

Computational Physics Approach



Computer Experiment

- Billion-atom reactive MD simulation of shock-induced nanobubble collapse in water near silica surface (67 million core-hours on 163,840 Blue Gene/P cores)



- Water nanojet formation and its collision with silica surface

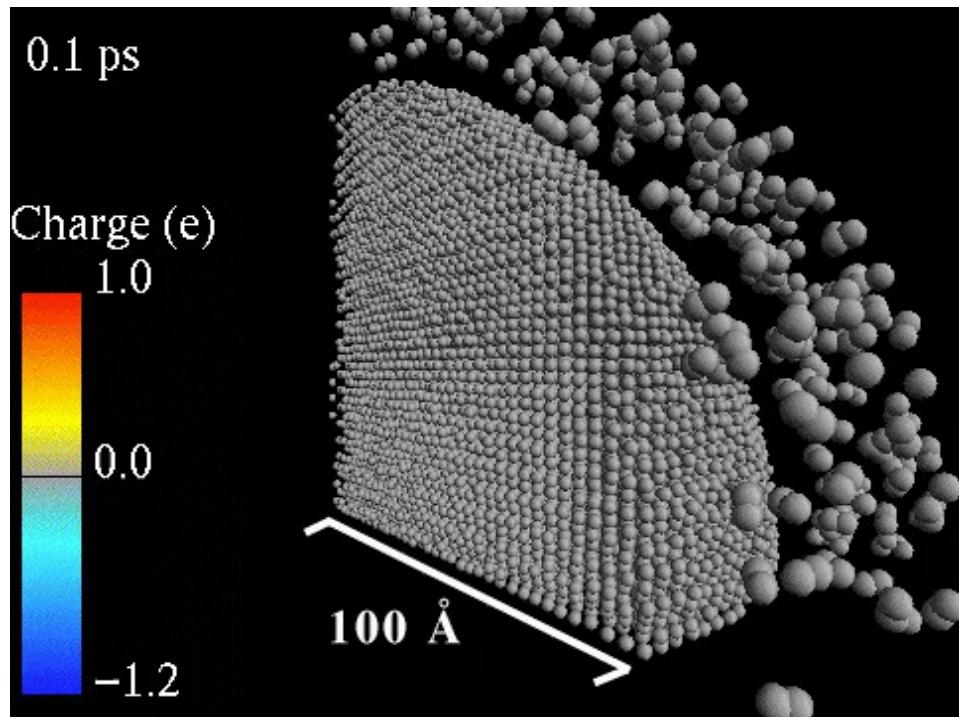
A. Shekhar et al., *Phys. Rev. Lett.* **111**, 184503 ('13)



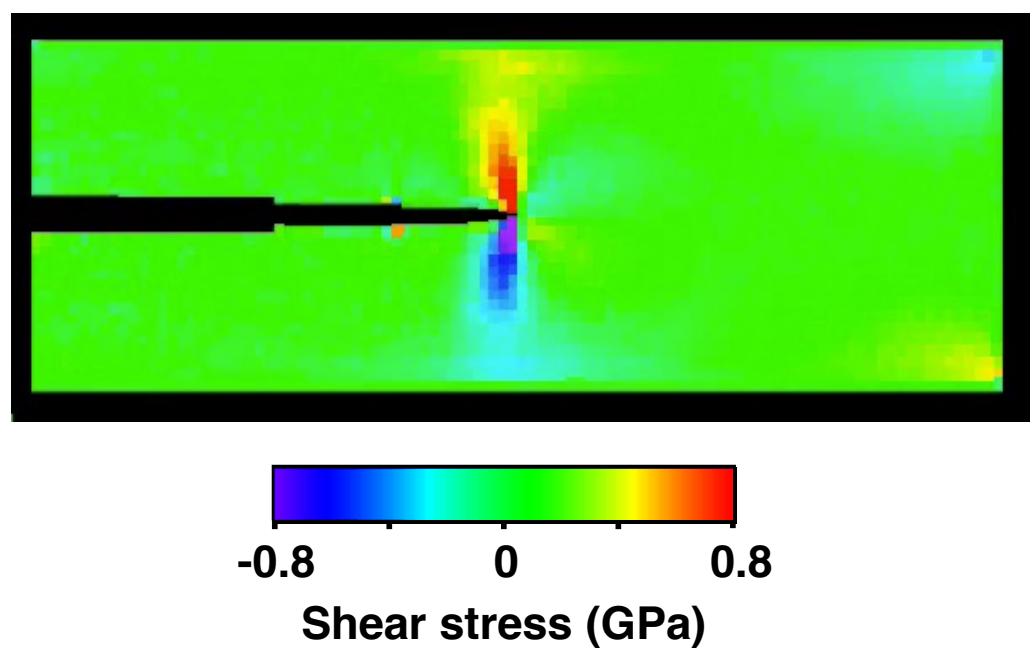
Type of Mathematical Models

	Discrete/particle model (ordinary differential equations)	Continuum model (partial differential equations)
Deterministic	molecular dynamics	computational fluid dynamics, continuum mechanics
Stochastic	Monte Carlo particle simulation	quantum Monte Carlo

Particle model of oxidation



Continuum model of fracture



Continuum Model: Quantum Mechanics

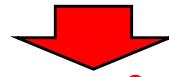
Challenge: Complexity of quantum N -body problem



Density functional theory (DFT)

(Walter Kohn, Nobel Chemistry Prize, '98)

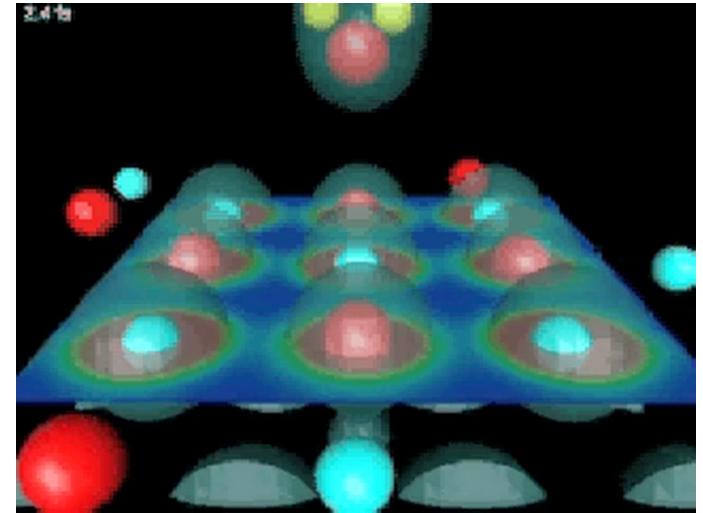
$$\psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_{N_{\text{el}}}) \mathcal{O}(C^N)$$



$$\{\psi_n(\mathbf{r}) | n = 1, \dots, N_{\text{el}}\} \mathcal{O}(N^3)$$

See DFT reading list:

<https://aiichironakano.github.io/phys516/DFT-seminar.tar.gz>



Constrained minimization problem:

