

Quantum Computing

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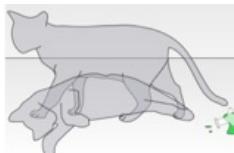
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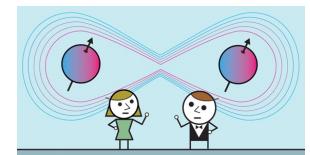
Goal: Quantum dynamics simulation on quantum circuits



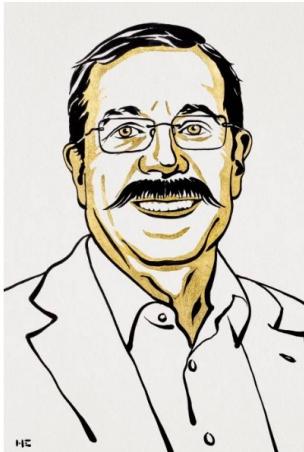
It's Timely: 2022 Nobel Physics Prize



Quantum computing utilizes quantum properties such as superposition & entanglement for computation



The Nobel Prize in Physics 2022



III. Niklas Elmehed © Nobel Prize Outreach

Alain Aspect

Prize share: 1/3



III. Niklas Elmehed © Nobel Prize Outreach

John F. Clauser

Prize share: 1/3

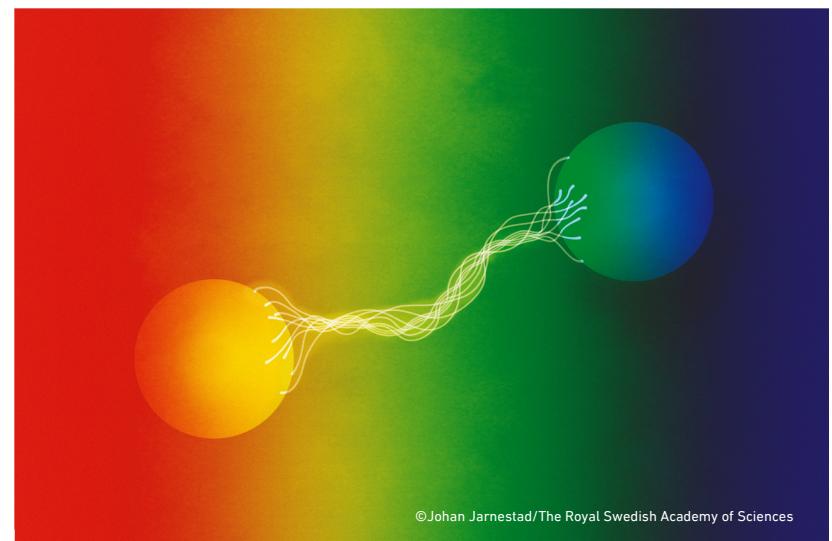


III. Niklas Elmehed © Nobel Prize Outreach

Anton Zeilinger

Prize share: 1/3

It's entanglement!

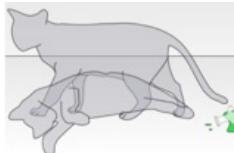


© Johan Jarnestad/The Royal Swedish Academy of Sciences

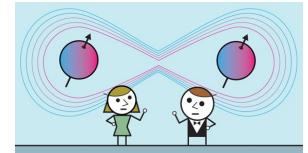
The Nobel Prize in Physics 2022 was awarded jointly to Alain Aspect, John F. Clauser and Anton Zeilinger "for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science"

cf. Microsoft Majorana 1 chip (Feb. 19, '25)
<https://news.microsoft.com/azure-quantum/>

Quantum Computing (QC) for Science



Quantum computing utilizes quantum properties such as superposition & entanglement for computation



- U.S. Congress (Dec. 21, '18) signed National Quantum Initiative Act to ensure leadership in quantum computing & its applications

- Quantum supremacy demonstrated by Google

F. Arute, *Nature* **574**, 505 ('19)

- Quantum computing for science:
Universal simulator of quantum many-body systems

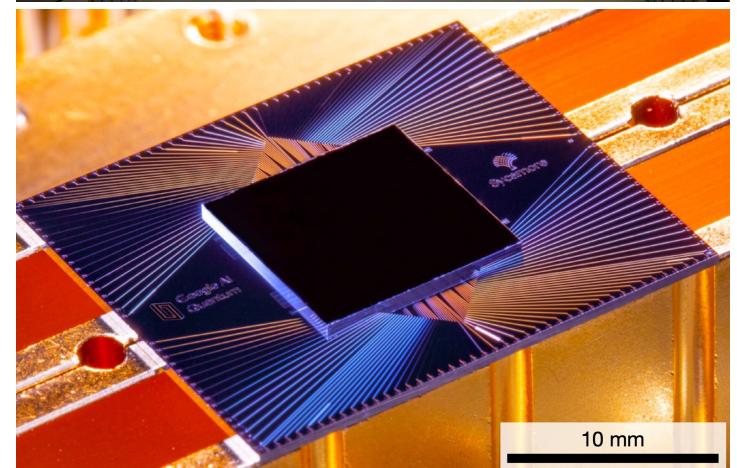
R. P. Feynman, *Int. J. Theo. Phys.* **21**, 467 ('82);
S. Lloyd, *Science* **273**, 1073 ('96)

- Success in simulating *static* properties of quantum systems (*i.e.*, ground-state energy of small molecules)

A. Aspuru-Guzik *et al.*, *Science* **309**, 1704 ('05)

- Challenge: Simulate quantum many-body *dynamics* on current-to-near-future noisy intermediate-scale quantum (NISQ) computers

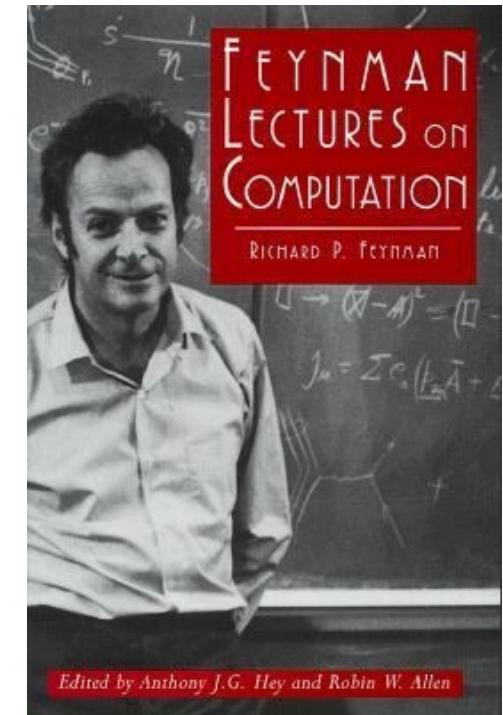
J. Preskill, *Quantum* **2**, 79 ('18)



54-qubit Google Sycamore

Quantum Dynamics Simulations

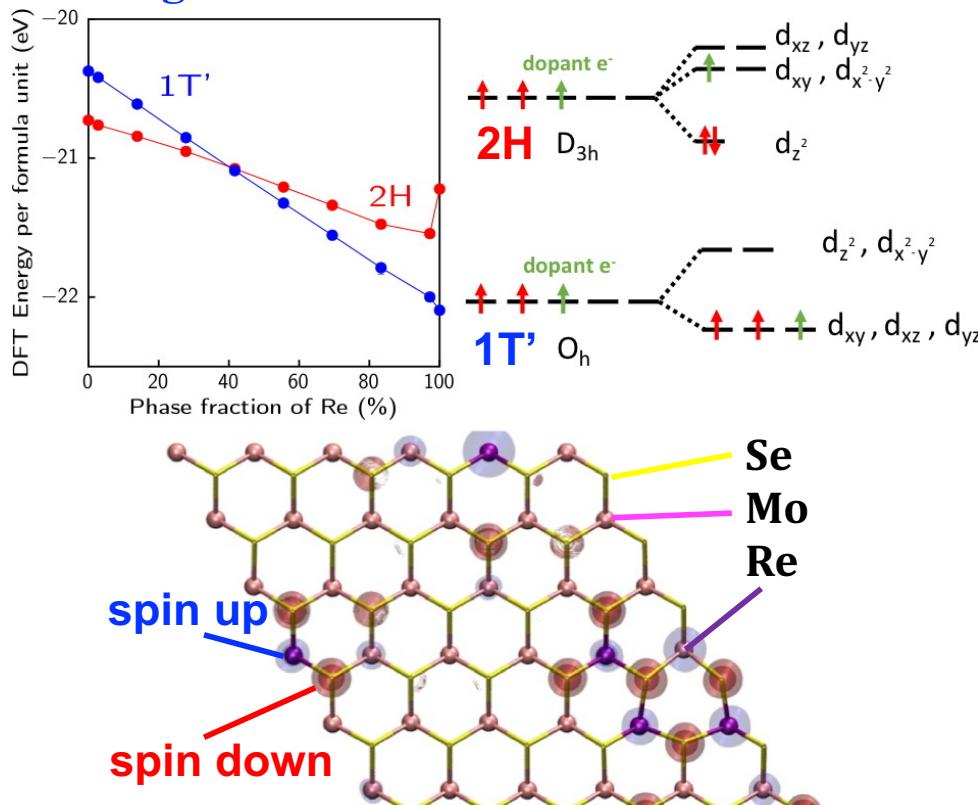
- An exciting scientific application of quantum computers is as a universal simulator of quantum many-body dynamics, as envisioned by Richard Feynman [*Int. J. Theor. Phys.* **21**, 467 ('82)]
- Seth Lloyd provided concrete algorithms and analysis [*Science* **273**, 1073 ('96)]
- Second edition of *Feynman Lectures on Computation* will add a section on “Simulating quantum dynamics” by John Preskill [*arXiv:2106.10522* ('21)]
- Simulated nontrivial quantum dynamics on publicly available IBM’s Q16 Melbourne & Rigetti’s Aspen NISQ computers, *i.e.*, ultrafast control of emergent magnetism by THz radiation in 2D material [*L. Bassman et al., Phys. Rev. B* **101**, 184305 ('20)]



