

Designing approximated Machine Learning models in Python for Homomorphic evaluation.

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Bachelor Thesis



SCIENCE
PASSION
TECHNOLOGY



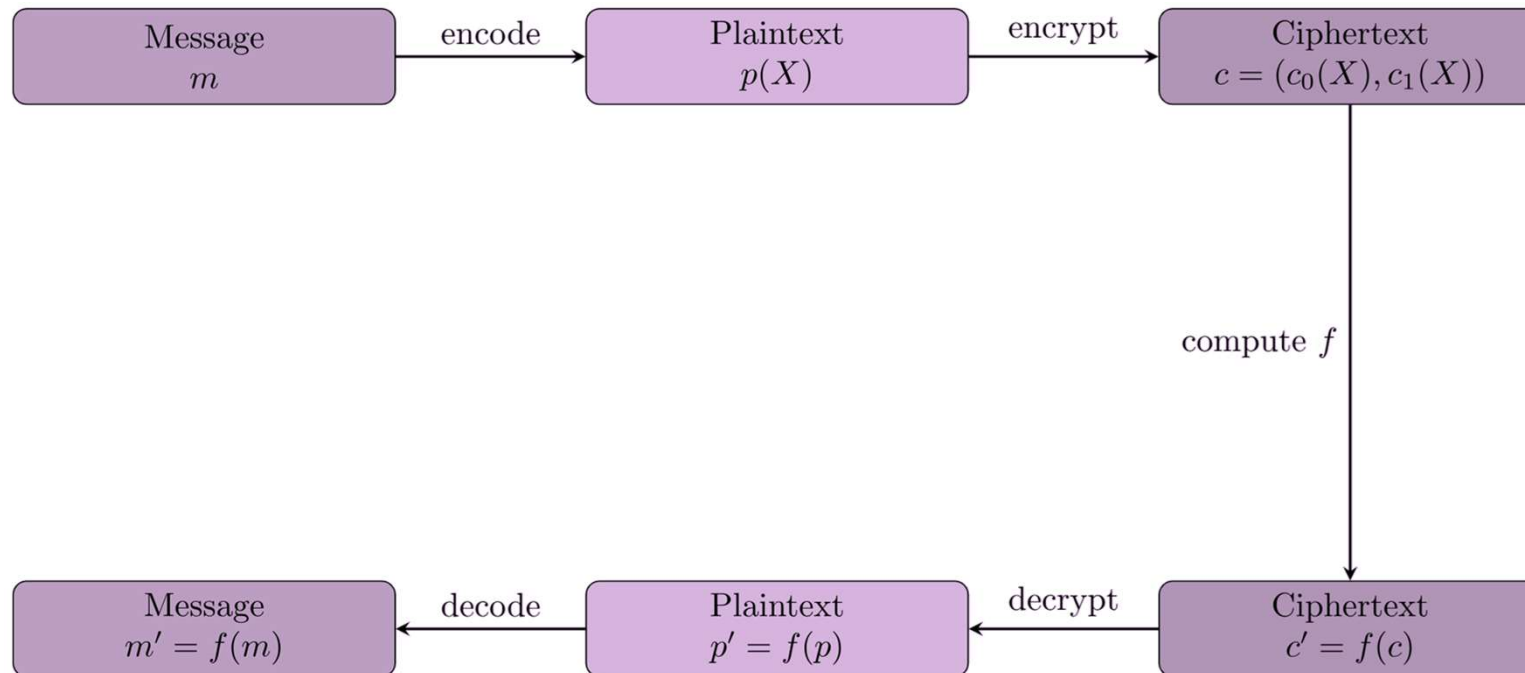
Motivation

- Privacy-Preserving Machine Learning
- Homomorphic Encryption
- Addition, Multiplication, Rotation
- Softmax function