Minimize

$$E[\{\psi_n\}] = \sum_{n=1}^{N_{\text{el}}} \int d^3r \psi_n^*(\mathbf{r}) \left(-\frac{\hbar^2}{2m_e} \frac{\partial^2}{\partial \mathbf{r}^2} + V_{\text{ion}}(\mathbf{r}) \right) \psi_n(\mathbf{r}) + \frac{e^2}{2} \iint d^3r d^3r' \frac{\rho(\mathbf{r})\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} + E_{\text{XC}}[\rho(\mathbf{r})]$$

with orthonormal constraints: $\int d^3r \psi_m^*(\mathbf{r}) \psi_n(\mathbf{r}) = \delta_{mn}$

$$\text{Charge density: } \rho(\mathbf{r}) = \sum_{n=1}^{N_{\text{el}}} |\psi_n(\mathbf{r})|^2$$

See PHYS 760 “Extreme-scale quantum simulations”:

<https://aiichironakano.github.io/phys760.html>

Walter Kohn (1923-2016)

Book chapter No access

DENSITY FUNCTIONAL THEORY OF SUPERCONDUCTORS REGARDED AS TWO-COMPONENT PLASMAS

Walter KOHN

Pages 331-335

 Purchase

Book chapter No access

GREEN'S FUNCTION AND DYNAMIC CORRELATIONS OF ELECTRONS IN METALS

Aiichiro NAKANO and Setsuo ICHIMARU

Pages 337-340

 Purchase

Strongly Coupled Plasma Physics

Proceedings of Yamada Conference XXIV on Strongly Coupled Plasma Physics, Lake Yamanaka, Japan, August 29–September 2, 1989



Multiscale Modeling

The Nobel Prize in Chemistry 2013



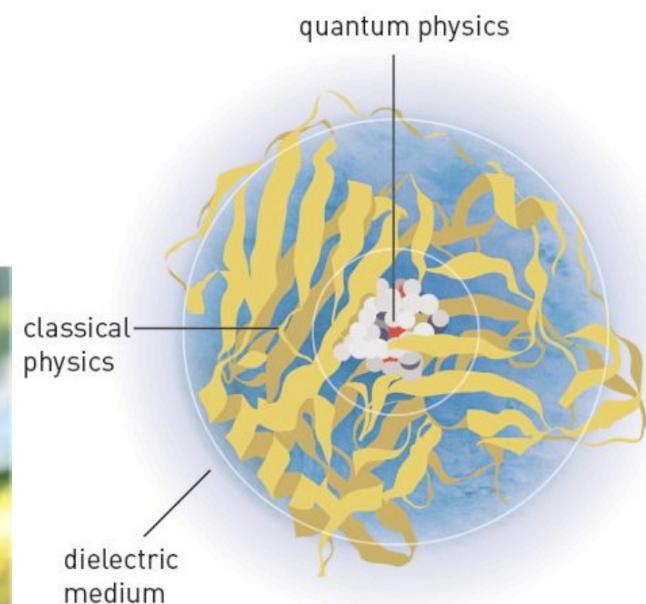
© Nobel Media AB
Martin Karplus



Photo: Keilana via
Wikimedia Commons
Michael Levitt



Photo: Wikimedia
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Arieh Warshel



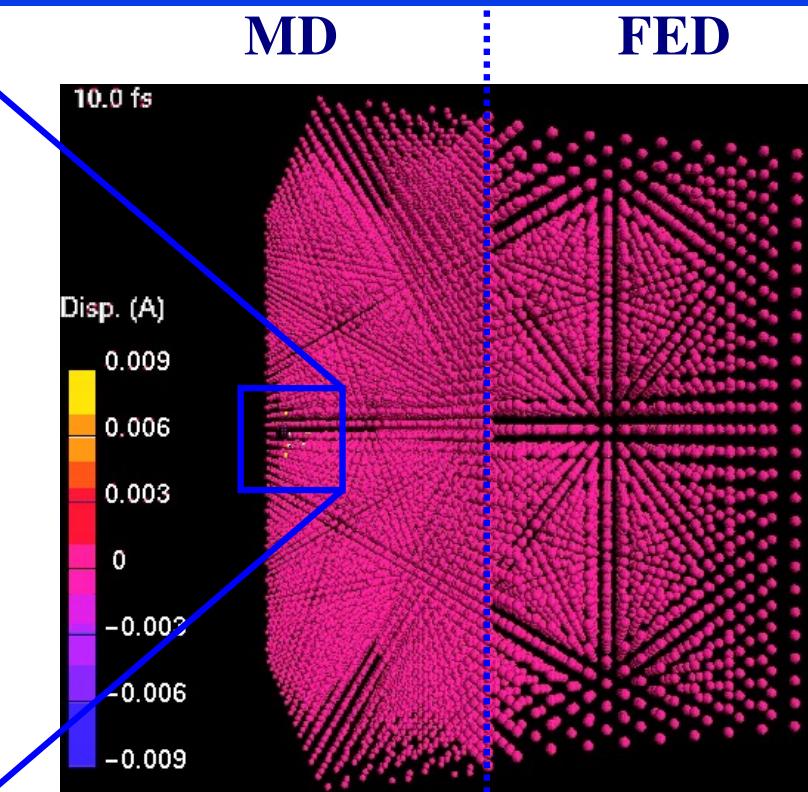
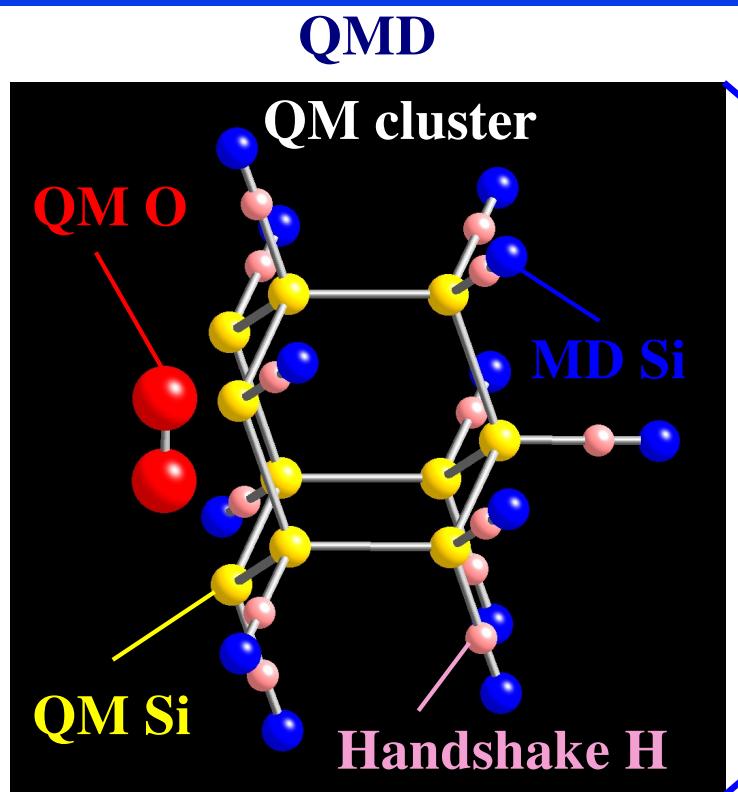
**QM/MM:
quantum-
mechanical/molecular-
mechanical modeling**

The Nobel Prize in Chemistry 2013 was awarded jointly to Martin Karplus, Michael Levitt and Arieh Warshel *"for the development of multiscale models for complex chemical systems"*.