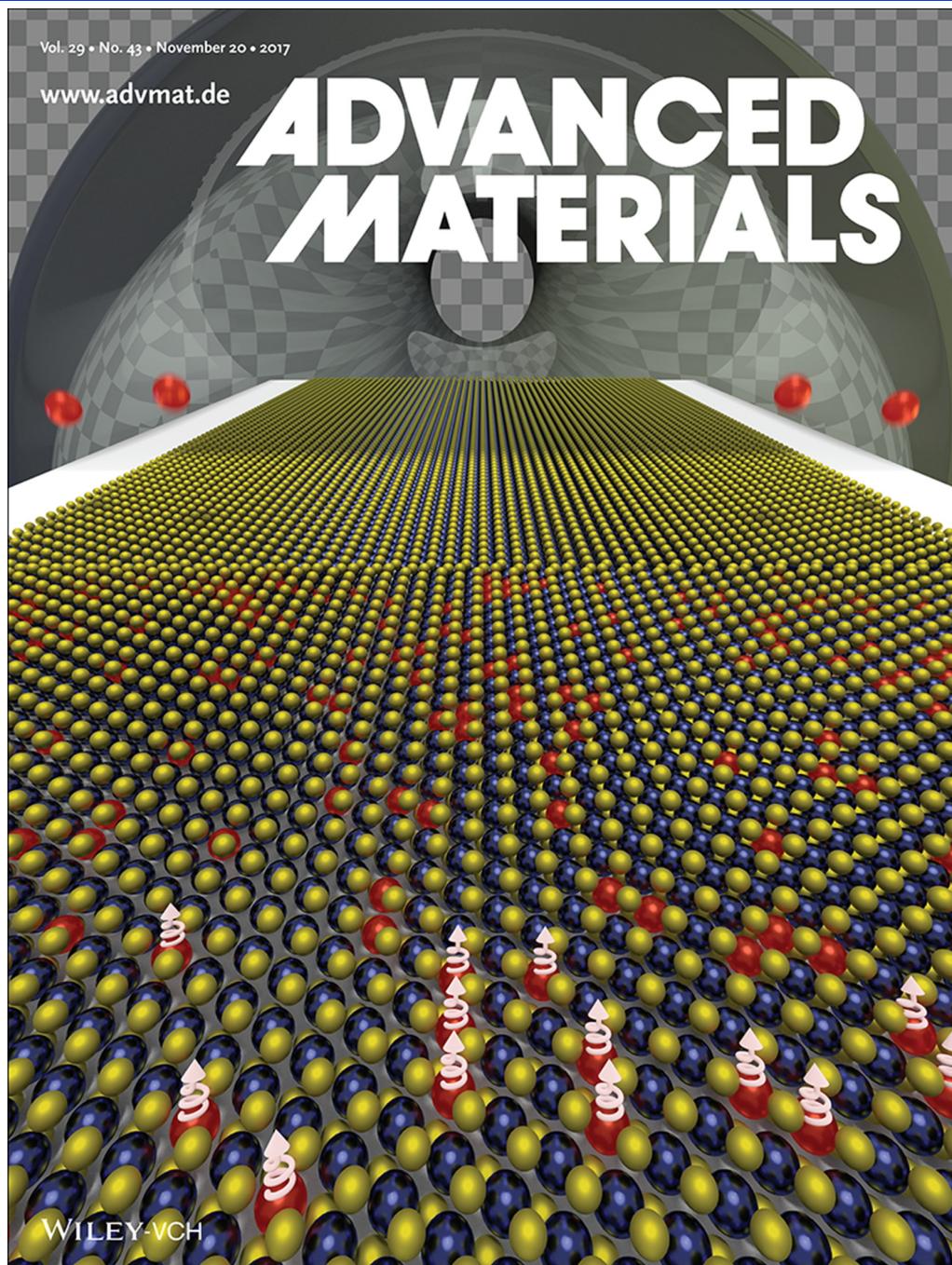
Do it yourself at <https://quantum-computing.ibm.com>

Emergent Magnetism: Structural Transition via Doping

- Experiment at Rice showed 2H-to-1T' phase transformation by alloying MoSe₂ with Re
- Simulations at USC elucidated its electronic origin
- Simulation & experiment showed novel magnetism centered at Re atoms



V. Kochat et al., Adv. Mater. 29, 1703754 ('17)



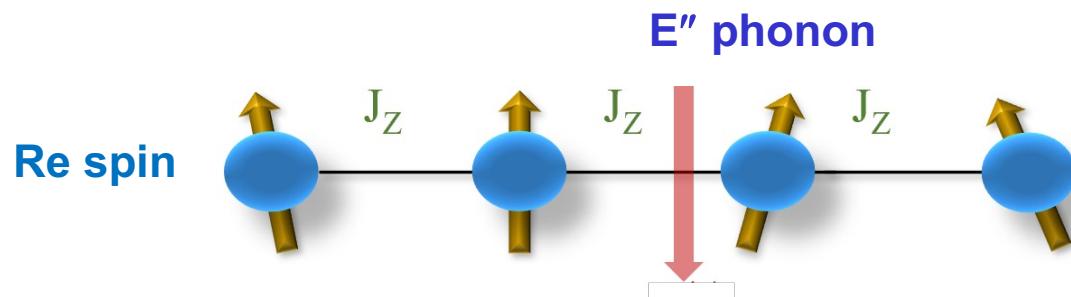
Transverse Field Ising Model

- Electromagnetic-field control of quantum states in a chain of rhenium-magnets in MoSe_2 monolayer to realize desired material properties on demand, thereby pushing the envelope of “quantum materials science”

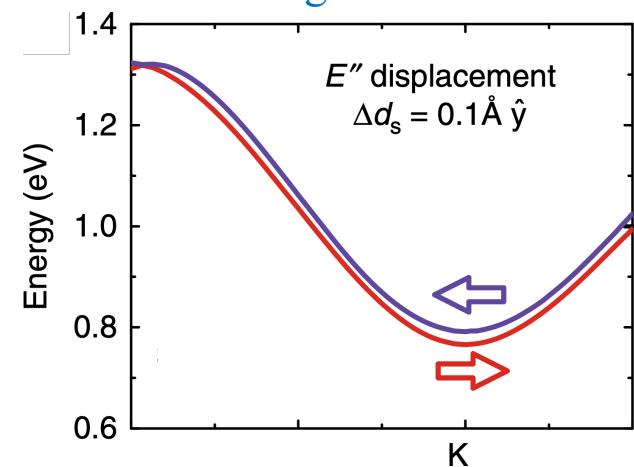
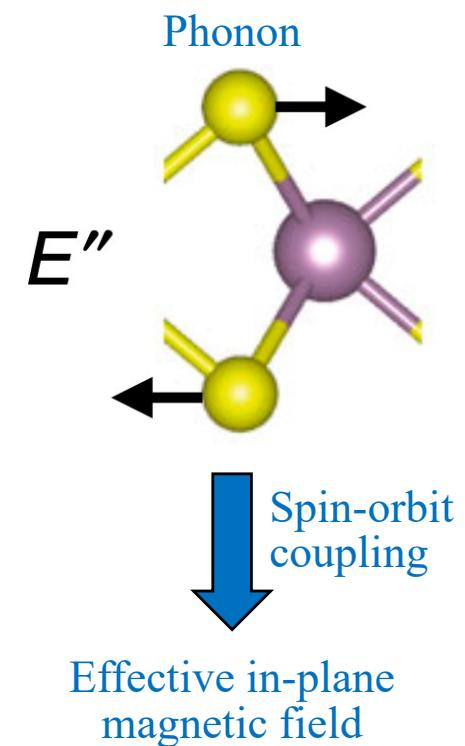
$$\begin{aligned}
 H(t) &= -J_z \sum_{j=1}^{N-1} \sigma_z^j \sigma_z^{j+1} - \varepsilon_{ph} \sin(\omega_{ph} t) \sum_{j=1}^N \sigma_x^j \\
 &= H_z + H_x(t)
 \end{aligned}$$

Phonon frequency
Phonon-induced energy split

$$\sigma_z^j = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}; \quad \sigma_x^j = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} // \text{Act on } j\text{-th qubit}$$



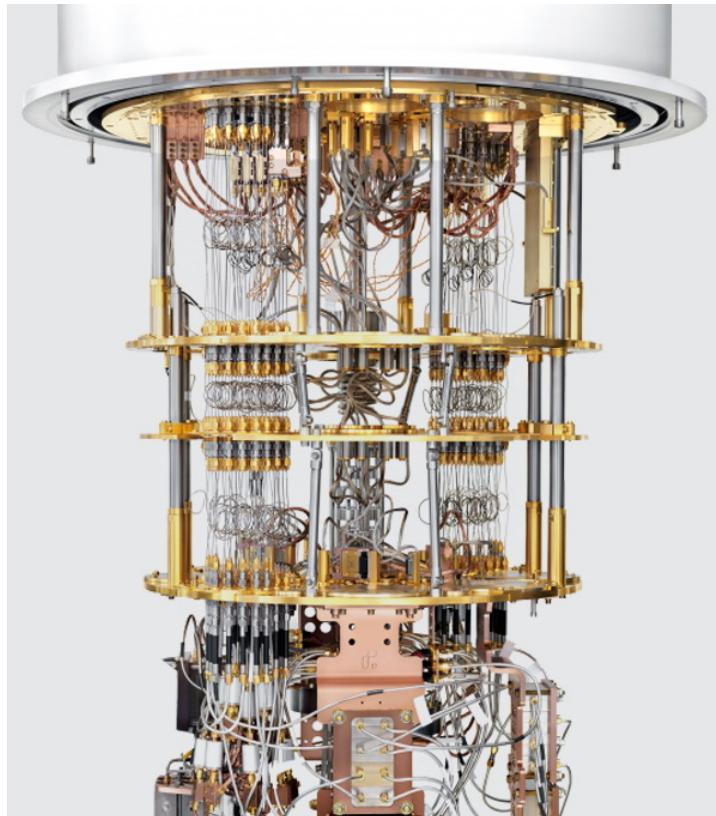
D. Shin et al., Nat. Commun. 9, 638 ('18)



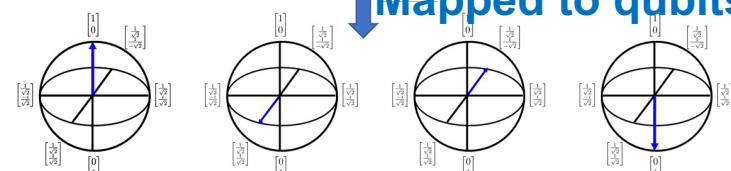
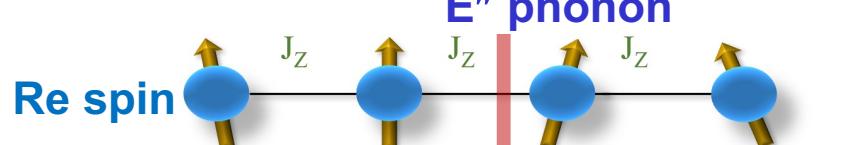
Quantum Computing of Magnetism

- Simulated quantum many-body dynamics on IBM's Q16 Melbourne & Rigetti's Aspen quantum processors

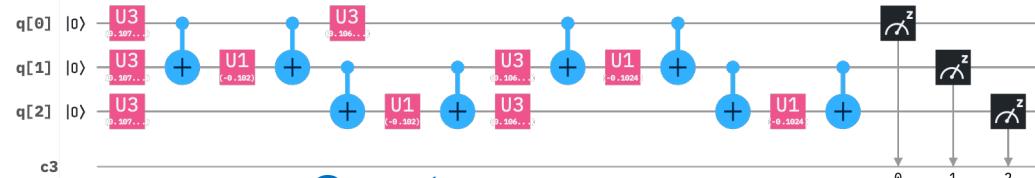
L. Bassman *et al.*, Phys. Rev. 101, 184305 ('20)



$$H(t) = -J_z \sum_{j=1}^{N-1} \sigma_z^j \sigma_z^{j+1} - \varepsilon_{ph} \sin(\omega_{ph} t) \sum_{j=1}^N \sigma_x^j$$



