Particle Filtering over Sets

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Outline

- The Problem
- The Approach
- Experimental Evaluation
- Future Work
- Conclusion and Discussion

The IMHT Problem

- Intention-Aware Multi-Human Tracking
    - Track multiple humans
    - Understand their motion intentions
- Human-Robot Interaction Tasks
    - Entering an elevator with human occupation
    - Following a human in crowded environments
    - Staying inside a team of moving humans

The Challenges

- Non-perfect human detectors
  - Inevitable false and missing detections
  - Can not distinguish different people
- Complex human dynamics
  - Dynamic number of humans
  - Unpredictable motion models
- The robot navigates from place to place
- Real-time constraints

The PFS Approach

- Hidden Markov Modelling
  - States: $S = \{h_i\}_{i=1:|S|}$, $h = (x, y, \dot{x}, \dot{y}, i, \rho)$ (or $h = (s, i, \rho)$ for short)
  - Observations: $O = \{(x_i, y_i)\}_{i=1:|O|}$
  - Motion model: $P(S' \mid S)$
  - Observation model: $P(O \mid S)$
- Particle filtering over sets
  - Observation function approximation
  - Particle refinement method
  - Kernel density estimation over sets
- Human identification process

A Set as a Random Variable

- Theorem
  - Let $S = \{X_i\}_{i=1:n}$
    - Observe $S = \{x_i\}_{i=1:n}$
  - $P(S) = \sum_{\sigma \in A_n} P(X_1 = x_{\sigma(1)}, X_2 = x_{\sigma(2)}, \dots, X_n = x_{\sigma(n)})$
  - A_n is the set of all permutations of $\{i\}_{i=1:n}$
- Example
  - Throw a dice for 3 times
  - Possible outputs: $\{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{3,1\}, \{1,2,3\}$
  - $P(\{1,2,3\}) = P(1,2,3) + P(1,3,2) + \cdots = 3!P(1)P(2)P(3) = \frac{1}{36}$

A Set as a Random Variable (cont'd)

- Corollary 1
  - $\mathcal{O} = \{o_i\}_{i=1:n}$
    - Sample without replacement for k times
  - Observe $S = \{o_{(i)}\}_{i=1:k}$
  - $P(S) = k! \frac{1}{n(n-1)\cdots(n-k+1)} = \frac{1}{\binom{n}{k}}$
- Corollary 2
  - Suppose $X \sim f_X(x)$
  - Let $S = \{X_i\}_{i=1:n}$, i.i.d. $X_i \sim f_X(x)$
  - Observe $S = \{x_i\}_{i=1:n}$
  - $P(S) = n! \prod_{1 \leq i \leq n} f_X(x_i)$

The Single Human Intention-Aware Motion Model

- Intention model
  - Select actions: $P(a \mid s, i)$
  - Change intentions: $P(i' \mid s, i)$
- Single human intention-aware motion function
  - $P(s', i' \mid s, i) = \sum_{a \in \mathcal{A}} P(s' \mid s, a) P(a \mid s, i') P(i' \mid s, i)$
  - $P(s', i', \rho' \mid s, i, \rho) = \mathbf{1}[\rho' = \rho] P(s', i' \mid s, i)$ (1)

The Joint Intention-Aware Motion Model

- Number of humans follows a birth-death process
  - Birth rate λ per second
  - Death rate $|S|\mu$ per second
  - Expected number of humans $\frac{\lambda}{|S|\mu}$
  - Expected survival time $\frac{1}{\mu}$
- Joint intention-aware motion function
  - Current state S, and next state S'
  - Newly appearing human set $B \subseteq S'$
  - Newly disappearing human set $D \subseteq S$
  - Update time interval τ
  - $P(S' \mid S) = (\lambda\tau)^{|B|} e^{-\lambda\tau} \prod_{h \in B} P_b(h) \frac{(|S|\mu\tau)^{|D|} e^{-|S|\mu\tau}}{|D|!} \frac{1}{\binom{|S|}{|D|}} \cdot P(S' - B \mid S - D)$

The Observation Model

- State S, and observation O
- Suppose no false and missing detections
- Let Ψ_S^O be the set of all possible assignments from S to O
- $P(O \mid S) = \sum_{\psi \in \Psi_S^O} \prod_{h \in S} P_o(\psi(h) \mid h)$ (2)

The Observation Model (cont'd)

- False and missing detections are mutually independent, following Poisson processes with parameters ν and $|S|\xi$ per second
- False detections $F = \{(x_i, y_i)\}_{i=1:|F|} \subseteq O$
- Missing detections $M = \{h_i\}_{i=1:|M|} \subseteq S$
- $|O - F| = |S - M|$
- Let $O \circ S = \{\langle F_i, M_i \rangle\}_{i=1:|O \circ S|}$ the set of all possible F-M pairs,
  $|O \circ S| = \sum_{0 \leq i \leq \min\{|O|,|S|\}} \binom{|O|}{i}\binom{|S|}{i} = \binom{|O|+|S|}{|O|}$
- $P(O \mid S) = \sum_{\langle F,M \rangle \in O \circ S} P(O - F \mid S - M) \cdot (\nu\tau)^{|F|} e^{-\nu\tau} \prod_{o \in F} P_f(o) \frac{(|S|\xi\tau)^{|M|} e^{-|S|\xi\tau}}{|M|!} \frac{1}{\binom{|S|}{|M|}}$ (3)

Observation Function Approximation

- Assignment pruning
  - Convert probability densities $P_o(o \mid h)$ to costs $c(h, o) = -\log(P_o(o \mid h))$
  - Find the assignments in cost-increasing order by following Murty's algorithm [**?**]
  - Stop when the density ratio of the last assignment to the best assignment is lower than a threshold
- False-missing detection pruning
  - Let $P_F(F) = (\nu\tau)^{|F|} e^{-\nu\tau} \prod_{o \in F} P_f(o)$, and $P_M(M) = \frac{(|S|\xi\tau)^{|M|} e^{-|S|\xi\tau}}{|M|!} \frac{1}{\binom{|S|}{|M|}}$
  - Find F-M pairs $\langle F, M \rangle$ in $P_F(F) P_M(M)$ probability decreasing order until a threshold

Particle Filtering

- Propose: for $1 \leq i \leq |\mathcal{P}_{t-1}|$, draw samples from the proposal distribution $\hat{X}_t^{(i)} \sim \pi(\cdot \mid X_{t-1}^{(i)}, O_t)$
- Usually propose directly from motion model, $\pi(\cdot \mid X_{t-1}^{(i)}, O_t) = P(\cdot \mid X_{t-1}^{(i)})$