- A. Warshel & M. Karplus, *J. Am. Chem. Soc.* **94**, 5612 ('72)
- A. Warshel & M. Levitt, *J. Mol. Biol.* **103**, 227 ('76)

[See A computational account of Nobel history \(Nature Computational Science, Sep. '22\)](#)

Adaptive Multiscale Dynamics



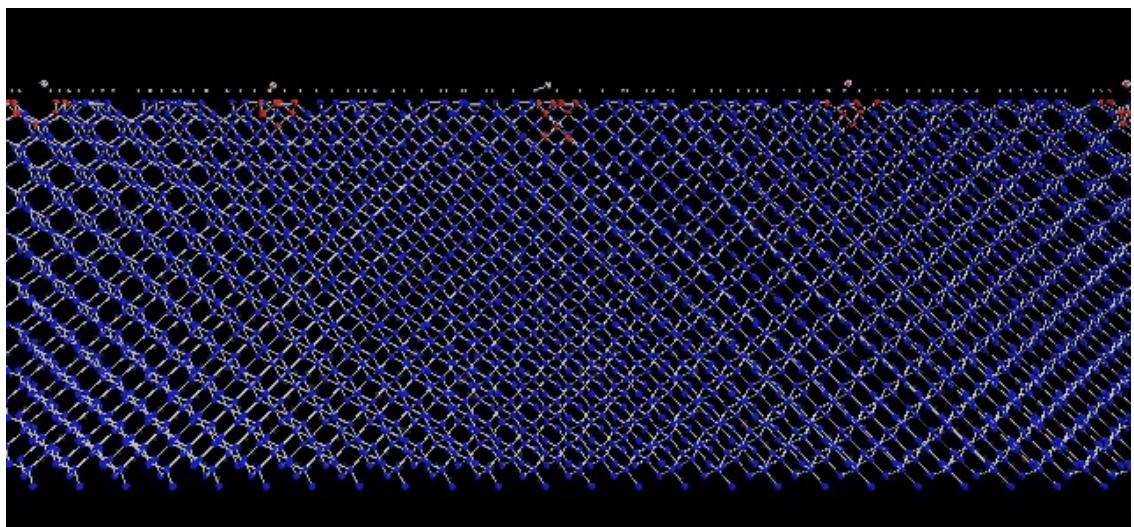
High-energy beam oxidation of Si (SIMOX)

