

Parallel Quantum Molecular Dynamics

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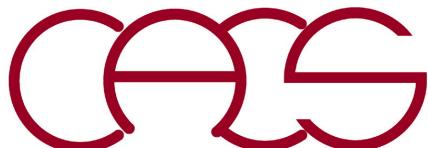
Email: anakano@usc.edu

For basic & advanced parallel computing, see:

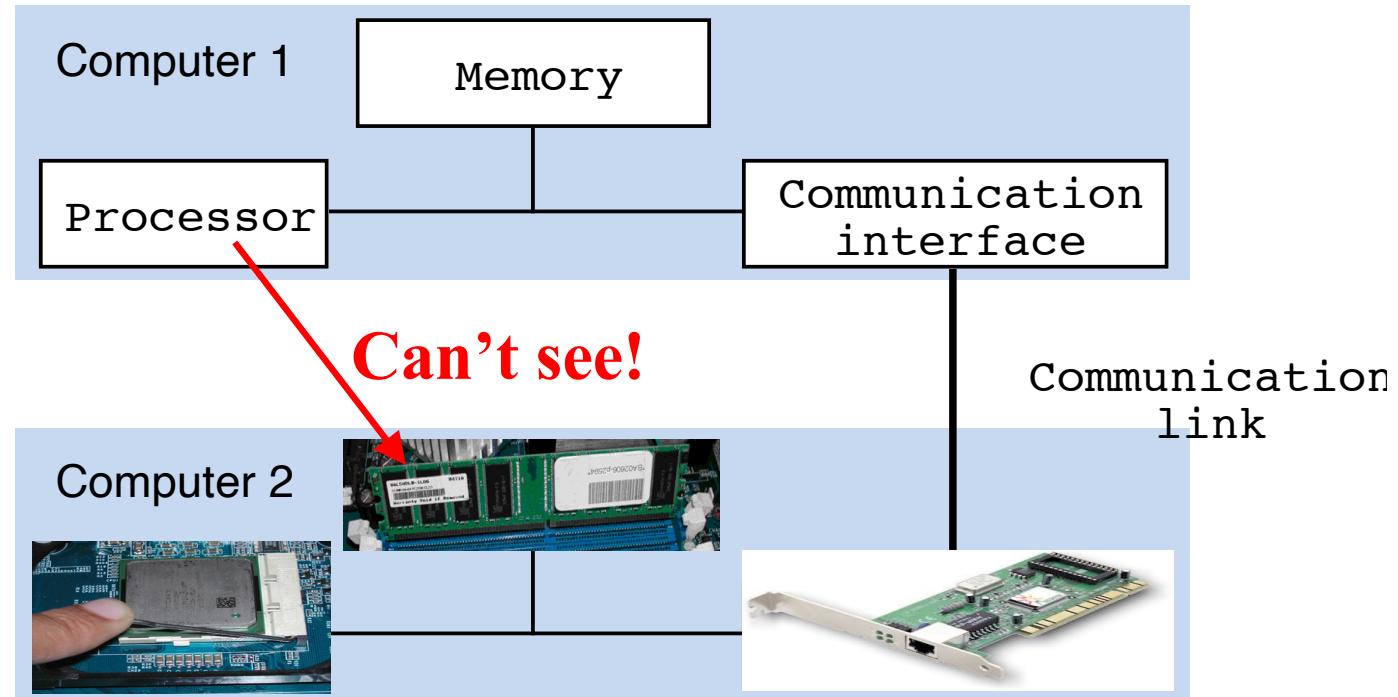
A. Grama *et al.*, [Introduction to Parallel Computing](#) (Addison-Wesley, '03)

[Extreme-scale-computing bootcamp at Argonne National Laboratory](#)

<https://aiichironakano.github.io/cs596.html>



Parallel Computing Hardware

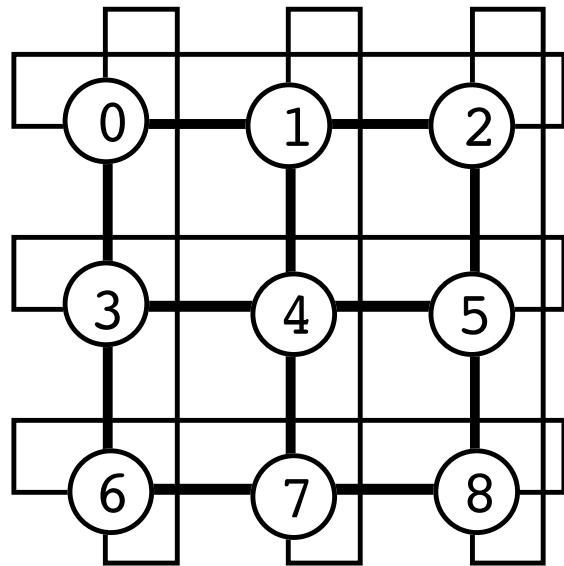


- **Processor:** Executes arithmetic & logic operations
- **Memory:** Stores program & data (stored program computer)
- **Communication interface:** Performs signal conversion & synchronization between communication link & a computer
- **Communication link:** A wire capable of carrying a sequence of bits as electrical (or optical) signals

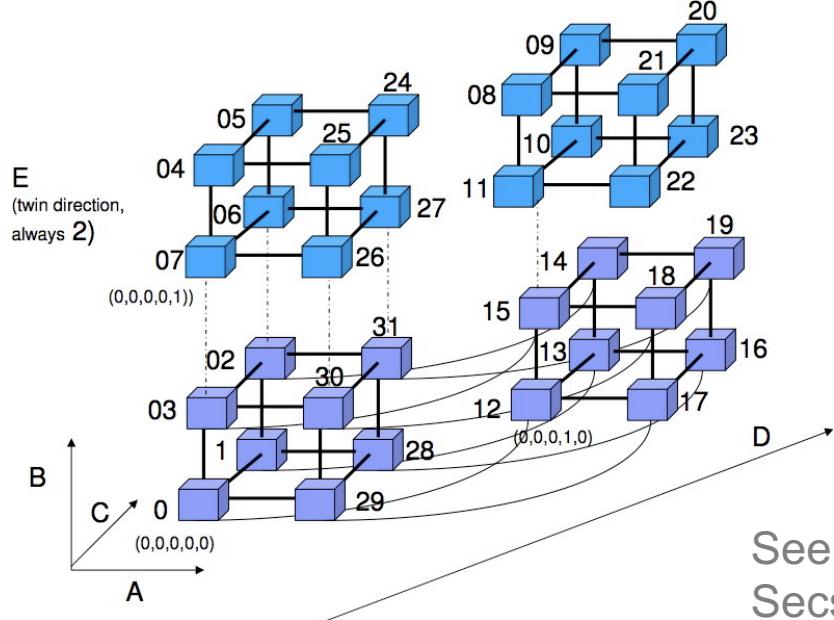
See <https://aiichironakano.github.io/cs596.html>

Communication Network

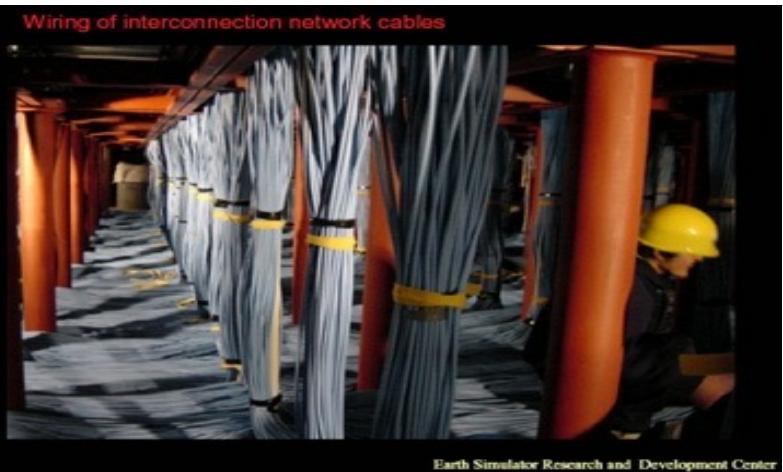
Mesh
(torus)



IBM Blue Gene/Q (5D torus)

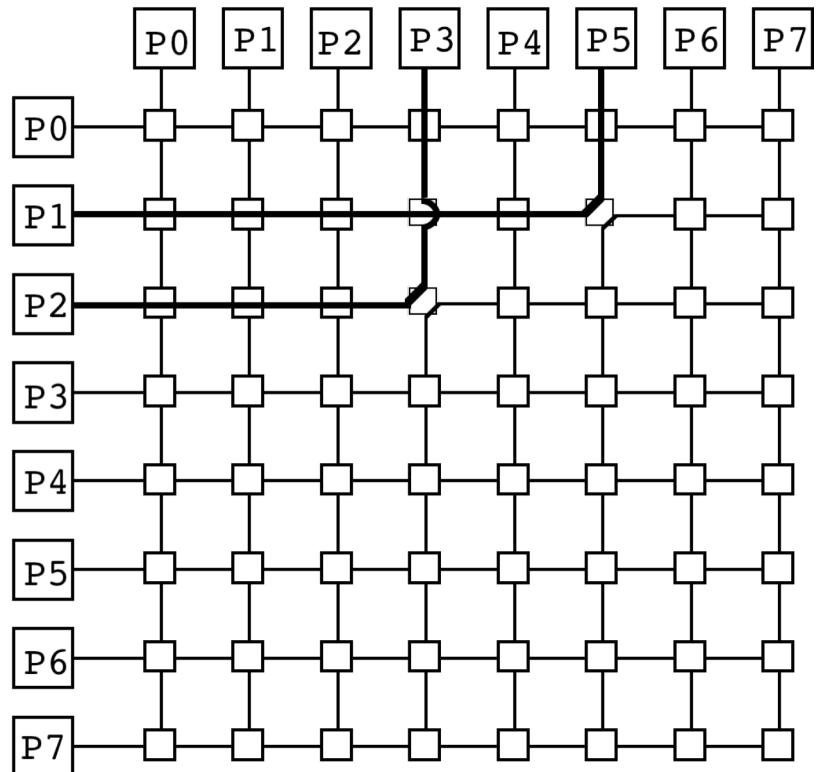


See [Gramma](#),
Secs. 2.4.2-2.4.4



NEC Earth Simulator (640x640 crossbar)

Crossbar
switch



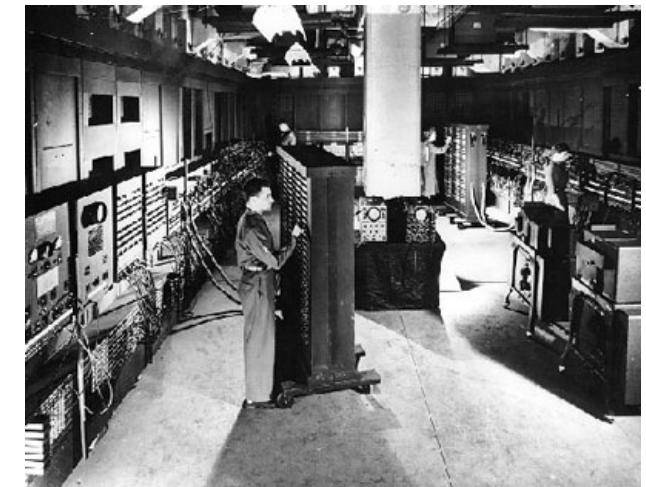
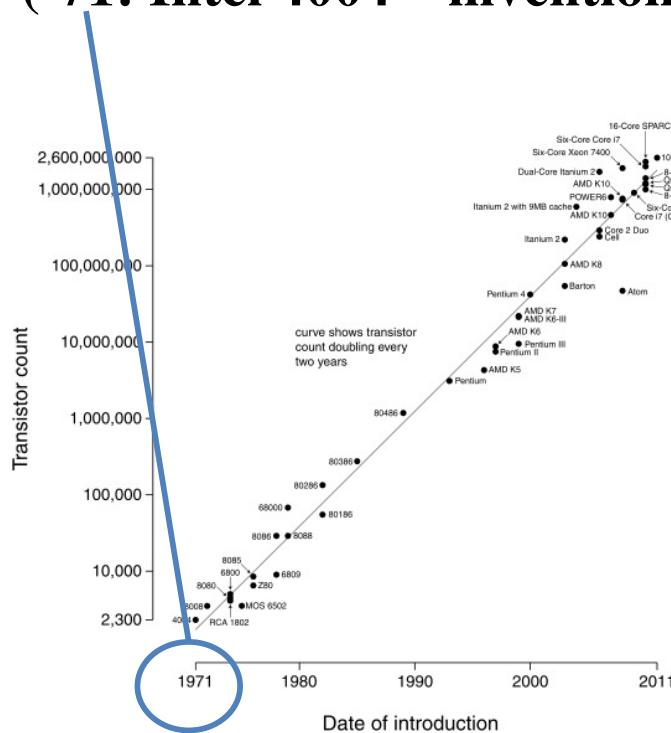
History of Supercomputers

Early '40s: ENIAC by Presper Eckert & John Mauchly at Univ. of Pennsylvania—First general-purpose electronic computer

'76: Cray 1 by Seymour Cray—beginning of vector supercomputer era

Late 80's: massively parallel computers such as the Thinking Machines CM-2

('71: Intel 4004—invention of microprocessor**)**



See lecture on [MD machines](#)

Merge of PC & Supercomputers

Rank	System	Cores	Rmax (PFlop/s)	Rpeak (PFlop/s)	Power (kW)
1	Frontier - HPE Cray EX235a, AMD Optimized 3rd Generation EPYC 64C 2GHz, AMD Instinct MI250X, Slingshot-11, HPE DOE/SC/Oak Ridge National Laboratory United States	8,699,904	1,206.00	1,714.81	22,786
2	Aurora - HPE Cray EX - Intel Exascale Compute Blade, Xeon CPU Max 9470 52C 2.4GHz, Intel Data Center GPU Max, Slingshot-11, Intel DOE/SC/Argonne National Laboratory United States	9,264,128	1,012.00	1,980.01	38,698
3	Eagle - Microsoft NDv5, Xeon Platinum 8480C 48C 2GHz, NVIDIA H100, NVIDIA Infiniband NDR, Microsoft Azure Microsoft Azure United States	2,073,600	561.20	846.84	
4	Supercomputer Fugaku - Supercomputer Fugaku, A64FX 48C 2.2GHz, Tofu interconnect D, Fujitsu RIKEN Center for Computational Science Japan	7,630,848	442.01	537.21	29,899
5	LUMI - HPE Cray EX235a, AMD Optimized 3rd Generation EPYC 64C 2GHz, AMD Instinct MI250X, Slingshot-11, HPE EuroHPC/CSC Finland	2,752,704	379.70	531.51	7,107

Theoretical performance
Measured performance
(in Pflop/s)

Flop/s =
floating-point
operations/second

M (mega) = 10^6
G (giga) = 10^9
T (Tera) = 10^{12}
P (Peta) = 10^{15}
X (Exa) = 10^{18}

<http://www.top500.org> (June '24)

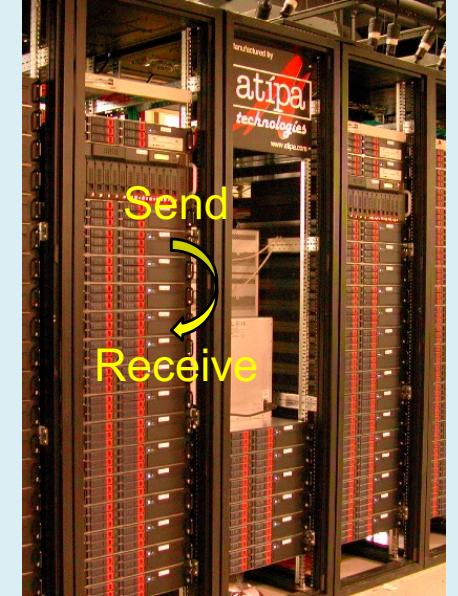
- **USC-CARC: 30,000 cores**
- **CACS: 4,096 cores**
- **CACS-INCITE: 4M node-hours/year on exaflop/s Aurora at Argonne Nat'l Lab**



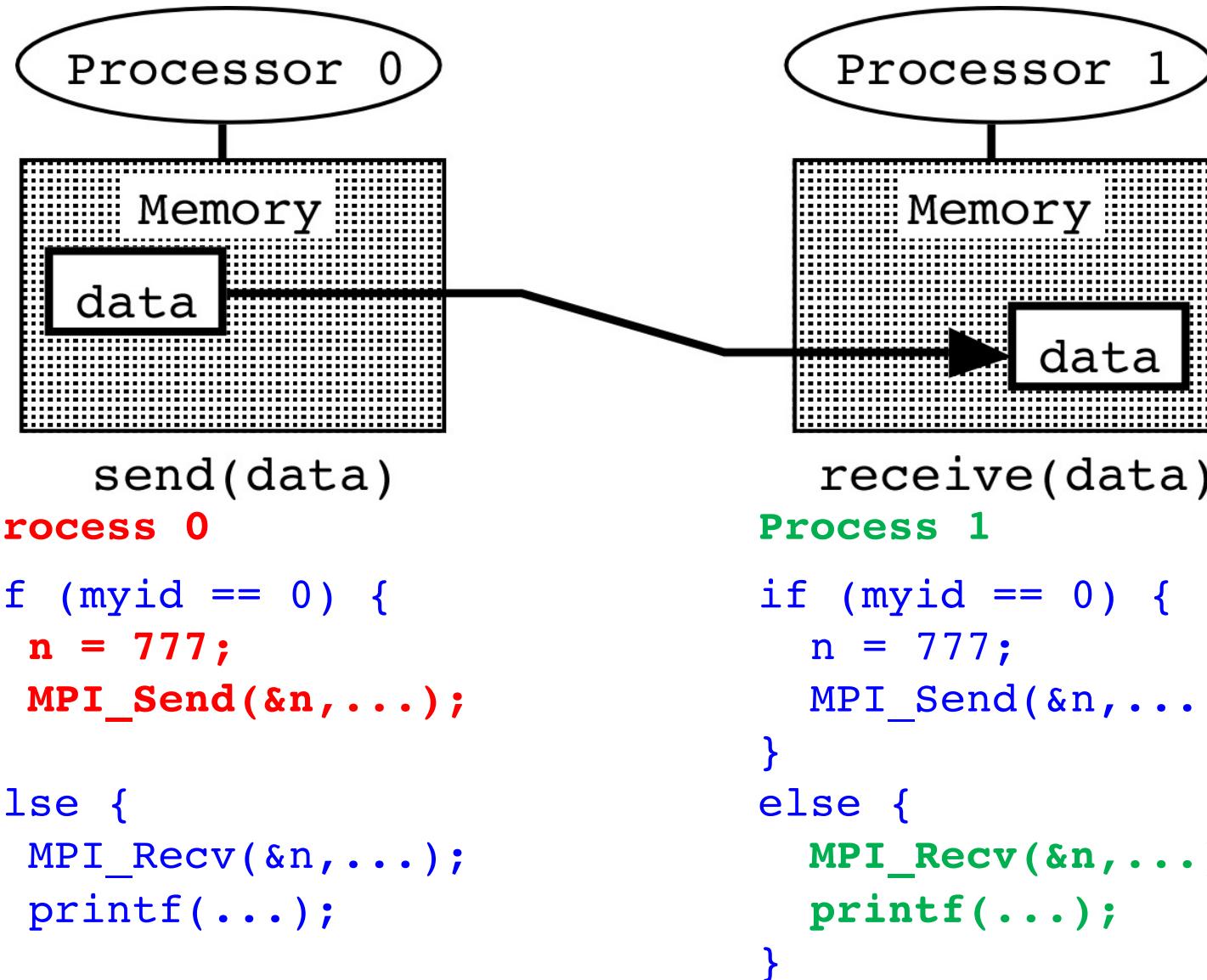
MPI Programming

```
#include "mpi.h"
#include <stdio.h>
main(int argc, char *argv[ ]) {
    MPI_Status status;
    int myid;
    int n;
    MPI_Init(&argc, &argv);
    MPI_Comm_rank(MPI_COMM_WORLD, &myid);
    if (myid == 0) {
        n = 777;
        MPI_Send(&n, 1, MPI_INT, 1, 10, MPI_COMM_WORLD);
    }
    else {
        MPI_Recv(&n, 1, MPI_INT, 0, 10, MPI_COMM_WORLD, &status);
        printf("n = %d\n", n);
    }
    MPI_Finalize();
}
```

Only need two (send & receive) functions!

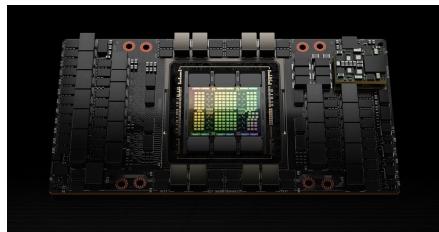
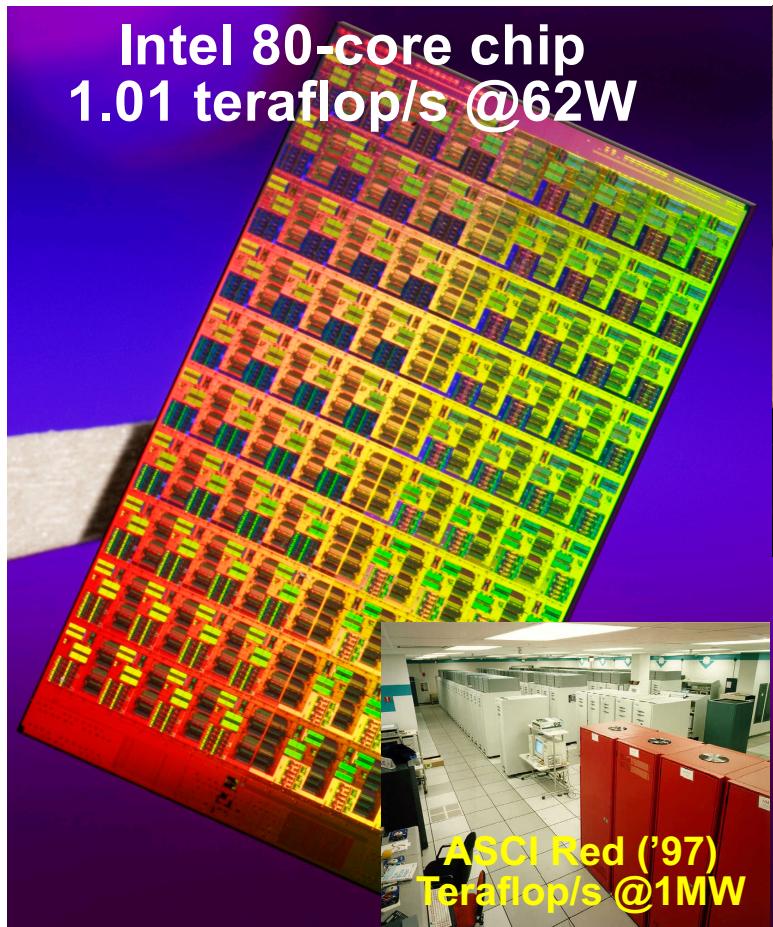


Single Program Multiple Data (SPMD)



Parallel computing: Specifies “Who does what”

Many-core CPU/GPU Computing



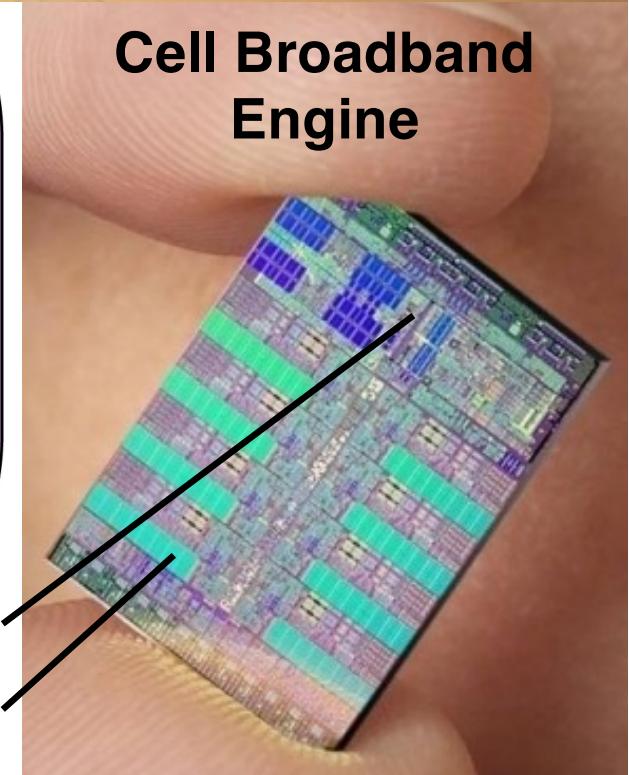
14,592 cores
67 teraflop/s
NVIDIA H100



10²-cluster Grid
10⁵-node cluster
10³-core node

64bit PowerPC

8 synergistic processing elements



Godson-T Many-core Architecture

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journal homepage: www.elsevier.com/locate/jpdc



Scalability study of molecular dynamics simulation on Godson-T many-core architecture

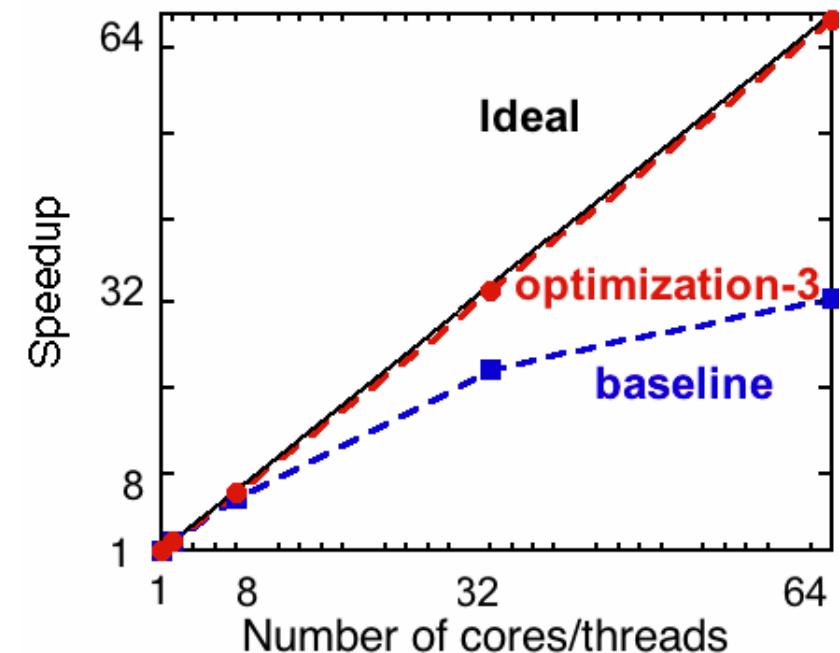
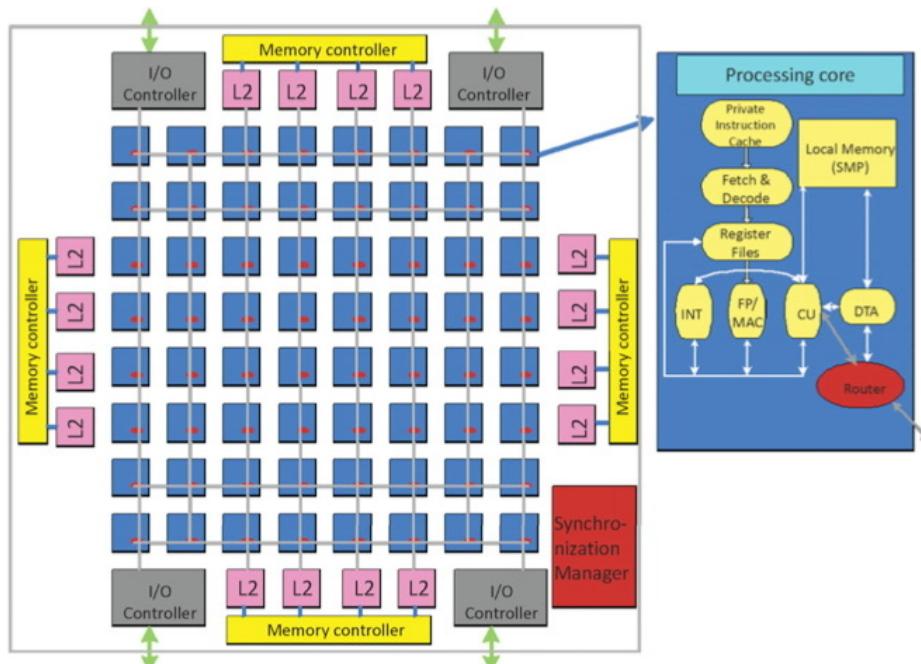
狗剩



Liu Peng^{a,*}, Guangming Tan^{b,*}, Rajiv K. Kalia^a, Aiichiro Nakano^a, Priya Vashishta^a, Dongrui Fan^b, Hao Zhang^b, Fenglong Song^b

^a Collaboratory for Advanced Computing and Simulations, University of Southern California, Los Angeles, CA, 90089, USA

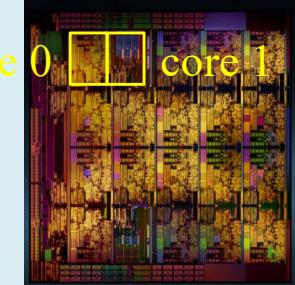
^b Key Laboratory of Computer System and Architecture, Institute of Computing Technology, Chinese Academy of Sciences, Beijing, 100190, China