Quantum circuit: $U(\Delta t) = \exp(-iH\Delta t)$



Quantum program

```
32: ...#define the two non-commuting terms that comprise the Hamiltonian-
33: ...Hz = PauliTerm("Z", 0, epsilon_0)-
34: ...Hy = PauliTerm("Y", 0, epsilon_ph*np.sin(w_ph*t))-#
35: ...#exponentiate the terms of the Hamiltonian for use in Trotter approx-
36: ...exp_Hz = exponential_map(Hz)(delta_t/(2.0*hbar))-#
37: ...exp_Hy = exponential_map(Hy)(delta_t/hbar)-
```

Will derive & implement the circuit in the hands-on session

Quantum Dynamics on NISQ Computers

- Time-evolution operator for wave function $|\Psi(t)\rangle$ for small time interval Δt (atomic unit, $\hbar = 1$)

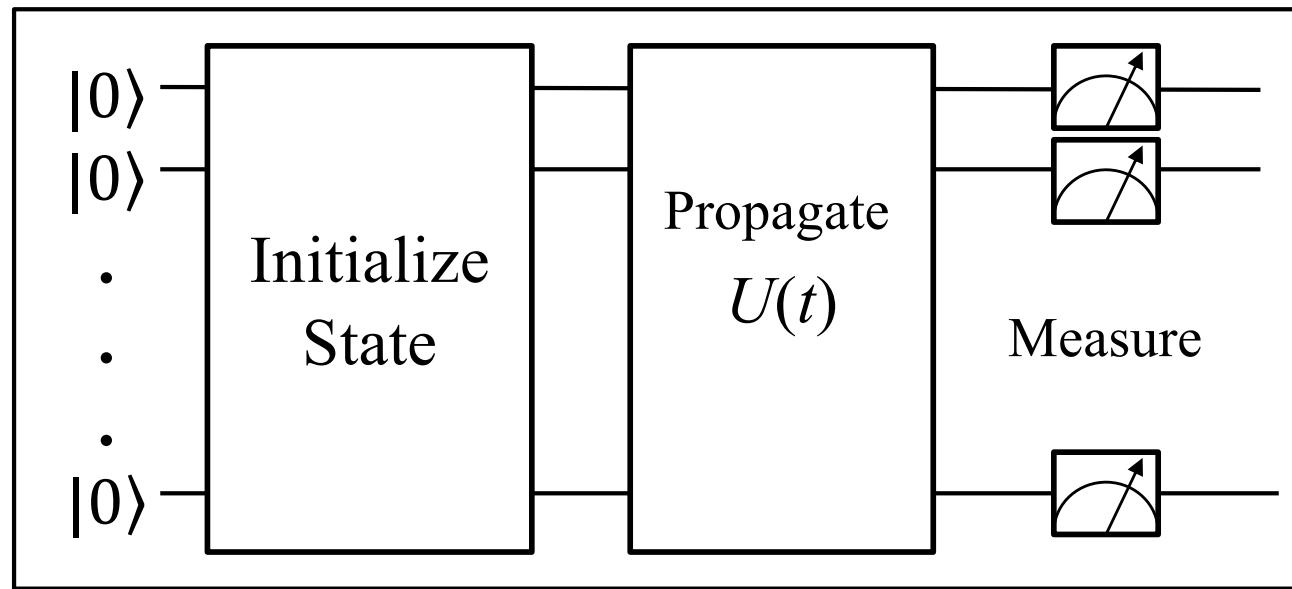
$$|\Psi(\Delta t)\rangle = U(\Delta t)|\Psi(t=0)\rangle$$
$$U(\Delta t) = \exp(-iH\Delta t)$$

- Time discretization with time-step Δt and Trotter expansion

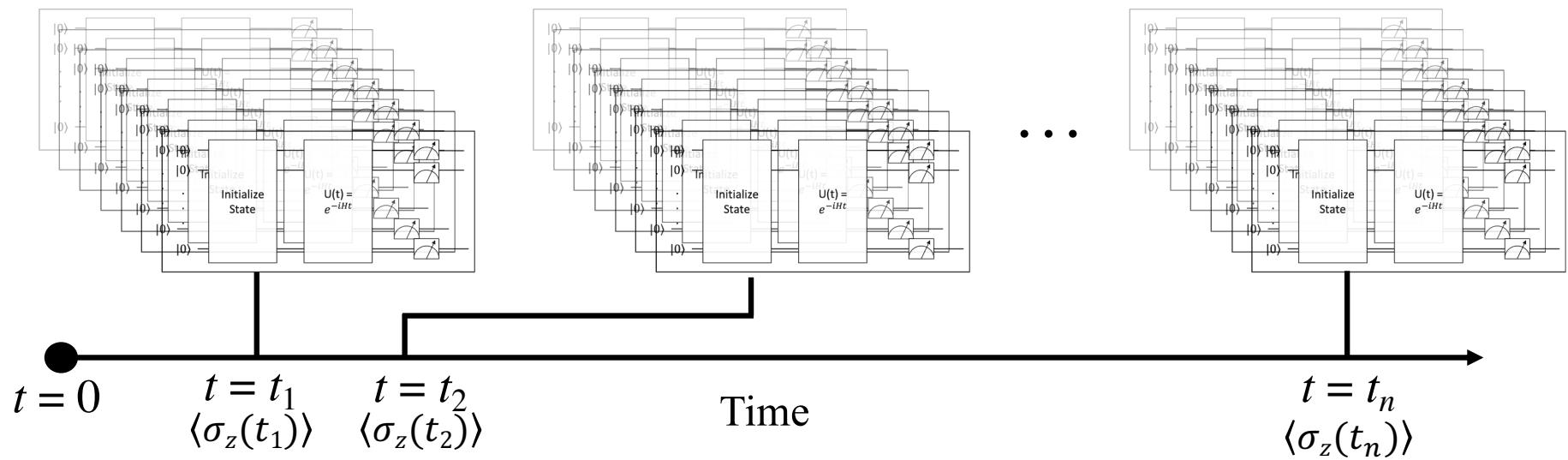
$$U(n\Delta t) \approx \prod_{k=0}^{n-1} \exp(-iH_z\Delta t) \exp(-iH_x((k + 1/2)\Delta t)\Delta t)$$

- One simulation run provides measurement for only one time instance ($t = n\Delta t$) — if you can see intermediate time steps, it's not quantum computing