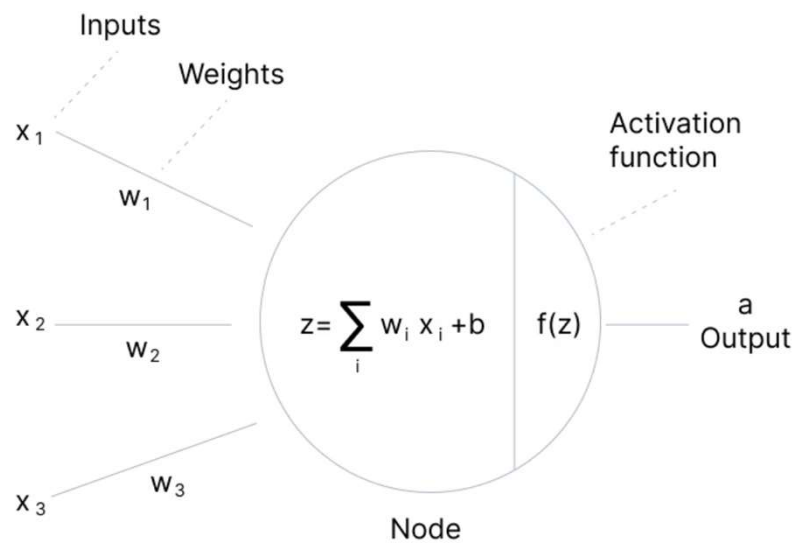
Implement an approximation of a vision transformer by Tin Nguyen, trained on the CIFAR 10 dataset.