OpenMP Programming

parallel section

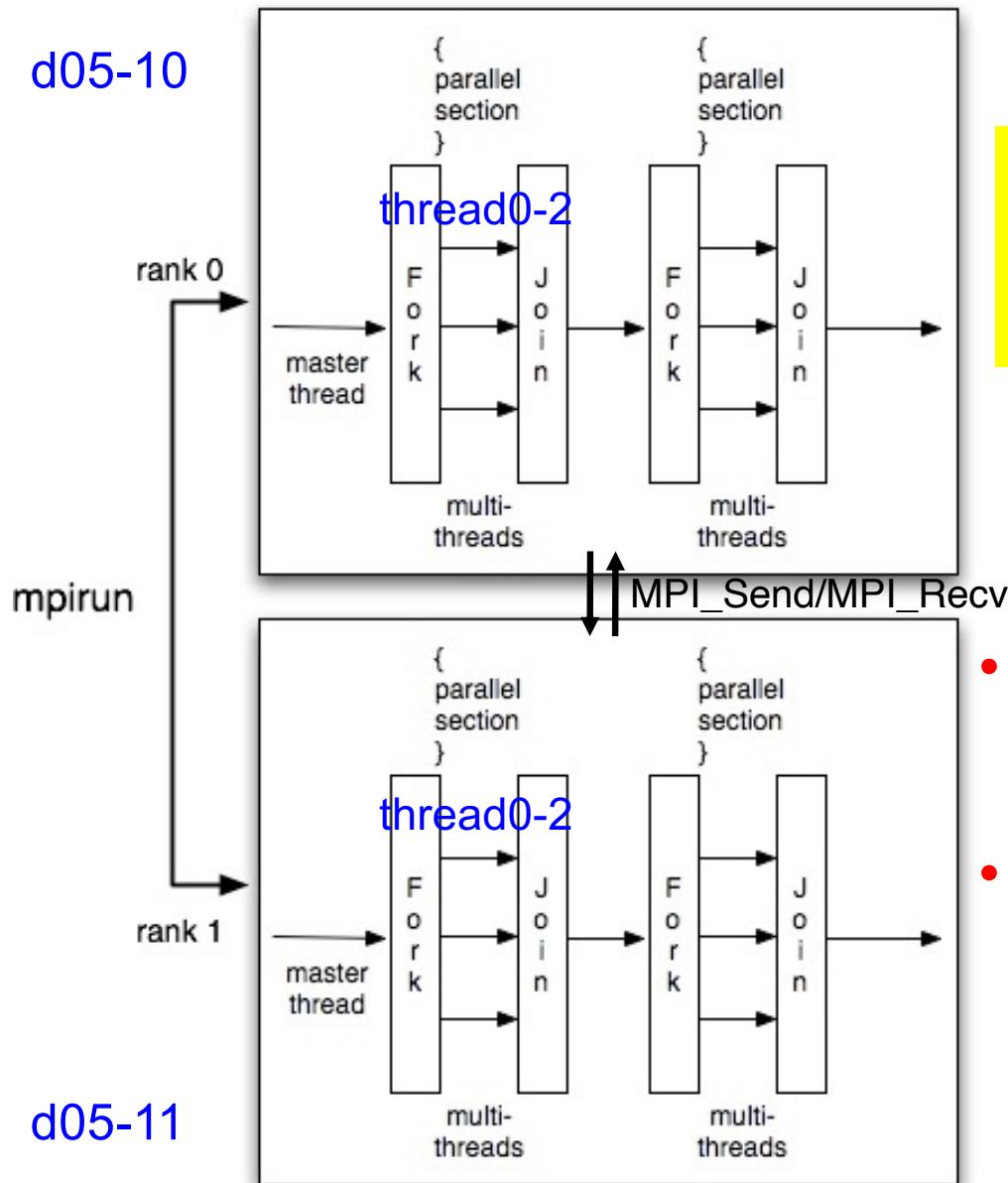
```
#include <stdio.h>
#include <omp.h>
void main () {
    int nthreads,tid;
    nthreads = omp_get_num_threads();
    printf("Sequential section: # of threads = %d\n",nthreads);
    /* Fork multi-threads with own copies of variable */
    #pragma omp parallel private(tid)
    {
        /* Obtain & print thread id */
        tid = omp_get_thread_num();
        printf("Parallel section: Hello world from thread %d\n",tid);
        /* Only master thread does this */
        if (tid == 0) {
            nthreads = omp_get_num_threads();
            printf("Parallel section: # of threads = %d\n",nthreads);}
        } /* All created threads terminate */
    }
```



- Obtain the number of threads & my thread ID
- By default, all variables are shared unless selectively changing storage attributes using private clauses

Hybrid MPI+OpenMP Programming

Each MPI process spawns multiple OpenMP threads



In a Slurm script:

`mpirun -n 2`

In the code:

`omp_set_num_threads(3);`

- **MPI processes communicate by sending/receiving messages**
- **OpenMP threads communicate by writing to/reading from shared variables**

SIMD Vectorization: MD

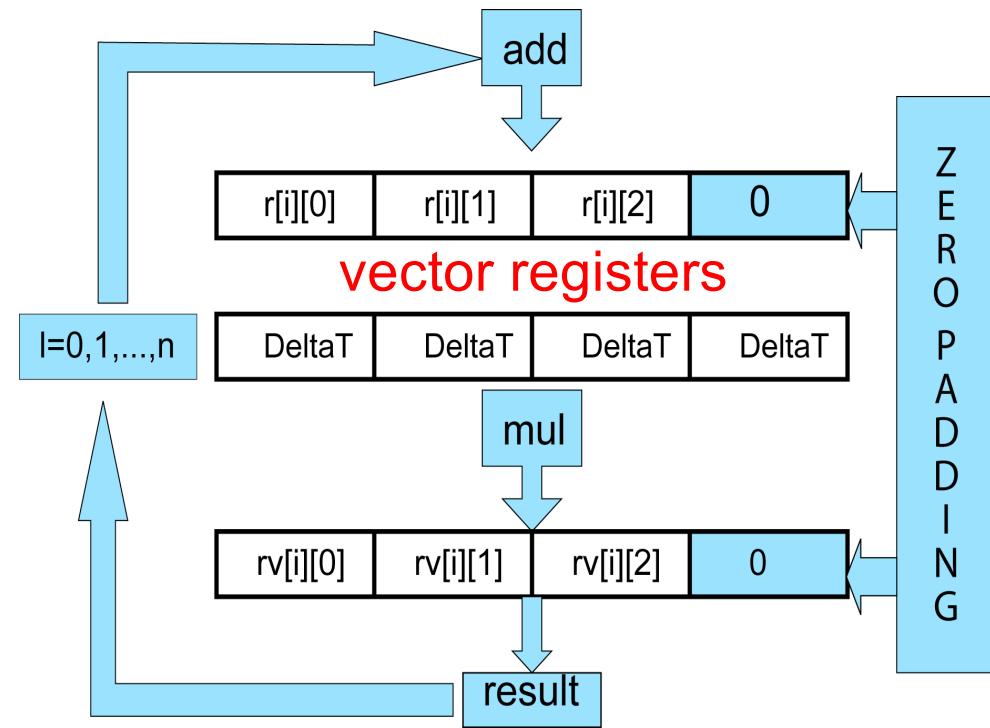
- Single-instruction multiple-data (SIMD) parallelism using vector registers

(Example) Zero padding to align complex data in molecular dynamics

Original solution

```
for (i=0; i<N; i++)
  for (a=0; a<3; a++)
    r[i][a] =
      r[i][a] +
      DeltaT*rv[i][a];
```

SIMD solution

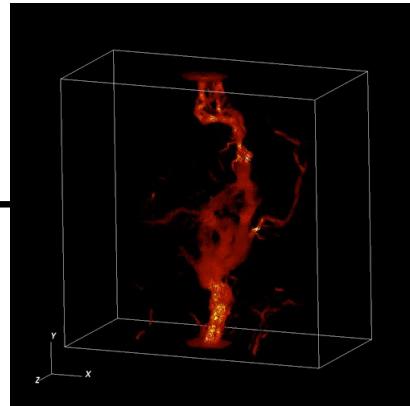


SIMD Vectorization: LBM

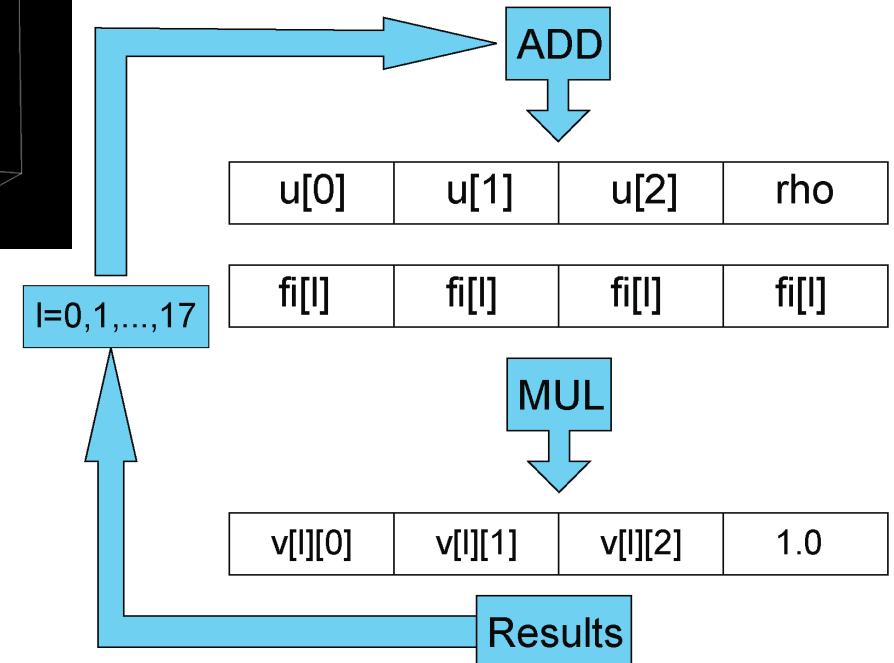
- Translocated statement fusion in lattice-Boltzmann (LBM) flow simulation

Original solution

```
for(i=0;i<3;i++){  
    u[i]=0.0; rho=0.0;  
    for(l=0;l<18;l++){  
        fi[l] = f[18*cnz+1];  
        u[i] += fi[l]*v[l][i];  
        rho += fi[l];  
    }  
}
```



SIMD solution



$3 \times 18 \times 5 = 270$ computation

SIMDizable mathematical formulations:

Special relativity, quaternion, etc.

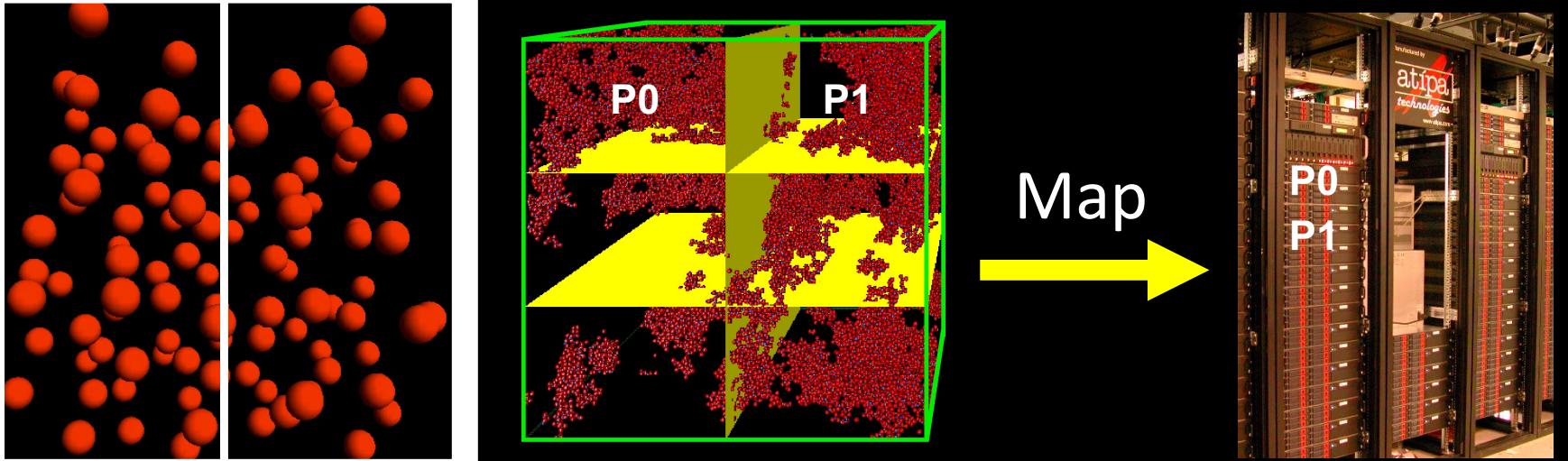
$$\begin{aligned} J^\alpha &= (c\rho, j^1, j^2, j^3) \\ A^\alpha &= (\phi/c, A^1, A^2, A^3) \\ \square A^\alpha &= \left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) A^\alpha = \frac{4\pi}{c} J^\alpha \end{aligned}$$

$18 \times 4 = 72$ computation

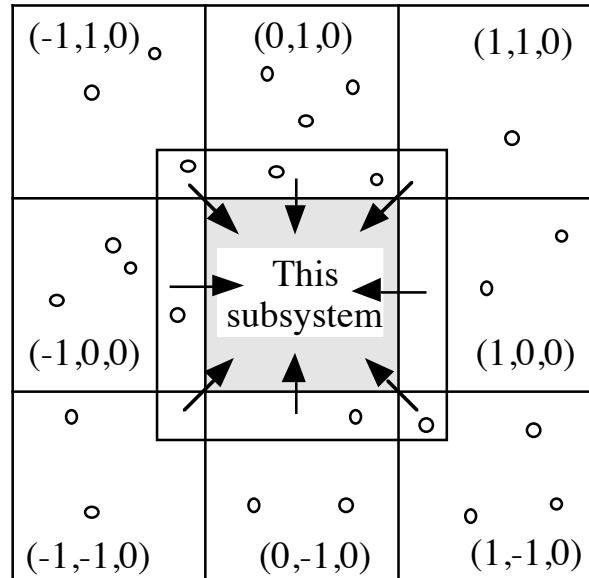
Ideal speedup: 3.5

Parallel Molecular Dynamics

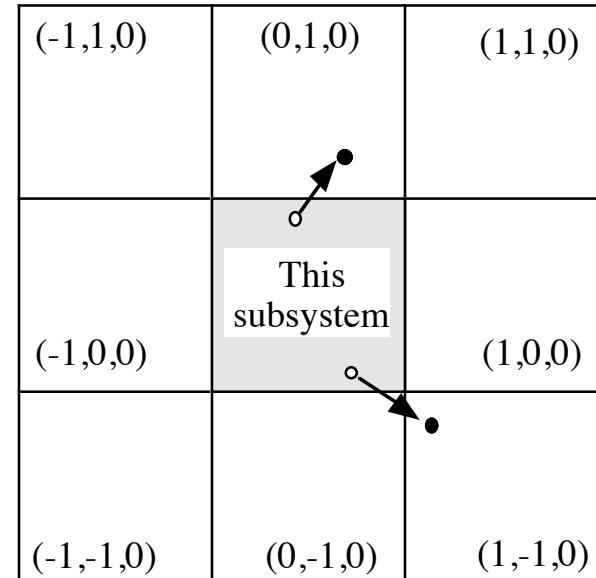
Spatial decomposition (short ranged): $O(N/P)$ computation



Atom caching: $O((N/P)^{2/3})$



Atom migration

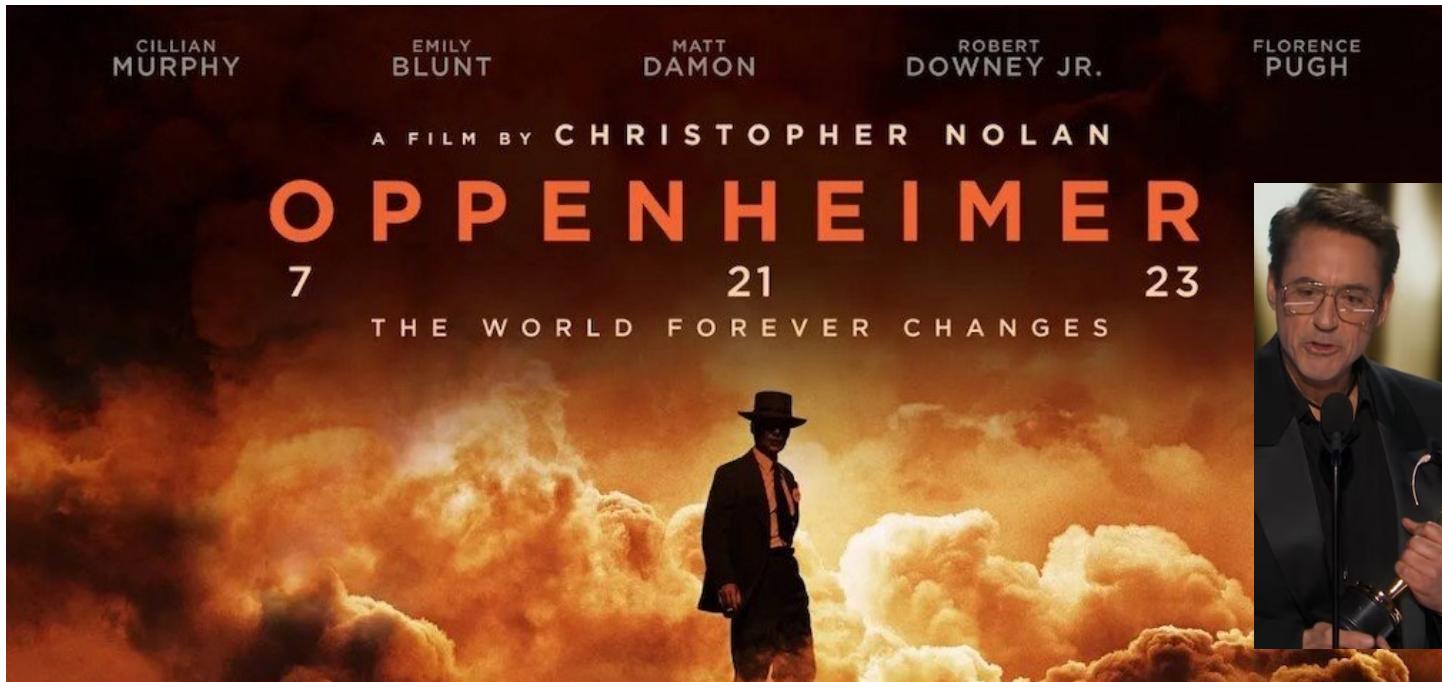


See also [parallel quantum dynamics lecture](#)

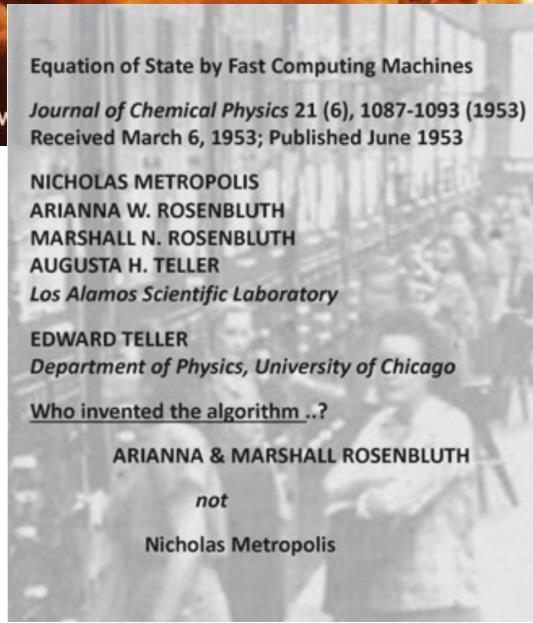
History of Particle Simulations

- '44 John von Neumann memo on a stored-program computer: "*Our present analytical methods seem unsuitable for the solution of the important problems arising in connection with nonlinear partial differential equations. The really efficient high-speed computing devices may provide us with those heuristic hints which are needed in all parts of mathematics for genuine progress*"
- '53 First Monte Carlo simulation of liquid by Metropolis, Rosenbluth, Rosenbluth, Teller, and Teller on MANIAC at Los Alamos Nat'l Lab
- '55 Enrico Fermi, John Pasta, and Stanislaw Ulam studied the dynamics of an one-dimensional array of particles coupled by anharmonic springs on MANIAC
- '56 Dynamics of hard spheres (billiards) studied by Alder and Wainwright at the Lawrence Livermore Nat'l Lab.
- '60 Radiation damage in crystalline Cu studied with short-range repulsion and uniform attraction toward the center by George Vineyard's group at Brookhaven Nat'l Lab
- '64 First MD simulation of liquid (864 argon atoms) using interatomic potentials by Aneesur Rahman at the Argonne Nat'l Lab on a CDC 3600

Simulation Pioneers in “Oppenheimer”



Benny Safdie
Edward Teller



STUDIES OF NON LINEAR PROBLEMS

E. FERMI, J. PASTA, and S. ULAM
Document LA-1940 (May 1955).

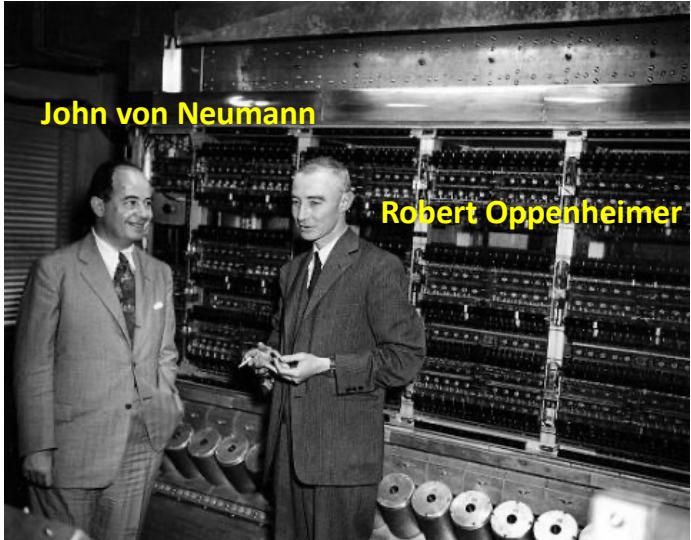
ABSTRACT.

A one-dimensional dynamical system of 64 particles with forces between neighbors containing nonlinear terms has been studied on the Los Alamos computer MANIAC I. The nonlinear terms considered are quadratic, cubic, and broken linear types. The results are analyzed into Fourier components and plotted as a function of time.



Danny Deferrari
Dr Enrico Fermi

And US Supercomputers



MANIAC computer at Los Alamos



Aiichiro Nakano of the University of Southern California uses the Argonne Leadership Computing Facility to study how light can change the structures and properties of atomically thin materials to create better, cheaper semiconductors. (Image by Argonne National Laboratory.)

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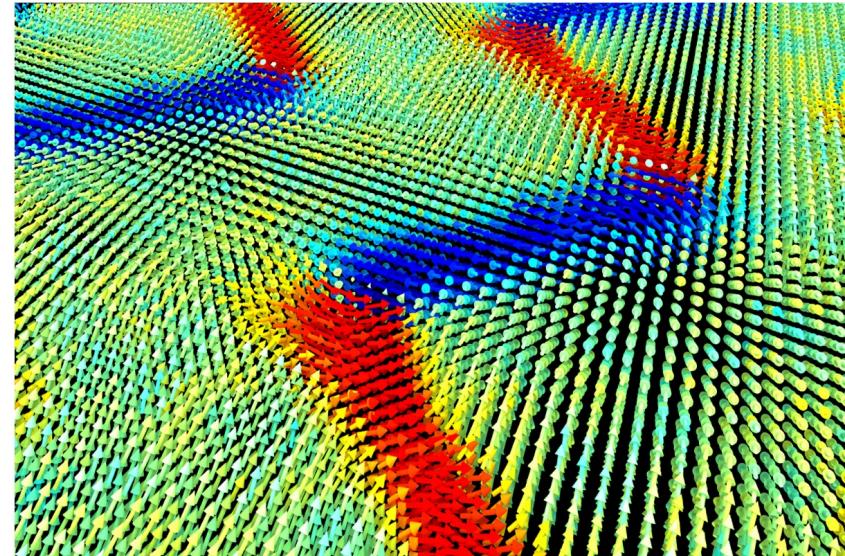
Nanomaterials, Quantum

APRIL 2024



Quanta in bulk

A USC computer scientist wants to produce quantum materials at scale, with help from Argonne supercomputers.



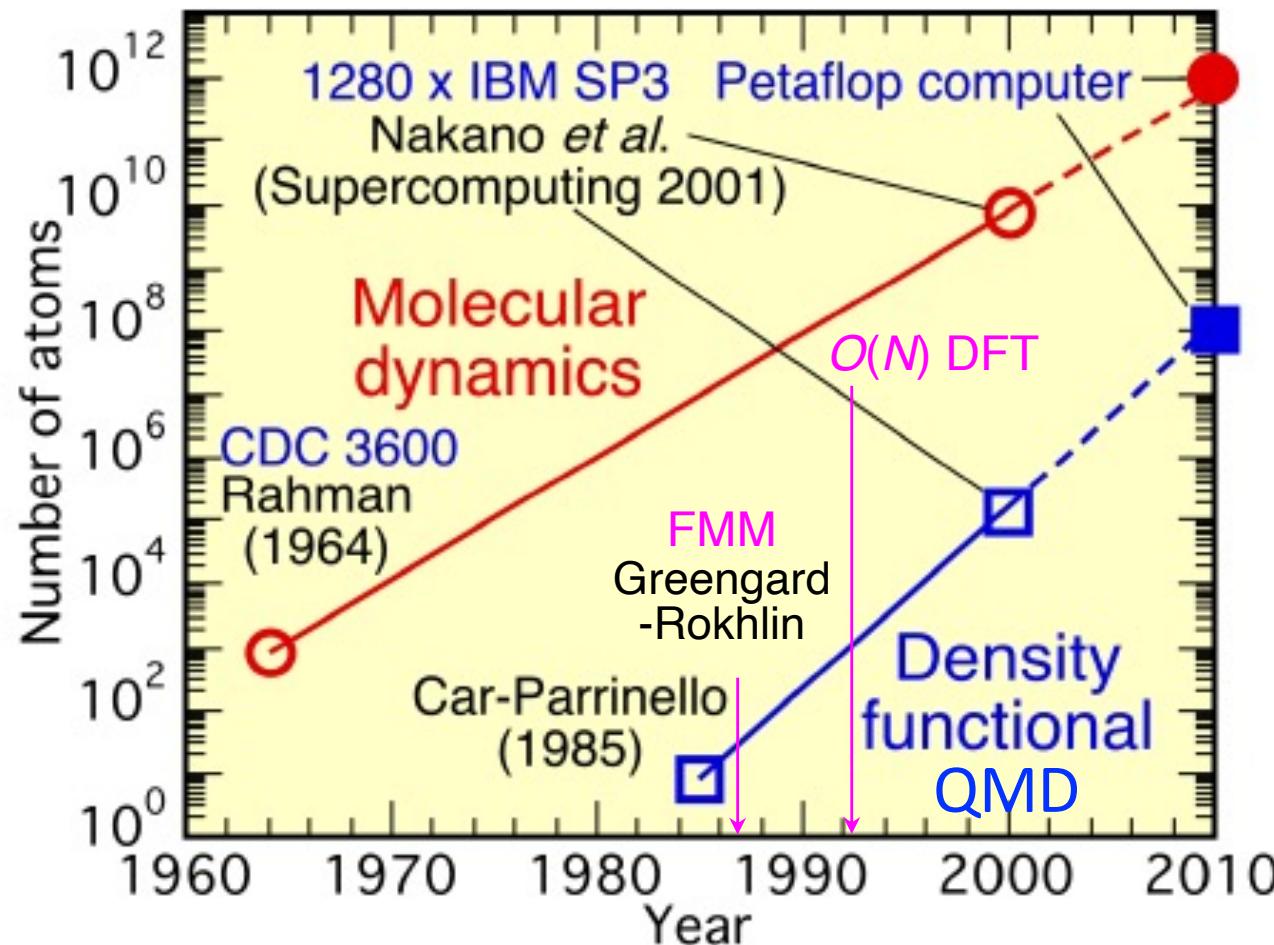
Topological structure during ferroelectric switching in PbTiO₃, where arrows represent electrical polarization colored according to its magnitude. Image courtesy of Thomas Linker and Ken-ichi Nomura/USC Viterbi School of Engineering.