Quantum Computing Runs

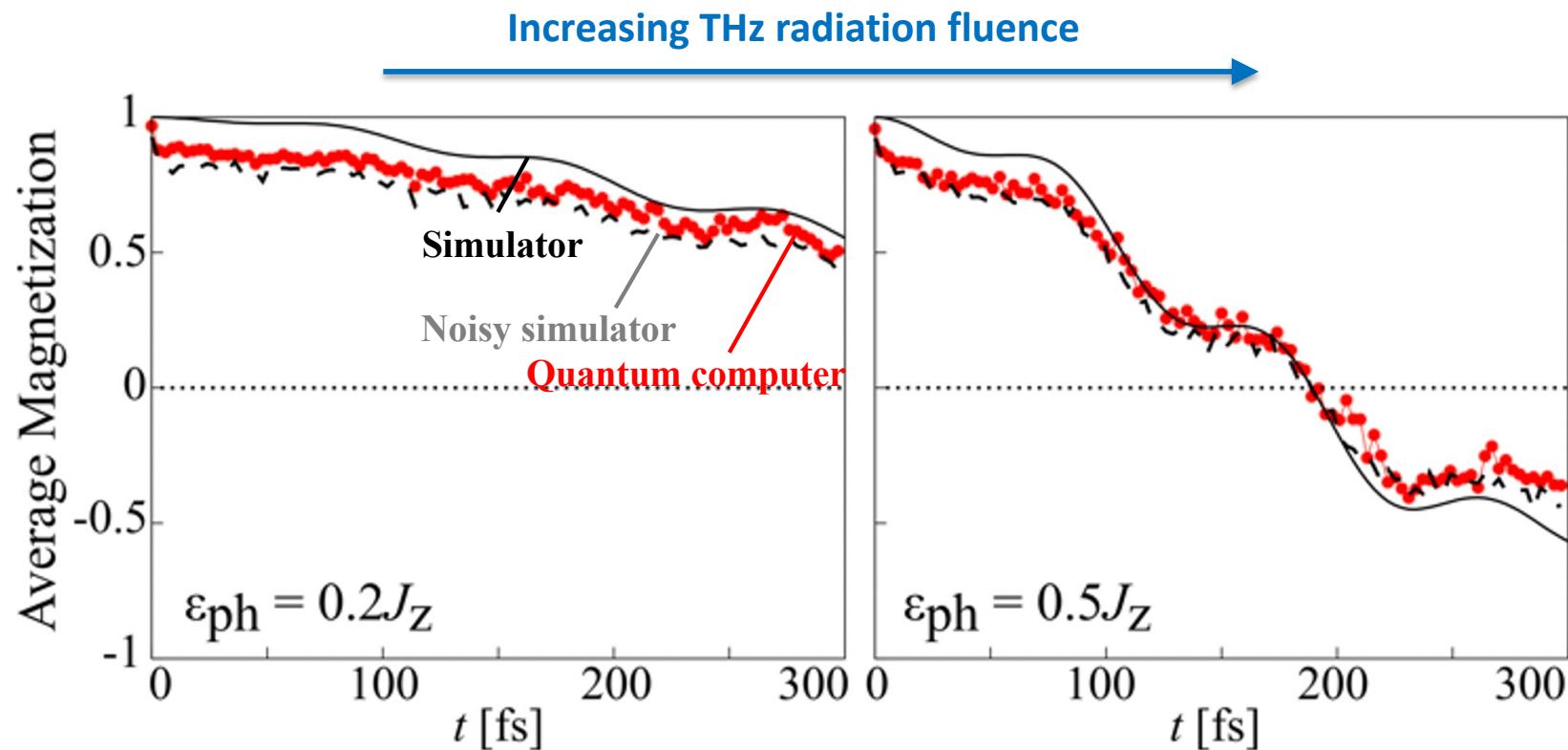


For each time instance, many runs to obtain statistics



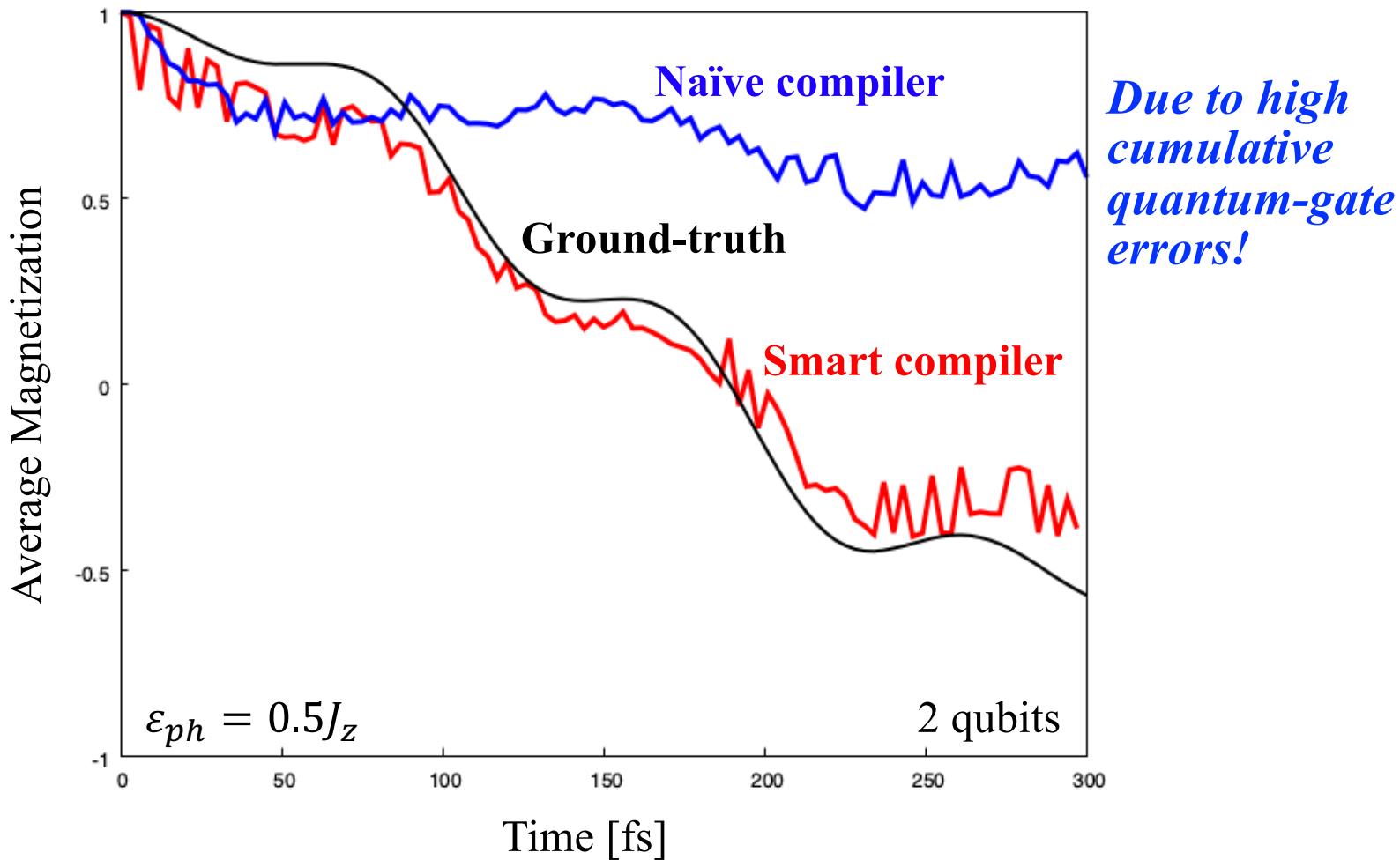
Quantum Computing Results

- Quantum-dynamics simulations on NISQ computers show dynamic suppression of magnetization by THz radiation



Circuit Size vs. Simulation Fidelity

- Reduced circuit size improves the fidelity of simulation



Naïve compiler: circuit size \propto time

Smart compile: Constant circuit size w.r.t. time

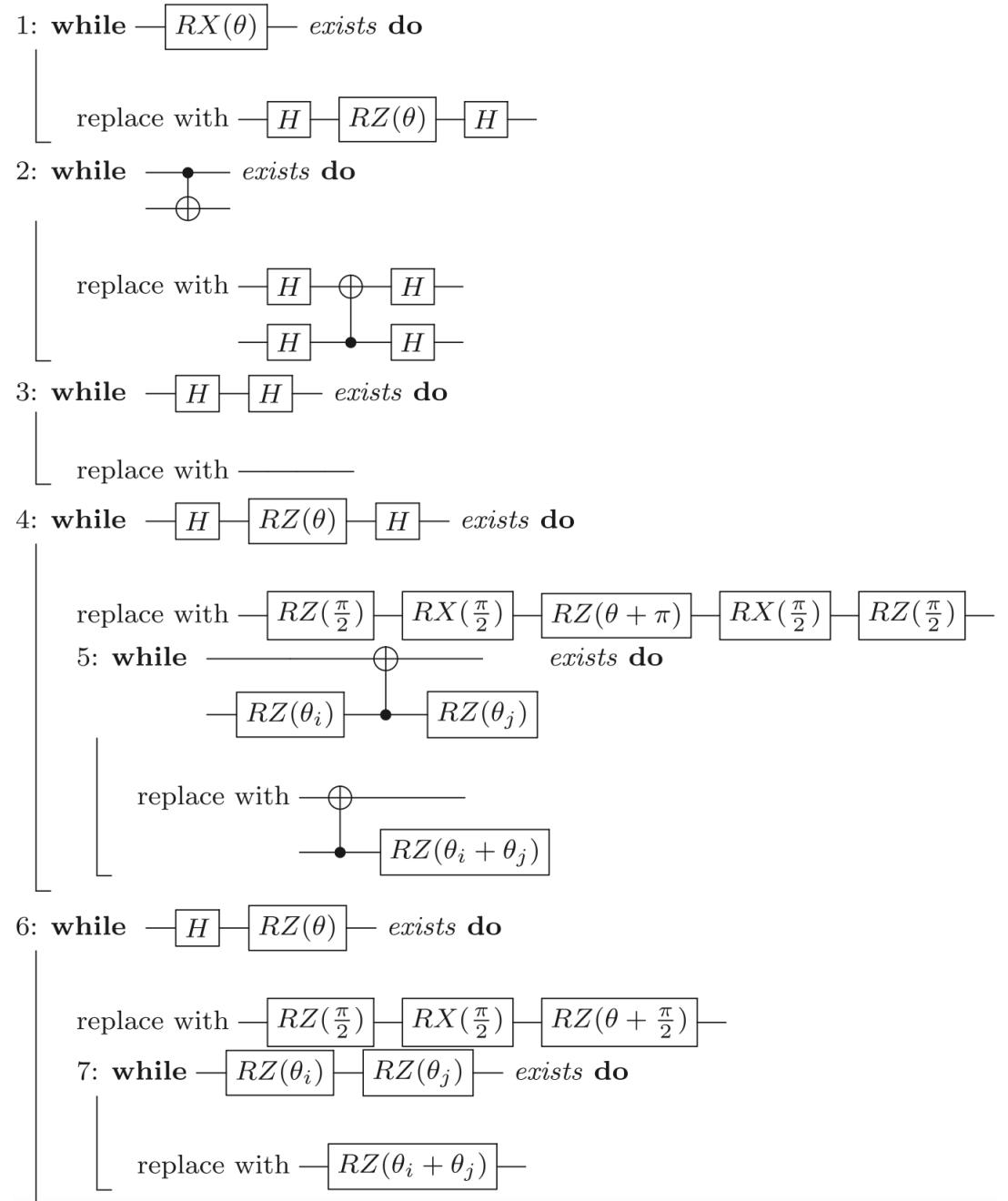
Quantum Compiler: Math

- **Problem:** High gate errors make long-time simulations impractical
- **Solution:** Domain-specific compiler = use algebraic identities to derive an equivalent circuit with reduced circuit size

No.	Common gate set in TFIM circuits	Equivalent
1	$\cdots RX(\theta) \cdots$	$\cdots H RZ(\theta) H \cdots$
2	$\cdots H \cdots$	$\cdots RZ(\frac{\pi}{2}) RX(\frac{\pi}{2}) RZ(\frac{\pi}{2}) \cdots$
3	$\cdots \bullet \oplus \cdots$	$\cdots Z \cdots$ $\cdots H \bullet H \cdots$
4	$\cdots \bullet \oplus \cdots$	$\cdots H \oplus H \cdots$ $\cdots H \bullet H \cdots$
5	$\cdots RX(\theta_i) RZ(\pi) \cdots$	$\cdots RZ(-\pi) RX(-\theta_i) \cdots$
6	$\cdots Z \cdots RX(\frac{\pi}{2}) RZ(\theta_i) RX(-\frac{\pi}{2}) \cdots RX(\frac{\pi}{2}) \cdots$	$\cdots RX(-\frac{\pi}{2}) Z RX(\frac{\pi}{2}) RZ(\theta_i) RX(-\frac{\pi}{2}) Z \cdots$
7	$\cdots H H \cdots$	—
8	$\cdots RX(\theta_i) RX(\theta_j) \cdots$	$\cdots RX(\theta_i + \theta_j) \cdots$
9	$\cdots RZ(\theta_i) RZ(\theta_j) \cdots$	$\cdots RZ(\theta_i + \theta_j) \cdots$
10	$\cdots RZ(\theta_i) \bullet RZ(\theta_j) \cdots$ $\cdots Z \cdots$	$\cdots \bullet RZ(\theta_i + \theta_j) \cdots$ $\cdots Z \cdots$
11	$\cdots RZ(\theta_i) \bullet RZ(\theta_j) \cdots$ $\cdots \oplus \cdots$	$\cdots \bullet RZ(\theta_i + \theta_j) \cdots$ $\cdots \oplus \cdots$