H. Takemiya et al.,
IEEE/ACM Supercomputing (SC06)

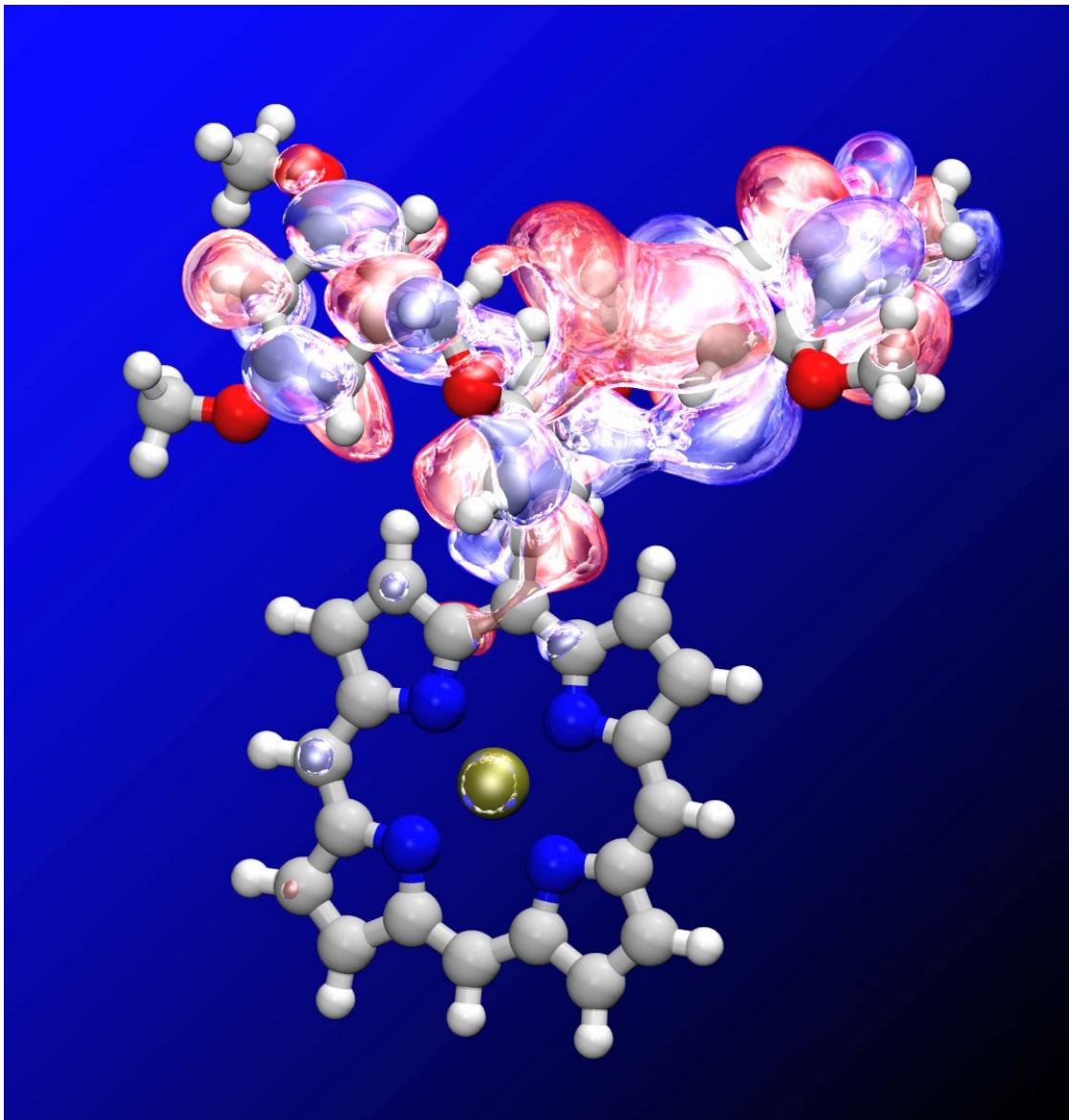
Oxidation of Si

S. Ogata et al.,
Comput. Phys. Commun. **138**, 143 ('01)
L. Lidorikis et al.,
Phys. Rev. Lett. **87**, 086104 ('01)

QM/MD/FED:
quantum mechanical/
molecular-dynamics/
finite-element dynamics



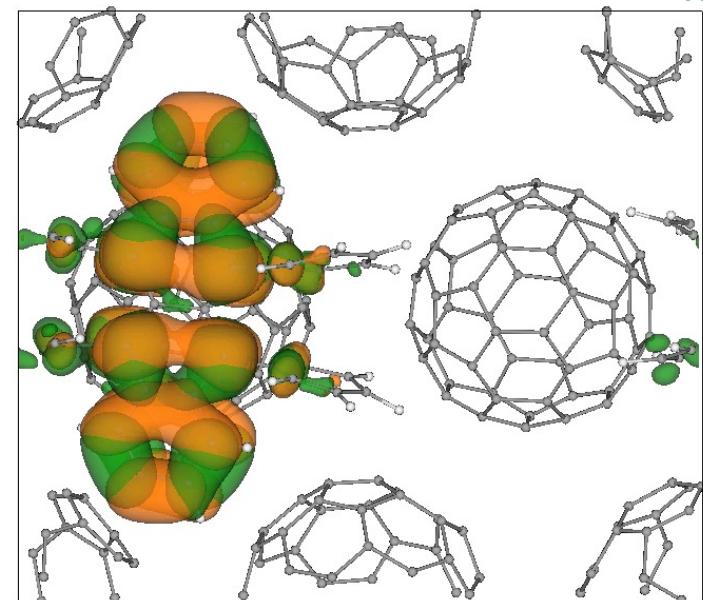
Nonadiabatic Quantum Molecular Dynamics



Appl. Phys. Lett. **98**, 113302 ('11); *ibid.* **100**, 203306 ('12); *ibid.* **102**, 173301 ('13); *Comput. Phys. Commun.* **184**, 1 ('13); *J. Chem. Phys.* **140**, 18A529 ('14); *IEEE Computer* **48**(11), 33 ('15); *Sci. Rep.* **5**, 19599 ('16); *Nature Commun.* **8**, 1745 ('17); *Nano Lett.* **18**, 4653 ('18); *Nature Photon.* **13**, 425 ('19); *Science Adv.* **8**, eabk2625 ('22)

Zn porphyrin

Rubrene/C₆₀

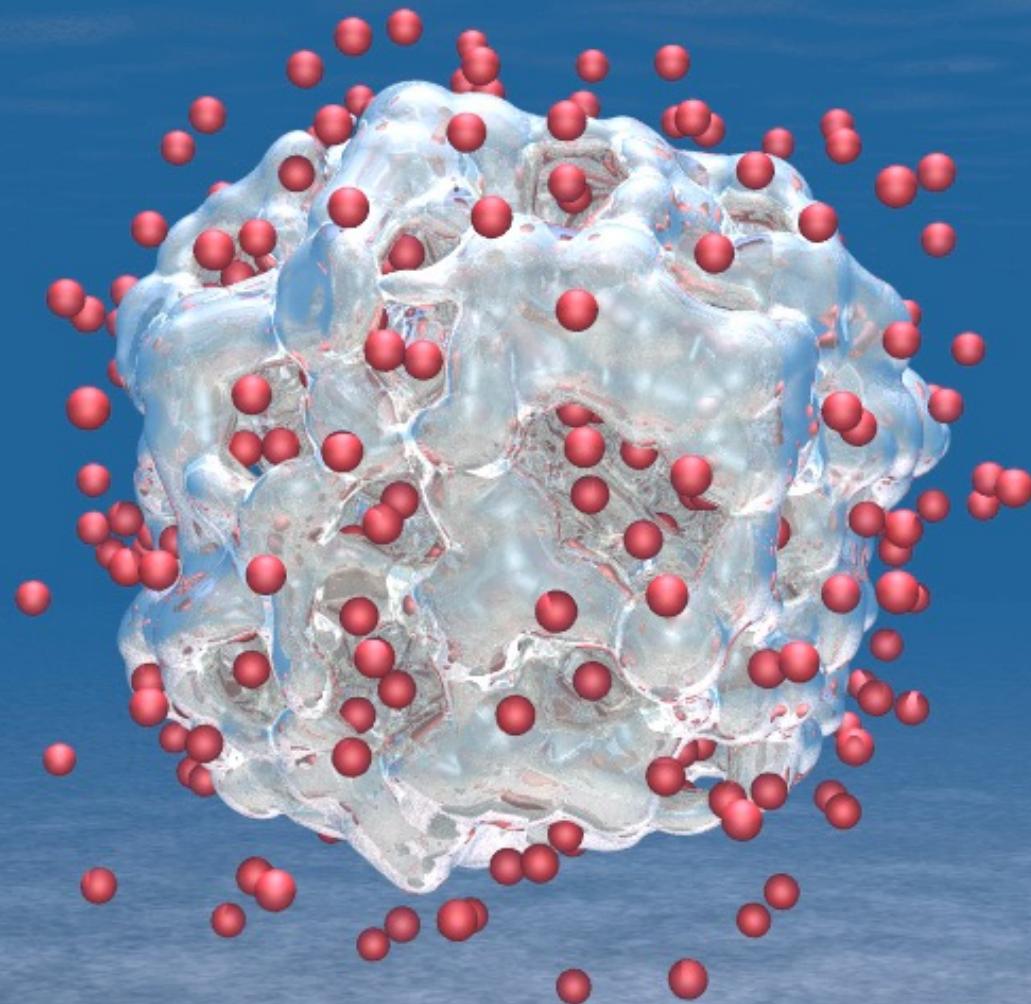


quasi-electron; quasi-hole

- **Excited states:** Linear-response time-dependent density functional theory [Casida, '95]
- **Interstate transitions:** Surface hopping [Tully, '90; Jaeger, Fisher & Prezhdo, '12]

Hydrogen Production from Water

- 16,611-atom quantum MD simulation of rapid H₂ production from water using a LiAl particle on 786,432 Blue Gene/Q cores