Main Goal

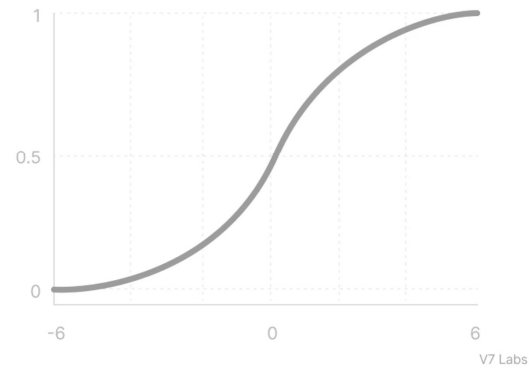
Cheon-Kim-Kim-Song Scheme



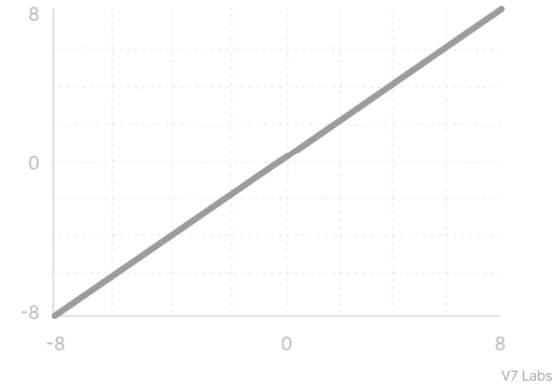
Artificial Neurons/Nodes



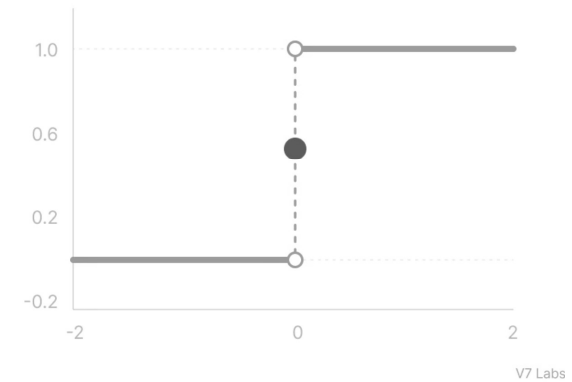
Sigmoid / Logistic



Linear Activation Function

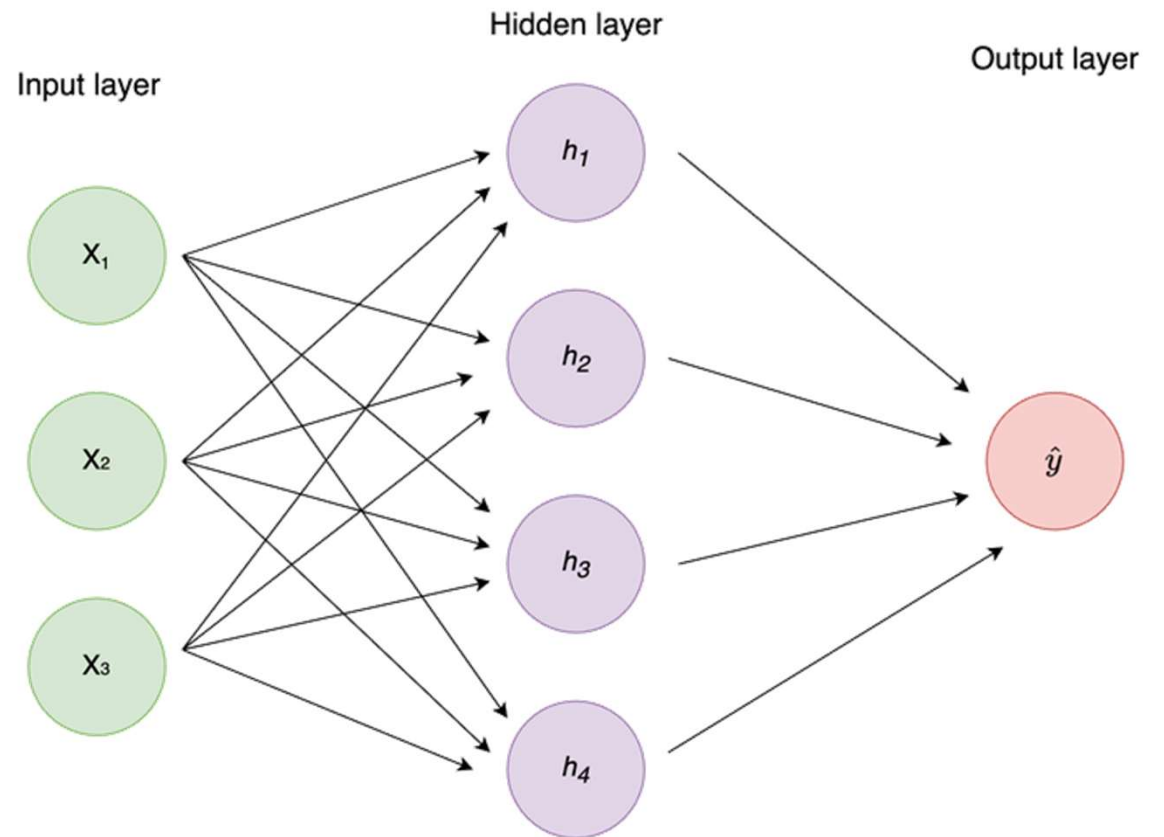


