Logistic Regression. Simplified.

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After the basics of Regression, it's time for basics of ***Classification***. And, what can be easier than Logistic Regression!

This is what Classification actually means:

As simple as dividing waste!

We saved the easy algorithms for the last. Happy Learning. :)

> ***Fear is the enemy of logic.***

. . .

What is Logistic Regression?

It’s a classification algorithm, that is used where the response variable is *categorical*. The idea of Logistic Regression is to find a **relationship between features and probability of particular outcome**.

E.g. When we have to predict if a student passes or fails in an exam when the number of hours spent studying is given as a feature, the response variable has two values, pass and fail.

This type of a problem is referred to as **Binomial Logistic Regression**, where the response variable has two values 0 and 1 or pass and fail or true and false. **Multinomial Logistic Regression** deals with situations where the response variable can have three or more possible values.

Why Logistic, not Linear?

With binary classification, let ***‘x’*** be some feature and ***‘y’*** be the output which can be either 0 or 1.
The probability that the output is 1 given its input can be represented as:

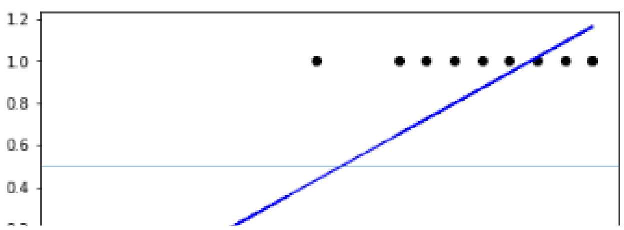
$$P(y = 1 \mid x)$$

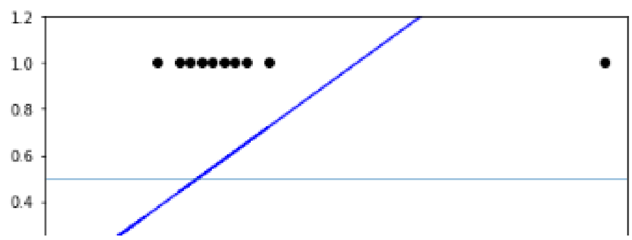
If we predict the probability via linear regression, we can state it as:

$$p(X) = \beta_0 + \beta_1 X.$$

where, p(x) = p(y=1|x)

Linear regression model can generate the ***predicted probability*** as any number ranging from negative to positive infinity, whereas probability of an outcome can only lie between 0< P(x)<1.





Really sad!

Also, Linear regression has a considerable effect on outliers.
To avoid this problem, ***log-odds*** function or ***logit*** function is used.

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Logit Function

Logistic regression can be expressed as:

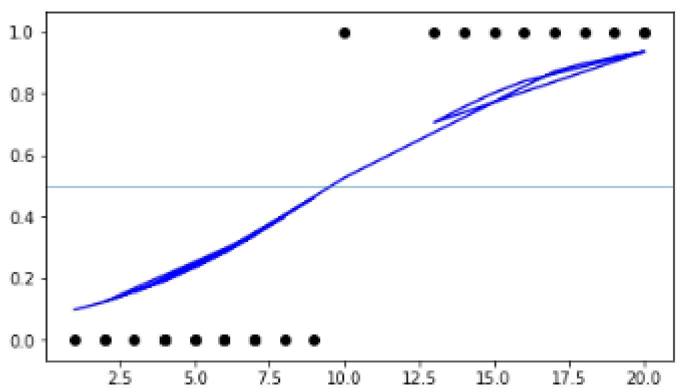
$$\log\left(\frac{p(X)}{1-p(X)}\right) = \beta_0 + \beta_1 X.$$

where, the left hand side is called the **logit** or log-odds function, and **p(x)/(1-p(x))** is called odds.

The *odds* signifies the ratio of probability of success to probability of failure. Therefore, in Logistic Regression, linear combination of inputs are mapped to the log(odds) - the output being equal to 1.
If we take an **inverse of the above function**, we get:

$$p(X) = \frac{e^{\beta_0+\beta_1 X}}{1+e^{\beta_0+\beta_1 X}}.$$

This is known as the *Sigmoid function* and it gives an S-shaped curve. It always gives a value of probability ranging from $0<p<1$.



Sigmoid Function.

Estimation of Regression Coefficients

Unlike linear regression model, that uses Ordinary Least Square for parameter estimation, we use **Maximum Likelihood Estimation**.
There can be infinite sets of regression coefficients. The maximum likelihood estimate is that set of regression coefficients for which the probability of getting the data we have observed is maximum.
If we have binary data, the probability of each outcome is simply π if it was a success, and 1−π otherwise. Therefore we have the likelihood function:

$$\mathcal{L}(\beta; \boldsymbol{y}) = \prod_{i=1}^{N} \left(\frac{\pi_i}{1-\pi_i}\right)^{y_i} (1-\pi_i)$$

To determine the value of parameters, log of likelihood function is taken, since it does not change the properties of the function.
The log-likelihood is *differentiated* and using **iterative** techniques like Newton method, values of parameters that maximise the log-likelihood are determined.