K. Shimamura *et al.*, *Nano Lett.* **14**, 4090 ('14)

Stochastic Model of Stock Prices

Fluctuation in stock price

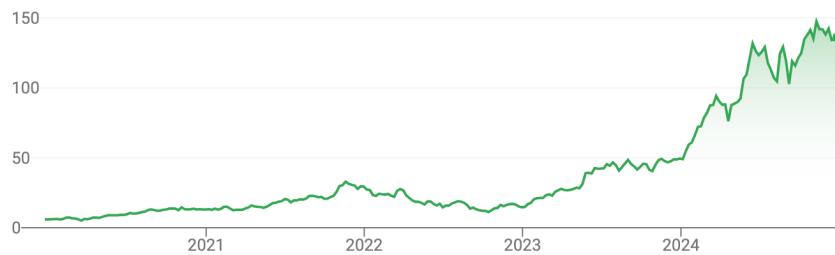
Market Summary > NVIDIA Corp

136.24 USD

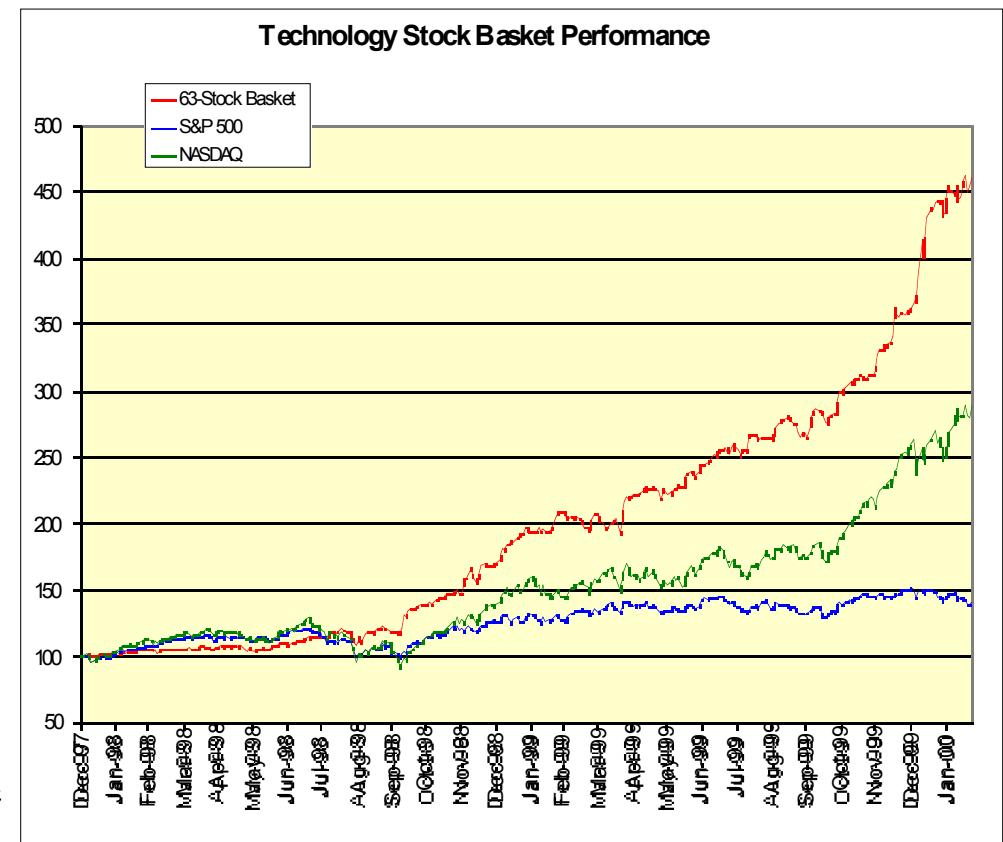
+130.32 (2,201.35%) ↑ past 5 years

Dec 27, 1:48 PM EST • Disclaimer

1D | 5D | 1M | 6M | YTD | 1Y | 5Y | Max



Computational stock portfolio trading



Basis of Black-Scholes analysis of option prices



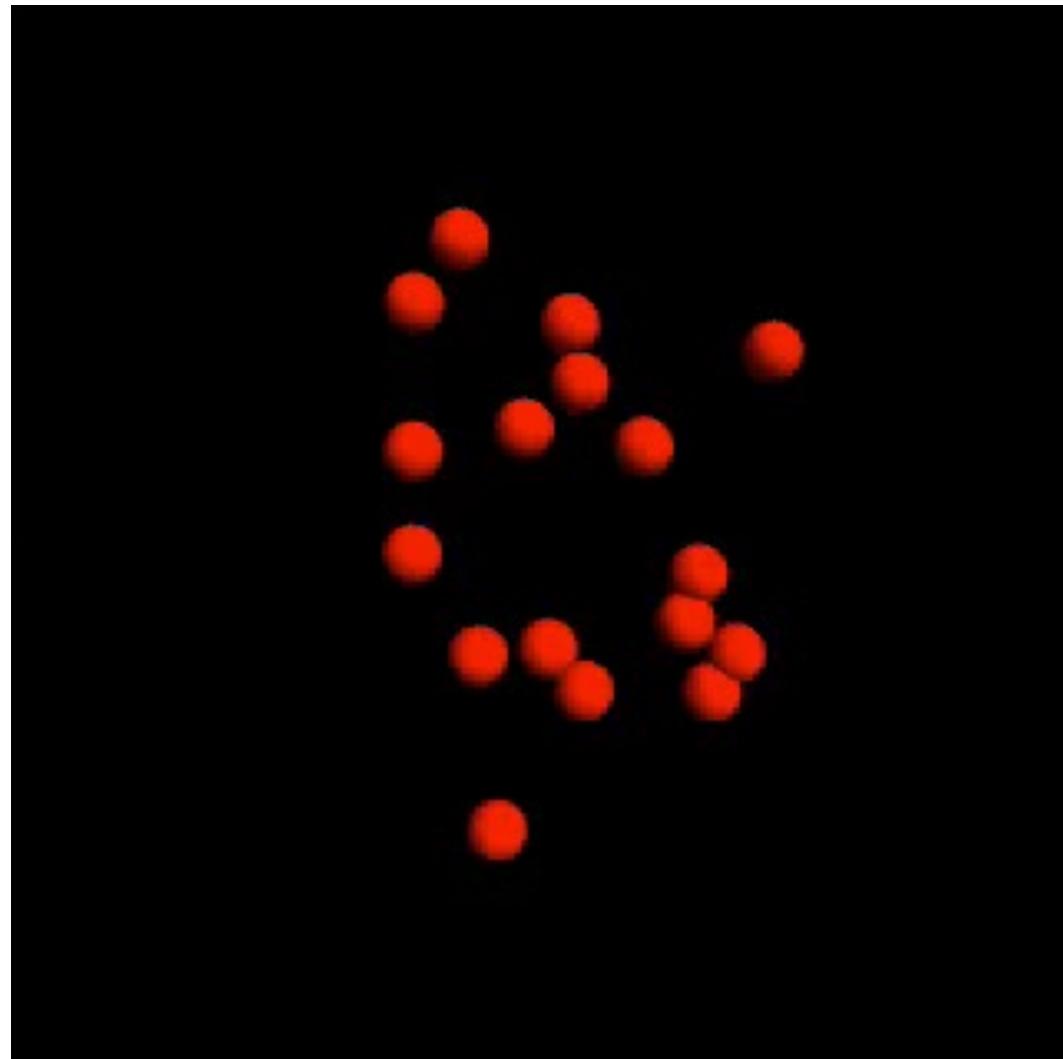
$$dS = \mu S dt + \sigma S \varepsilon \sqrt{dt}$$

(1997 Nobel Economy Prize to Myron Scholes)

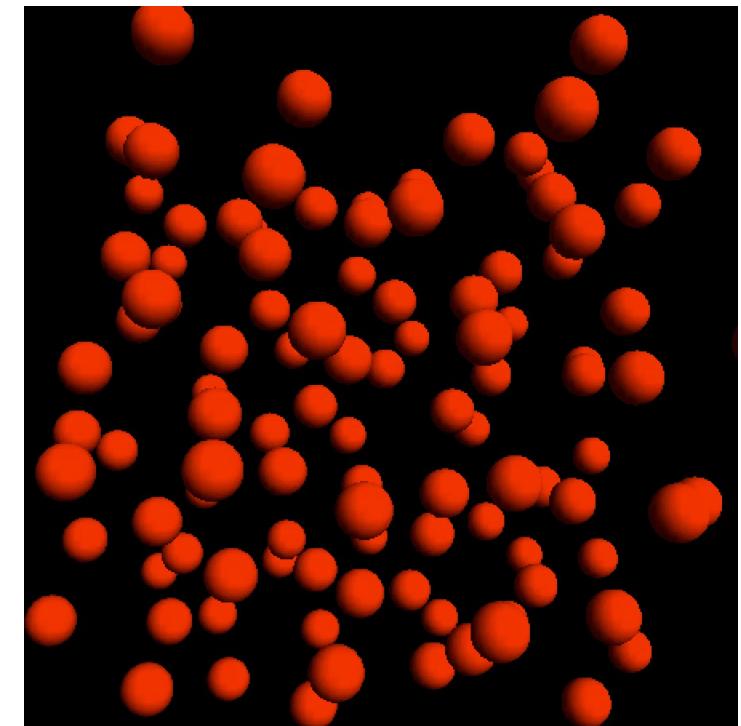
Andrey Omelchenko (Quantlab)

<http://www.convexitymaven.com/themavensclassroom.html>

Monte Carlo Simulation



Monte Carlo



Molecular
dynamics

Random trial → acceptance by a cost criterion

Phys516: What You Will Learn

Nature to math to computing!