Supercomputing at Argonne National Laboratory now

Moore's Law in Scientific Computing

Number of particles in MD simulations has doubled:

- Every 19 months in years for classical MD
- Every 22 months in 30 years for quantum MD (QMD)

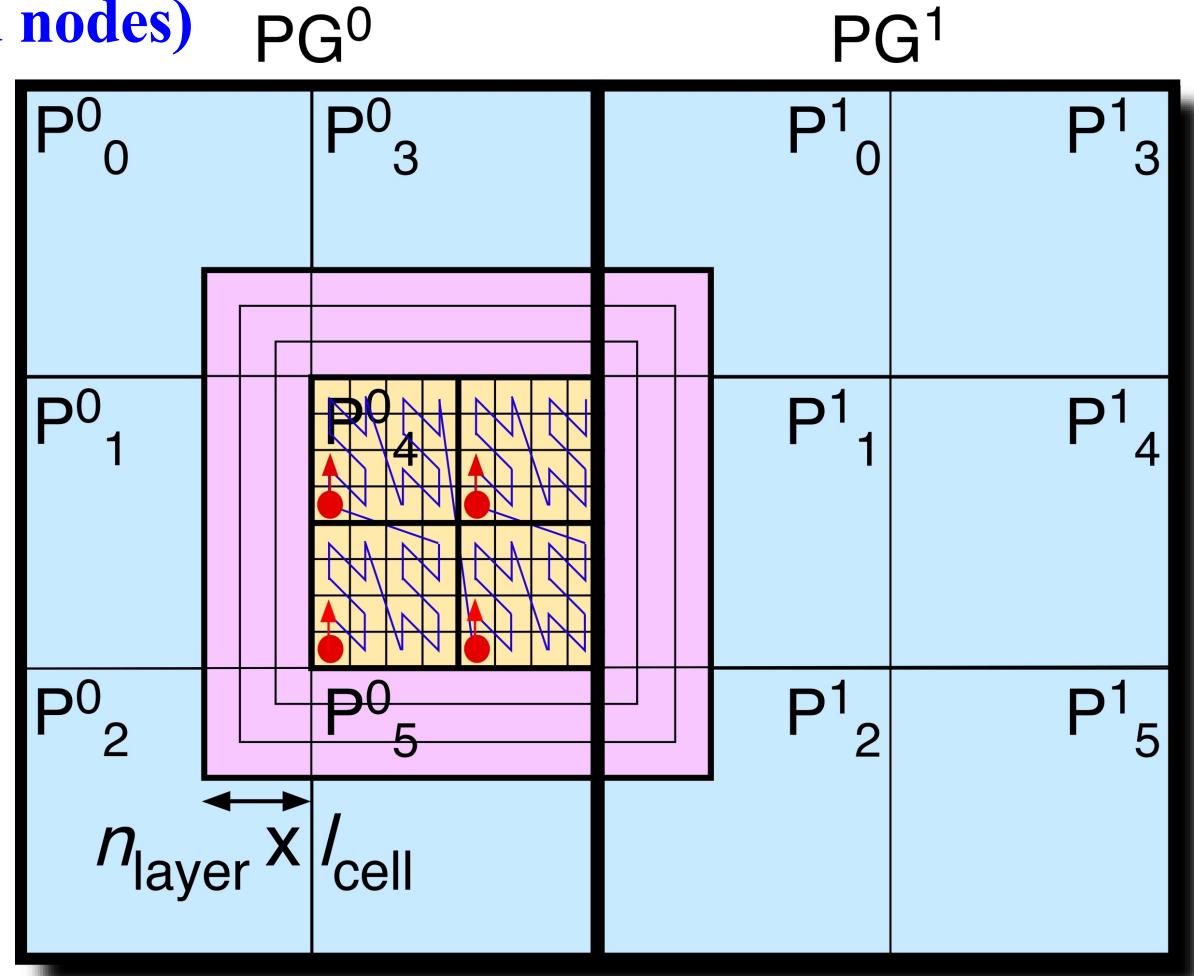


2014: 10^{12} -atom MD & 10^8 -electron QMD on a 10 petaflop/s Blue Gene/Q with advances in algorithmic & parallel-computing techniques

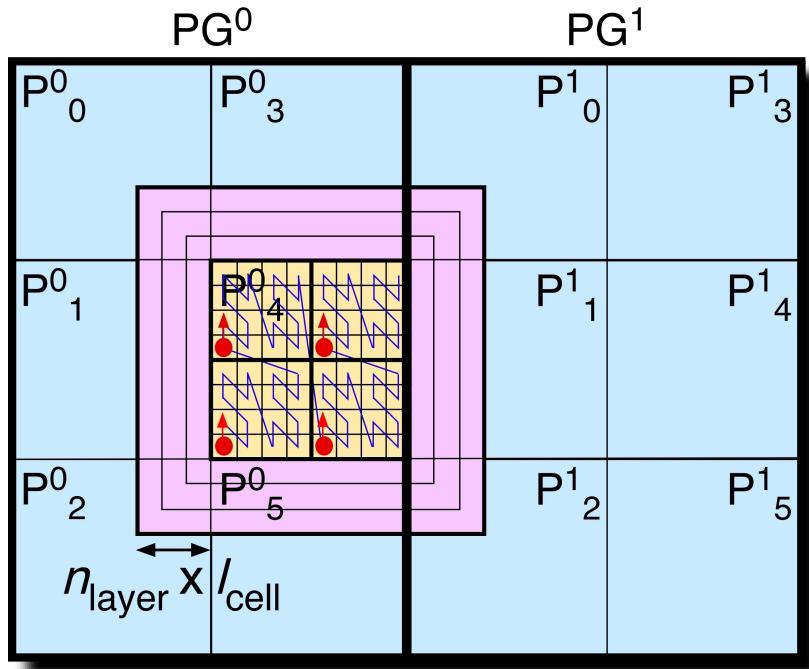
Tunable Hierarchical Cellular Decomposition

Mapping $O(N)$ divide-&-conquer algorithms onto memory hierarchies

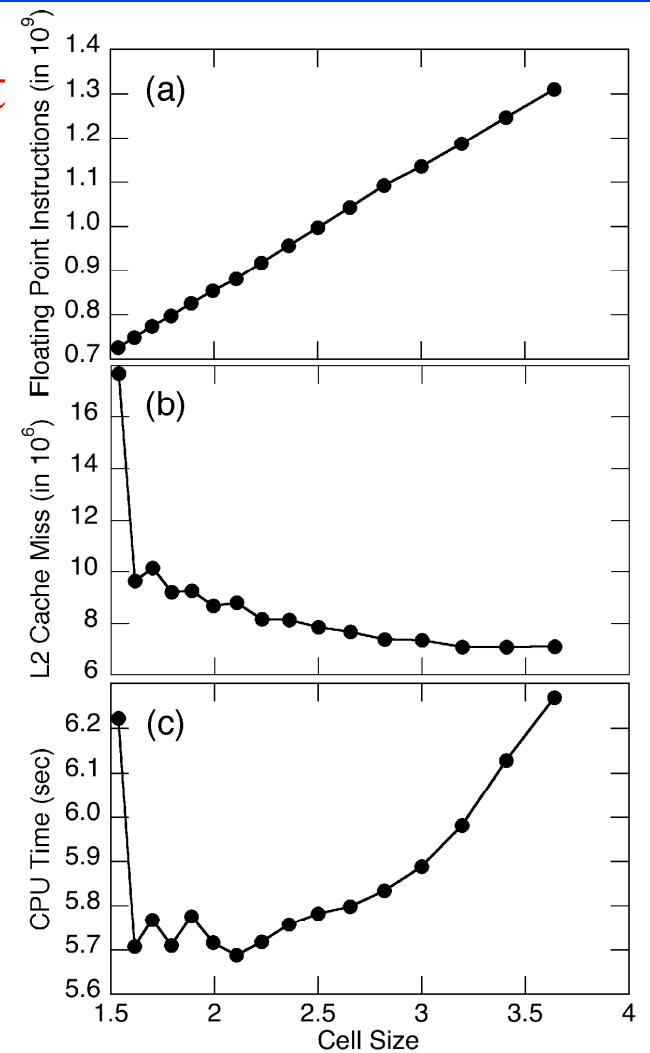
- Spatial decomposition with data “caching” & “migration”
- Computational cells (*e.g.* linked-list cells in MD) < cell blocks (threads) < processes (P^{γ}_{π} , spatial decomposition subsystems) < process groups (P^{γ} , Grid nodes)
- Multilayer cellular decomposition (MCD) for n -tuples ($n = 2\text{--}6$)
- Tunable cell data & computation structures: Data/computation re-ordering & granularity parameterized at each decomposition level
- Tunable hybrid MPI + OpenMP + SIMD implementation



Performance Tunability



Floating-point operation/L2 cache miss trade-off:
331,776-atom silica MRMD on 1.4GHz Pentium III



MPI/OpenMP parallelism trade-off:
8,232,000-atom silica MRMD &
290,304-atom RDX F-ReaxFF on 8-way 1.5 GHz Power4

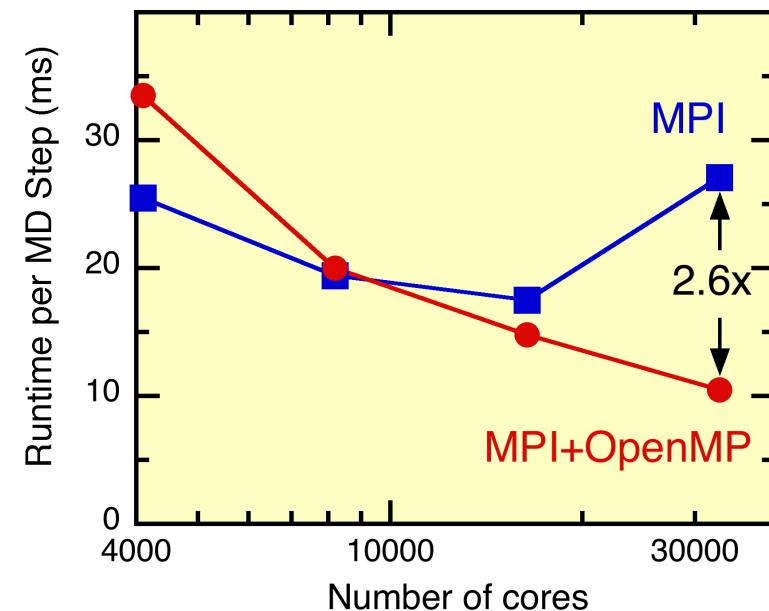
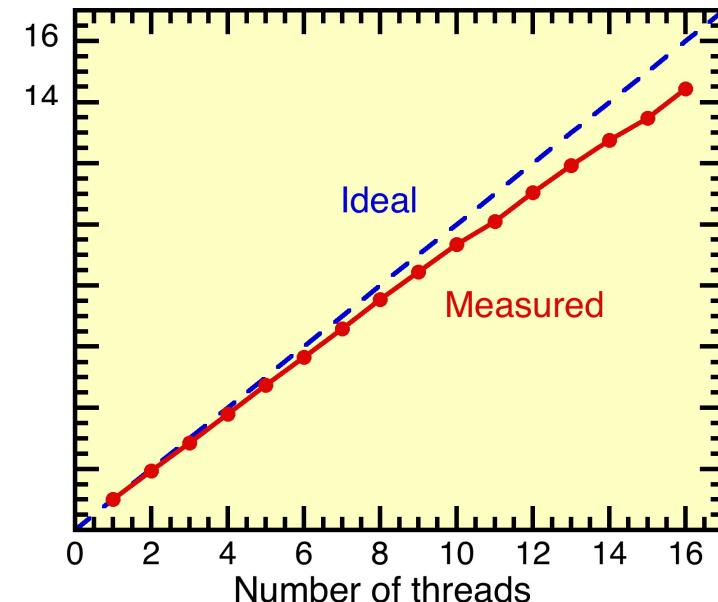
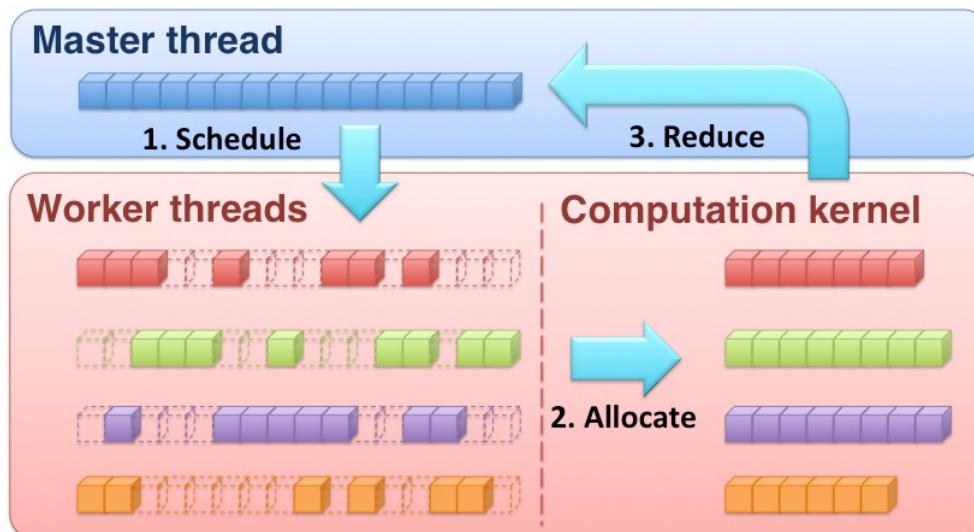
Number of OpenMP threads, n_{td}	Number of MPI processes, n_p	Execution time/MD time step (sec)	
		MRMD	P-ReaxFF
1	8	4.19	62.5
2	4	5.75	58.9
4	2	8.60	54.9
8	1	12.5	120

Spatially Compact Thread Scheduling

Concurrency-control mechanism:

Data privatization

- Reduced memory: # of threads $\Theta(nq) \rightarrow \Theta(n+n^{2/3}q^{1/3})$
- Strong scaling parallel efficiency 0.9 on quad quad-core AMD Opteron
- 2.6x speedup over MPI by hybrid MPI+OpenMP on 32,768 IBM Blue Gene/P cores



Concurrency-Control Mechanisms

A number of concurrency-control mechanisms (CCMs) are provided by OpenMP to coordinate multiple threads:

- Critical section: Serialization
- Atomic update: Expensive hardware instruction
- Data privatization: Requires large memory $\Theta(nq)$
- Hardware transactional memory: Rollbacks (on IBM Blue Gene/Q)

CCM performance varies:

- Depending on computational characteristics of each program
- In many cases, CCM degrades performance significantly

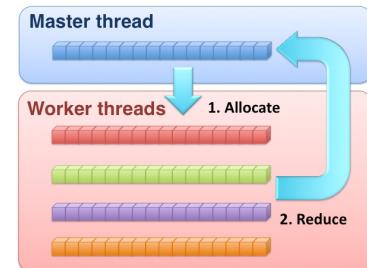
HTM/critical section

```
#pragma omp <critical|tm_atomic>
{
    ra[i][0] += fa*dr[0];
    ra[i][1] += fa*dr[1];
    ra[i][2] += fa*dr[2];
}
```

Atomic update

```
#pragma omp atomic
ra[i][0] += fa*dr[0];
#pragma omp atomic
ra[i][1] += fa*dr[1];
#pragma omp atomic
ra[i][2] += fa*dr[2];
```

Data privatization



Goal: Provide a guideline to choose the “right” CCM

Hardware Transactional Memory

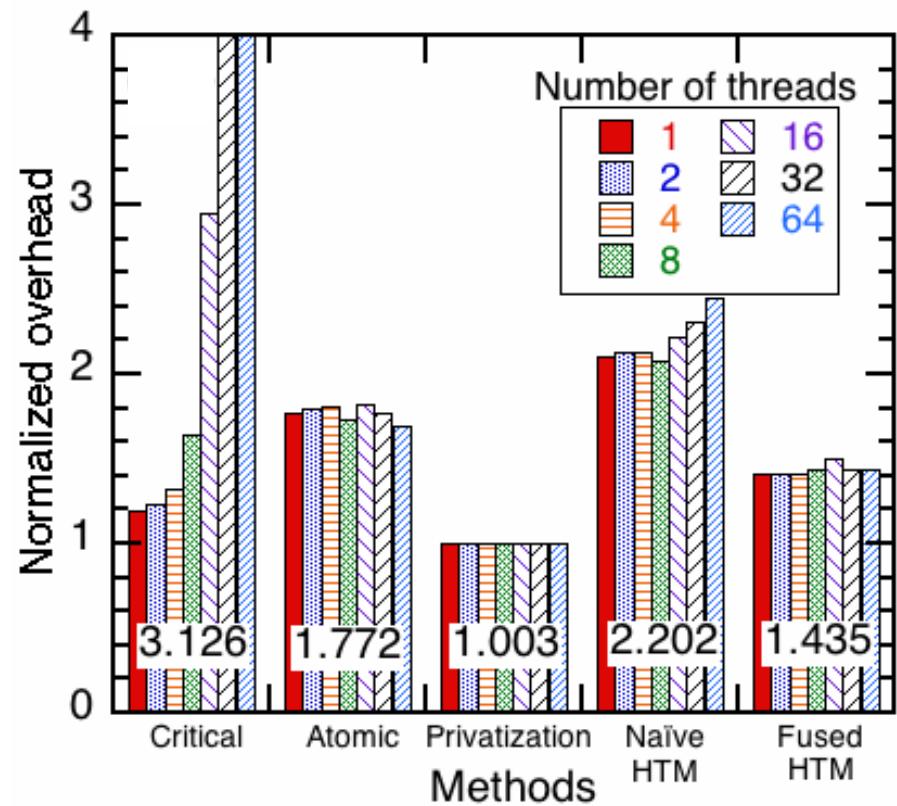
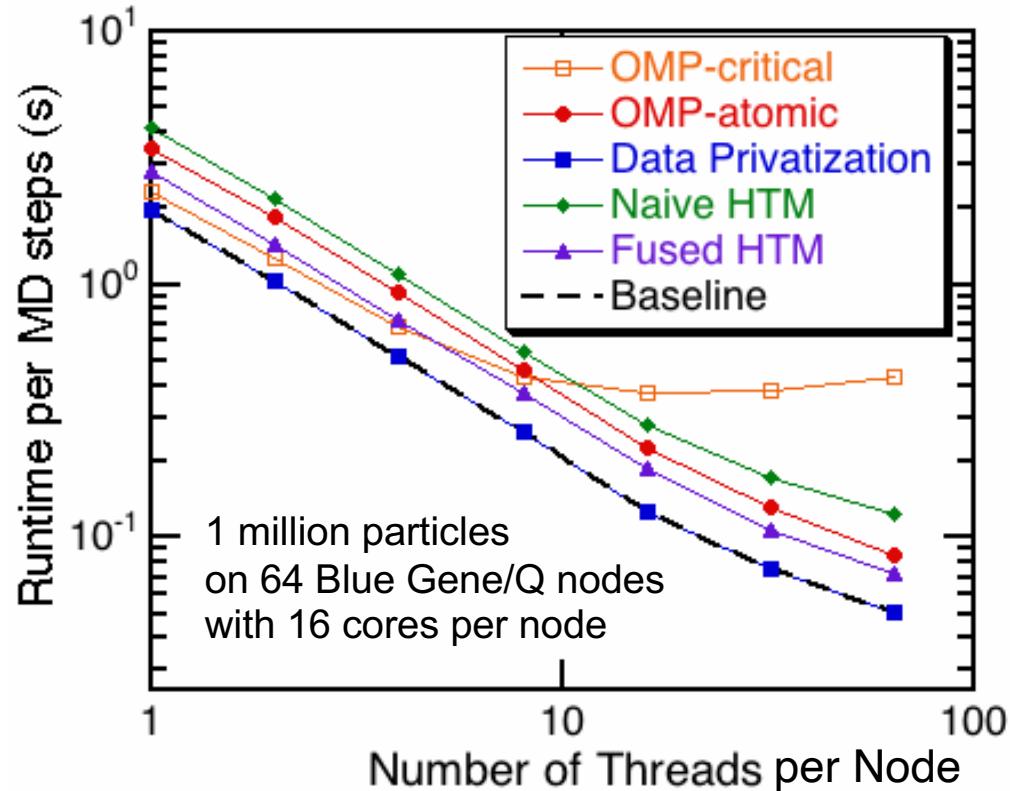
Transactional memory (TM): An opportunistic CCM

- Avoids memory conflicts by monitoring a set of speculative operations (*i.e.* transaction)
- If two or more transactions write to the same memory address, transaction(s) will be restarted—a process called **rollback**
- If no conflict detected in the end of a transaction, operations within the transaction becomes permanent (*i.e.* committed)
- Software TM usually suffers from large overhead

Hardware TM on IBM Blue Gene/Q:

- The first commercial platform implementing TM support at hardware level *via* multiversioned L2-cache
- Hardware support is expected to reduce TM overhead
- Performance of HTM on molecular dynamics has not been quantified

Strong-Scaling Benchmark for MD

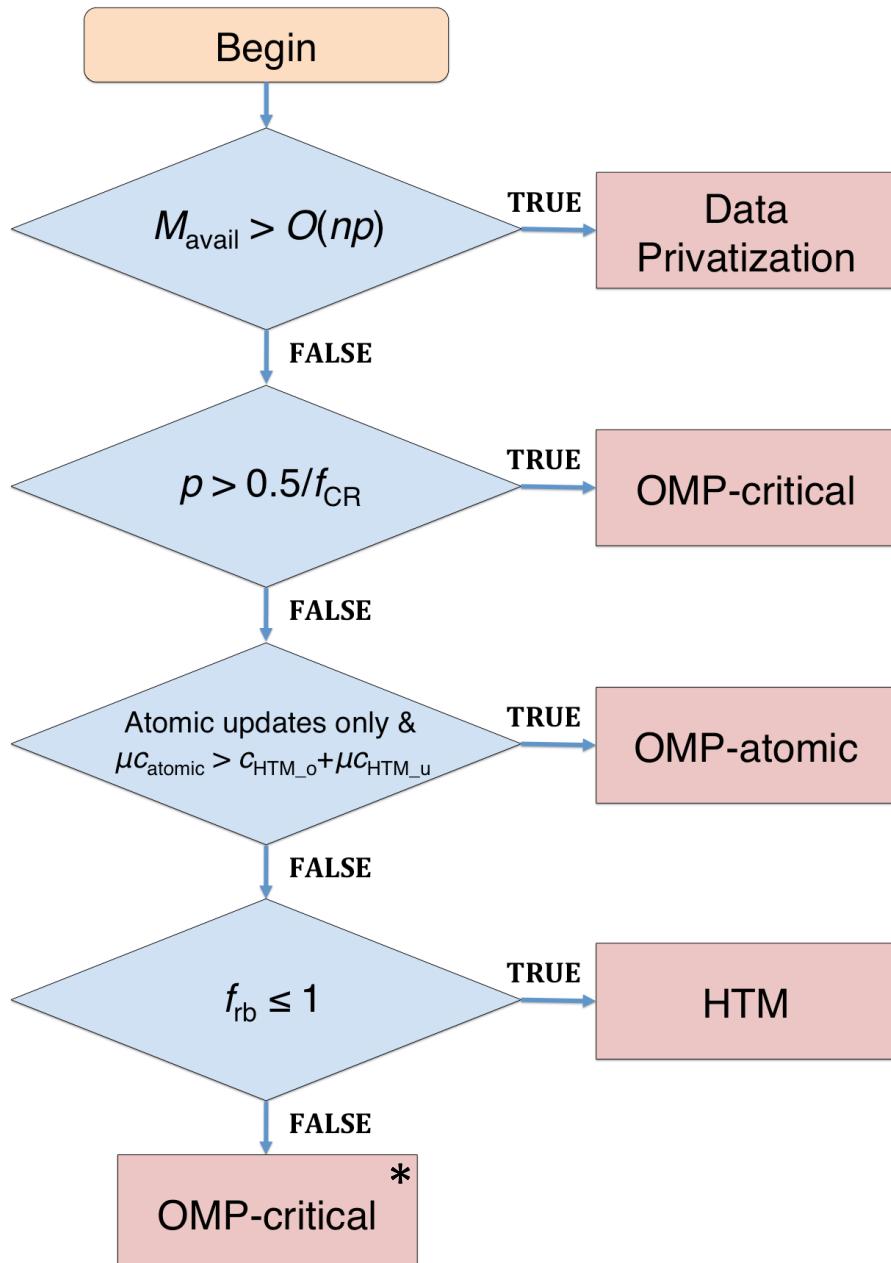


*Baseline: No CCM; the result is wrong

Developed a fundamental understanding of CCMs:

- OMP-critical has limited scalability on larger number of threads ($q > 8$)
- Data privatization is the fastest, but it requires $\Theta(nq)$ memory
- Fused HTM performs the best among constant-memory CCMs

Threading Guideline for Scientific Programs



**Focus on minimizing runtime
(best performance):**

- Have enough memory → data privatization
- Conflict region is small → OMP-critical
- Small amount of updates → OMP-atomic
- Conflict rate is low → HTM
- Other → OMP-critical* (poor performance)

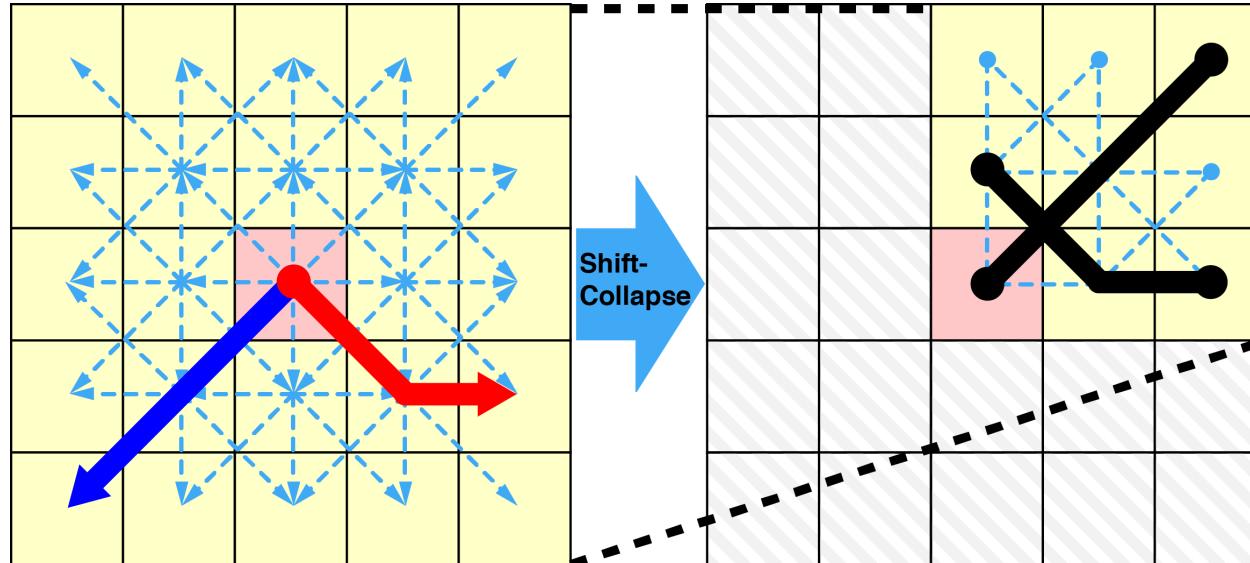
Concurrency control mechanism	Parallel efficiency
OMP-critical	$e = \min\left(\frac{1}{pf_{\text{CR}}}, 1\right)$
OMP-atomic	$e = \frac{t_{\text{total}}}{t_{\text{total}} + m\mu c_{\text{atomic}}}$
Data privatization	$e = \frac{t_{\text{total}}}{t_{\text{total}} + c_{\text{reduction}} n \log p}$
HTM	$e = \frac{t_{\text{total}}}{t_{\text{total}} + m(c_{\text{HTM_overhead}} + \mu c_{\text{HTM_update}})}$

IEEE PDSEC Best Paper & Beyond

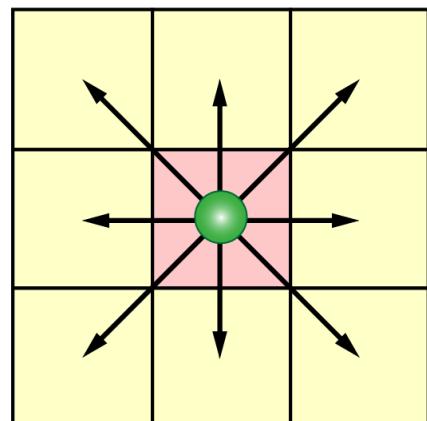


Shift-Collapse (SC) Algorithm

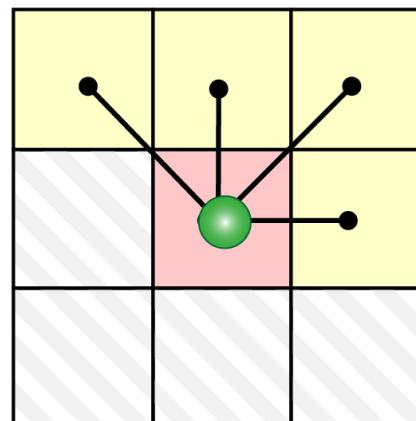
- Generalization of Shaw's eighth-cell method (**non-owner-compute method** on high-latency cluster) for pair computation to general dynamic range-limited n -tuples



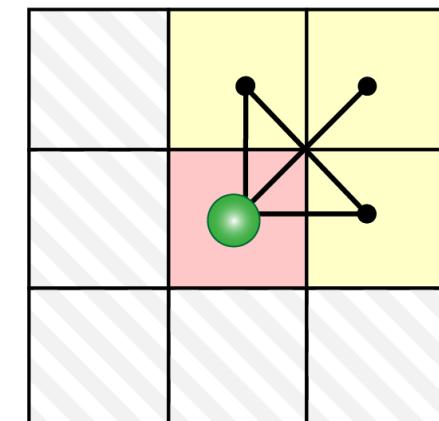
M. Kunaseth et al., *IEEE/ACM Supercomputing (SC13)*