Binary Step Function



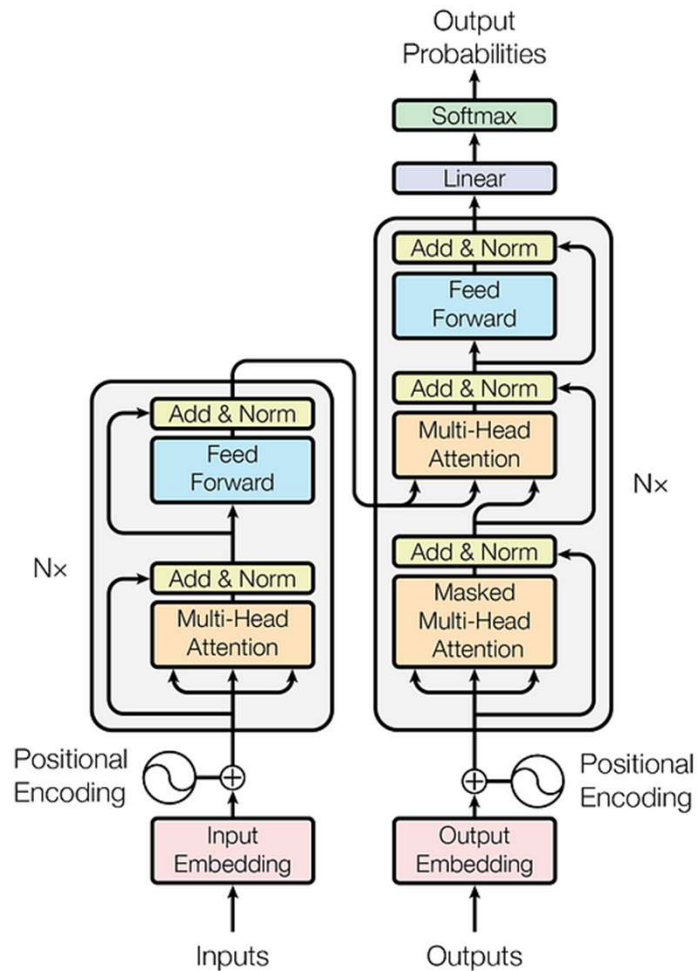
Artificial Neuronal Network

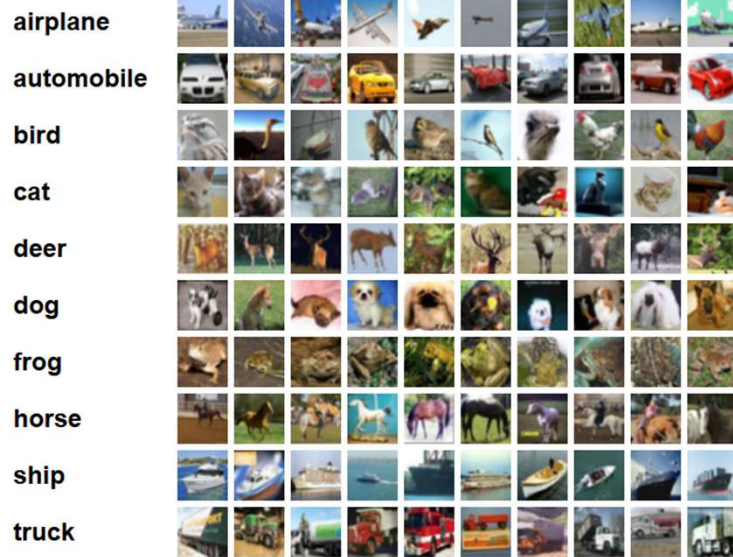
- Feed forward
- Recurrent



Transformer Model

- Attention is all you need - Google 2017
- Feedforward
- Attention