The ability to implement the solution of mathematically formulated problems on a computer

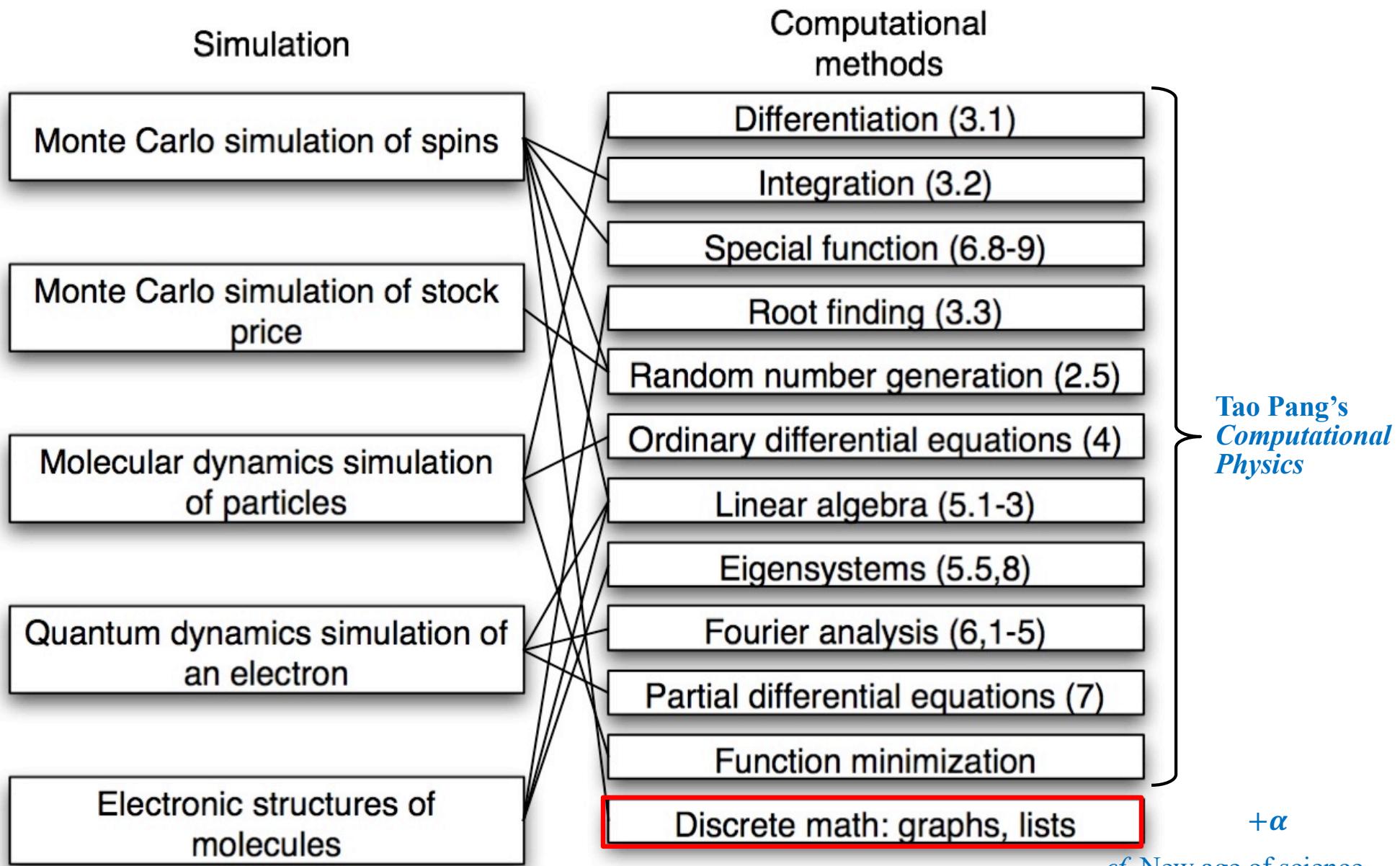
You understand it = you can program it

Computational physicists' survival kit

- Mathematical methods in physics: Any book you are familiar with, e.g., G. B. Arfken & H. J. Weber, *Mathematical Methods for Physicists*, 7th Ed. (Academic Press, '12)
- Numerical algorithms: W. H. Press, B. P. Flannery, S. A. Teukolsky, & W. T. Vetterling, *Numerical Recipes*, 3rd Ed. (Cambridge U Press, '07)—available online

<https://numerical.recipes/>

Phys516: Computational Methods in the Context of Simulations



Changing Computing Landscape for Science

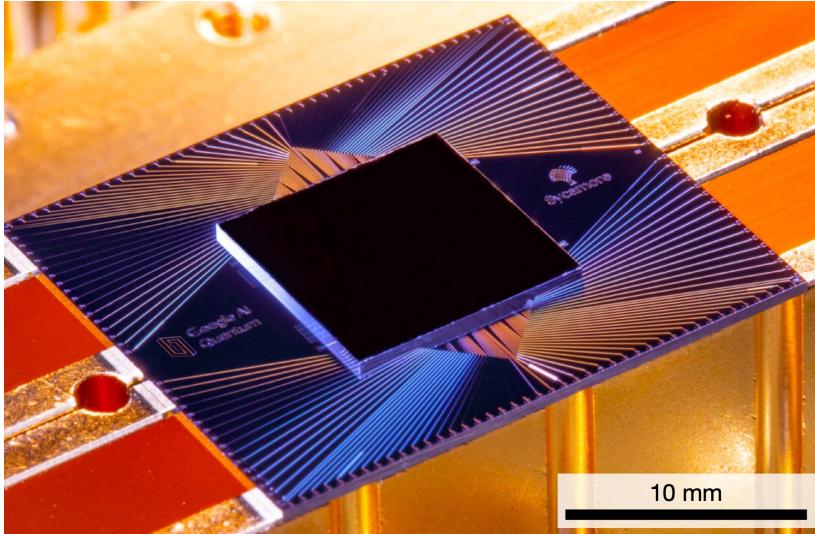
Post-exascale Computing for Science



Compute Cambrian explosion



Quantum Computing for Science



AI for Science

DOE readies multibillion-dollar AI push

U.S. supercomputing leader is the latest big backer in a globally crowded field

By Robert F. Service, in Washington, D.C.