Full-shell (FS) method
e.g., Rappaport, '88



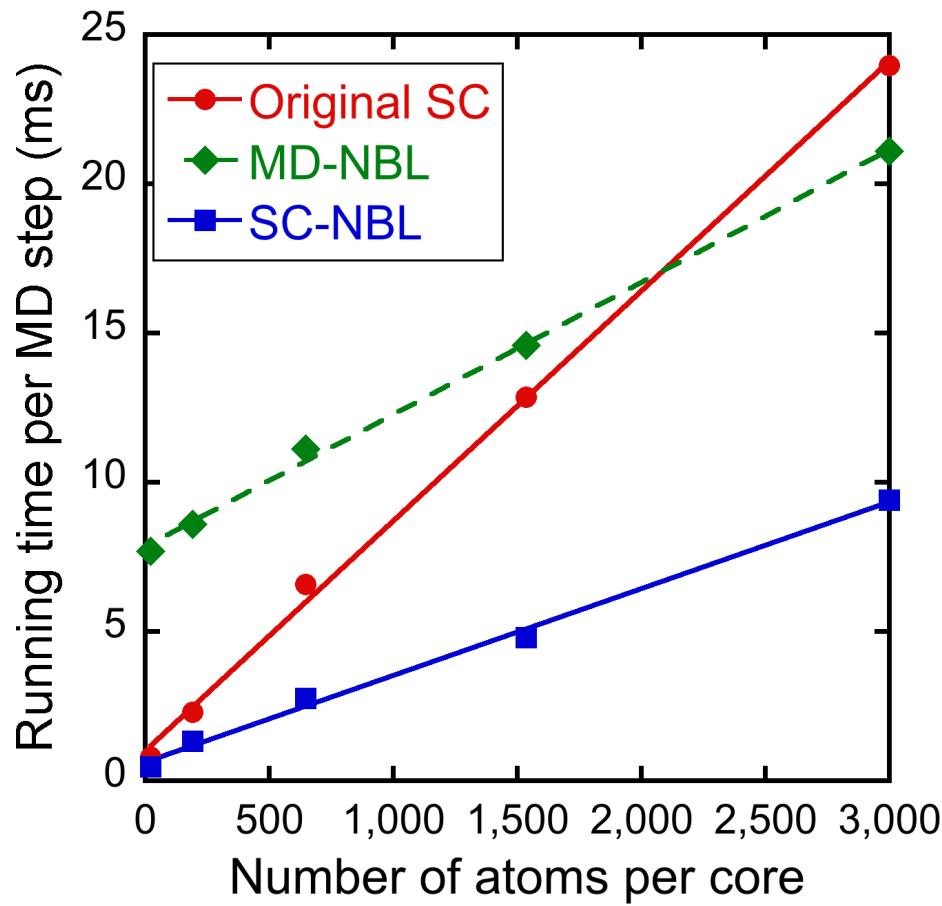
Half-shell (HS) method
e.g., Rappaport, '88



Eighth-shell (ES) method
Bower et al., '06

Shift-Collapse on Neighbor List (SC-NBL)

- Apply shift-collapse operations to the hybrid linked-cell & neighbor list code (best of both)



Shift/collapse on neighbor list (SC-NBL): fast evaluation of dynamic many-body potentials in molecular dynamics simulations, M. Kunaseth, S. Hannongbua, & A. Nakano, *Comput. Phys. Commun.* 235, 88 (2019)

Challenge: Expose massive data parallelism for SC on graphics processing unit (GPU)

GPU Programming: CUDA

- Compute Unified Device Architecture
- Integrated host (CPU) + device (GPU) application programming interface based on C language developed at NVIDIA
- CUDA homepage

http://www.nvidia.com/object/cuda_home_new.html

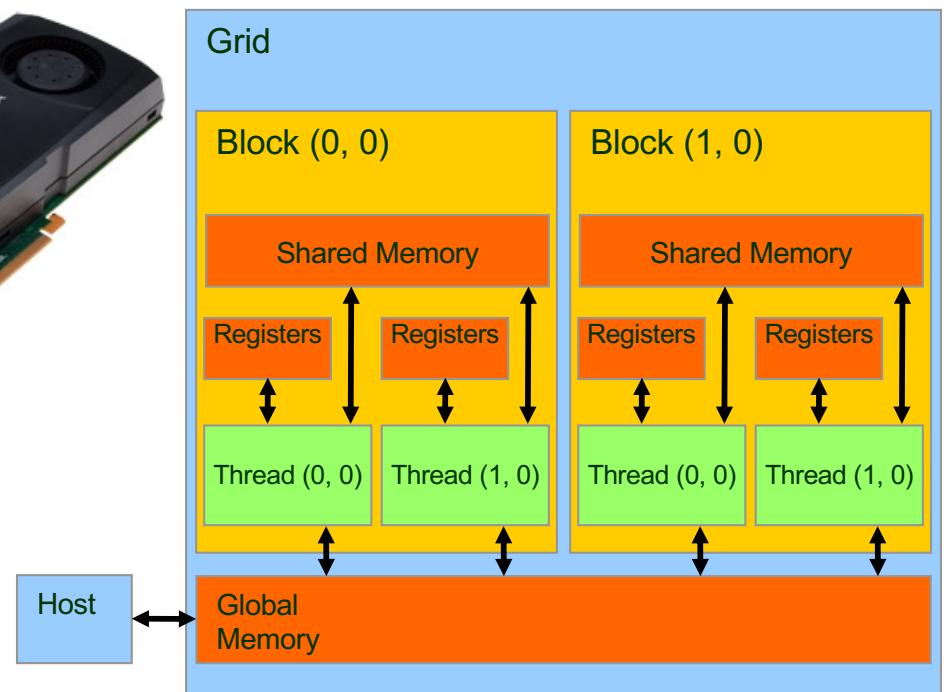
- Compilation

\$ nvcc pi.cu

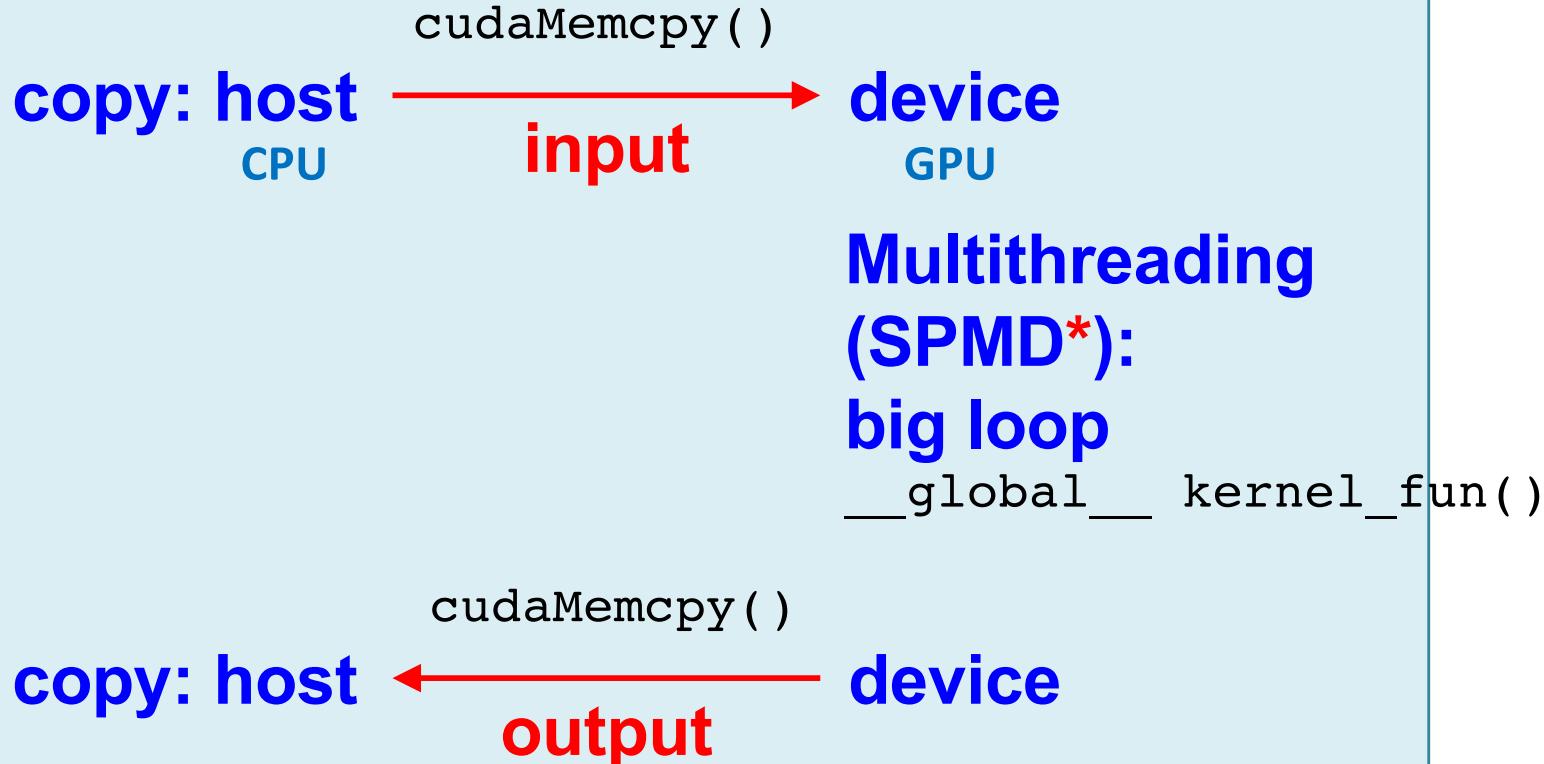
- Execution

\$ a.out

PI = 3.141593



Summary: CUDA Computing



- * Single program multiple data we have learned is called **single instruction multiple threads (SIMT)** in GPU terminology

See <https://aiichironakano.github.io/cs596/CUDA.pdf>

Exaflop/s Supercomputing

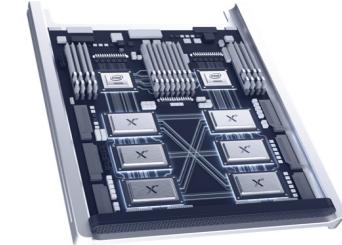
- Diverse exaflop/s supercomputing platforms



Summit (0.2 Exaflop/s)
IBM CPU/NVIDIA GPU



1 HPC and AI Optimized
AMD EPYC CPU
4 Purpose Built AMD
Radeon Instinct GPU

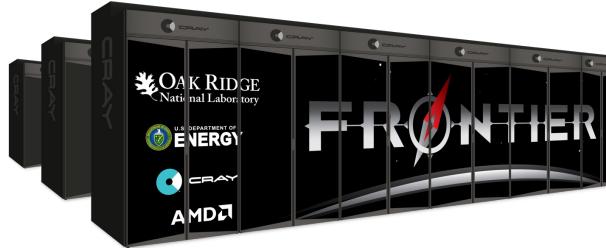


GPU Architecture

X^e arch-based “Ponte Vecchio”
GPU Tile-based, chiplets, HBM
stack, Foveros 3D integration, 7nm

On-Node Interconnect

CPU-GPU: PCIe
GPU-GPU: X^e Link



Frontier (1.7 Exaflop/s)
AMD CPU/AMD GPU



Aurora (2.0 Exaflop/s)
Intel CPU/Intel GPU

- Need an *open* programming model for *heterogeneous* (e.g., GPU-accelerated) clusters (note CUDA is a proprietary language by NVIDIA)

See <https://extremecomputingtraining.anl.gov/agenda-2024/>

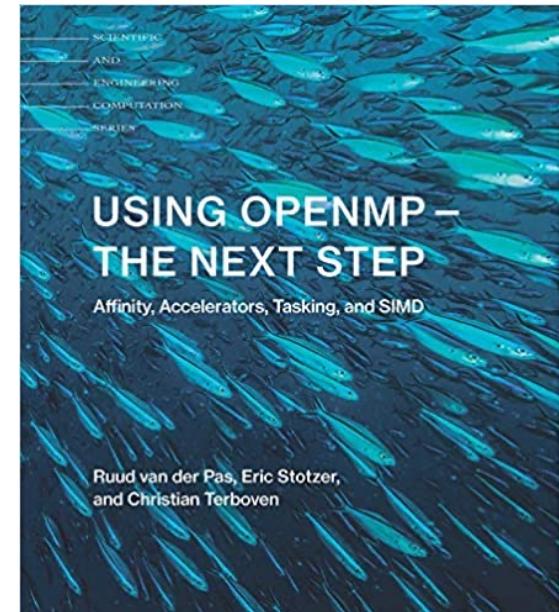
Open Programming Models: Easy Way

- **OpenACC (Open Computing Language)**
Open standard for directive-based programming of heterogeneous devices

<https://www.openacc.org/>

- **OpenMP 4.5/5**
Starting specification version 4.5, OpenMP allows offloading the execution of code & data to heterogeneous devices

<https://www.openmp.org/specifications/>



OpenMP Target Offload

- Simple example

```
main() {  
    float a[1000],b[1000],c,d;  
    ...  
    #pragma omp target map(a,b,c,d)  
    {  
        int i;  
        #pragma omp parallel for  
        for (i=0; i<N; i++)  
            a[i] = b[i]*c+d;  
    }  
    ...  
}
```

See <https://aiichironakano.github.io/cs596/OMPtarget.pdf>

- When a host thread encounter the `#pragma omp target` directive, the target region specified by it will be executed by a new thread running on an accelerator, *cf.* CUDA GPU kernel
- Before the new thread starts executing the target region, the variable in the `map()` clause are mapped onto accelerator memory, which often is disjunct from the host memory, *cf.* `cudaMemcpy()`
- The offloaded code is usually a data-parallel structured block, which can be handled by multiple threads on accelerator using standard OpenMP constructs like `#pragma omp parallel for`

Open Programming Models: Hard Way

- **OpenCL (Open Computing Language)**

Open standard for programming heterogeneous devices

<https://www.khronos.org/opencl/>

- **SYCL**

High-level programming standard (or abstraction layer) for single-source C++ based language on heterogeneous computer architectures

<https://www.khronos.org/sycl/>

- **Data parallel C++ (DPC++)**

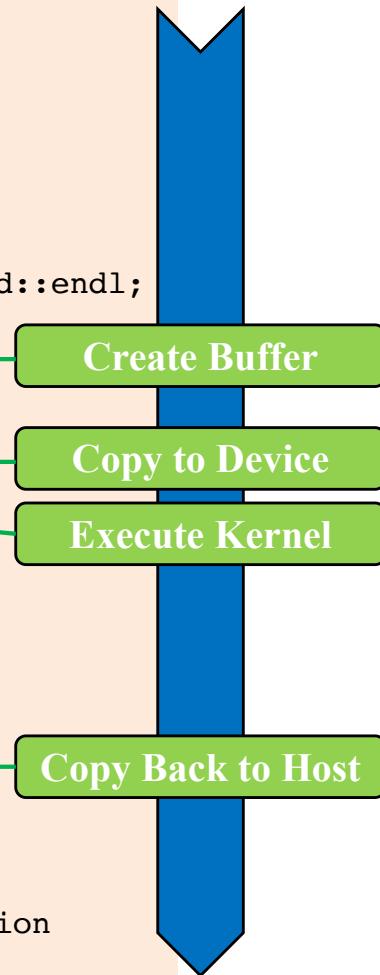
Extension of C++ programming language, incorporating SYCL & other features, initially created by Intel; an open-source compiler is available on GitHub

<https://intel.github.io/llvm-docs/index.html>

See <https://aiichironakano.github.io/cs596/SYCL.pdf>

DPC++ Program Pattern

```
#include <CL/sycl.hpp>
#include <iostream>
#include <array>
using namespace cl::sycl;
#define NBIN 1000000 // # of bins for quadrature
#define NTRD 512     // # of threads
int main() {
    float step = 1.0f/NBIN;
    std::array<float, NTRD> sum;
    for (int i=0; i<NTRD; ++i) sum[i] = 0.0f;
{
    queue q(cpu_selector{});
    std::cout << "Running on: " <<
        q.get_device().get_info<info::device::name>() << std::endl;
    range<1> sizeBuf{NTRD};
    buffer<float, 1> sumBuf(sum.data(), sizeBuf);
    q.submit([&](handler &h){
        auto sumAccessor =
            sumBuf.get_access<access::mode::read_write>(h);
        h.parallel_for(sizeBuf, [=](id<1> tid) {
            for (int i=tid; i<NBIN; i+=NTRD) {
                float x = (i+0.5f)*step;
                sumAccessor[tid] += 4.0f/(1.0f+x*x);
            }
        });
    });
}
float pi=0.0f;
for (int i=0; i<NTRD; i++) // Thread reduction
    pi += sum[i];
pi *= step; // Multiply bin width to complete integration
std::cout << "Pi = " << pi << std::endl;
return 0;
}
```



GPU: Easy & Hard Ways

Serial: pi.c

```
#include <stdio.h>
#define NBIN 100000000
int main() {
    double step, x, sum=0.0, pi;
    step = 1.0/NBIN;
    for (long long i=0; i<NBIN; i++) {
        x = (i+0.5)*step;
        sum += 4.0/(1.0+x*x);
    }
    pi = sum*step;
    printf("PI = %f\n",pi);
    return 0;
}
```

OpenMP: omp_target_pi.c



```
#include <omp.h>
#include <stdio.h>
#define NBIN 1000000
int main() {
    float step,sum=0.0,pi;
    step = 1.0/(float)NBIN;
    #pragma omp target map(step,sum)
    {
        # pragma omp parallel for reduction(+:sum)
        for (long long i=0; i<NBIN; i++) {
            float x = (i+0.5)*step;
            sum += 4.0/(1.0+x*x);
        }
        pi = sum*step;
        printf("PI = %f\n",pi);
        return 0;
    }
}
```



DPC++: pi.cpp

```
#include <CL/sycl.hpp>
#include <iostream>
#include <array>

using namespace cl::sycl;

#define NBIN 1000000 // # of bins for quadrature
#define NTRD 512      // # of threads

int main()
{
    float step = 1.0f/NBIN;
    std::array<float, NTRD> sum;
    for (int i=0; i<NTRD; ++i) sum[i] = 0.0f;

    queue q(cpu_selector{});

    std::cout << "Running on: " <<
    q.get_device().get_info<info::device::name>() << std::endl;

    range<1> sizeBuf{NTRD};

    {
        buffer<float, 1> sumBuf(sum.data(), sizeBuf);
        q.submit([&](handler &h){
            auto sumAccessor =
            sumBuf.get_access<access::mode::read_write>(h);
            h.parallel_for(sizeBuf, [=](id<1> tid) {
                for (int itid: i<NBIN; i+=NTRD) {
                    float x = (i+0.5f)*step;
                    sumAccessor[tid] += 4.0f/(1.0f+x*x);
                }
            });
        });
    }

    float pi=0.0f;
    for (int i=0; i<NTRD; i++) // Inter-thread reduction
        pi += sum[i];
    pi *= step; // Multiply bin width to complete integration

    std::cout << "Pi = " << pi << std::endl;

    return 0;
}
```

<https://aiichironakano.github.io/cs596-code.html>

CUDA: pi.cu

```
// Using CUDA device to calculate pi
#include <stdio.h>
#include <cuda.h>

#define NBIN 10000000 // Number of bins
#define NUM_BLOCK 13 // Number of thread blocks
#define NUM_THREAD 192 // Number of threads per block
int tid;
float pi = 0;

// Kernel that executes on the CUDA device
__global__ void cal_pi(float *sum, int nbin, float step, int nthreads, int nblocks) {
    int i;
    float x;
    int idx = blockIdx.x*blockDim.x+threadIdx.x; // Sequential thread index across the blocks
    for (i=idx; i< nbin; i+=nthreads*nblocks) {
        x = (i+0.5)*step;
        sum[idx] += 4.0/(1.0+x*x);
    }
}

// Main routine that executes on the host
int main(void) {
    dim3 dimGrid(NUM_BLOCK,1,1); // Grid dimensions
    dim3 dimBlock(NUM_THREAD,1,1); // Block dimensions
    float *sumHost, *sumDev; // Pointer to host & device arrays

    float step = 1.0/NBIN; // Step size
    size_t size = NUM_BLOCK*NUM_THREAD*sizeof(float); // Array memory size
    sumHost = (float *)malloc(size); // Allocate array on host
    cudaMalloc((void **)&sumDev, size); // Allocate array on device
    // Initialize array in device to 0
    cudaMemset(sumDev, 0, size);
    // Do calculation on device
    cal_pi(<<dimGrid, dimBlock>> (sumDev, NBIN, step, NUM_THREAD, NUM_BLOCK)); // call CUDA kernel
    // Retrieve result from device and store it in host array
    cudaMemcpy(sumHost, sumDev, size, cudaMemcpyDeviceToHost);
    for(tid=0; tid<NUM_THREAD*NUM_BLOCK; tid++)
        pi += sumHost[tid];
    pi *= step;

    // Print results
    printf("PI = %f\n",pi);

    // Cleanup
    free(sumHost);
    cudaFree(sumDev);

    return 0;
}
```

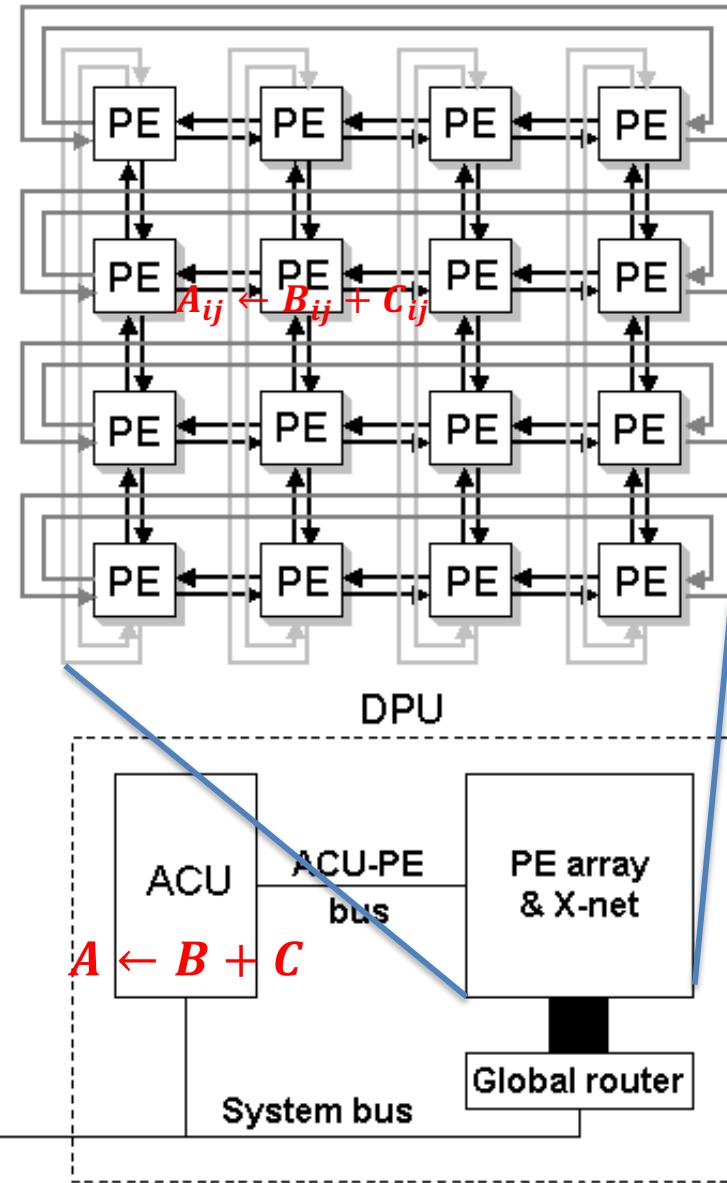
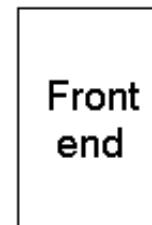
Massive SIMD Data-Parallel Accelerator



SIMD: single-instruction multiple data

**Quantum dynamics on 8,192-processor
(128×64) MasPar 1208B**

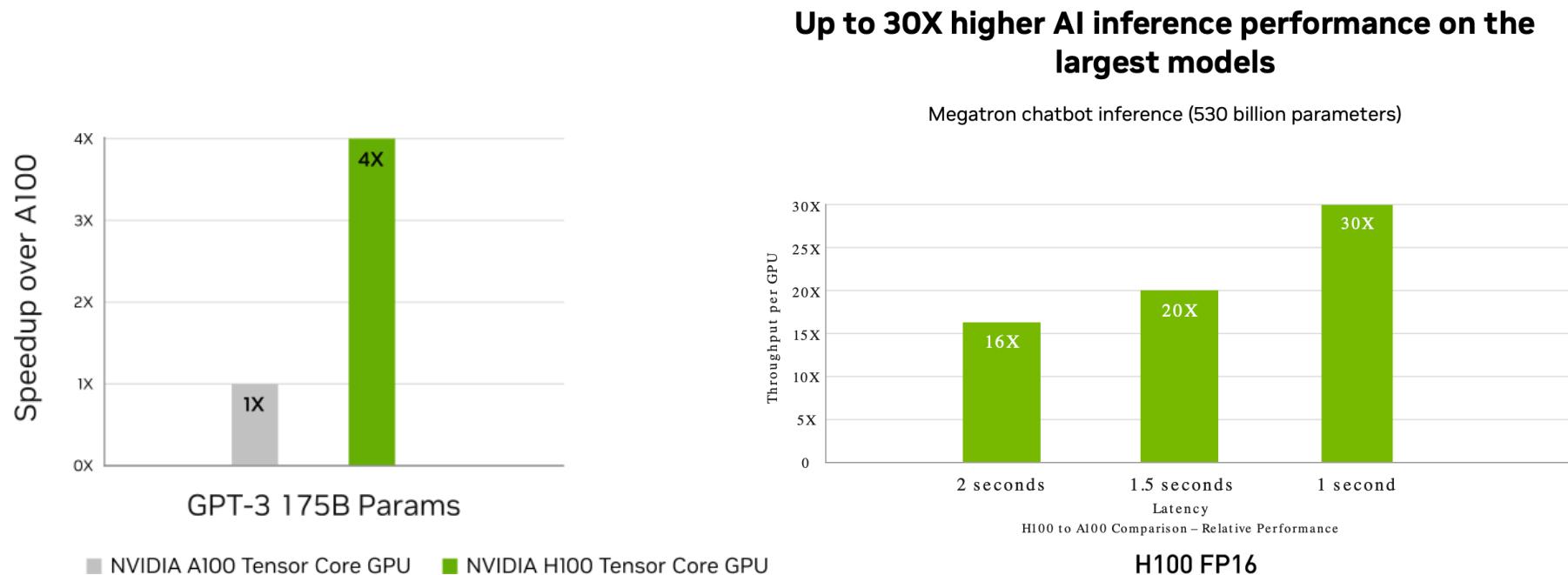
Nakano,
Comput. Phys. Commun.
83, 181 ('94)



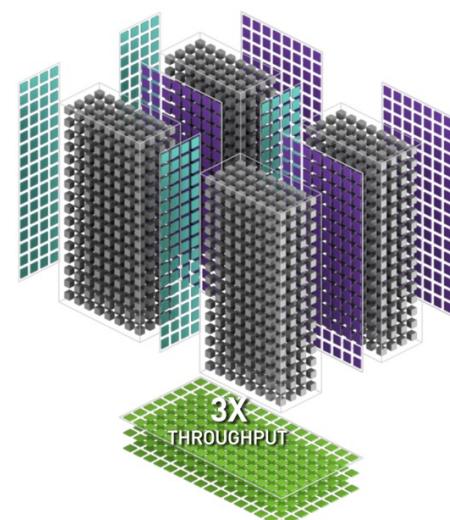
See lecture on [pre-Beowulf parallel computing](#)

New Generation of GPU

- H100 is here: 18,432 CUDA cores & 640 tensor cores



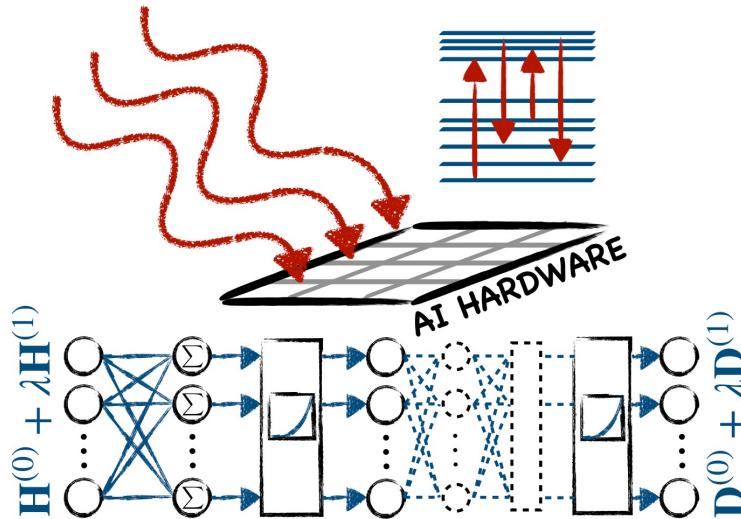
- Unlike general-purpose CUDA cores, tensor cores are specialized processing units designed for (mixed-precision) matrix operations in deep learning



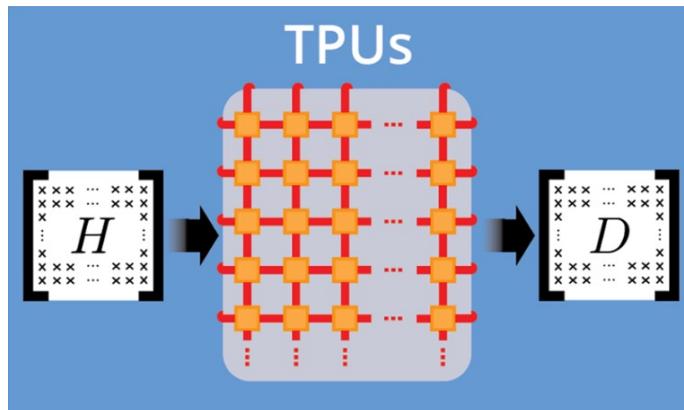
Scientific Tensor Computing?