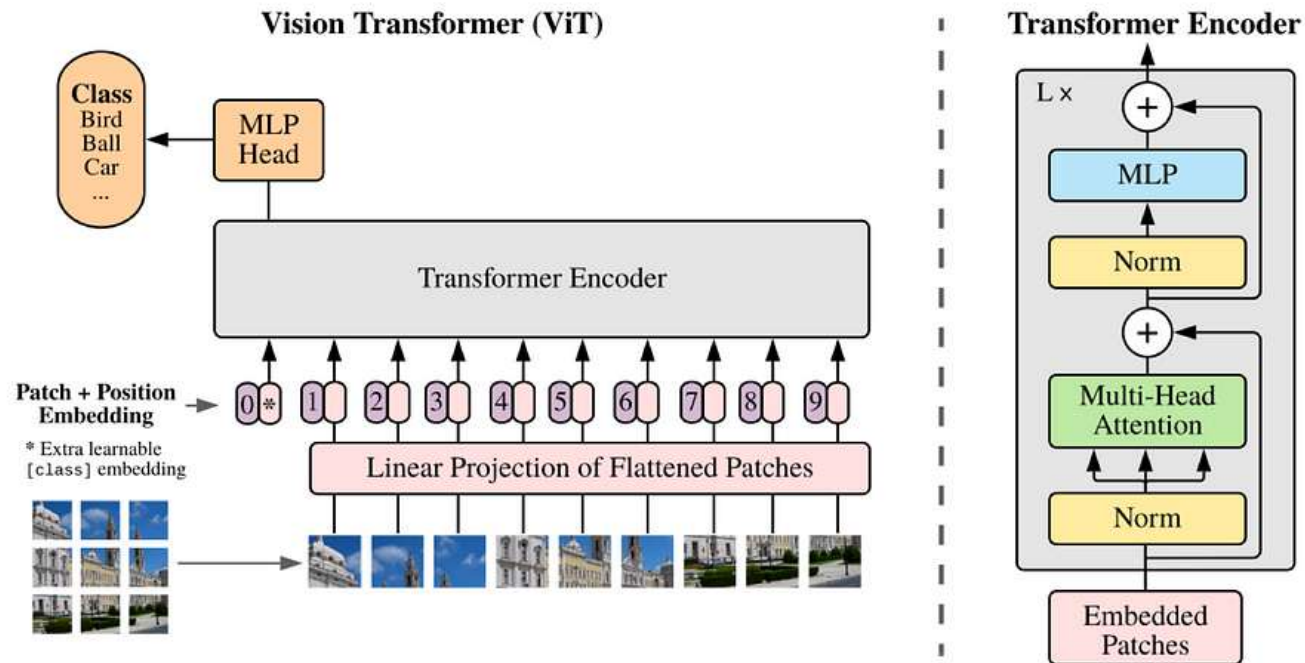




Vision Transformer

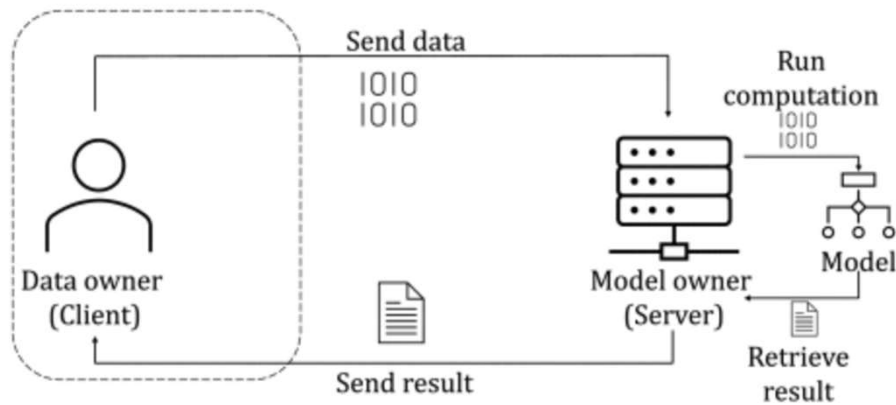
- Tin Nguyen - „Vision Transformer from Scratch“
- Inspired by “An Image is worth 16x16 Words: Transformers for Image Recognition at Scale”
- CIFAR-10 dataset