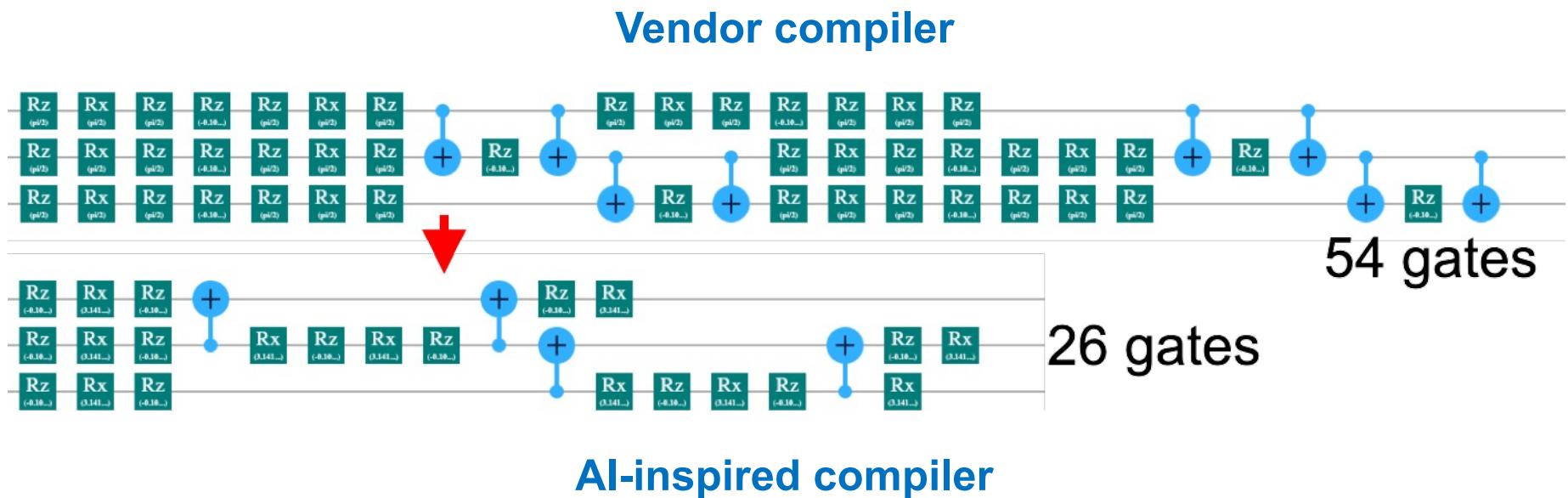
Algorithm for IBM Native Gates

- Heuristic algorithm similar to unit propagation in artificial intelligence (AI)
- The heuristic order & types of identities applied are specific to the particular quantum dynamics we simulated



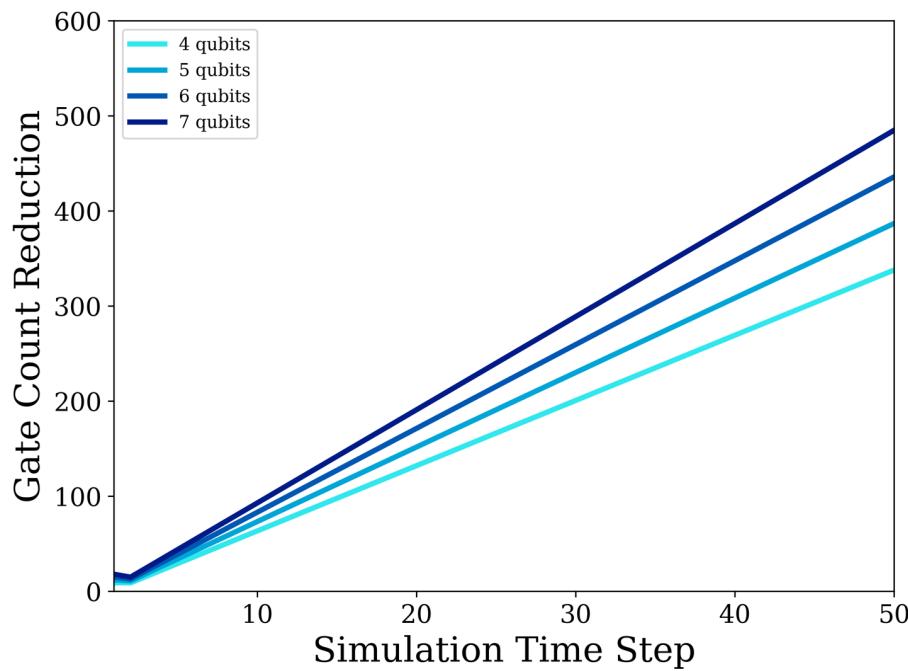
Domain-Specific Quantum Compiler

- Take advantage of specific problem structure
- AI-inspired quantum compiler reduced the circuit size by 30% to mitigate environmental noise

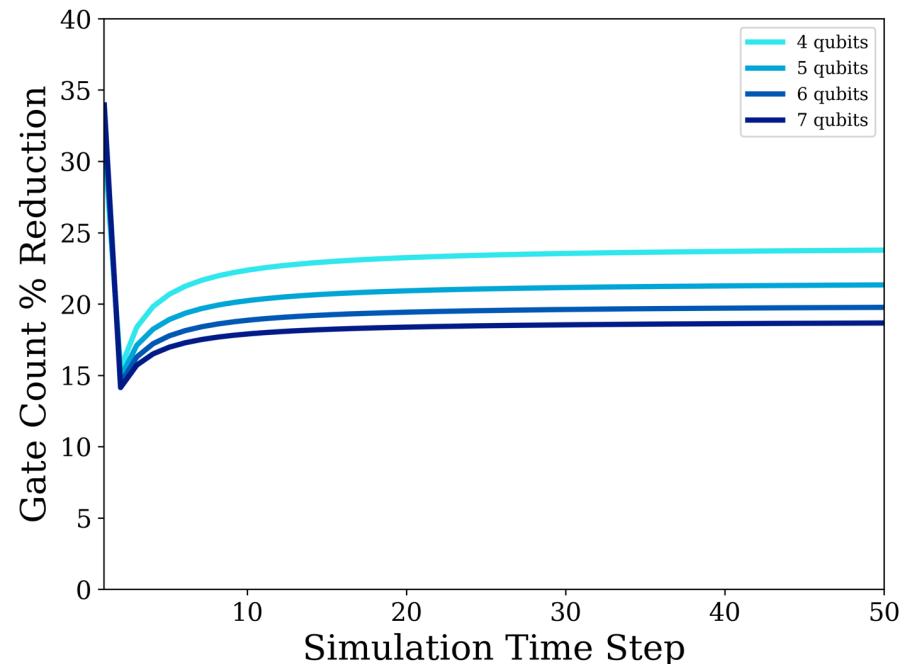


Performance of Domain-Specific Compiler

Absolute gate count difference
(IBM Compiler - DS Compiler)

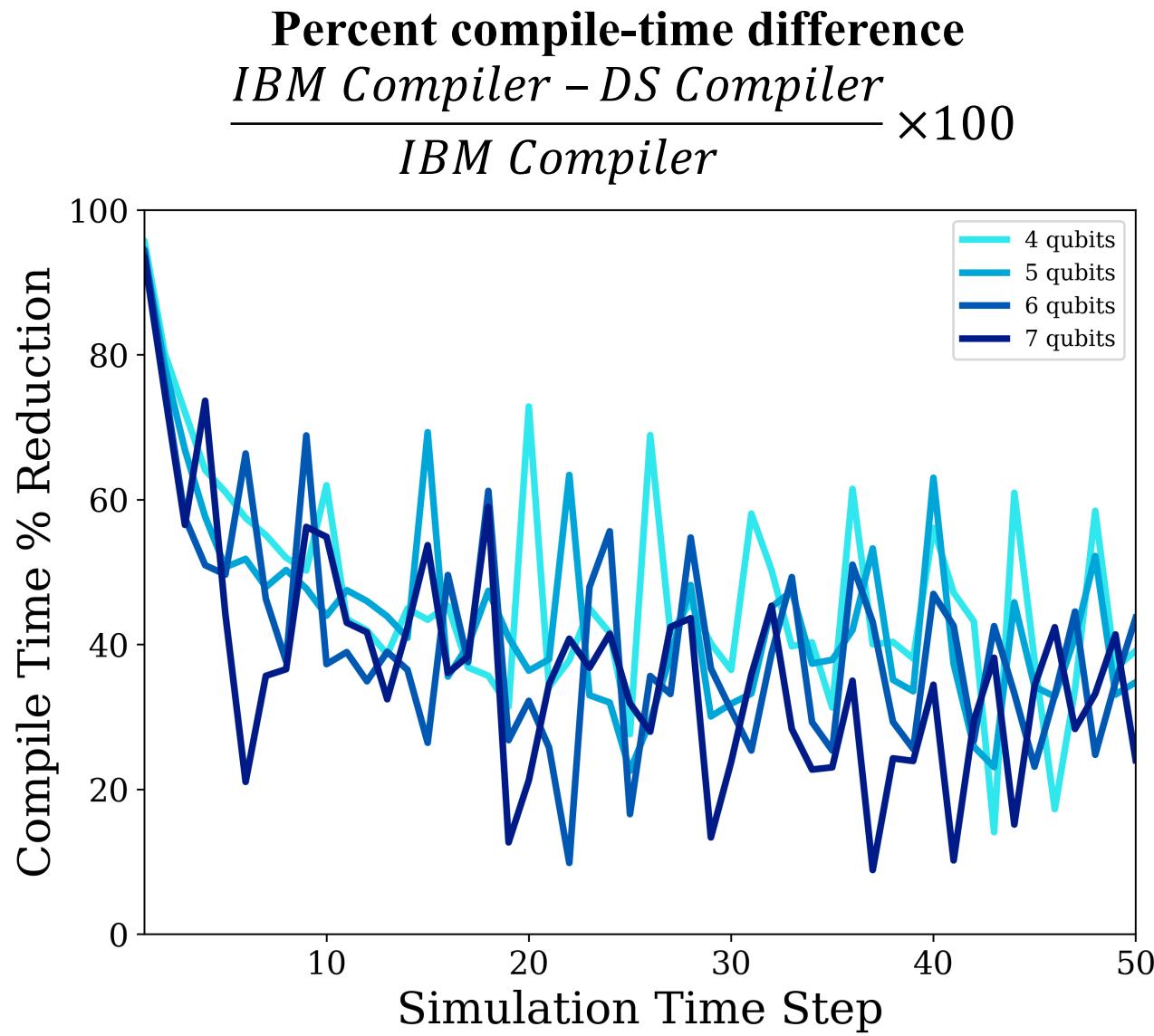


Percent gate count difference
$$\frac{IBM\ Compiler - DS\ Compiler}{IBM\ Compiler} \times 100$$