NVIDIA tensor cores to Google tensor processing unit (TPU) & beyond

- Joshua Finkelstein *et al.*, “Quantum perturbation theory using tensor cores and a deep neural network,” *J. Chem. Theo. Comput.* 18, 4255 (’22)



- Ryan Pederson *et al.*, “Large scale quantum chemistry with tensor processing units,” *J. Chem. Theo. Comput.* 19, 25 (’23)



Tensor processing unit (TPU) is an AI accelerator developed by Google for neural-network machine learning, using Google’s own TensorFlow software

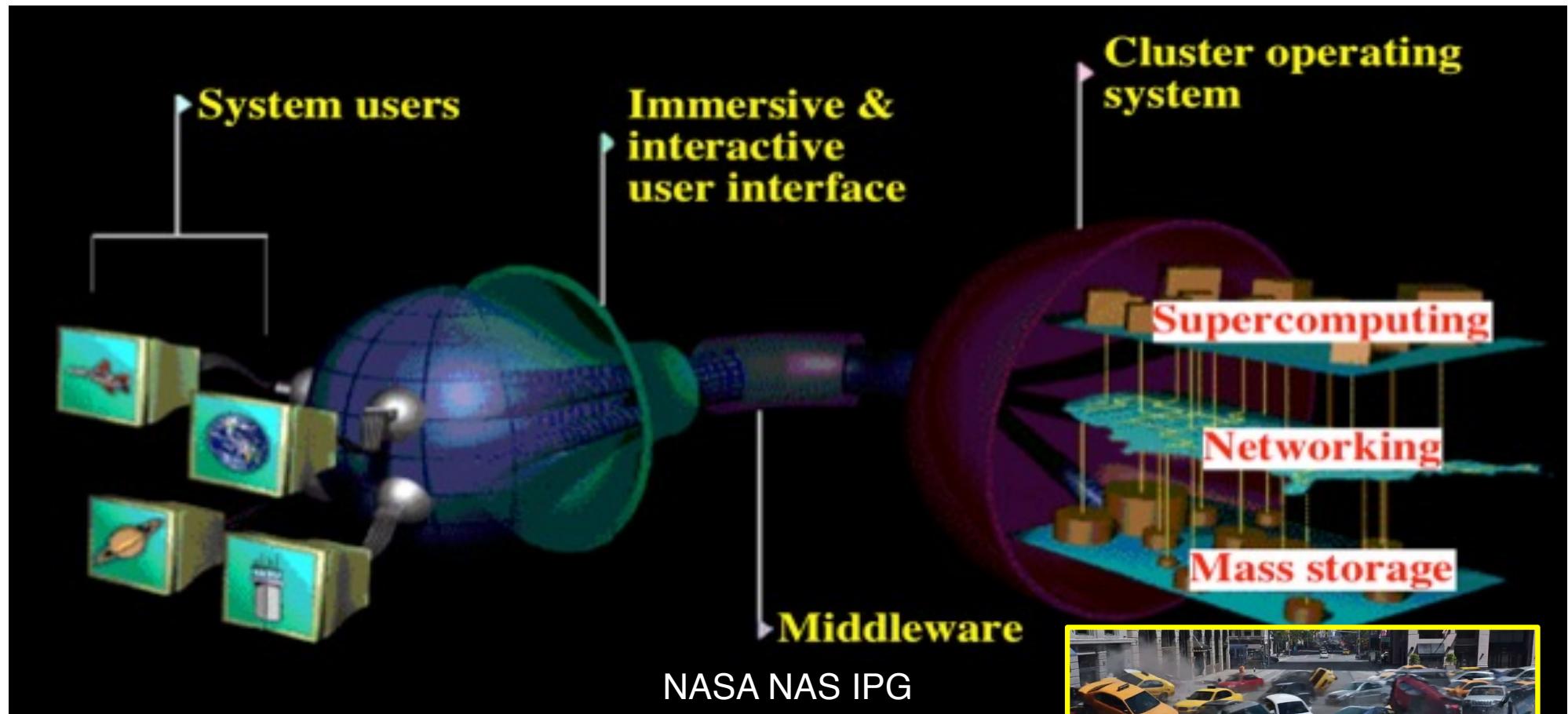
Google Cloud Says TPU-Powered Machine Learning Cluster Delivers 9 Exaflops Aggregate Power

May 12, 2022 by [Doug Black](#)

<https://insidehpc.com>

Grid Computing

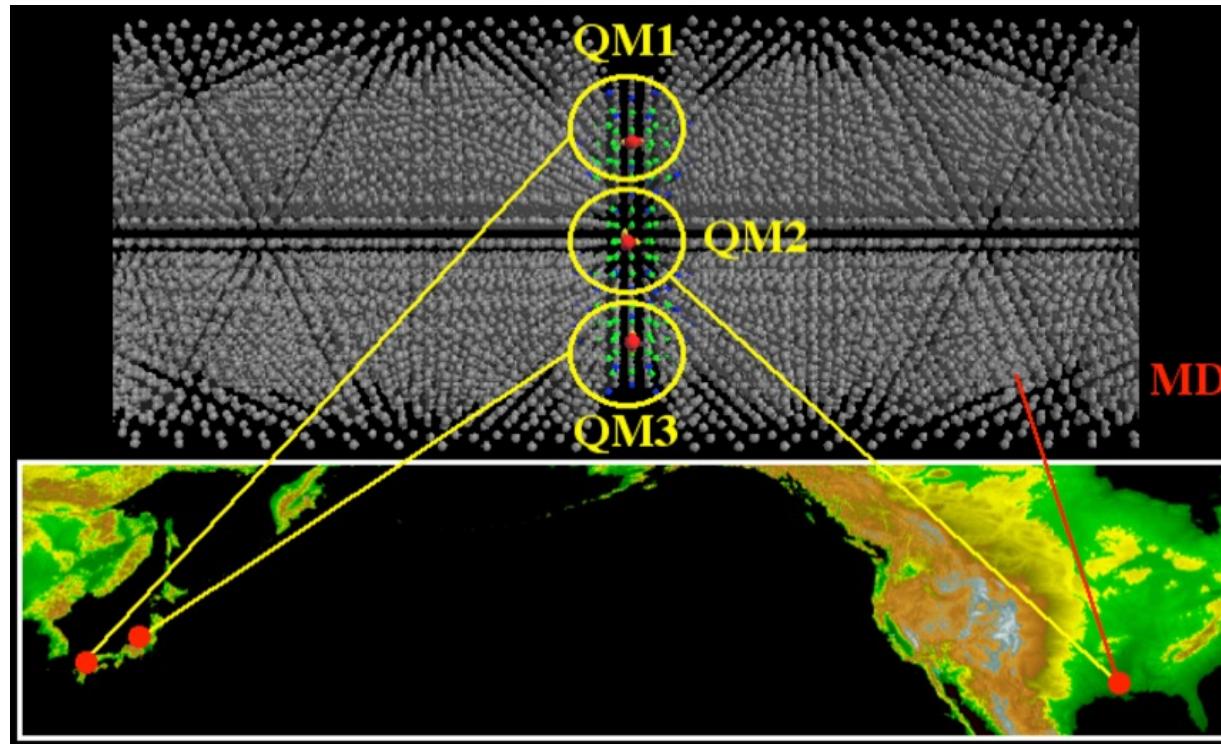
- **World Wide Web:** Universal interface to digital library on the Internet
- **Information Grid:** Pervasive (from any place in the world at any time) access to everything (computing, mass storage, experimental equipments, distributed sensors, etc., on high-speed networks)



Global Collaborative Simulation

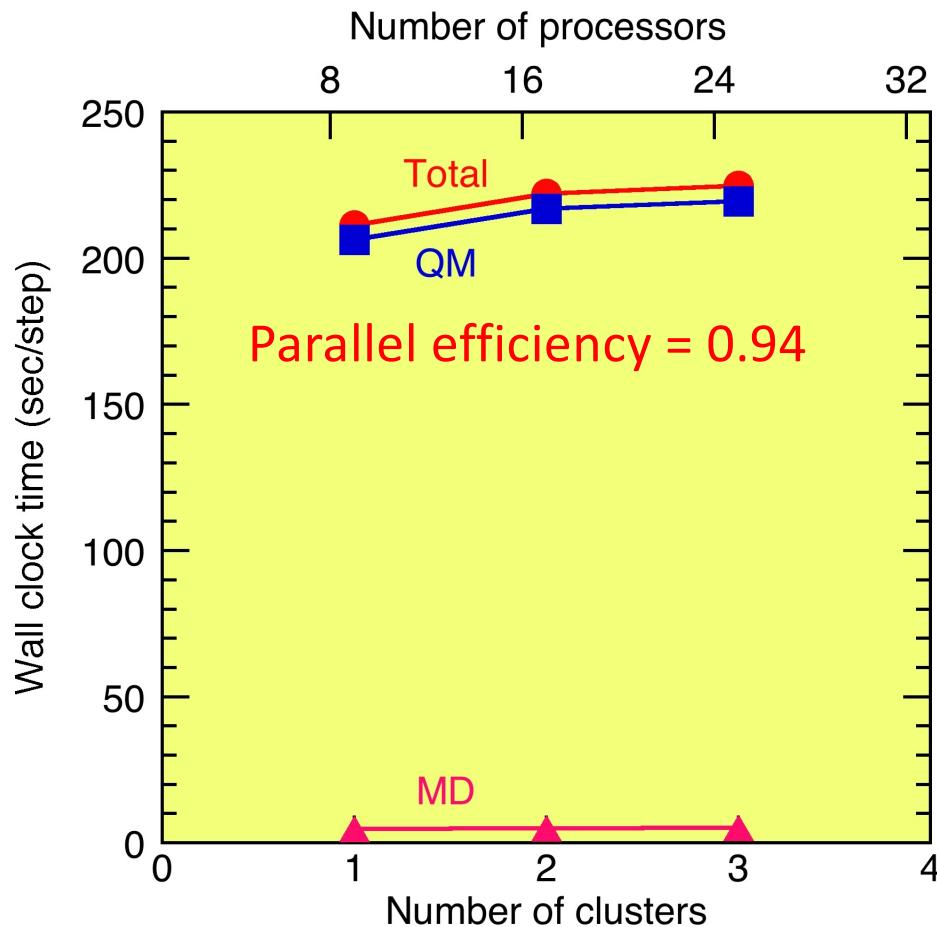
Multiscale MD/QMD simulation on
a Grid of distributed PC clusters in the US & Japan

- Task decomposition (**MPI Communicator**) + spatial decomposition
- **MPICH-G2/Globus**



H. Kikuchi et al., "Collaborative simulation Grid: multiscale quantum-mechanical/classical atomistic simulations on distributed PC clusters in the US & Japan, IEEE/ACM SC02

Computation-Communication Overlap



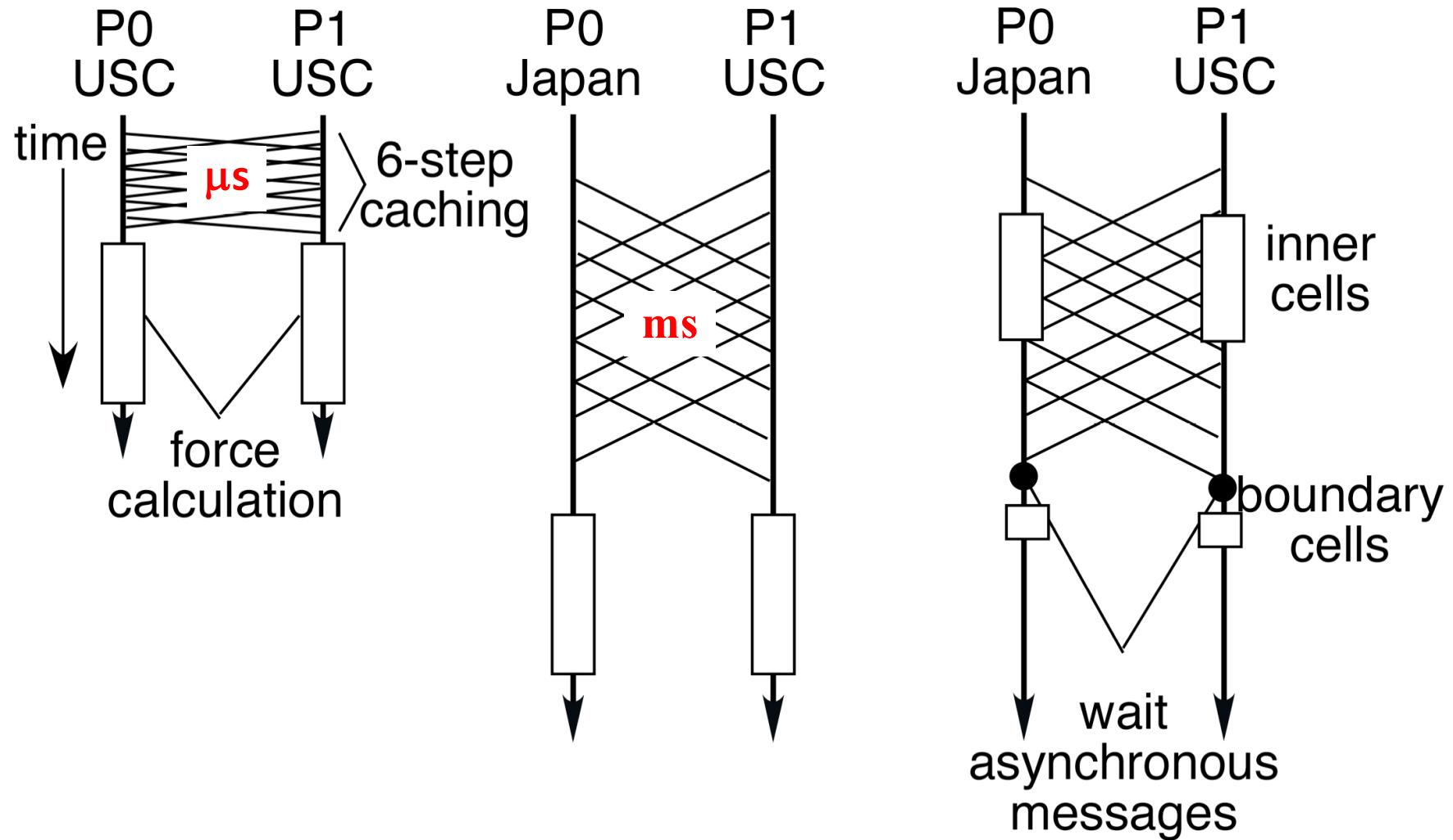
$$\frac{\text{Earth's circumference}}{\text{Light speed}} = \frac{40,000 \text{ [km]}}{3 \times 10^8 \text{ [m/s]}} = 4 \times 10^{-7} \text{ [m]} = 0.1 \text{ s} = 100 \text{ ms}$$

Try on Discovery:
traceroute www.u-tokyo.ac.jp
vs. ping hpc-transfer.usc.edu

- How to overcome 200 ms latency & 1 Mbps bandwidth?
- Computation-communication overlap: To hide the latency, the communications between the MD & QM processors have been overlapped with the computations using asynchronous messages

Internode Optimization

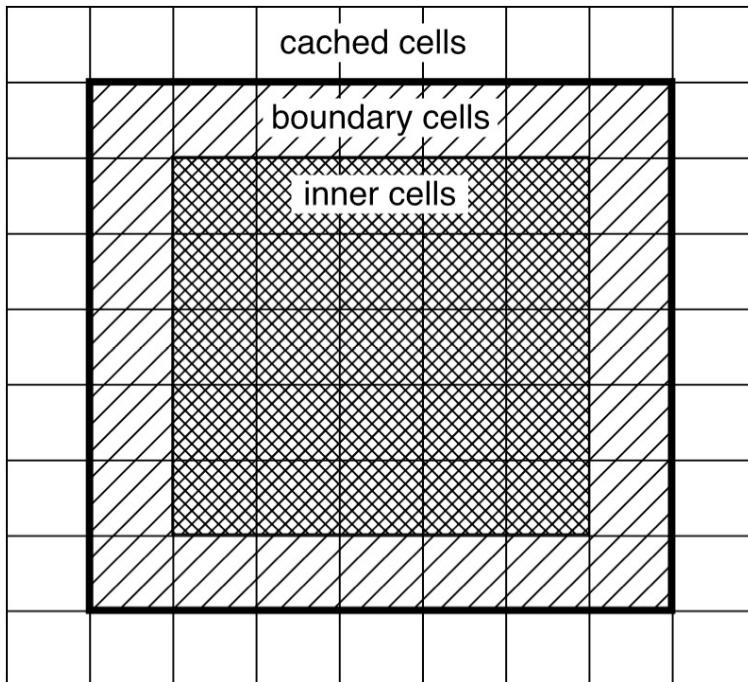
- Communication bottleneck in metacomputing on a Grid



Grid-Enabled MD Algorithm

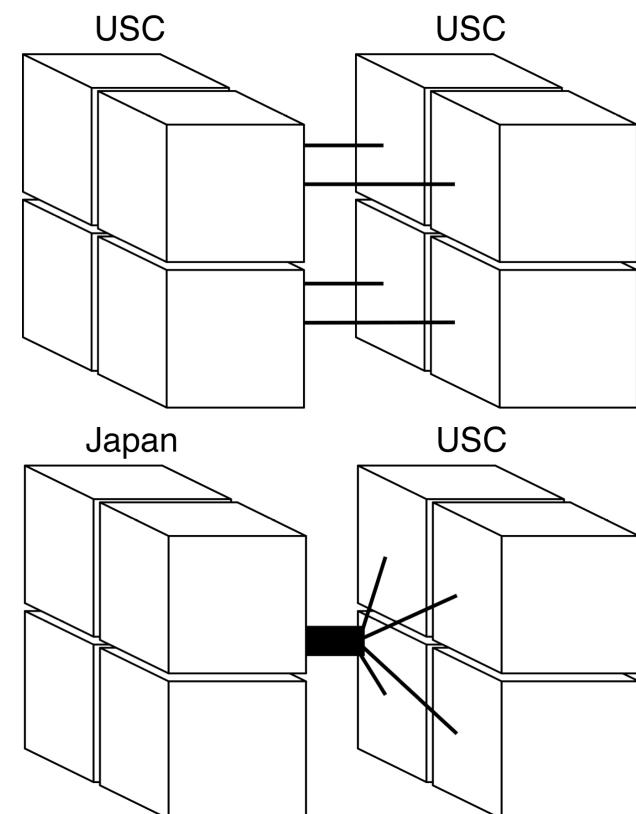
Grid MD algorithm:

1. asynchronous receive of cells to be cached `MPI_Irecv()`
2. send atomic coordinates in the boundary cells
3. compute forces for atoms in the inner cells
4. wait for the completion of the asynchronous receive `MPI_Wait()`
5. compute forces for atoms in the boundary cells



Renormalized Messages:

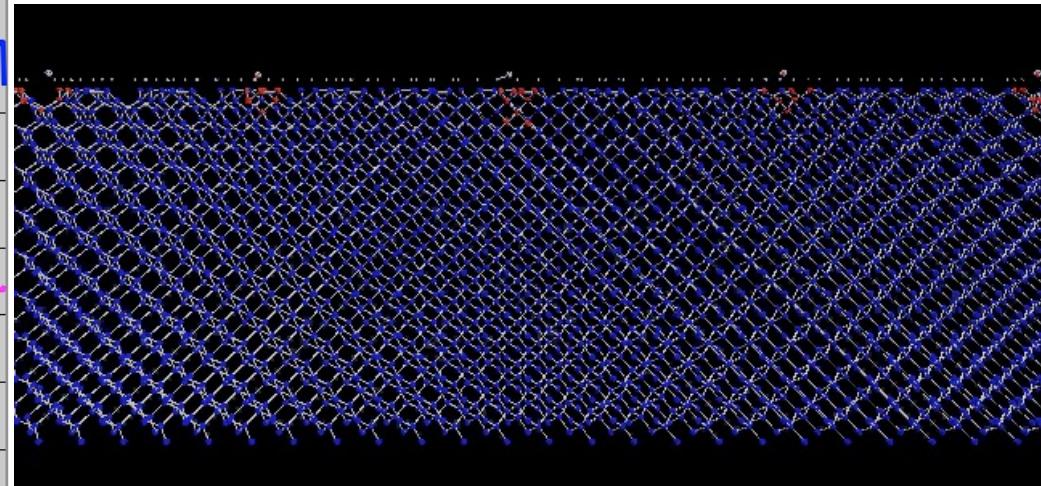
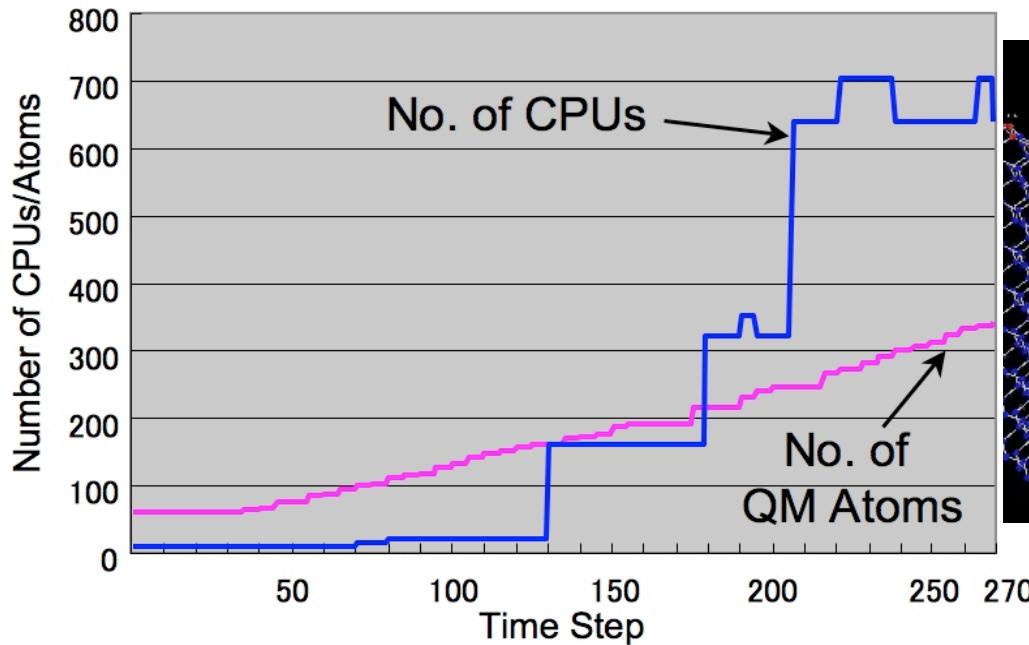
Latency can be reduced by composing a large cross-site message instead of sending all processor-to-processor messages



Sustainable Grid Supercomputing

- Sustained ($>$ months) supercomputing ($> 10^3$ CPUs) on a Grid of geographically distributed supercomputers
- Hybrid Grid remote procedure call (GridRPC) + message passing (MPI) programming
- Dynamic allocation of computing resources on demand & automated migration due to reservation schedule & faults

Ninf-G GridRPC: ninf.apgrid.org; MPICH: www.mcs.anl.gov/mpi



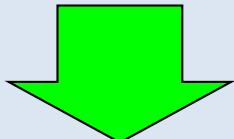
Takemiya et al., IEEE/ACM SC06
Song et al., IJCS ('09)

Multiscale QM/MD simulation of high-energy beam oxidation of Si

Grid Remote Procedure Call (RPC)

- Simple RPC API (application program interface)
- Existing libraries & applications into Grid applications
- IDL (interface definition language) embodying call information, with minimal client-side management

```
double A[n][n],B[n][n],C[n][n]; /* Data Declaration */  
dmmul(n,A,B,C); /* Call local function */
```

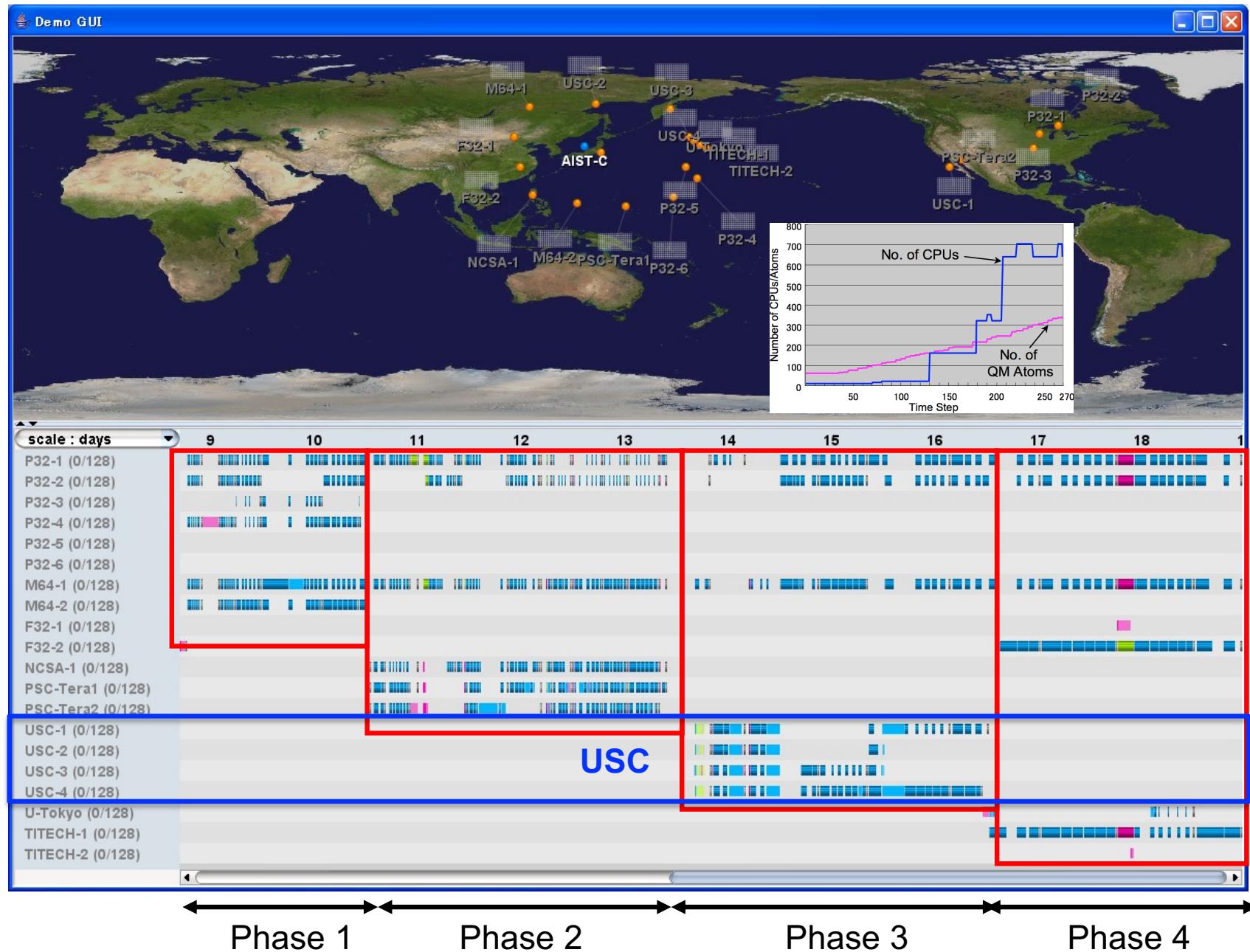


```
grpc_function_handle_default(&hdl, "dmmul");  
grpc_call(hdl,n,A,B,C); /* Call server side routine */
```

- **Ninf–G Grid RPC system**
<http://ninf.apgrid.org>

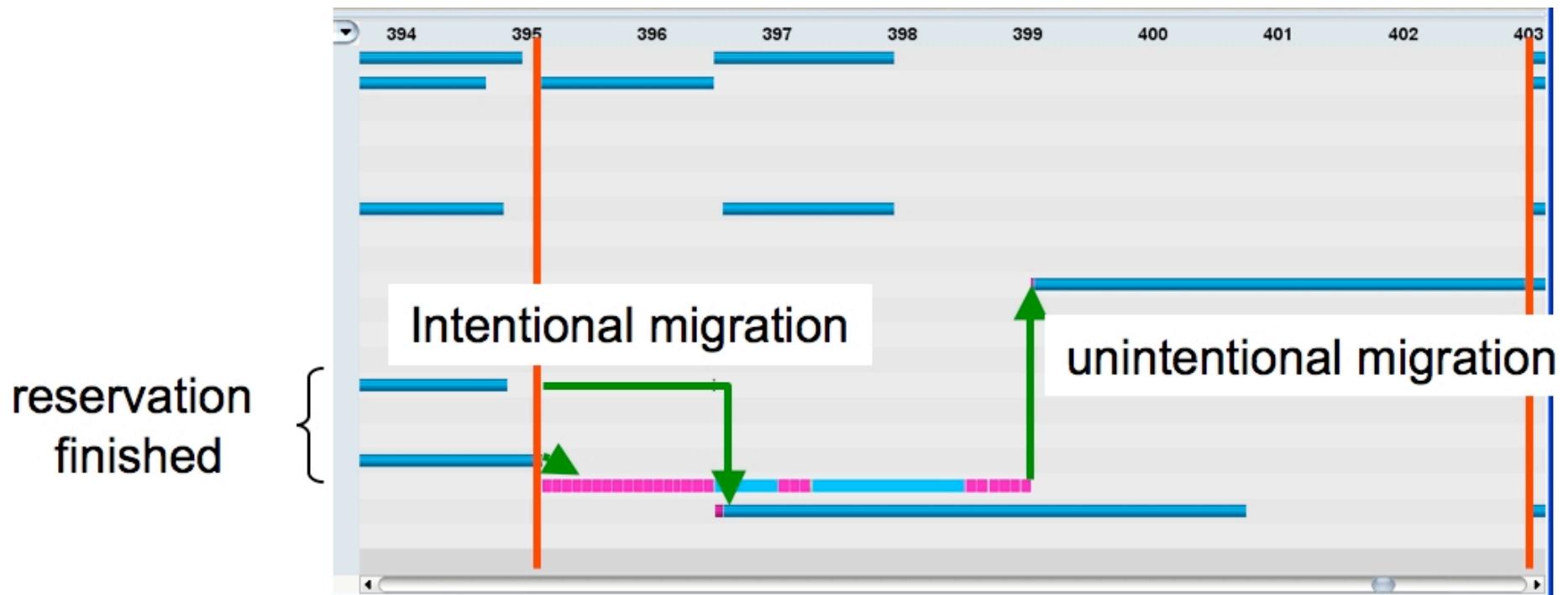


US-Japan Grid Testbed



Fault Tolerance

- Automated migration in response to unexpected faults



Current & Future Supercomputing

- Won two DOE supercomputing awards to develop & deploy metascalable (“design once, scale on future platforms”) simulation algorithms