Vision Transformer Model



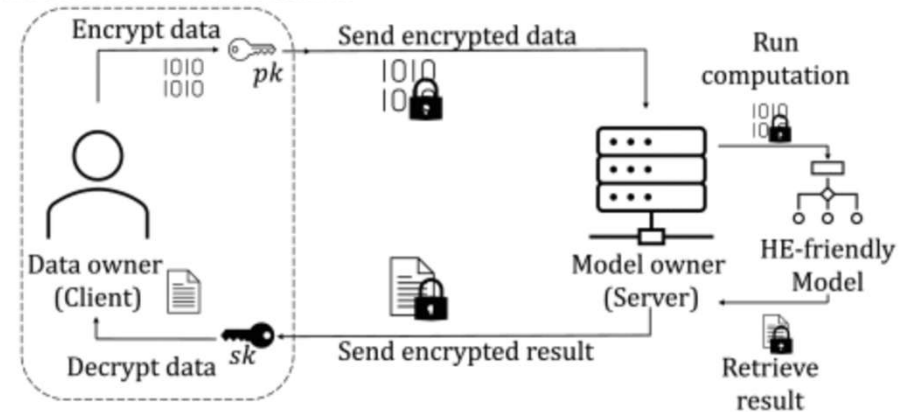
Machine Learning

Data owner's domain of control



(a) Without privacy preservation

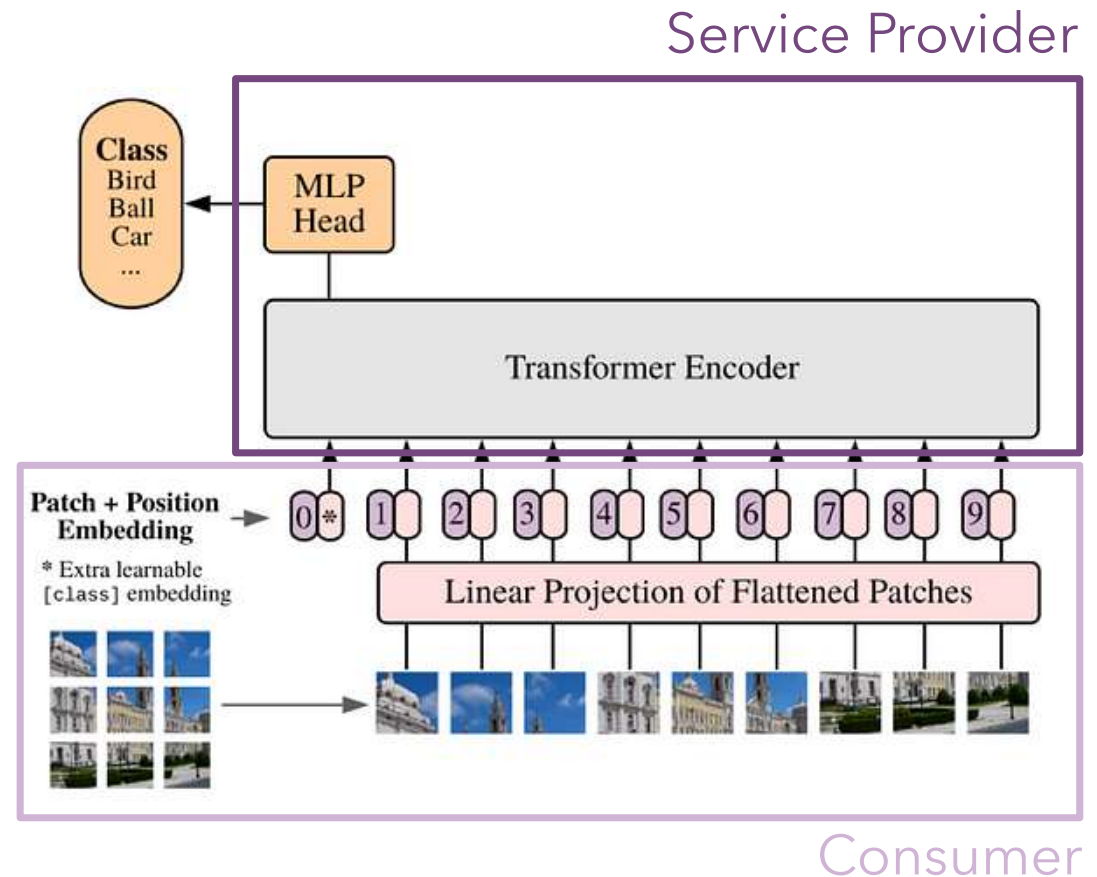
Data owner's domain of control



(b) With privacy preservation

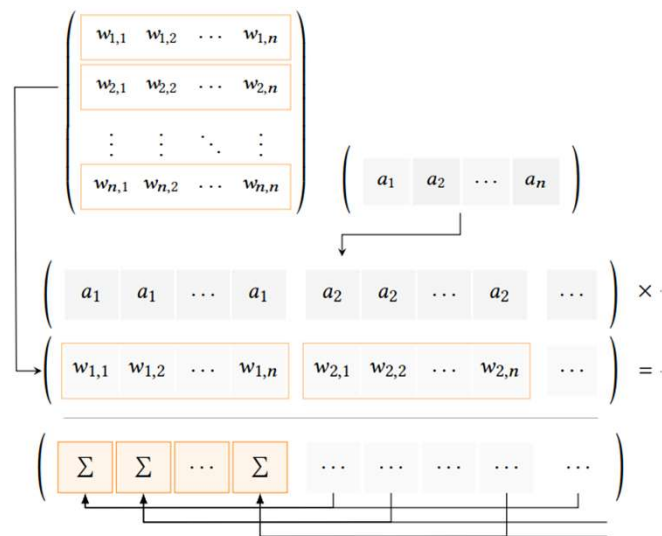
HE Vit Setting

- **Input:** Encrypted Input Embeddings
- **Output:** Encrypted CLS Token

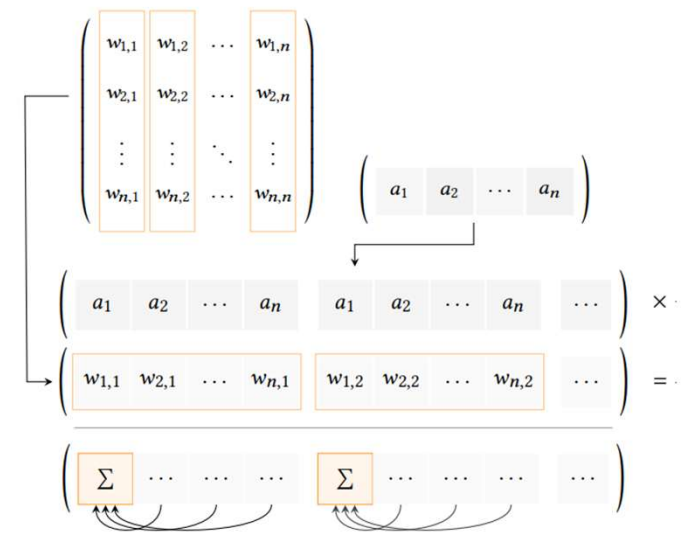


Multiplication

Expanded Vector x Row-major Matrix



Repeated Vector x Column-major Matrix



Wrap Up

Wrap Up Expanded

$$\begin{aligned}
 a : & \left(\boxed{a_1} \text{ } a_1 \text{ } a_1 \text{ } \boxed{a_2} \text{ } a_2 \text{ } a_2 \text{ } \boxed{a_3} \text{ } a_3 \text{ } a_3 \right) \\
 b : & \left(\text{ } b_1 \text{ } \boxed{b_1} \text{ } b_1 \text{ } \text{ } b_2 \text{ } \boxed{b_2} \text{ } b_2 \text{ } \text{ } b_3 \text{ } \boxed{b_3} \text{ } b_3 \right) \\
 c : & \left(\text{ } \text{ } c_1 \text{ } \text{ } c_2 \text{ } \boxed{c_2} \text{ } \text{ } c_3 \text{ } \text{ } c_3 \text{ } \boxed{c_3} \right) \\
 \hline
 M : & \left(\boxed{a_1} \text{ } \boxed{b_1} \text{ } \boxed{c_1} \text{ } \boxed{a_2} \text{ } \boxed{b_2} \text{ } \boxed{c_2} \text{ } \boxed{a_3} \text{ } \boxed{b_3} \text{ } \boxed{c_3} \right)
 \end{aligned}$$

Wrap Up Repeated

$$\begin{aligned}
 a : & \left(\boxed{a_1} \text{ } \boxed{a_2} \text{ } \boxed{a_3} \text{ } \text{ } a_1 \text{ } a_2 \text{ } a_3 \text{ } \text{ } a_1 \text{ } a_2 \text{ } a_3 \right) \\