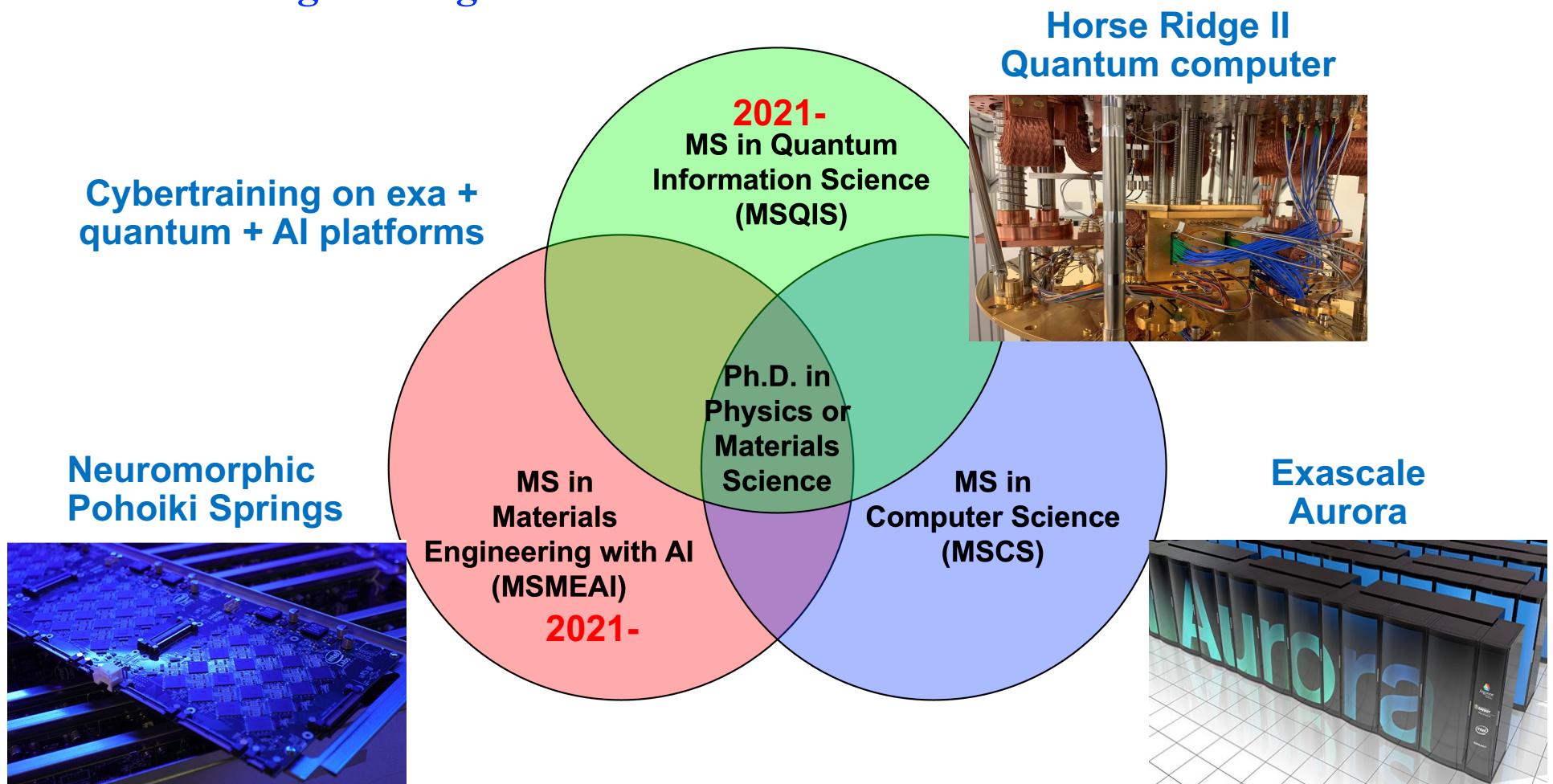
Science 366, 559 (Nov. 1, '19)

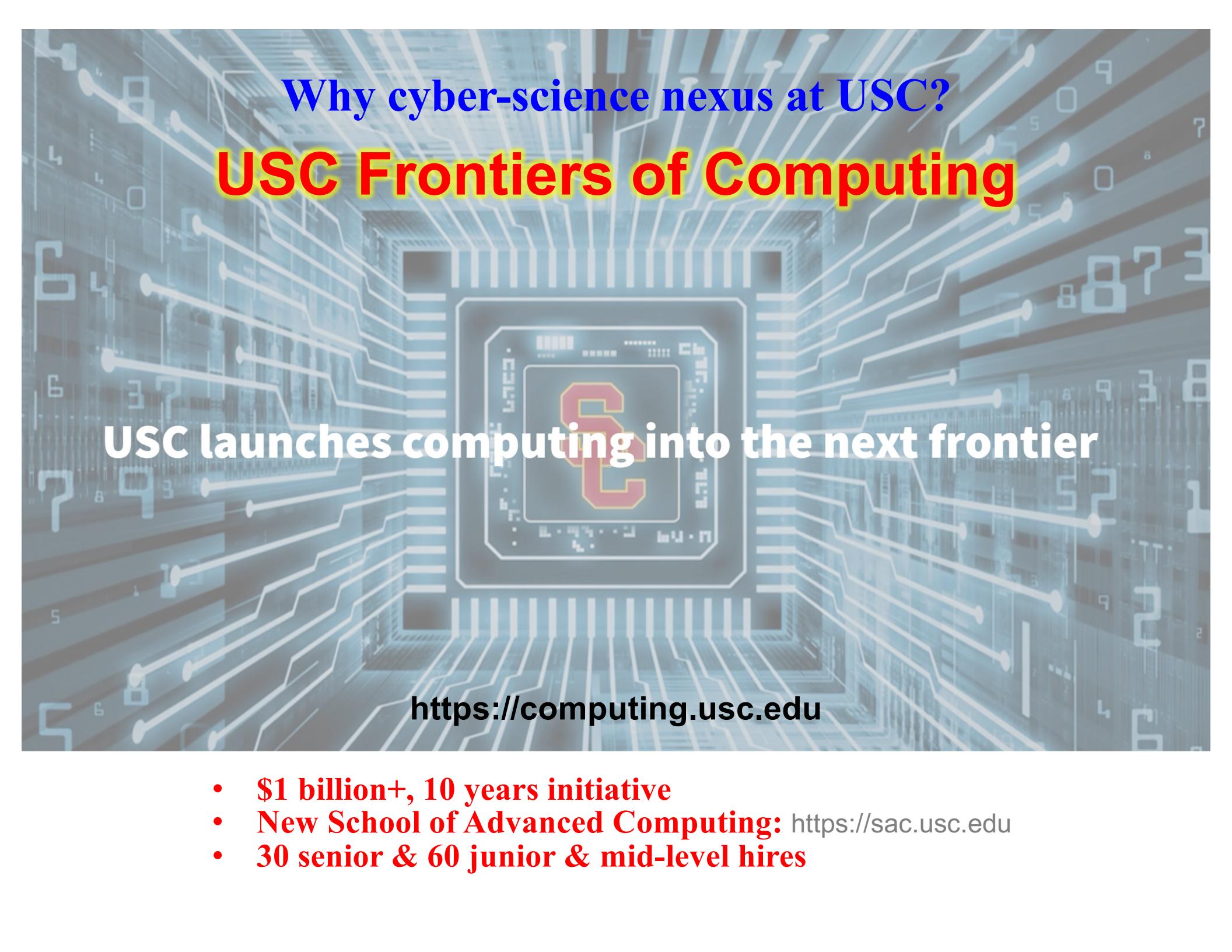


Use all to advance science!

Training Cyber-Science Workforce

- New generation of computational scientists at the **nexus of post-exascale computing, quantum computing & AI**
- Unique dual-degree program: Ph.D. in science or engineering, along with MS in computer science, MS in quantum information science or MS in materials engineering with AI





Why cyber-science nexus at USC?