Innovative & Novel Computational Impact on Theory & Experiment

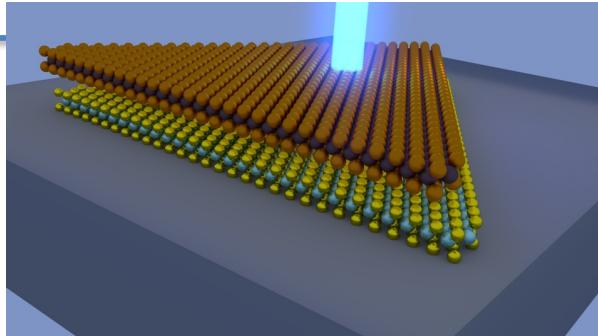
Title: AI-Guided Exascale Simulations of Quantum Materials Manufacturing and Control

PI and Co-PIs: Aiichiro Nakano—PI, Rajiv K. Kalia, Ken-ichi Nomura, Priya Vasishta

- Atomistic simulations on million cores (pre-exascale)



786,432-core IBM Blue Gene/Q
281,088-core Intel Xeon Phi
560-node (2,240-GPU) AMD/NVIDIA Polaris



Early Science Projects for Aurora

Supercomputer Announced

Metascalable layered materials genome

Investigator: Aiichiro Nakano, University of Southern California

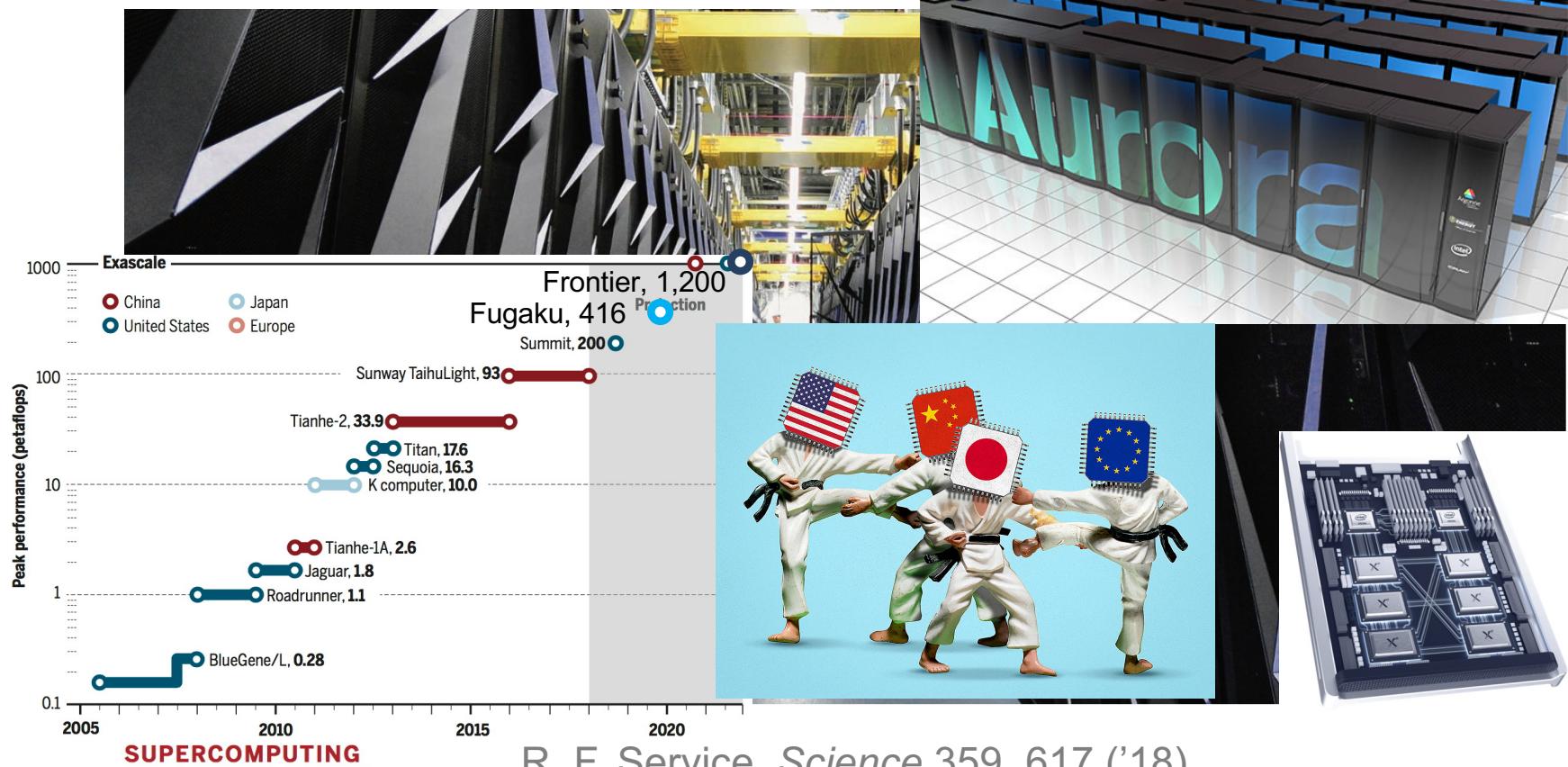


1.01 exaflop/s
Intel Aurora

exaflop/s = 10^{18} mathematical operations per second

- One of the initial simulation users of the next-generation DOE supercomputer

CACS@Aurora in the Global Exascale Race



R. F. Service, Science 359, 617 ('18)

Design for U.S. exascale computer takes shape

Competition with China accelerates plans for next great leap in supercomputing power

Exa(peta)flop/s = 10^{18} (10^{15}) floating-point operations per second

By Robert F. Service

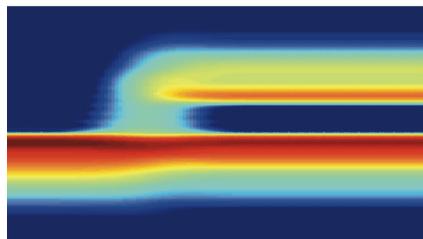
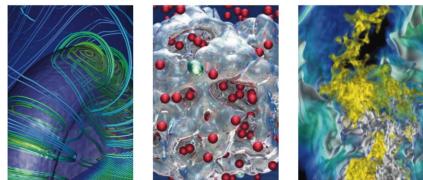
In 1957, the launch of the Sputnik satellite vaulted the Soviet Union to the lead in the space race and galvanized the United States. U.S. supercomputer researchers are today facing their own

Lemont, Illinois. That's 2 years earlier than planned. "It's a pretty exciting time," says Aiichiro Nakano, a physicist at the University of Southern California in Los Angeles who uses supercomputers to model materials made by layering stacks of atomic sheets like graphene.

pace reflects a change of strategy by DOE officials last fall. Initially, the agency set up a "two lanes" approach to overcoming the challenges of an exascale machine, in particular a potentially ravenous appetite for electricity that could require the output of a small nuclear plant.

<https://www.tomshardware.com/news/two-chinese-exascale-supercomputers>

BES



NOVEMBER 3-5, 2015

ROCKVILLE, MARYLAND

Exa-leadership

BASIC ENERGY SCIENCES

EXASCALE REQUIREMENTS REVIEW

An Office of Science review sponsored jointly by
Advanced Scientific Computing Research and Basic Energy Sciences

16,661-atom QMD

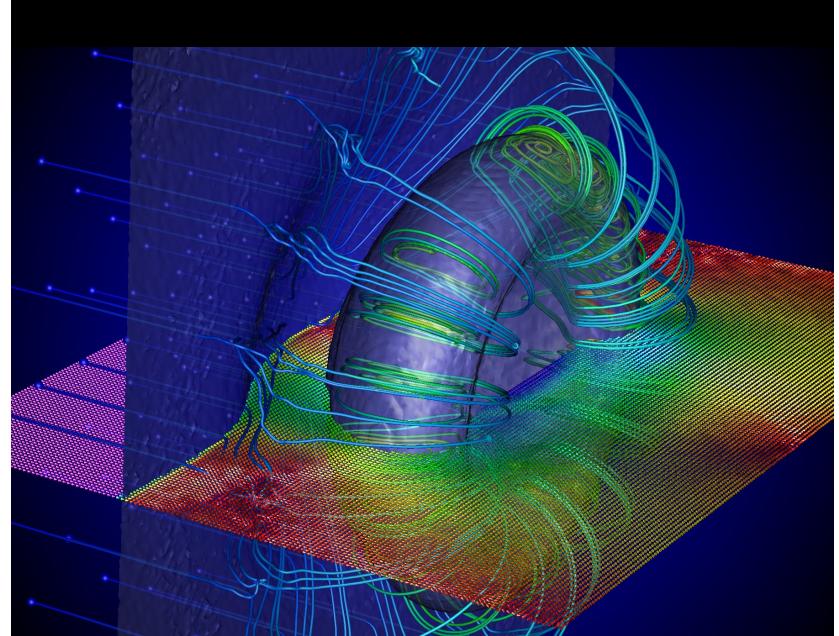
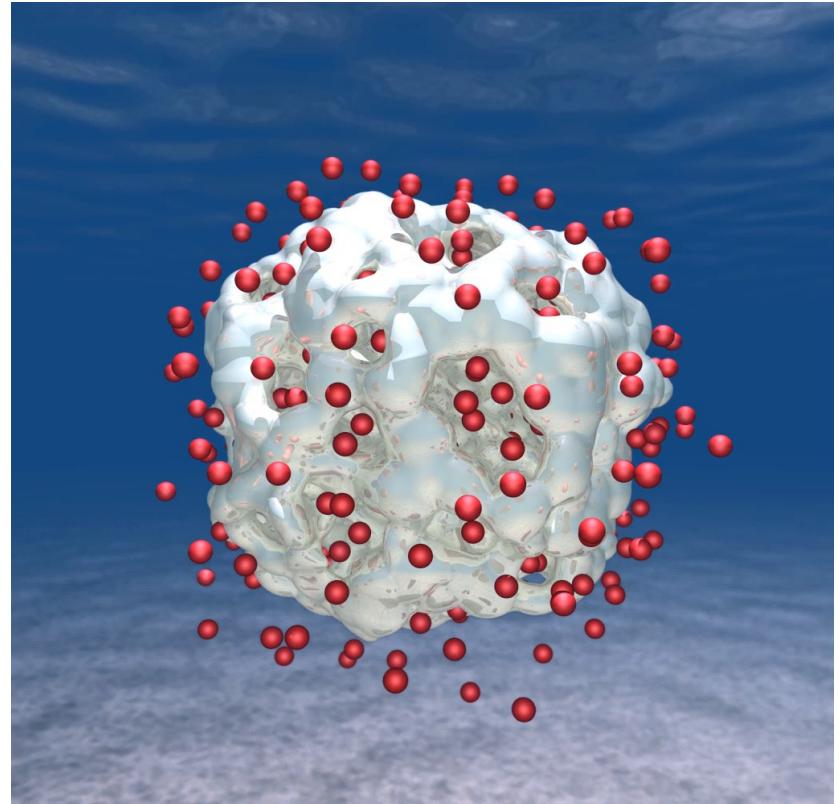
Shimamura *et al.*,
Nano Lett.
14, 4090 ('14)

*On-demand hydrogen
production from water*

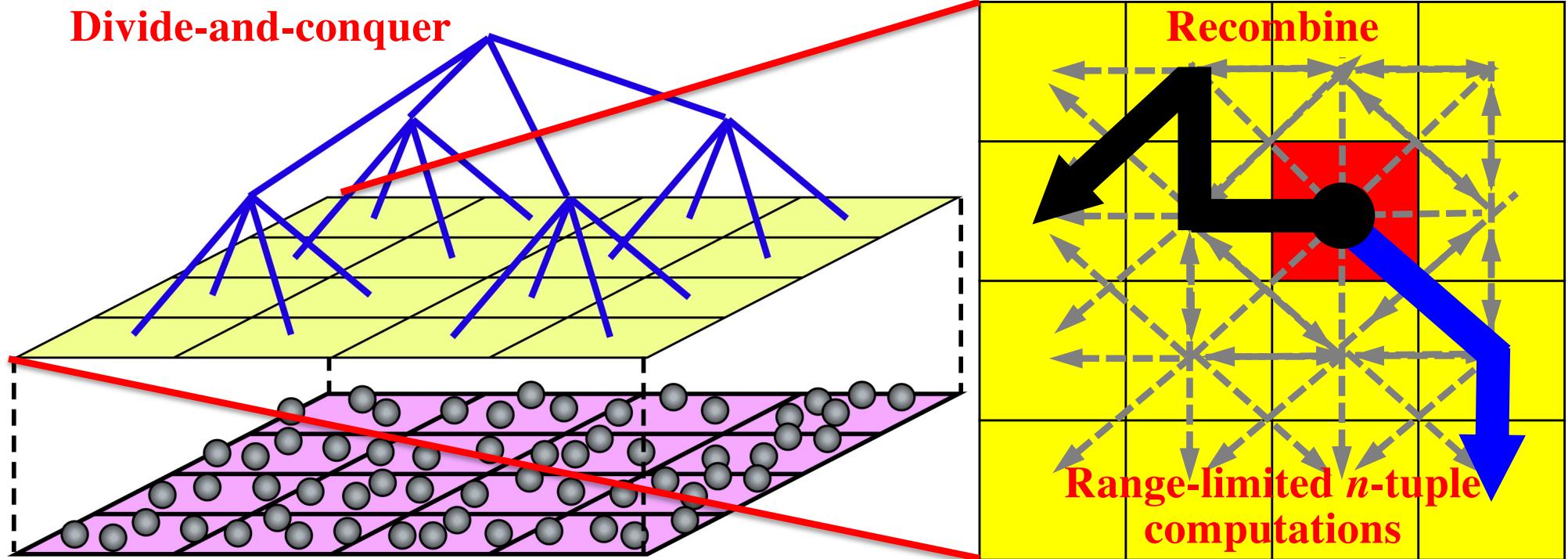
10⁹-atom RMD

Shekhar *et al.*,
Phys. Rev. Lett.
111, 184503 ('13)

*Fluid dynamics
atom-by-atom*



Divide-Conquer-Recombine (DCR) Engines



M. Kunaseth et al., ACM/IEEE SC13

See lecture on “shift-collapse” algorithm

- Lean divide-&-conquer density functional theory (LDC-DFT) algorithm minimizes the prefactor of $O(N)$ computational cost

F. Shimojo et al., *J. Chem. Phys.* 140, 18A529 ('14); K. Nomura et al., *IEEE/ACM SC14*

- Extended-Lagrangian reactive molecular dynamics (XRMD) algorithm eliminates the speed-limiting charge iteration

K. Nomura et al., *Comput. Phys. Commun.* 192, 91 ('15)

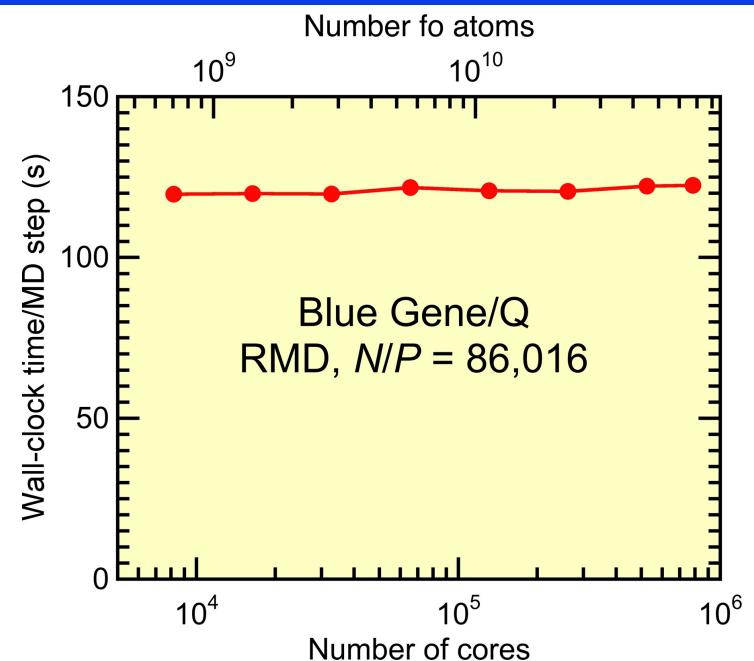
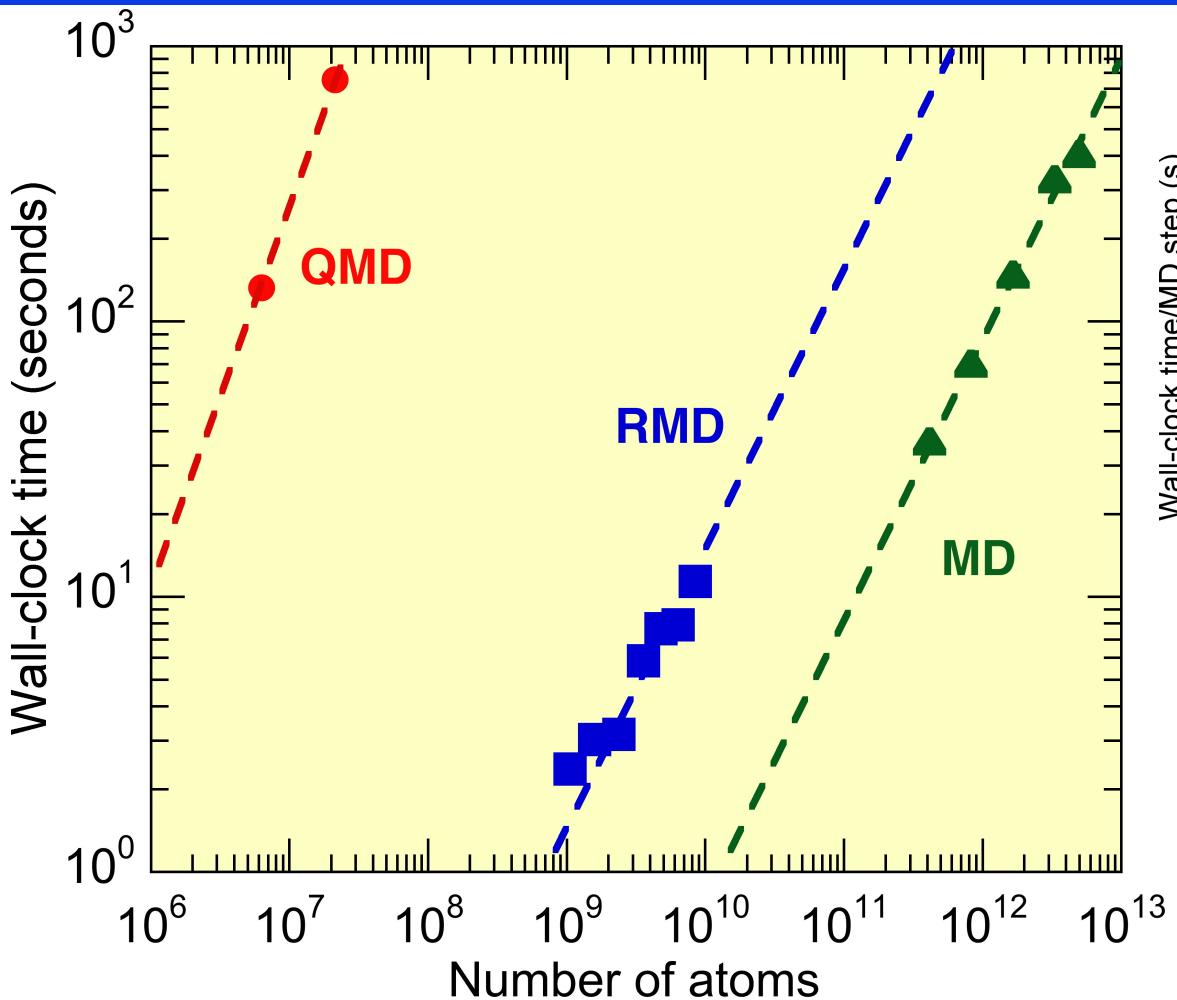
Divide-Conquer-(Re)combine

- “The first was to never accept anything as true which I could not accept as obviously true. The second was to divide each of the problems in as many parts as I should to solve them. The third, beginning with the simplest and easiest to understand matters, little by little, to the most complex knowledge. And the last resolution was to make my enumerations so complete and my reviews so general that I could be assured that I had not omitted anything.” (René Descartes, *Discourse on Method*, 1637)
- 「モデルの分割一再統合の方法の優れた点は、分割した要素的概念を、モデルの理解に役立つように再構成することができ、そこに創造の入り込む余地があるという点にある。」(福井謙一学問の創造、1987)
room for creativity

Kenichi Fukui [Nobel Chemistry Prize, '81]



Scalable Simulation Algorithm Suite



QMD (quantum molecular dynamics): DC-DFT

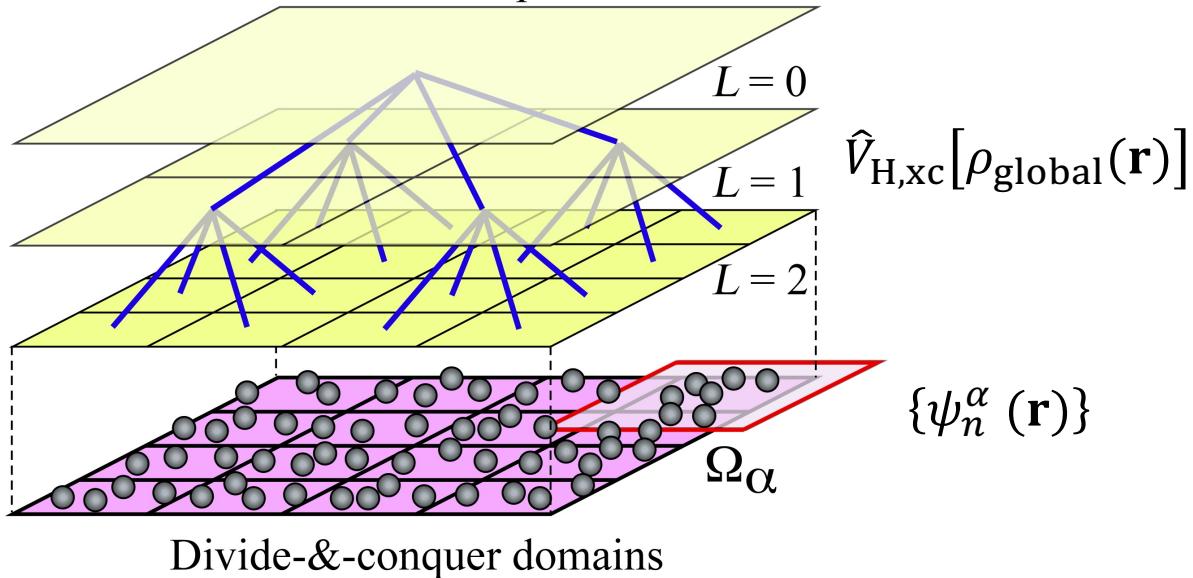
RMD (reactive molecular dynamics): F-ReaxFF

MD (molecular dynamics): MRMD

- 4.9 trillion-atom space-time multiresolution MD (MRMD) of SiO_2
 - 67.6 billion-atom fast reactive force-field (F-ReaxFF) RMD of RDX
 - 39.8 trillion grid points (50.3 million-atom) DC-DFT QMD of SiC
- parallel efficiency 0.984 on 786,432 Blue Gene/Q cores

Divide-&-Conquer Density Functional Theory

Global Kohn-Sham potential



Divide-&-conquer domains

- Overlapping spatial domains: $\Omega = \bigcup_{\alpha} \Omega_{\alpha}$
- Domain Kohn-Sham equations

Global-local
self-consistent
field (SCF)
iteration

$$\left(-\frac{1}{2} \nabla^2 + \hat{V}_{\text{ion}} + \hat{V}_{\text{H,xc}}[\rho_{\text{global}}(\mathbf{r})] \right) \psi_n^\alpha(\mathbf{r}) = \epsilon_n^\alpha \psi_n^\alpha(\mathbf{r})$$

- Global & domain electron densities

$$\rho_{\text{global}}(\mathbf{r}) = \sum_{\alpha} p_{\alpha}(\mathbf{r}) \rho_{\alpha}(\mathbf{r}) \quad \rho_{\alpha}(\mathbf{r}) = \sum_n [\psi_n^\alpha]^2 \Theta(\mu - \epsilon_n^\alpha)$$

Domain support function

$$\sum_{\alpha} p_{\alpha}(\mathbf{r}) = 1$$

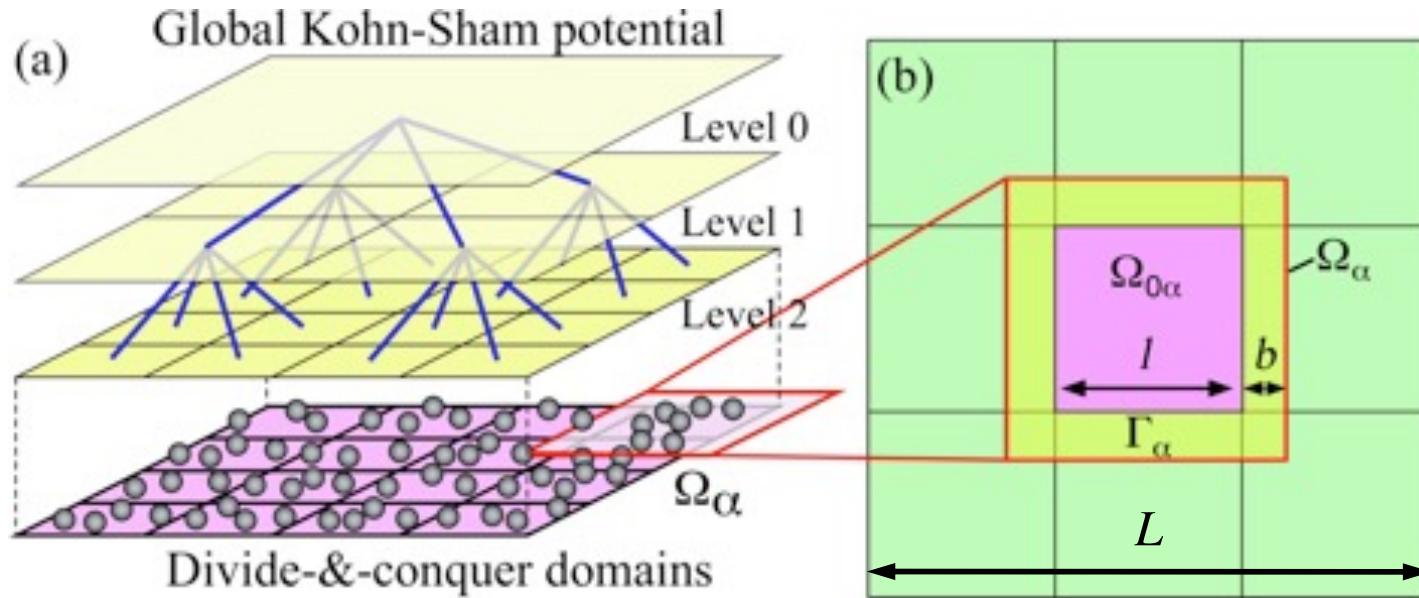
Global chemical potential

$$N = \int d\mathbf{r} \rho_{\text{global}}(\mathbf{r})$$

cf. subsystem DFT [W. Mi et al., *Comput. Phys. Commun.* **269**, 108122 ('21)]

Optimization of Divide-&-Conquer DFT

- Computational parameters of DC-DFT = domain size (l) + buffer thickness (b)



- Complexity analysis to optimize the domain size l

$$l_* = \operatorname{argmin}(T_{\text{comp}}(l)) = \operatorname{argmin} \left(\left(\frac{L}{l}\right)^3 (l + 2b)^{3\nu} \right) = \frac{2b}{\nu - 1}$$

Per-domain computational complexity of DFT = $O(n^\nu)$: $\nu = 2$ or 3 ($n <$ or $> 10^3$)

- Error analysis: Buffer thickness b is dictated by the accuracy requirement

$$b = \lambda \ln (\max \{ |\Delta \rho_\alpha(\mathbf{r})| \mid \mathbf{r} \in \partial \Omega_\alpha \}) / \varepsilon \langle \rho_\alpha(\mathbf{r}) \rangle \quad |\Delta \rho| e^{-b/\lambda} = \varepsilon \langle \rho \rangle$$