 b : & \left(\text{ } b_1 \text{ } b_2 \text{ } b_3 \text{ } \boxed{b_1} \text{ } \boxed{b_2} \text{ } \boxed{b_3} \text{ } \text{ } b_1 \text{ } b_2 \text{ } b_3 \right) \\
 c : & \left(\text{ } \text{ } c_1 \text{ } c_2 \text{ } c_3 \text{ } \text{ } c_1 \text{ } c_2 \text{ } c_3 \text{ } \text{ } \boxed{c_1} \text{ } \boxed{c_2} \text{ } \boxed{c_3} \right) \\
 \hline
 M : & \left(\boxed{a_1} \text{ } \boxed{a_2} \text{ } \boxed{a_3} \text{ } \boxed{b_1} \text{ } \boxed{b_2} \text{ } \boxed{b_3} \text{ } \boxed{c_1} \text{ } \boxed{c_2} \text{ } \boxed{c_3} \right)
 \end{aligned}$$

Input

$$\begin{pmatrix} \boxed{l_{1,1}} & \boxed{l_{1,1}} & \boxed{l_{1,1}} & \dots & \boxed{l_{1,1}} & \boxed{l_{1,2}} & \boxed{l_{1,2}} & \boxed{l_{1,2}} & \dots & \boxed{l_{1,2}} & \dots & \boxed{l_{1,64}} & \boxed{l_{1,64}} & \boxed{l_{1,64}} & \dots & \boxed{l_{1,64}} \end{pmatrix}$$
$$\begin{pmatrix} \boxed{l_{2,1}} & \boxed{l_{2,1}} & \boxed{l_{2,1}} & \dots & \boxed{l_{2,1}} & \boxed{l_{2,2}} & \boxed{l_{2,2}} & \boxed{l_{2,2}} & \dots & \boxed{l_{2,2}} & \dots & \boxed{l_{2,64}} & \boxed{l_{2,64}} & \boxed{l_{2,64}} & \dots & \boxed{l_{2,64}} \end{pmatrix}$$
$$\vdots$$
$$\begin{pmatrix} \boxed{l_{64,1}} & \boxed{l_{64,1}} & \boxed{l_{64,1}} & \dots & \boxed{l_{64,1}} & \boxed{l_{64,2}} & \boxed{l_{64,2}} & \boxed{l_{64,2}} & \dots & \boxed{l_{64,2}} & \dots & \boxed{l_{64,64}} & \boxed{l_{64,64}} & \boxed{l_{64,64}} & \dots & \boxed{l_{1,64}} \end{pmatrix}$$

Norm

$$y = \alpha * \left(\frac{x - mean}{\sqrt{variance + \epsilon}} \right) + bias$$