Domain-specific compiler reduces gate count compared to IBM compiler

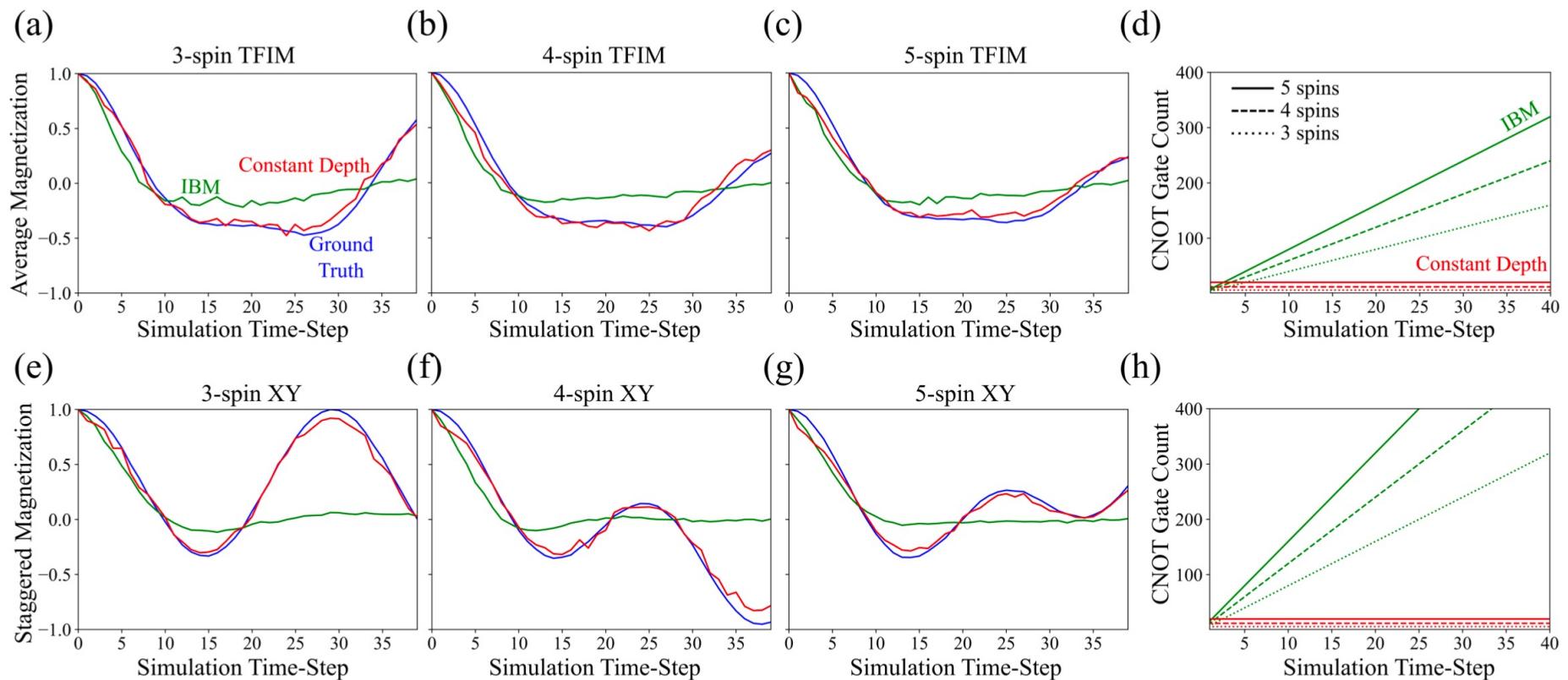
Speed of Domain-Specific Compiler



... and does it faster

Extension: Constant Circuit-Depth Algorithm

- Mathematical identities allow constant circuit depth independent of the number of time steps n for arbitrary number of spins N in a linear spin chain

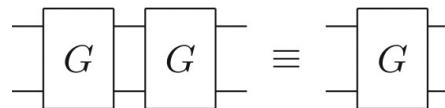


Exception for the “no-fast-forwarding theorem”

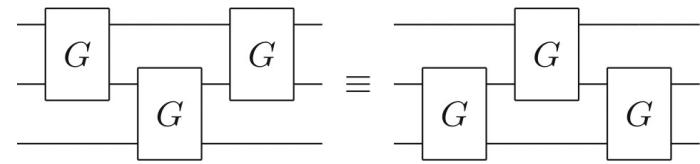
Constant Circuit-Depth: How It is Done

- Matchgate (shifted 2×2 block diagonal)

$$G(A, B) = \begin{bmatrix} p & q \\ w & x \\ y & z \\ r & s \end{bmatrix}$$



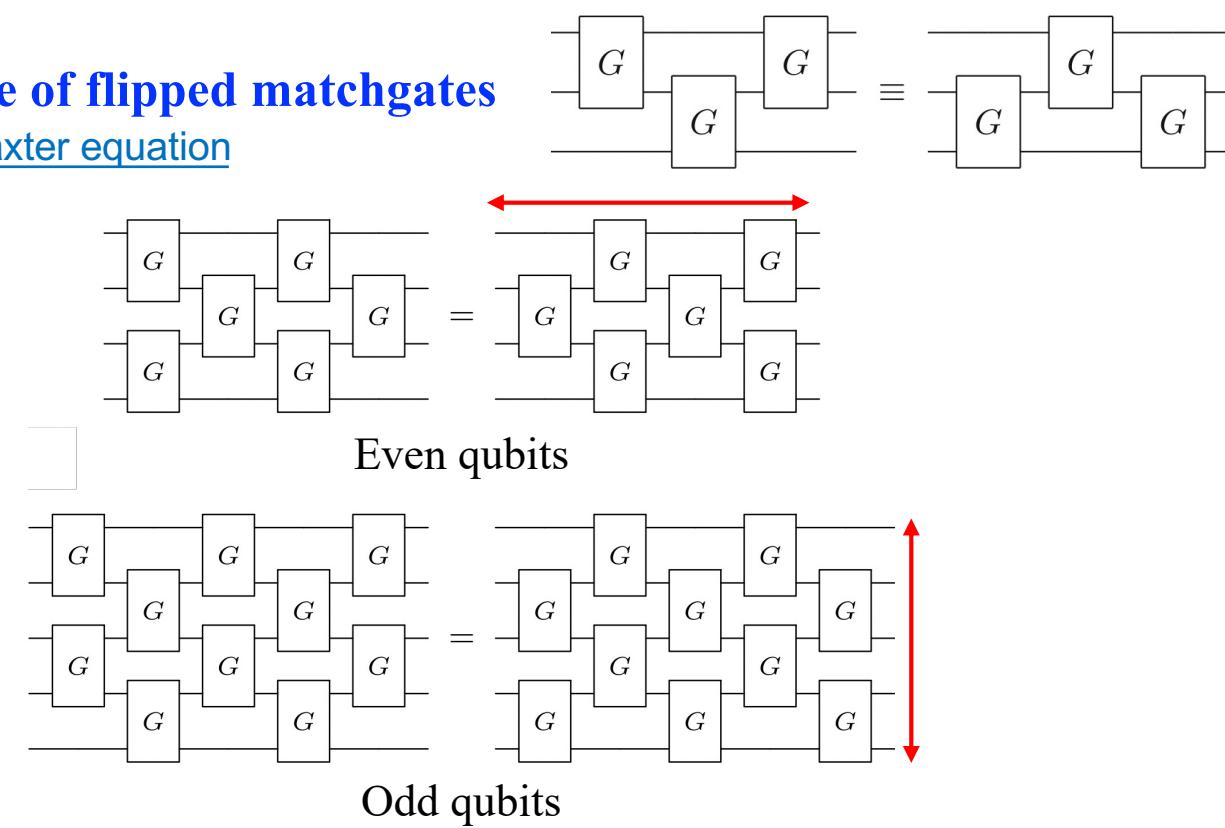
- Contraction: product of matchgates is a matchgates



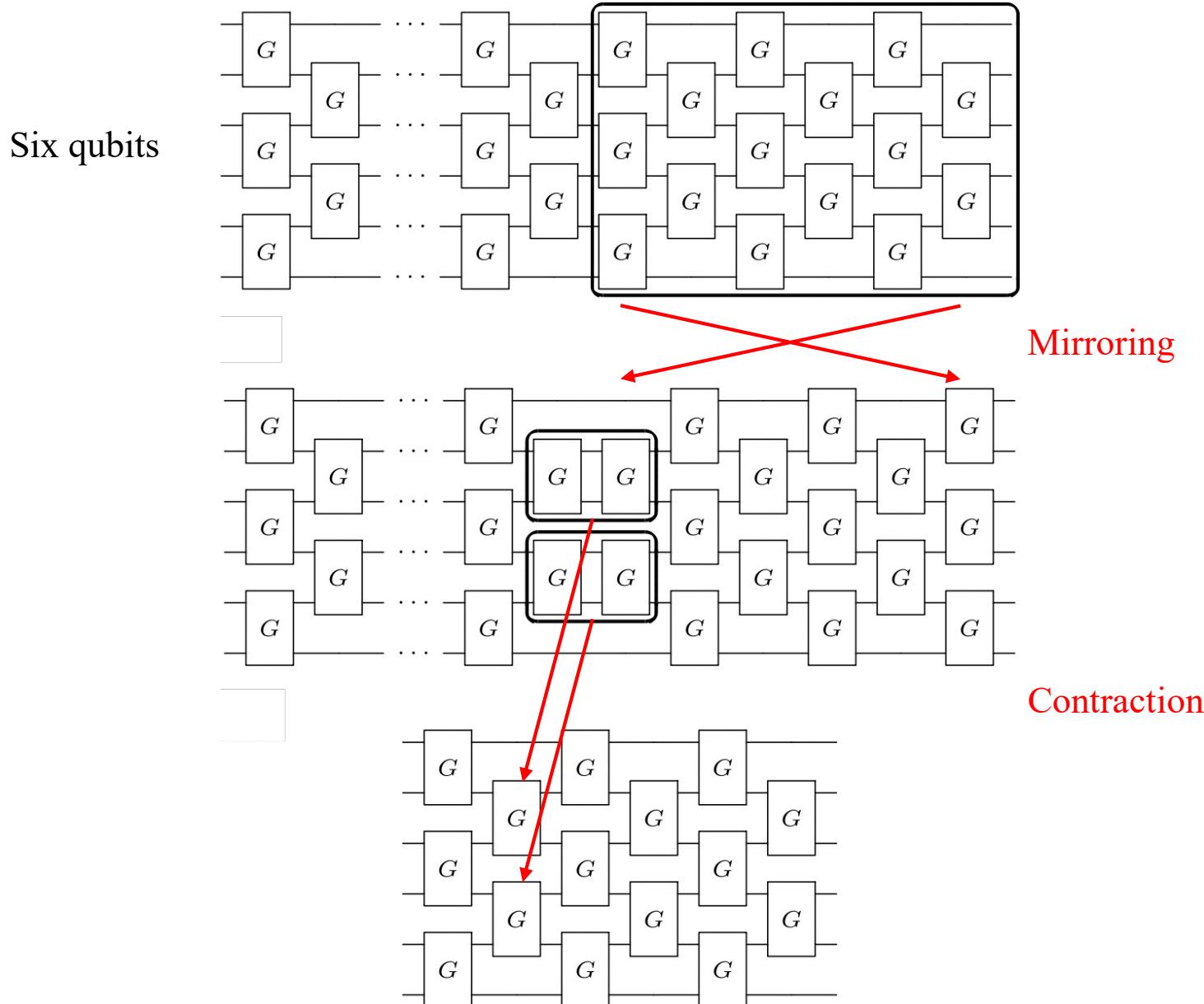
- Conjecture: existence of flipped matchgates