Decay length

$\rho_\alpha(\mathbf{r}) - \rho_{\text{global}}(\mathbf{r})$

Error tolerance

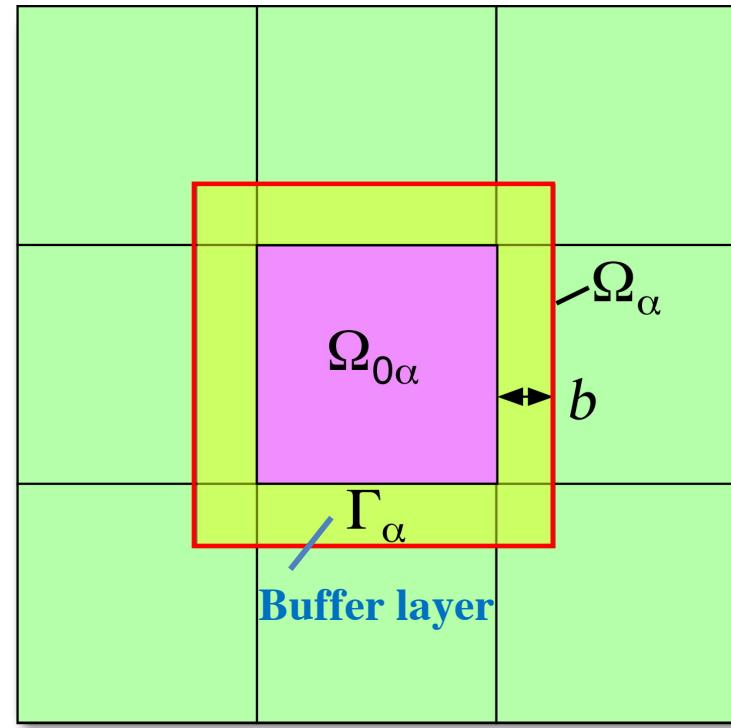
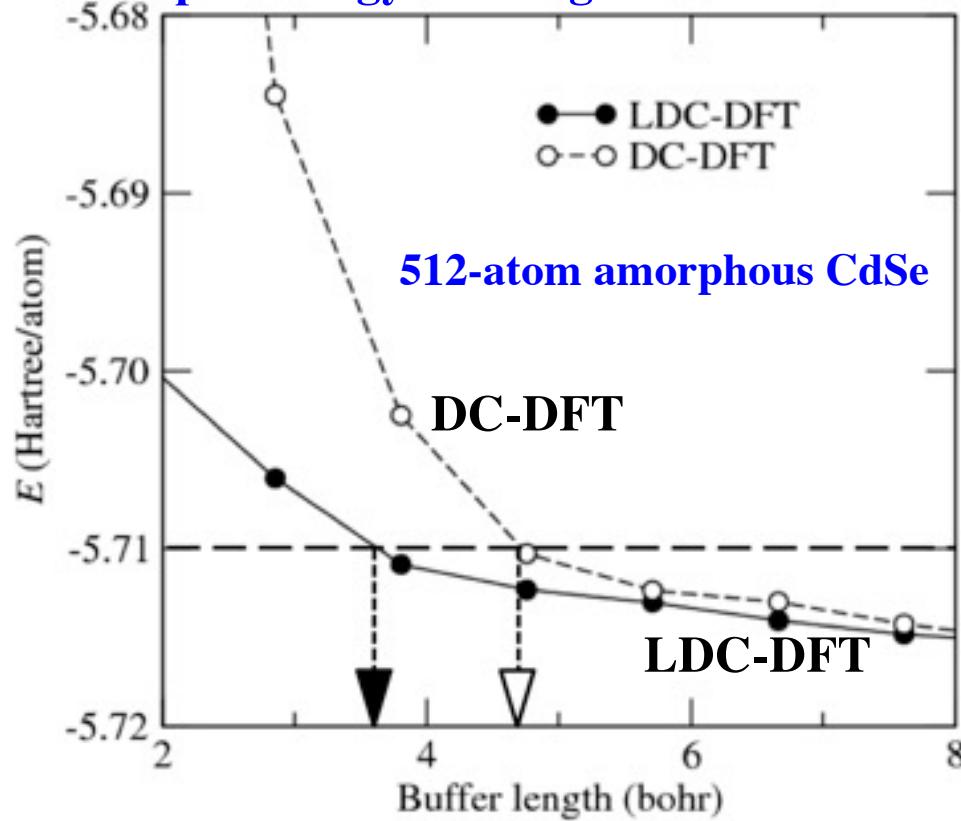
cf. quantum nearsightedness [Kohn, Phys. Rev. Lett. **76**, 3168 ('96); Prodan & Kohn, P. Nat. Acad. Sci. **102**, 11635 ('05)]

Lean Divide-&-Conquer (LDC) DFT

- Density-adaptive boundary potential to reduce the $O(N)$ prefactor local approximation

$$v_{\alpha}^{\text{bc}}(\mathbf{r}) = \int d\mathbf{r}' \frac{\partial v(\mathbf{r})}{\partial \rho(\mathbf{r}')}\left(\rho_{\alpha}(\mathbf{r}') - \rho_{\text{global}}(\mathbf{r}')\right) \cong \frac{\rho_{\alpha}(\mathbf{r}) - \rho_{\text{global}}(\mathbf{r})}{\xi}$$

- More rapid energy convergence of LDC-DFT compared with nonadaptive DC-DFT

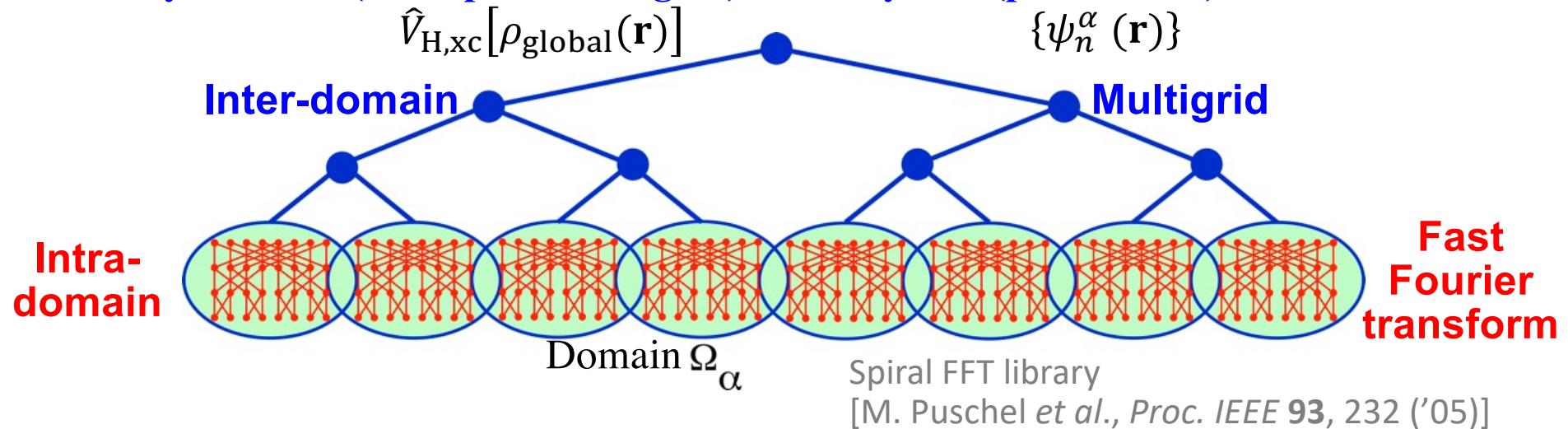


- Factor 2.03 (for $\nu = 2$) ~ 2.89 (for $\nu = 3$) reduction of the computational cost with an error tolerance of 5×10^{-3} a.u. (per-domain complexity: n^{ν})

F. Shimojo et al., *J. Chem. Phys.* **140**, 18A529 ('14);
Phys. Rev. B **77**, 085103 ('08); *Comput. Phys. Commun.* **167**, 151 ('05)

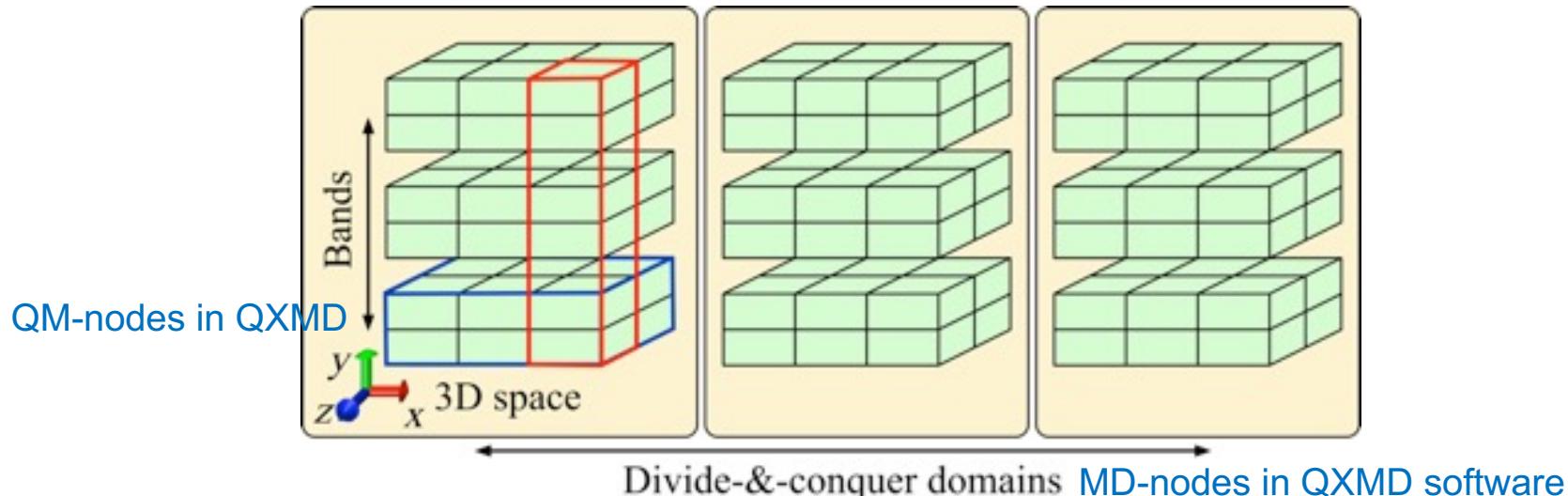
Hierarchical Computing

- Globally scalable (real-space multigrid) + locally fast (plane wave) electronic solver



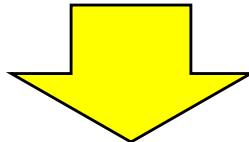
cf. globally- sparse-yet-locally-dense eigensolver [J. H. Lam et al., *Nature Commun.* **15**, 3479 ('24)]

- Hierarchical band (*i.e.*, Kohn-Sham orbital) + space + domain (BSD) decomposition



Key Skill: Scalability Analysis

- Parallel computing = Solving a big problem (W) in a short time (T) using many processors (P)



- How W , T & P scale with each other?
- How to define the efficiency of a parallel program?

See [Gramma et al.](#),
Chap. 5—Analytical modeling of parallel programs

Parallel Efficiency

- Execution time: $T(W,P)$
 W : Workload
 P : Number of processors

- Speed: $S(W,P) = \frac{W}{T(W,P)}$

- Speedup: $S_P = \frac{S(W_P, P)}{S(W_1, 1)} = \frac{W_P T(W_1, 1)}{W_1 T(W_P, P)}$

- Efficiency: $E_P = \frac{S_P}{P} = \frac{W_P T(W_1, 1)}{P W_1 T(W_P, P)}$

Ideal speedup

How to scale W_P with P ?

See <https://aiichironakano.github.io/cs596.html>

Fixed Problem-Size Scaling

$W_P = W$ —constant (strong scaling)

- Speedup:

$$S_P = \frac{T(W,1)}{T(W,P)}$$

- Efficiency:

$$E_P = \frac{T(W,1)}{PT(W,P)}$$

$$S_P = \frac{S(W_P, P)}{S(W_1, 1)} = \frac{W_P T(W_1, 1)}{W_1 T(W_P, P)}$$

$$E_P = \frac{S_P}{P} = \frac{W_P T(W_1, 1)}{P W_1 T(W_P, P)}$$

Solving the same problem faster using more processors!

- Amdahl's law: f (= sequential fraction of the workload) limits the asymptotic speedup

$$S_P = \frac{T(W,1)}{T(W,P)} \leq P$$

$$\begin{aligned} T(W,P) &= fT(W,1) + \frac{(1-f)T(W,1)}{P} \\ \therefore S_P &= \frac{T(W,1)}{T(W,P)} = \frac{1}{f + (1-f)/P} \\ \therefore S_P &\rightarrow \frac{1}{f} \quad (P \rightarrow \infty) \end{aligned}$$

Isogranular Scaling

$W_P = Pw$ (weak scaling)

w = constant workload per processor (granularity)

- Speedup:
$$S_P = \frac{S(P \bullet w, P)}{S(w, 1)} = \frac{P \bullet w / T(P \bullet w, P)}{w / T(w, 1)} = \frac{P \bullet T(w, 1)}{T(P \bullet w, P)}$$

- Efficiency:
$$E_P = \frac{S_P}{P} = \frac{T(w, 1)}{T(P \bullet w, P)}$$

$$S_P = \frac{S(W_P, P)}{S(W_1, 1)} = \frac{W_P T(W_1, 1)}{W_1 T(W_P, P)}$$

$$E_P = \frac{S_P}{P} = \frac{W_P T(W_1, 1)}{P W_1 T(W_P, P)}$$

*Solving larger problems within the same time
using more processors!*

$$E_P = \frac{T(w, 1)}{T(Pw, P)} \leq 1$$

Analysis of Parallel MD

- Parallel execution time:

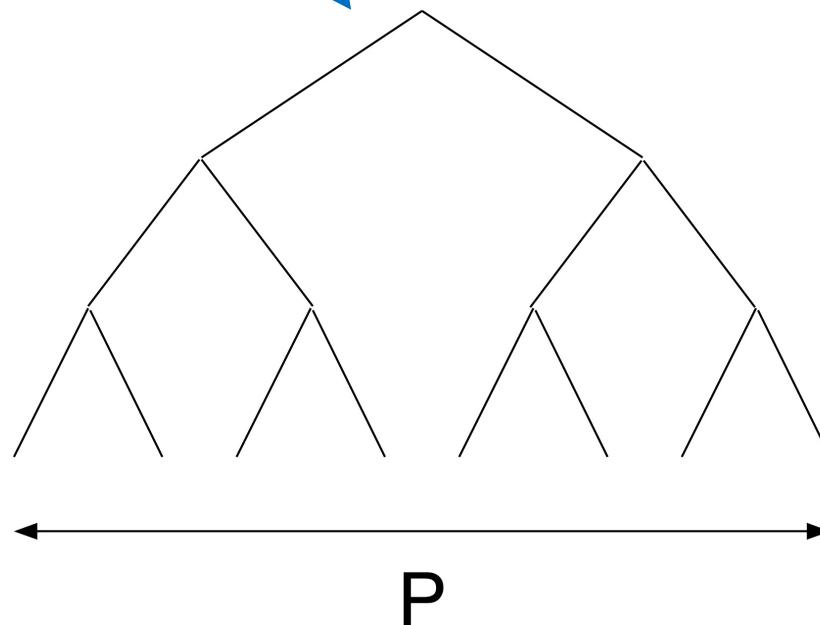
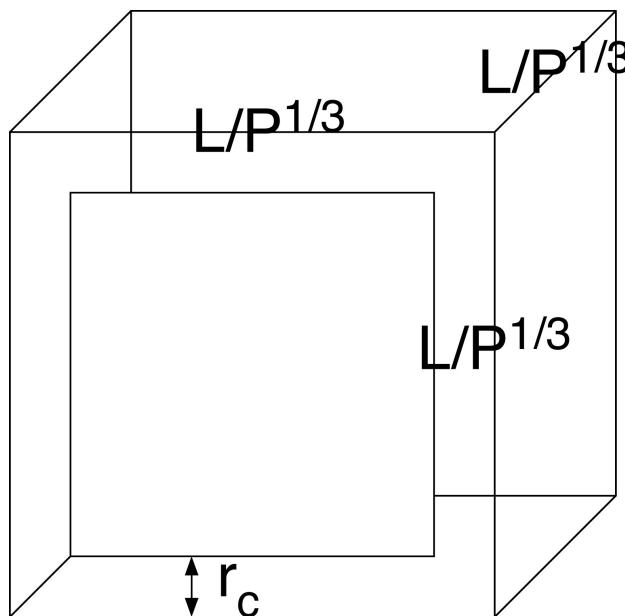
Workload \propto Number of atoms, N (linked-list cell algorithm)

$$T(N, P) = T_{\text{comp}}(N, P) + T_{\text{comm}}(N, P) + T_{\text{global}}(P)$$

$$= a \frac{N}{P} + b \left(\frac{N}{P} \right)^{2/3} + c \log P$$

`MPI_Allreduce()`

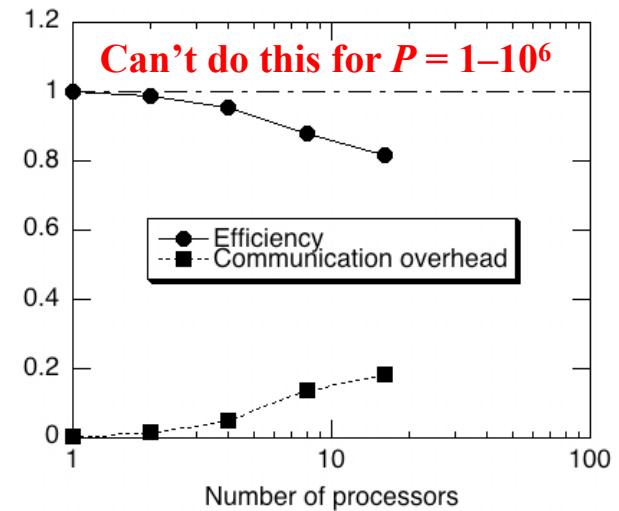
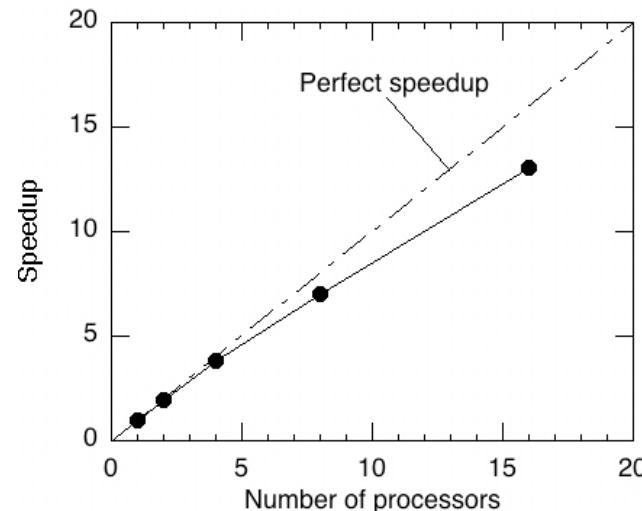
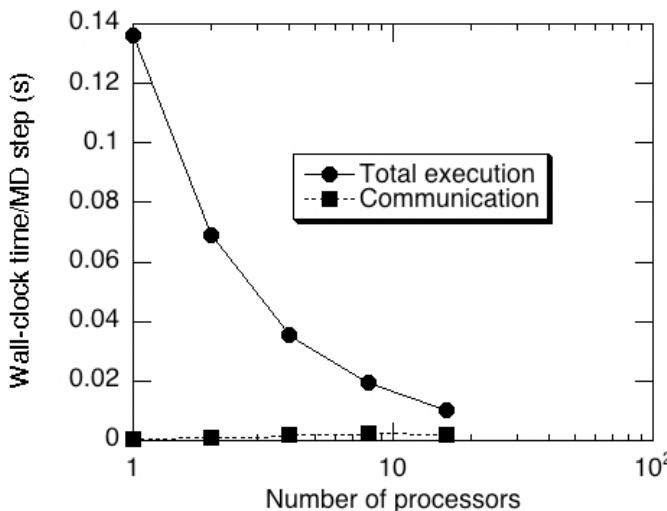
$$\begin{aligned} & \text{facets } \overbrace{6}^{\text{cached volume}} \times \overbrace{\frac{L^2}{P^{2/3}} r_c}^{\text{atom density}} \\ & = 6r_c \frac{N^{2/3}/\rho^{2/3}}{P^{2/3}} \rho \\ & = 6r_c \rho^{1/3} \left(\frac{N}{P} \right)^{2/3} \end{aligned}$$



Fixed Problem-Size Scaling

- Speedup:

$$S_P = \frac{T(N,1)}{T(N,P)} = \frac{\frac{aN}{P}}{aN/P + b(N/P)^{2/3} + c \log P}$$
$$= \frac{P}{1 + \frac{b}{a} \left(\frac{P}{N} \right)^{1/3} + \frac{c}{a} \frac{P \log P}{N}}$$
$$E_P = \frac{S_P}{P} = \frac{1}{1 + \frac{b}{a} \left(\frac{P}{N} \right)^{1/3} + \frac{c}{a} \frac{P \log P}{N}}$$

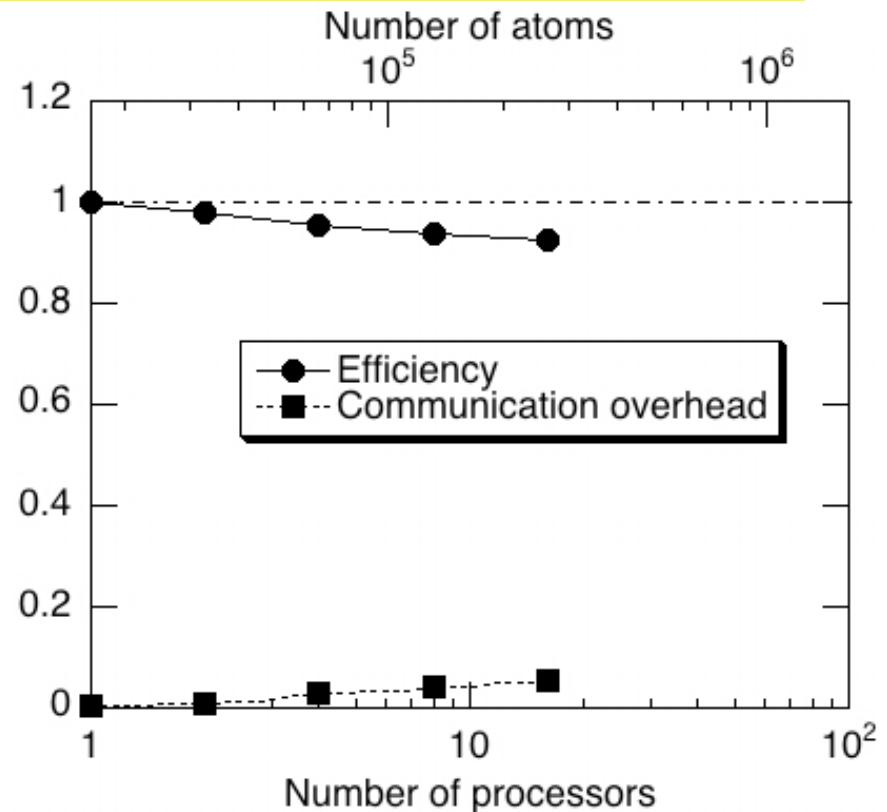
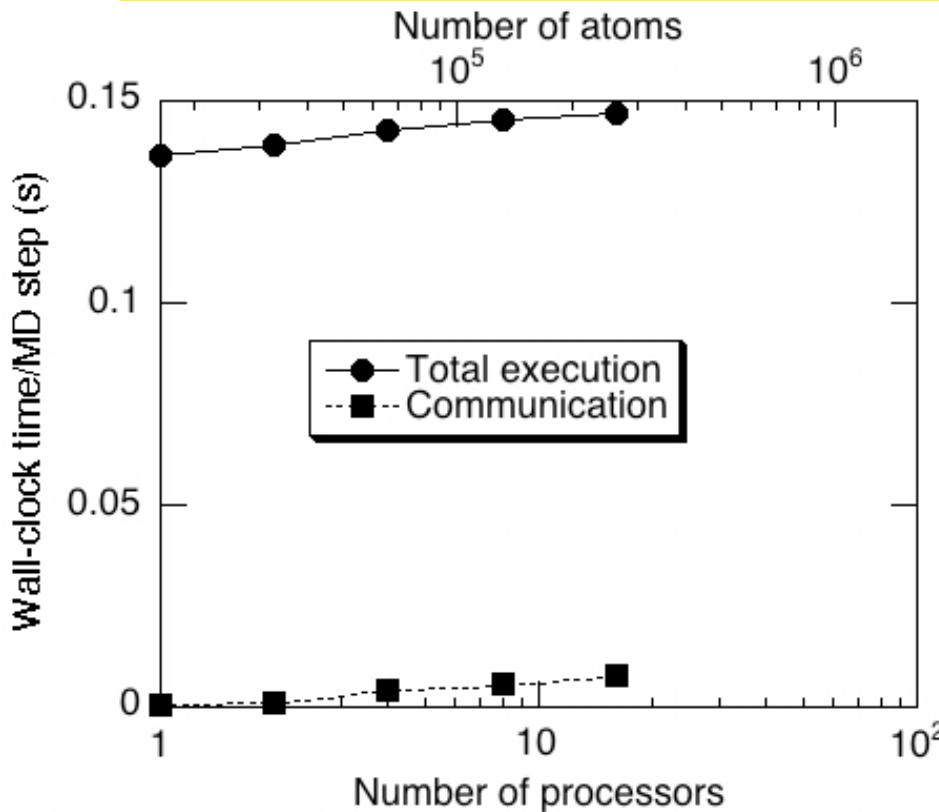


pmd.c: $N = 16,384$, on HPC (predecessor of CARC)

Isogranular Scaling of Parallel MD

- $n = N/P = \text{constant}$: doable for arbitrarily large P
- Efficiency:

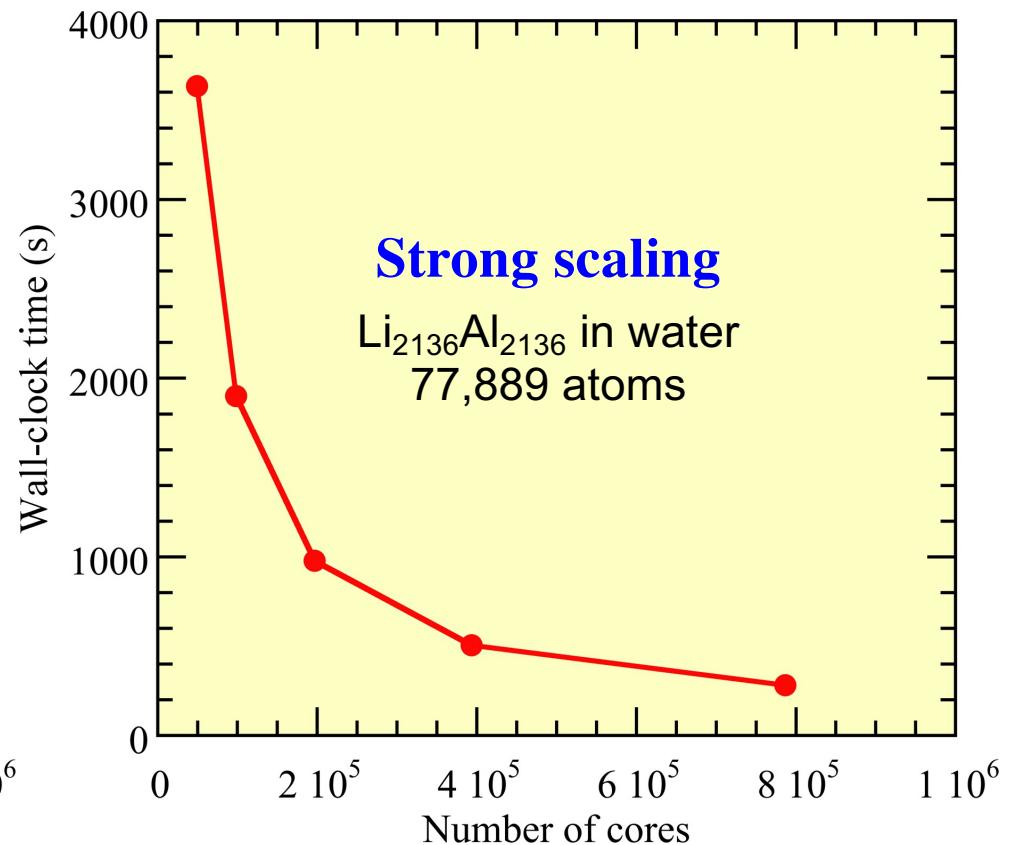
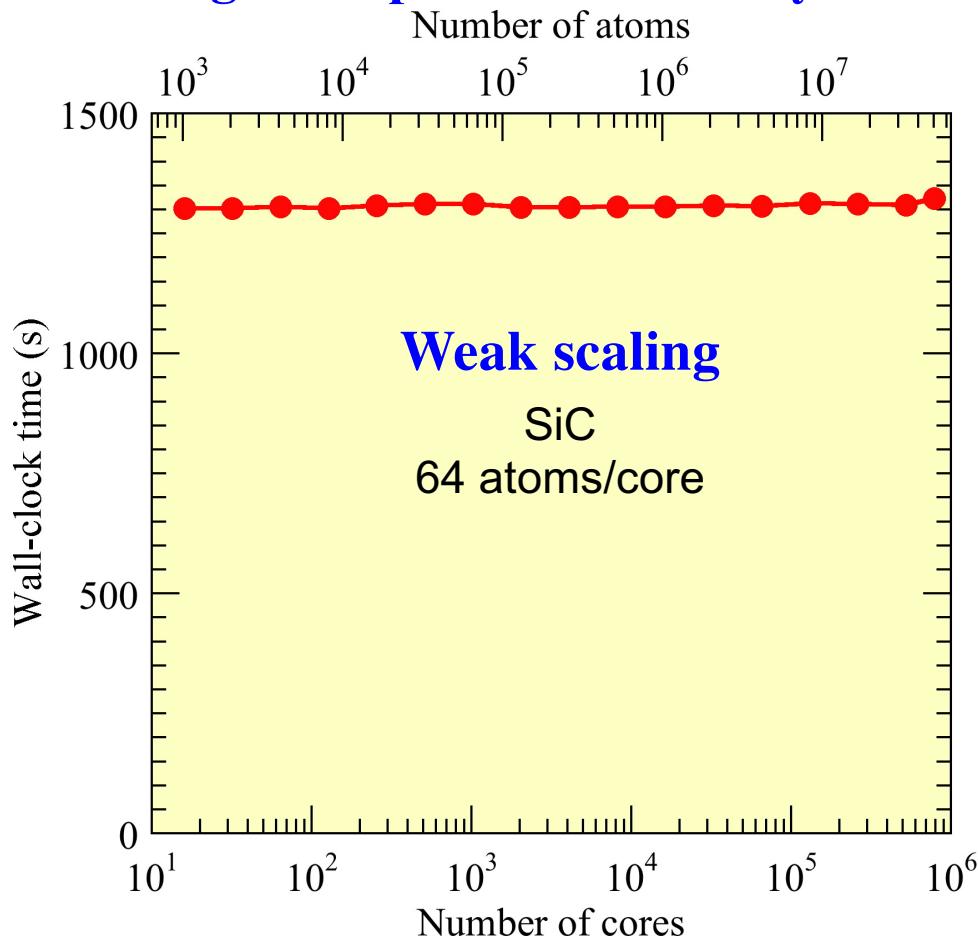
$$E_P = \frac{T(n,1)}{T(nP,P)} = \frac{an}{an + bn^{2/3} + c \log P} = \frac{1}{1 + \frac{b}{a} n^{-1/3} + \frac{c}{an} \log P}$$



pmd.c: $N/P = 16,384$, on HPC (predecessor of CARC)