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Performance of Logistic Regression model:

To evaluate the performance of a logistic regression model, **Deviance** is used in lieu of sum of squares calculations.

- ***Null*** Deviance indicates the response predicted by a model with nothing but an intercept.
- ***Model*** deviance indicates the response predicted by a model on adding independent variables. If the model deviance is significantly smaller than the null deviance, one can conclude that the parameter or set of parameters significantly improved model fit.
- Another way to find the accuracy of model is by using **Confusion Matrix**.

	p' (Predicted)	n' (Predicted)
p (Actual)	True Positive	False Negative
n (Actual)	False Positive	True Negative

Matrix!

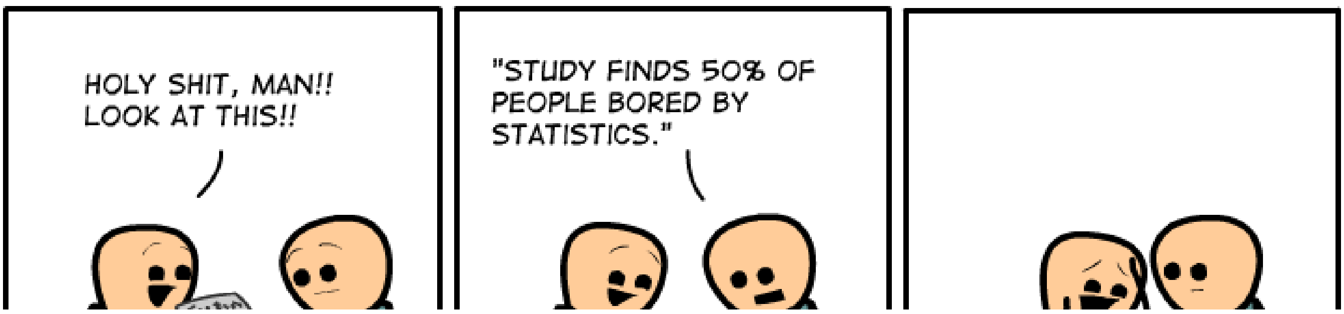
The ***accuracy*** of the model is given by:

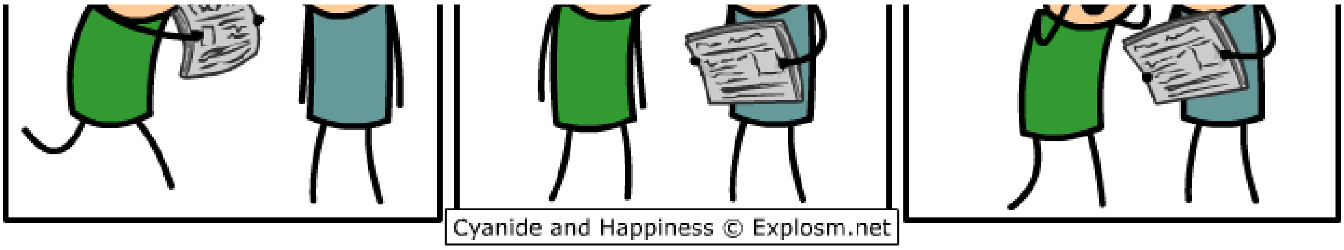
$$\frac{\text{True Positive} + \text{True Negatives}}{\text{True Positive} + \text{True Negatives} + \text{False Positives} + \text{False Negatives}}$$

Multi-class Logistic Regression

The basic intuition behind Multi-class and binary Logistic regression is same. However, for multi-class problem we follow a ***one v/s all approach***.

Eg. If we have to predict whether the weather is sunny, rainy, or windy, we are dealing with a Multi-class problem. We turn this problem into three binary classification problem i.e whether it is sunny or not, whether it is rainy or not and whether it is windy or not. We run all three classifications **independently** on input. The classification for which the value of probability is maximum relative to others, is the solution.





Can't afford this!

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Is it really that good?

As simple it seems, does it even solve any purpose? Let's check!

> *Pros*

- Simple and efficient.
- Low variance.
- It provides **probability** score for observations.

> *Cons:*

- Doesn't handle **large** number of categorical features/variables well.
- It requires transformation of non-linear features.

Implementation in Python

I applied the model on the data set of **Tic-tac-toe game**. This database encodes the complete set of possible board configurations at the end of tic-tac-toe games, where ***x*** is assumed to have played first. The target concept is ***win for x*** i.e., when ***x*** has one of 8 possible ways to create a "three-in-a-row" sequence.

Click ***HERE*** for the full code. Anybody can code, trust me!

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References

1. Statistics Solutions Blog
2. Sklearn module

3. Machine Learning Mastery Blog

Footnotes

You are aware of the most common ML Algorithms in the industry by now. I am thankful to all of you for being with us all this while.
Wait for the last one!

Thanks for reading. :)
And, ❤ if this was a good read. Enjoy!

Editor: Akhil Gupta

Thanks to Roopkishor Singh.

Machine Learning Data Science Algorithms Logistic Regression