USC Frontiers of Computing

USC launches computing into the next frontier

<https://computing.usc.edu>

- \$1 billion+, 10 years initiative
- New School of Advanced Computing: <https://sac.usc.edu>
- 30 senior & 60 junior & mid-level hires

Current & Future Supercomputing

- Won two DOE supercomputing awards to develop & deploy metascalable (“design once, scale on future platforms”) simulation algorithms



Innovative & Novel Computational Impact on Theory & Experiment

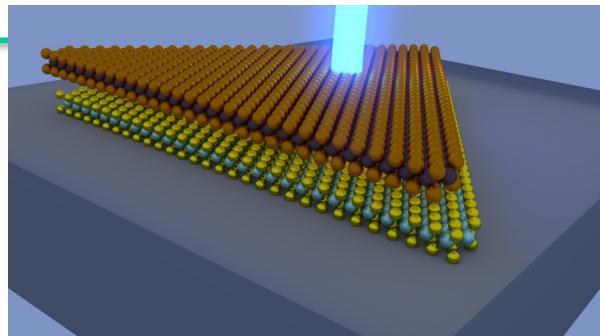
Title: AI-Guided Exascale Simulations of Quantum Materials Manufacturing and Control

PI and Co-PIs: Aiichiro Nakano—PI, Rajiv K. Kalia, Ken-ichi Nomura, Priya Vasishta

- Atomistic simulations on million cores (pre-exascale)



786,432-core IBM Blue Gene/Q
281,088-core Intel Xeon Phi
560-node (2,240-GPU) AMD/NVIDIA Polaris



Early Science Projects for Aurora

Supercomputer Announced

Metascalable layered materials genome

Investigator: Aiichiro Nakano, University of Southern

California

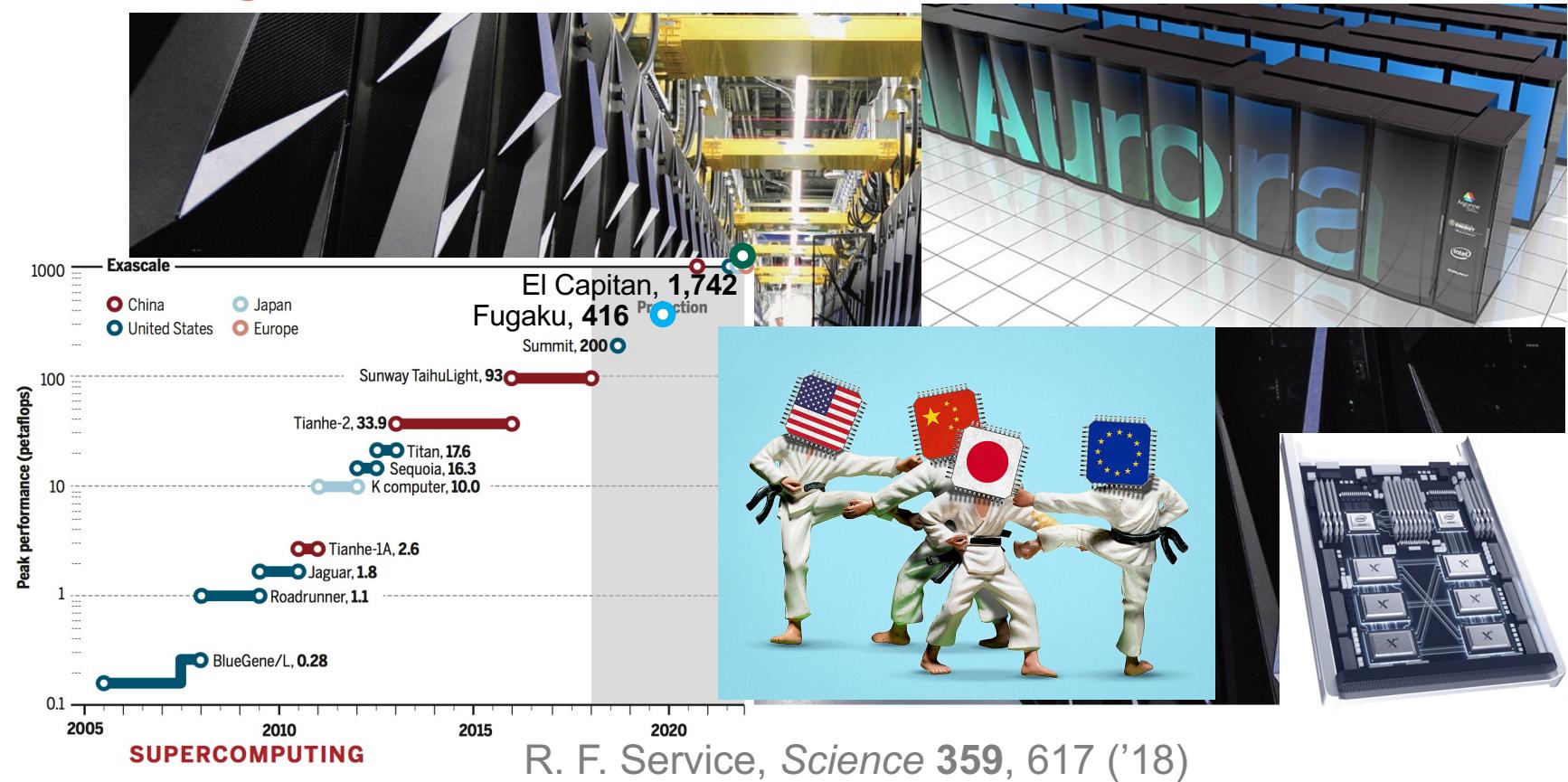


1.01 exaflop/s
Intel Aurora

exaflop/s = 10^{18} mathematical operations per second

- One of the initial simulation users of the next-generation DOE supercomputer

CACS@Aurora in the Global Exascale Race



Design for U.S. exascale computer takes shape

Competition with China accelerates plans for next great leap in supercomputing power

Exa(peta)flop/s = 10^{18} (10^{15}) floating-point operations per second

By Robert F. Service

In 1957, the launch of the Sputnik satellite vaulted the Soviet Union to the lead in the space race and galvanized the United States. U.S. supercomputer researchers are today facing their own

Lemont, Illinois. That's 2 years earlier than planned. "It's a pretty exciting time," says Aiichiro Nakano, a physicist at the University of Southern California in Los Angeles who uses supercomputers to model materials made by layering stacks of atomic sheets like graphene.

pace reflects a change of strategy by DOE officials last fall. Initially, the agency set up a "two lanes" approach to overcoming the challenges of an exascale machine, in particular a potentially ravenous appetite for electricity that could require the output of a small nuclear plant.

<https://www.tomshardware.com/news/two-chinese-exascale-supercomputers>

MS in Quantum Information Science

- New MS degree in Quantum Information Science (MSQIS) started in 2021
- Required foundational courses
 1. EE 520: Introduction to Quantum Information Processing
 2. EE 514: Quantum Error Correction
 3. Phys 513: Applications of Quantum Computing
- Core—at least two courses from
 1. EE 589: Quantum Information Theory
 2. Phys 550: Open Quantum Systems
 3. Phys 559: Quantum Devices
 4. Phys 660: Quantum Information Science & Many-Body Physics
- Phys 513: Application of Quantum Computing (co-taught with Prof. Rosa Di Felice)—quantum simulations on quantum circuits & adiabatic quantum annealer (syllabus)
- Phys 516 (this course): Elective for MSQIS

NSF CyberTraining

CyberMAGICS - Cyber Training on Materials Genome Innovation for Computational Software (2021-2025)

A. Nakano, K. Nomura, P. Vashishta (*University of Southern California*)
P. Dev, T. Wei (*Howard University*)

AI and Quantum-Computing Enabled Exa-scale Materials Simulator