Parallel Performance of QXMD

- Weak-scaling parallel efficiency is 0.984 on 786,432 Blue Gene/Q cores for a 50,331,648-atom SiC system
- Strong-scale parallel efficiency is 0.803 on 786,432 Blue Gene/Q cores



- 62-fold reduction of time-to-solution [441 s/SCF-step for 50.3M atoms] from the previous state-of-the-art [55 s/SCF-step for 102K atoms, Osei-Kuffuor *et al.*, PRL '14]

BLASification

- Transform from band-by-band to all-band computations to utilize a matrix-matrix subroutine (DGEMM) in the level 3 basic linear algebra subprograms (BLAS3) library
- Algebraic transformation of computations

Example: Nonlocal pseudopotential operation

D. Vanderbilt, *Phys. Rev. B* **41**, 7892 ('90)

$$\hat{v}_{\text{nl}}|\psi_n^\alpha\rangle = \sum_I^{N_{\text{atom}}} \sum_{ij}^{L_{\max}} |\beta_{i,I}\rangle D_{ij,I} \langle \beta_{j,I}| \psi_n^\alpha \rangle \quad (n = 1, \dots, N_{\text{band}})$$



$$\Psi = [|\psi_1^\alpha\rangle, \dots, |\psi_{N_{\text{band}}}^\alpha\rangle] \quad \widetilde{\mathbf{B}}(i) = [|\beta_{i,1}\rangle, \dots, |\beta_{i,N_{\text{atom}}}\rangle] \quad [\widetilde{\mathbf{D}}(i,j)]_{I,J} = D_{ij,I} \delta_{IJ}$$

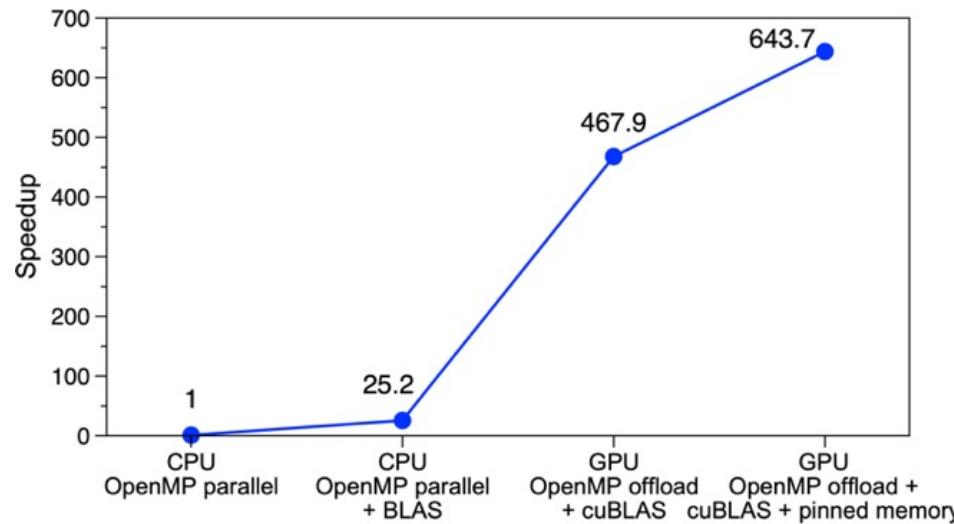
$$\hat{v}_{\text{nl}}\Psi = \sum_{i,j}^L \widetilde{\mathbf{B}}(i) \widetilde{\mathbf{D}}(i,j) \widetilde{\mathbf{B}}(j)^T$$

- **50.5% of the theoretical peak FLOP/s performance on 786,432 Blue Gene/Q cores (entire Mira at the Argonne Leadership Computing Facility)**
- **55% of the theoretical peak FLOP/s on Intel Xeon E5-2665**

BLASified Local Field Dynamics (LFD)

- BLASified nonlocal LFD on graphics processing unit (GPU): Operation of nonlocal potential is projected onto *a vector space spanned by Kohn-Sham orbitals* at time 0 within the real-time scissor approximation [Wang et al., *J. Phys. Condens. Mat.* **31**, 214002 ('19)], making it dense matrix operations implemented with highly optimized level3 (or matrix-matrix) BLAS (basic linear algebra subprogram) library on GPU

$$\hat{v}_{\text{nl}}|\psi_n(t)\rangle \cong \Delta_{\text{sci}} \sum_{m \geq \text{LUMO}} |\psi_m\rangle\langle\psi_m|\psi_n(t)\rangle$$

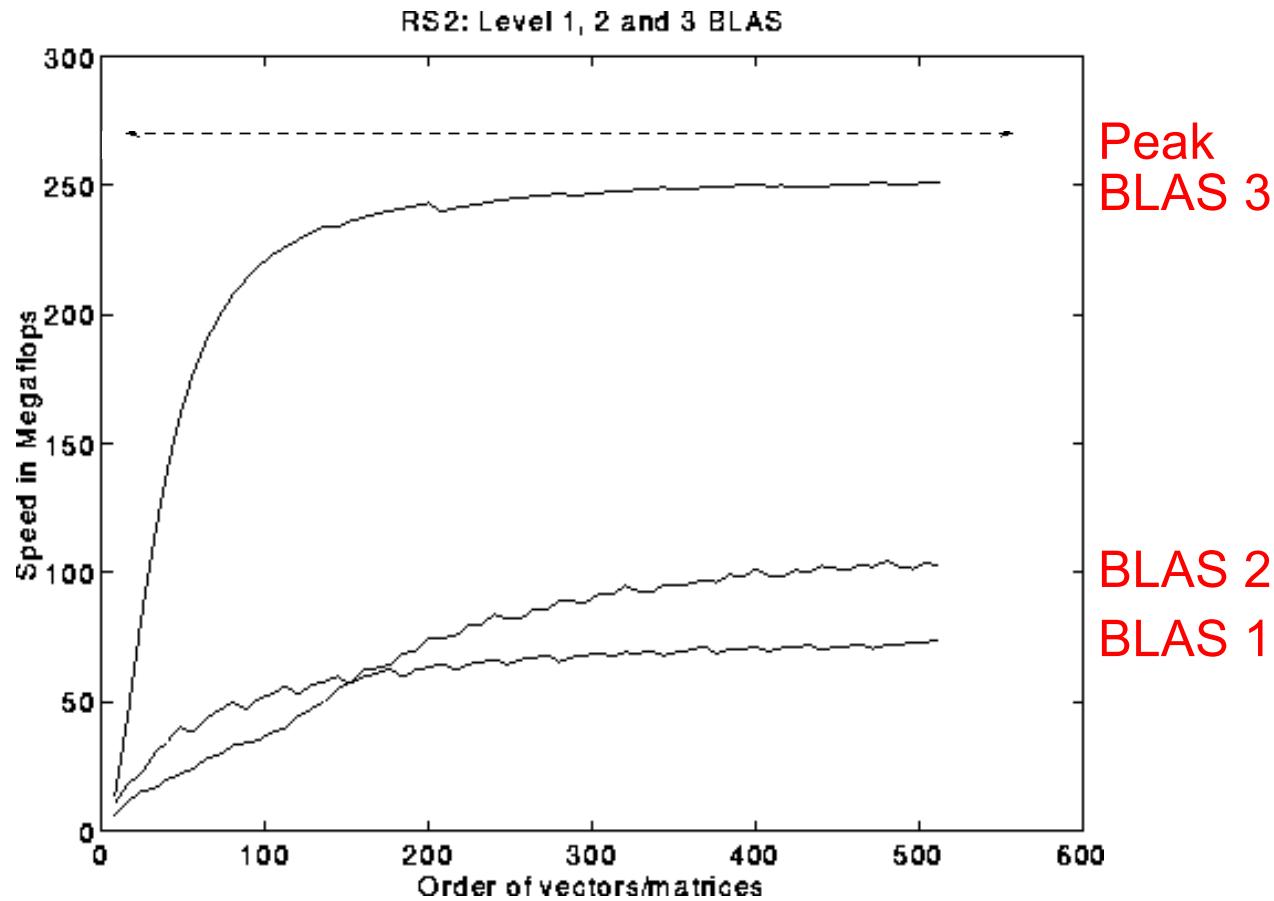


- See lecture on divide-&conquer Maxwell-Ehrenfest-surface hopping simulation

Razakh et al., PDSEC (IEEE, '24); Piroozan et al., PMBS (IEEE, '24)

BLAS3-Performance Molecular Dynamics?

- BLAS3: $q = \text{flop}/\text{memory access} = (\text{block size})^{1/2}$



- Molecular dynamics: $q = O(n^2)/O(n) = O(n)$: block size
 - > Use of SIMD (single instruction multiple data) instructions on Cell, multicore (SSE)?

Quantum MD@Scale

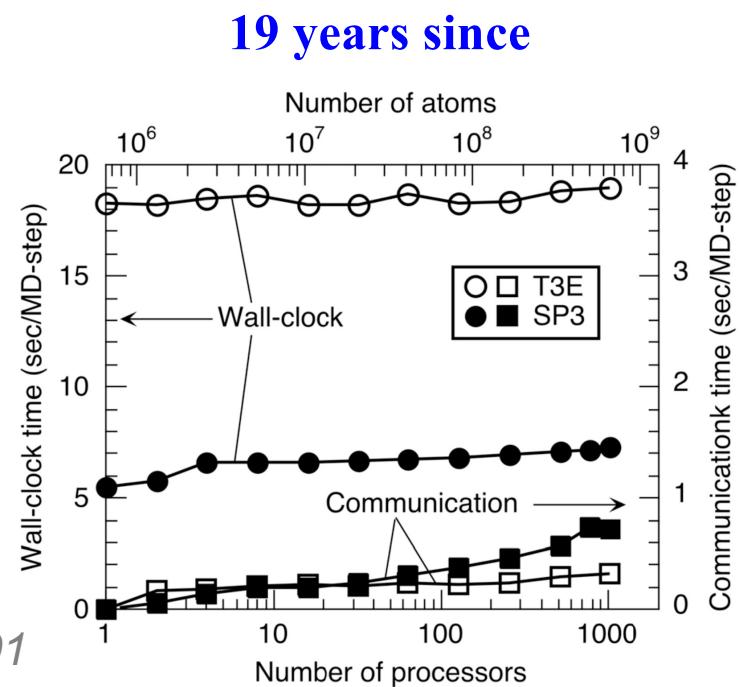
Quantum dynamics at scale: ultrafast control of emergent functional materials

S. C. Tiwari, P. Sakdhnagool, R. K. Kalia, A. Krishnamoorthy, M. Kunaseth,
A. Nakano, K. Nomura, P. Rajak, F. Shimojo, Y. Luo & P. Vashishta

Best Paper in *ACM HPC Asia 2020*



Scalable atomistic simulation algorithms
for materials research, A. Nakano et al.,
Best Paper, IEEE/ACM Supercomputing 2001, SC01



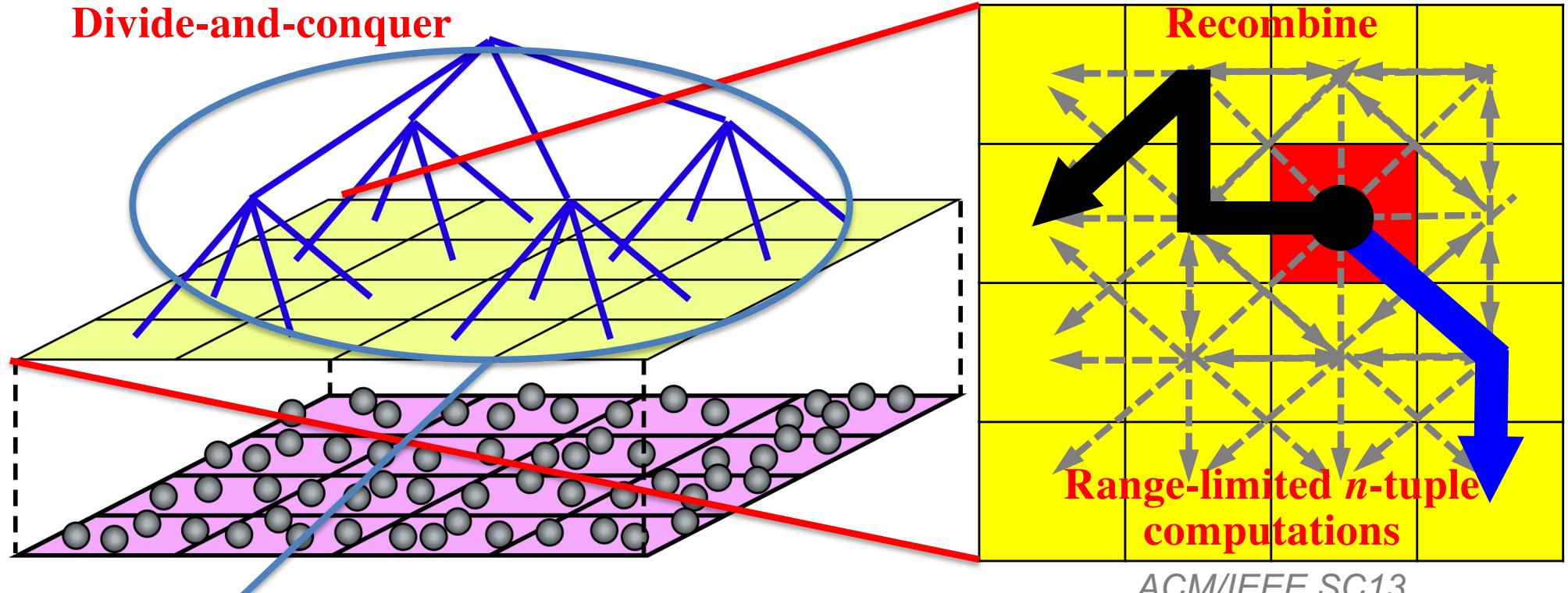
Exascale Computing Challenge

1. Scalability for billion-way parallelism

J. Chem. Phys. 140, 18A529 ('14)
IEEE/ACM SC14
IEEE Computer 48(11), 33 ('15)

Divide-conquer-recombine (DCR) algorithmic framework

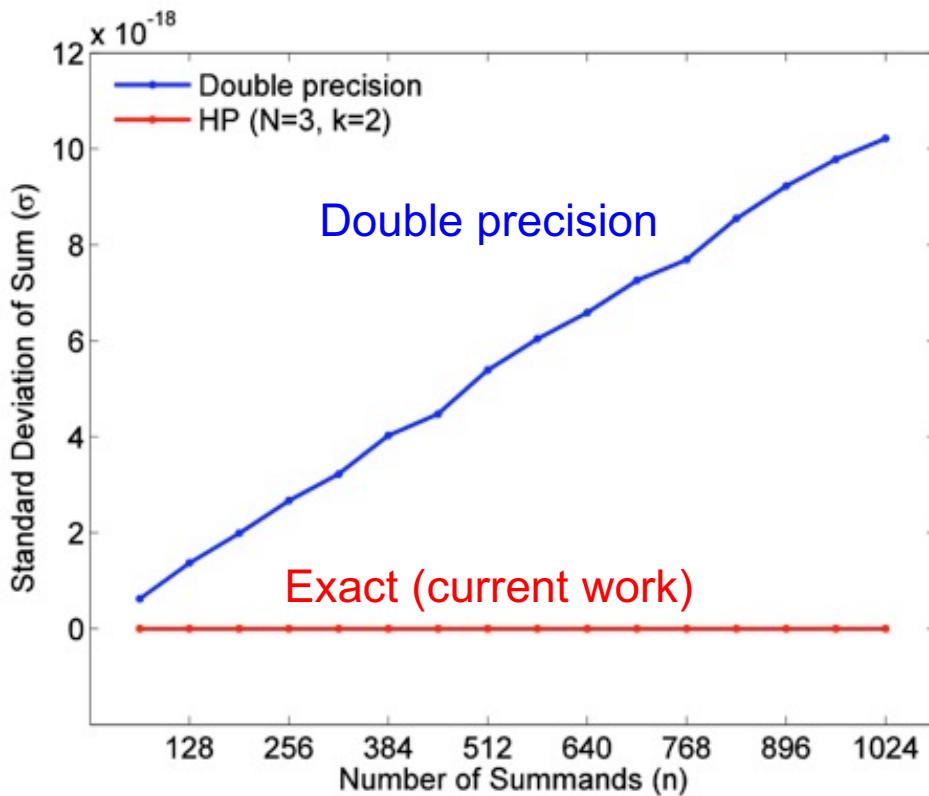
Metascalable (“design once, scale on future architectures”)



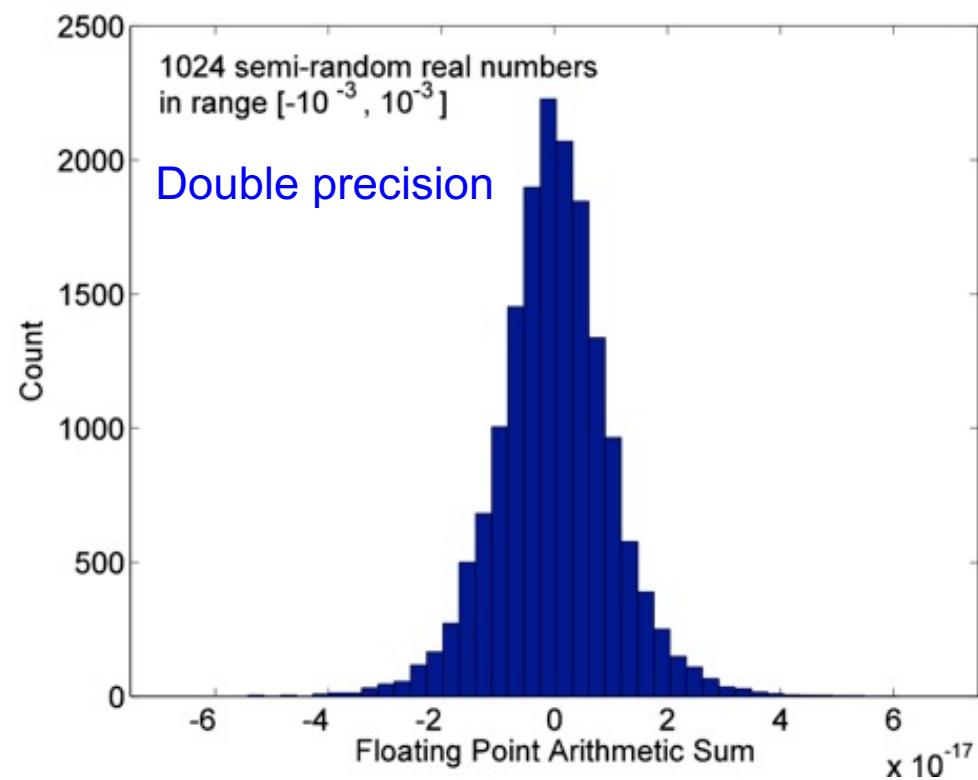
2. Reproducibility of real-number summation for multibillion summands in the global sum; double-precision arithmetic began to produce different results on different high-end architectures

Reproducibility Challenge

- Rounding (truncation) error makes floating-point addition non-associative



Standard deviation of sum with
random summation orders



Distribution of sum with random
summation orders

- Sum becomes a random walk across the space of possible rounding error

High-Precision (HP) Method

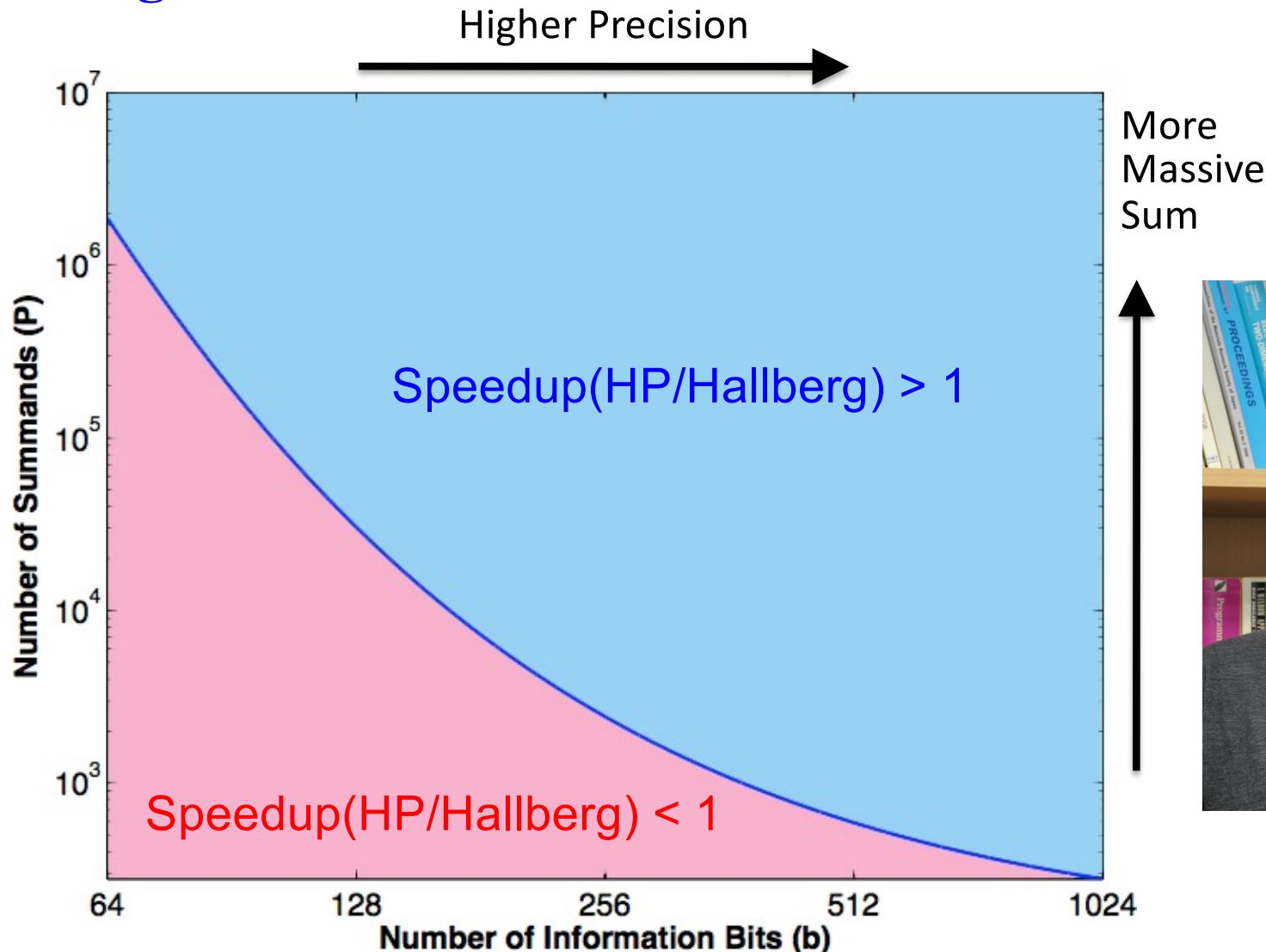
- Propose an extension of the order-invariant, higher-precision intermediate-sum method by Hallberg & Adcroft [*Par. Comput.* **40**, 140 ('14)]
- The proposed variation represents a real number r using a set of N 64-bit unsigned integers, a_i ($i = 0, N-1$)

$$r = \sum_{i=0}^{N-1} a_i 2^{64(N-k-i-1)} \\ = \underbrace{a_0 2^{64(N-k-1)} + \cdots + a_{N-k-1} 2^{-64}}_{N-k} + \underbrace{a_{N-k} 2^{-64} + \cdots + a_{N-1} 2^{-64k}}_k$$

- k is the number of 64-bit unsigned integers assigned to represent the fractional portion of r ($0 \leq k \leq N$), whereas $N-k$ integers represent the whole-number component
- Negative number is represented by two's complement in integer representation, using only 1 bit

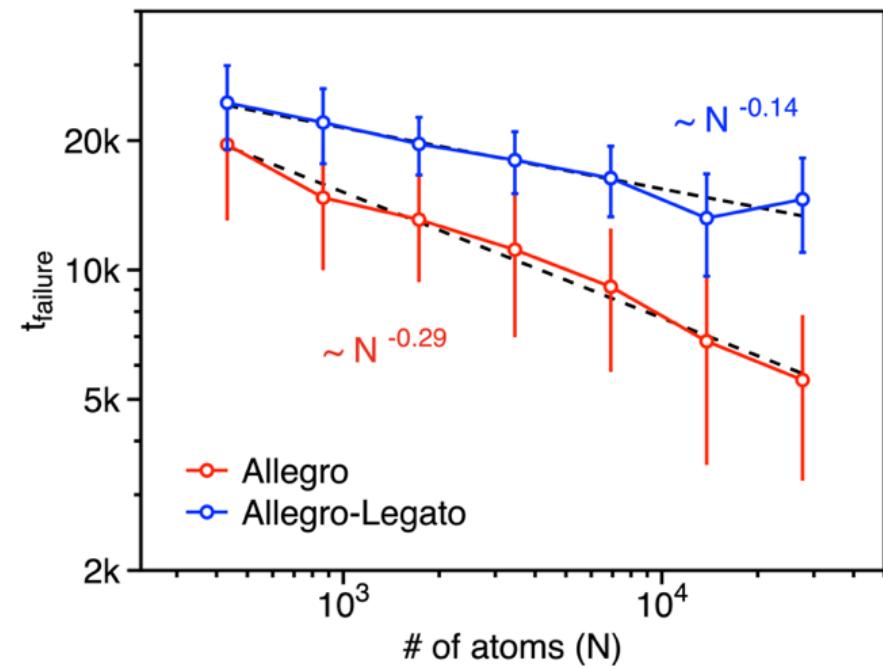
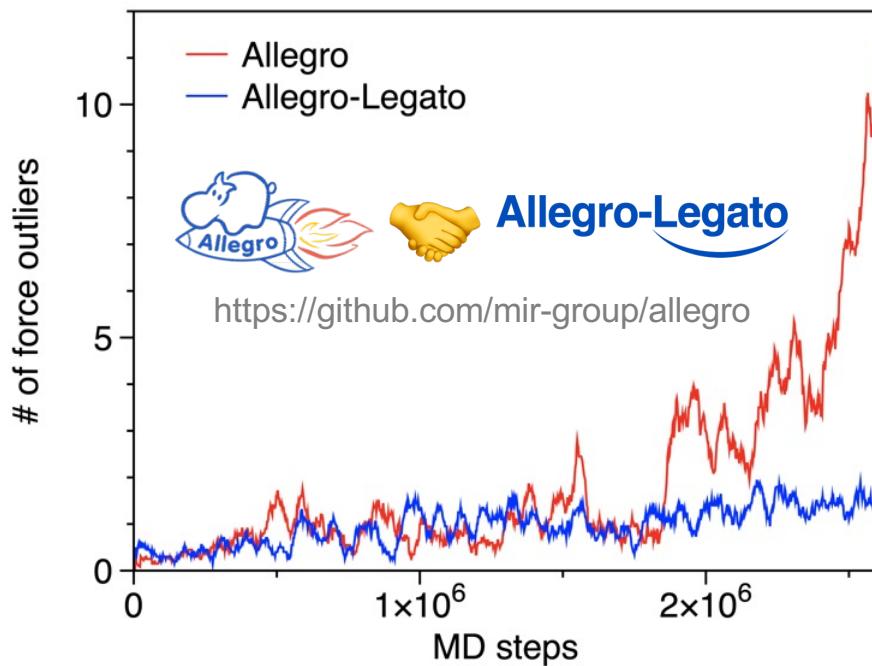
Performance Projection

- HP sum is faster than Hallberg sum for higher precision & larger numbers of summands



Fidelity Scaling Challenge

- **Allegro (fast) NNQMD:** State-of-the-art *accuracy & speed* founded on group-theoretical equivariance & local descriptors [Musaelian et al., *Nat. Commun.* **14**, 579 ('23)]
- **Fidelity-scaling problem:** On massively parallel computers, growing number of unphysical (adversarial) force predictions prohibits simulations involving larger numbers of atoms for longer times
NNQMD: neural-network quantum molecular dynamics
- **Allegro-Legato (fast and “smooth”):** *Sharpness aware minimization (SAM)* enhances the *robustness* of Allegro through improved smoothness of loss landscape
 $w_* = \operatorname{argmin}_w [L(w) + \max_{\|\epsilon\|_2 \leq \rho} \{L(w + \epsilon) - L(w)\}]$ (L : loss; w : model parameters)
- **Elongated time-to-failure scaling, $t_{\text{failure}} = O(N^{-\beta})$,** without sacrificing accuracy or speed, thereby achieving spectroscopically stable long-time Hamiltonian trajectory



Gordon Bell Prizes

aka Nobel prize of supercomputing

Gordon Bell winners in extreme-scale quantum simulations

- F. Gygi *et al.*, “Large-scale electronic structure calculations of high-Z metals on the BlueGene/L platform” ('06)
- M. Eisenbach *et al.*, “A scalable method for ab initio computation of free energies in nanoscale systems” ('09)
- Y. Hasegawa *et al.*, “First-principles calculations of electron states of a silicon nanowire with 100,000 atoms on the K computer” ('11)
- A. N. Ziegas *et al.*, “A data-centric approach to extreme-scale ab initio dissipative quantum transport simulations” ('19)
- S. Das *et al.*, “Large-scale materials modeling at quantum accuracy: *Ab initio* simulations of quasicrystals and interacting extended defects in metallic alloys” ('23)

See the [reading list](#)