cf. Yang-Baxter equation

- Mirroring identities



Downfolding to Constant-Depth



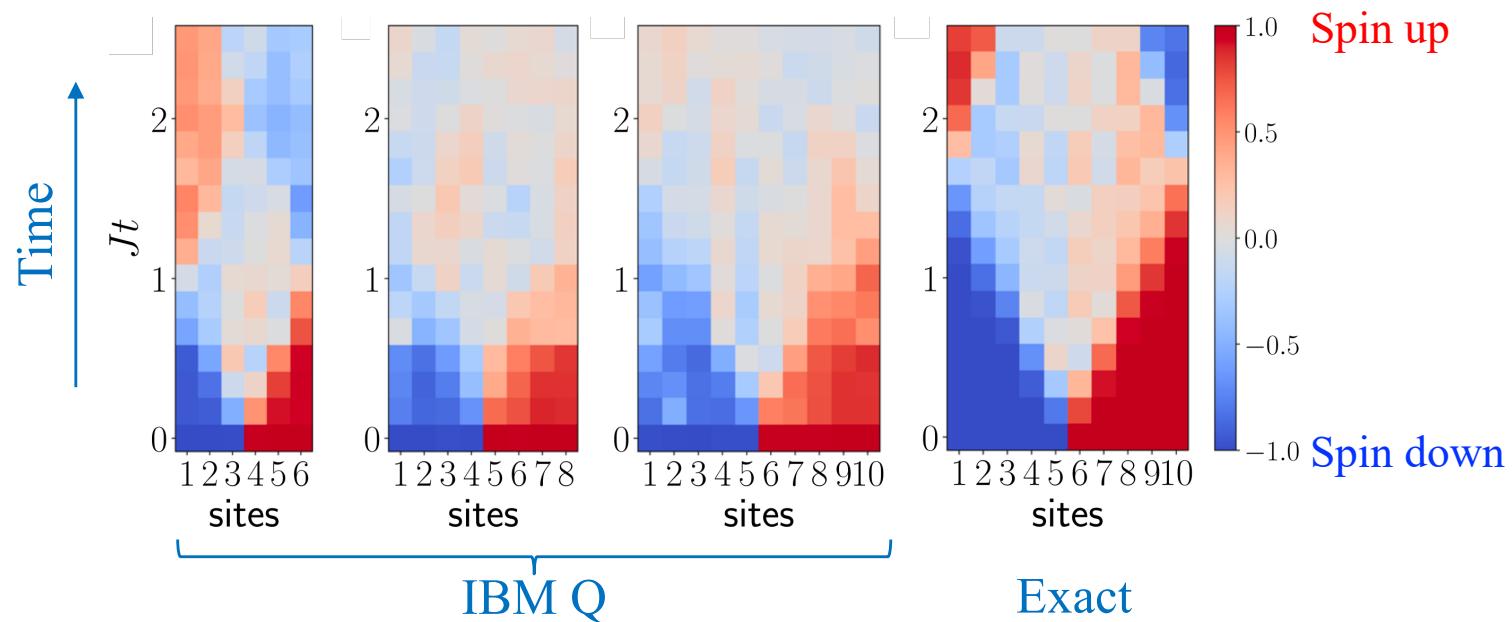
Richer Physics: Heisenberg Model

$$H = - \underbrace{\sum_{j=1}^{N-1} (J_x \sigma_x^j \sigma_x^{j+1} + J_y \sigma_y^j \sigma_y^{j+1} + J_z \sigma_z^j \sigma_z^{j+1})}_{\text{Exchange coupling}} - h \underbrace{\sum_{j=1}^N \sigma_z^j}_{\text{Magnetic field}}$$

Pauli spin-1/2 matrices

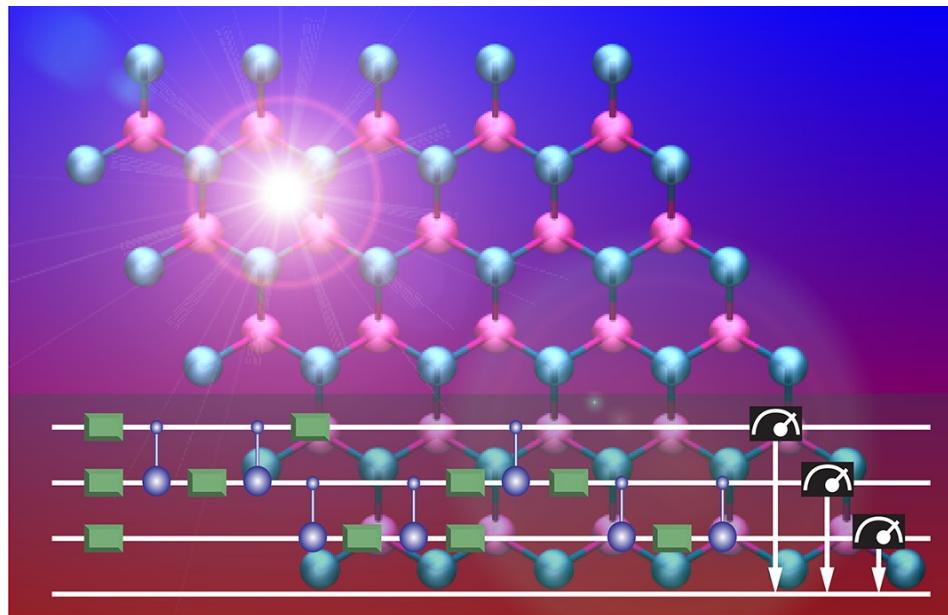
$$\sigma_x^j = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \sigma_y^j = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \sigma_z^j = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} // \text{Act on } j\text{-th qubit}$$

Domain-wall dynamics (6-, 8- & 10-site spin chains)



Open-Source Quantum Software

- Full-stack, cross-platform software for quantum dynamics simulations on NISQ computers was made available open-source



MISTIQS

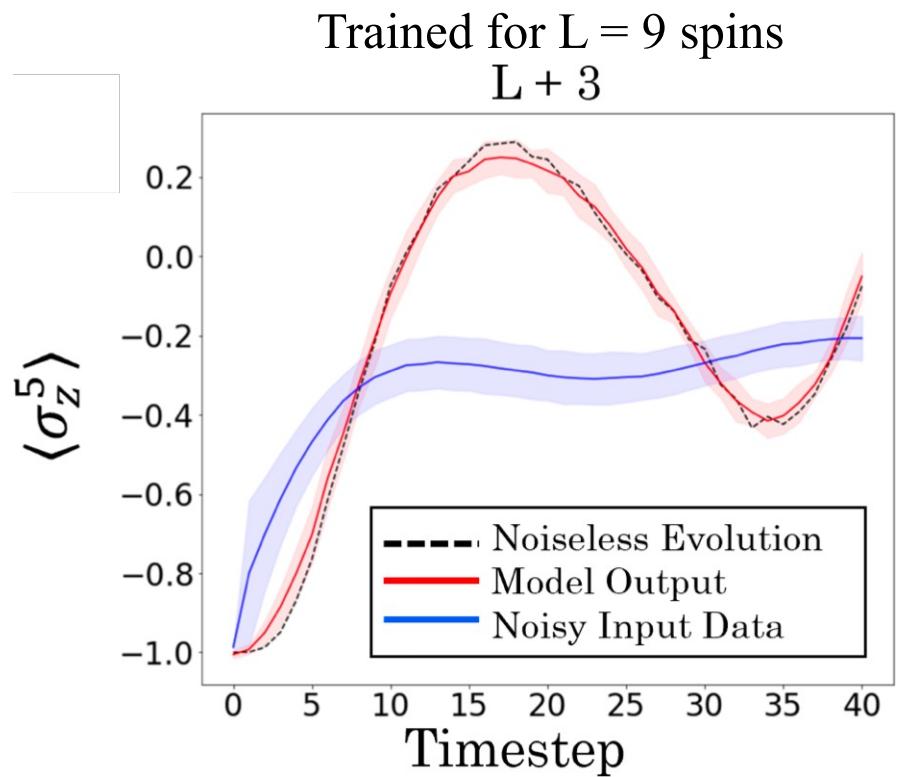
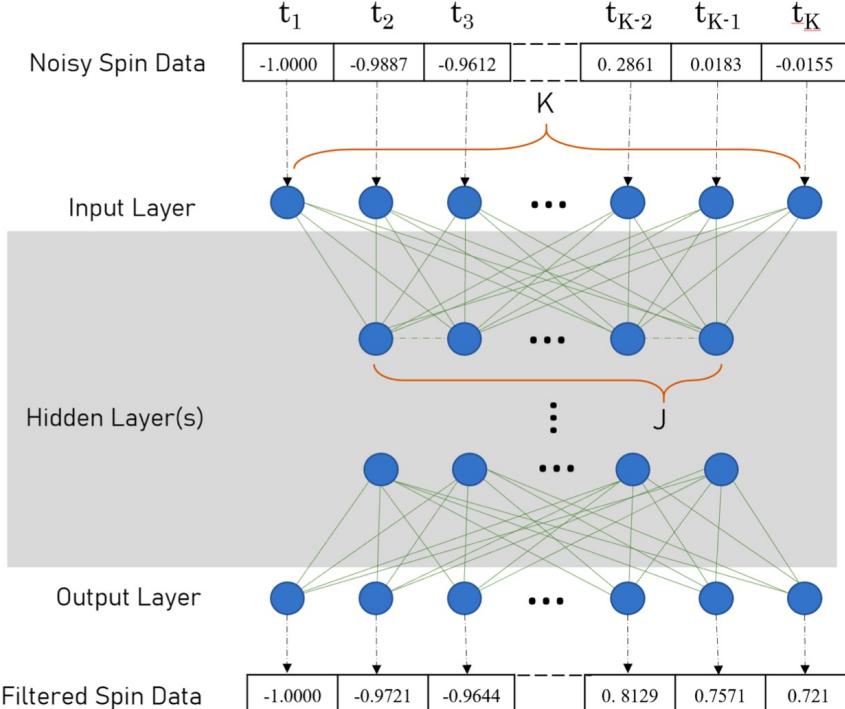
Multiplatform
Software for
Time-dependent
Quantum
Simulation

Paper: C. Powers et al., SoftwareX 14, 100696 ('21)

Software: <https://github.com/USCCACS/MISTIQS>

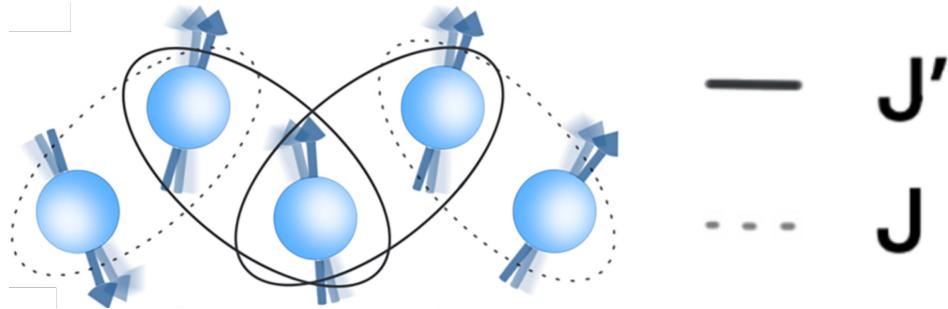
Extension: Machine Learning

- Alternative noise mitigation using machine learning:
Autoencoder, trained with quantum simulations of small systems, is capable of filtering noise from dynamic simulations of larger systems run on quantum computers

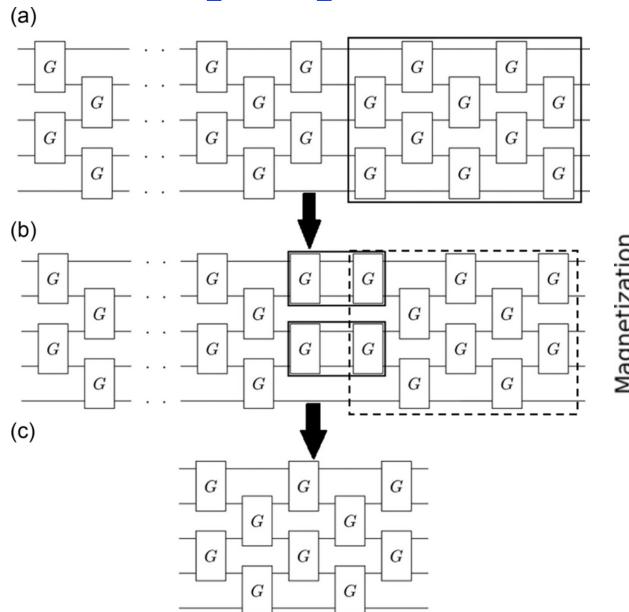


Topological Quantum Dynamics

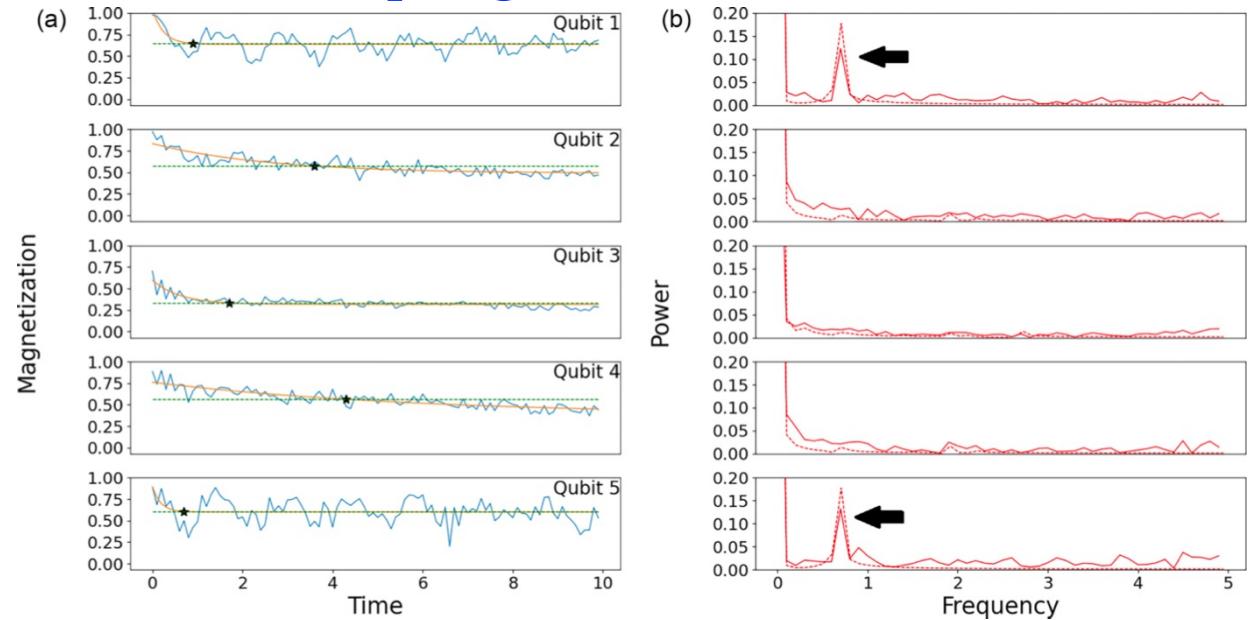
Quantum spin chain



Constant-depth quantum circuit



Topological surface mode



M. Mercado et al., *Phys. Rev. B* **110**, 075116 ('24)

New: Periodically-driven Floquet topology simulated

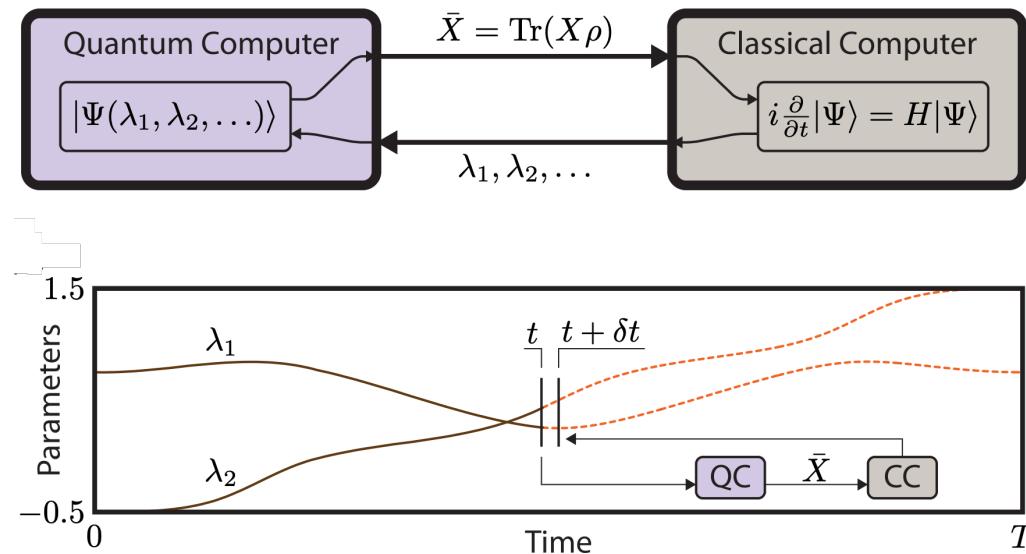
Variational Quantum Simulator

Hybrid quantum/classical approach: Boost the power of a classical supercomputer using a quantum co-processor

- A variational approach similar to variational quantum eigensolver (VQE) can be applied to quantum dynamics

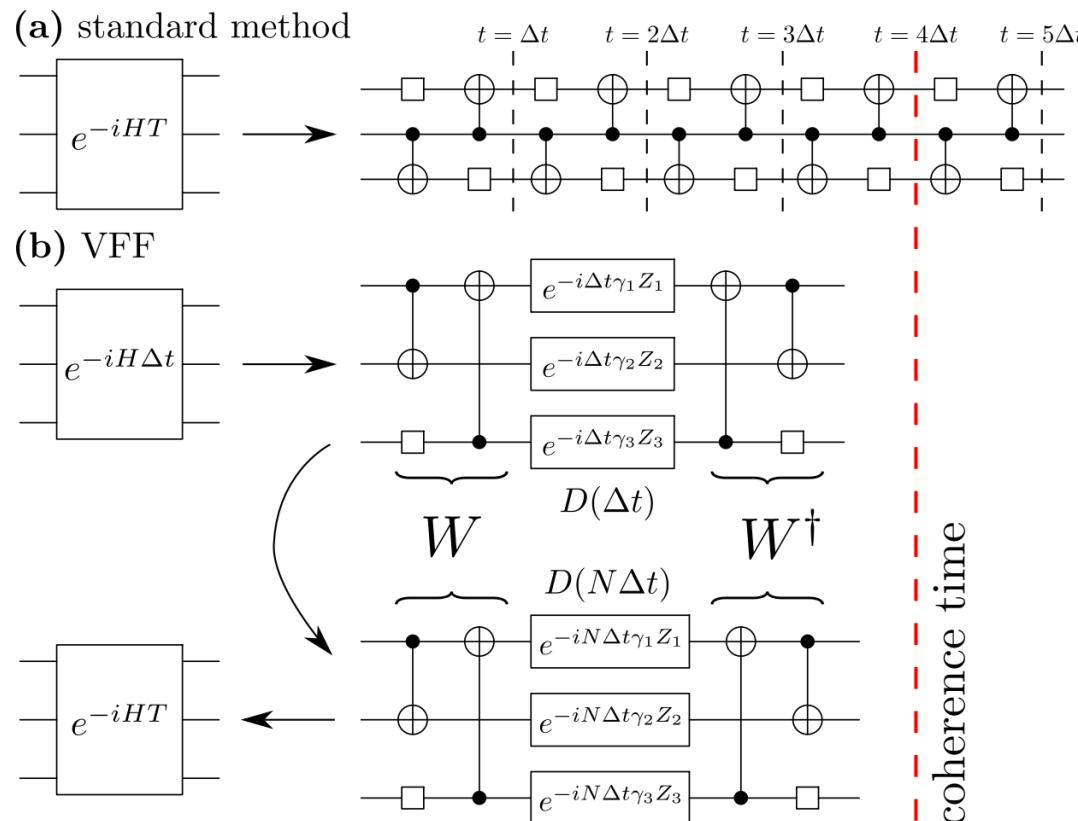
$$\delta \int_{t_i}^{t_f} dt \left\langle \psi(t) \right| \left(i \frac{\partial}{\partial t} - H \right) \left| \psi(t) \right\rangle = 0$$

- Short-time propagation of a many-body wave function on a quantum computer is mapped back to a parameterized variational wave function, $|\Psi(t)\rangle = |\Psi(\lambda_1(t), \dots, \lambda_p(t))\rangle$, which is tractable on a classical computer



Variational Fast Forwarding

- A variational approximation of the diagonalization of a Trotterized time evolution
- The optimal equivalent circuit with a fixed number of gates is obtained by quantum-assisted quantum compiling



C. Cirstoiu et al., *npjQI* 6, 82 ('20)

See B. Fauseweh, *Nat. Commun.* 15, 2133 ('24) for a review

Where to Go from Here

- New MS degree in Quantum Information Science ([MSQIS](#)) started in 2021
- Required foundational courses
 1. EE 520: Introduction to Quantum Information Processing
 2. EE 514: Quantum Error Correction
 3. Phys 513: Applications of Quantum Computing
- Core — at least two courses from
 1. EE 589: Quantum Information Theory
 2. Phys 550: Open Quantum Systems
 3. Phys 559: Quantum Devices
 4. Phys 660: Quantum Information Science & Many-Body Physics
- Phys 513: Application of Quantum Computing (co-taught with Prof. Rosa Di Felice) — quantum simulations on quantum circuits & adiabatic quantum annealer ([syllabus](#))
- Phys 516 (this course), CSCI 596, CSCI 653: Elective for MSQIS