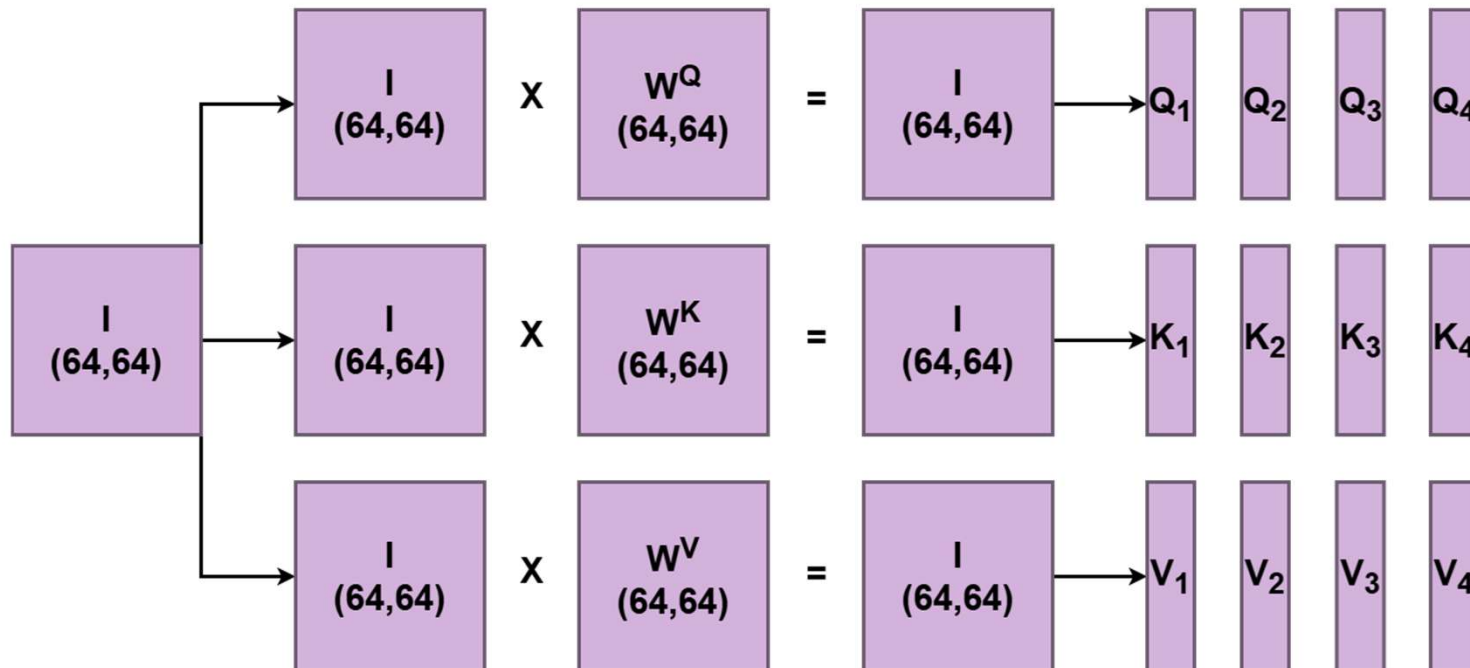
$$y' = (x_i - pre.mean) * (pre.std * \alpha) + b$$

Attention

$$Attention(Q, K, V) = SoftMax\left(\frac{QK^T}{\sqrt{features}}\right) * V$$

$$SoftMax(x) = \frac{e^{x_i}}{\sum_{j=1}^n e^{x_j}}$$

Multi Head Attention



Multi Head Attention

$$\text{Head}(Q_i, K_i, V_i) = \text{SoftMax}\left(\frac{Q_i K_i^T}{\sqrt{\text{values per head}}}\right) * V_i$$

$$\begin{aligned} \text{MultiHead}(Q, K, V) \\ = \text{Concat}(\text{head}_1, \text{head}_2, \dots, \text{head}_n) * W^O + b^O \end{aligned}$$

Multi Head Attention - Softmax

$$\begin{aligned}
 & \frac{1}{64} + \frac{63}{64^2} * x_j + \frac{63 * 62}{64^3} * x_j^2 - \frac{1}{64^2} * \sum_{i=1, i \neq j}^n x_i - \frac{62}{64^3} * \sum_{i=1, i \neq j}^n x_i^2 \\
 & - \frac{2 * 62}{64^3} * x_j * \sum_{i=1, i \neq j}^n x_i + \frac{2}{64^3} * \sum_{i=1, i \neq k, i \neq j}^n x_i * \sum_{k=1, k \neq i, k \neq j}^n x_k
 \end{aligned}$$

Multi Head Attention

$$Head(Q_i, K_i, V_i) = SoftMax(\frac{Q_i K_i^T}{\sqrt{values\ per\ head}}) * V_i$$

$$MultiHead(Q, K, V) = Concat(head_1, head_2, \dots, head_n) * W^O + b^O$$

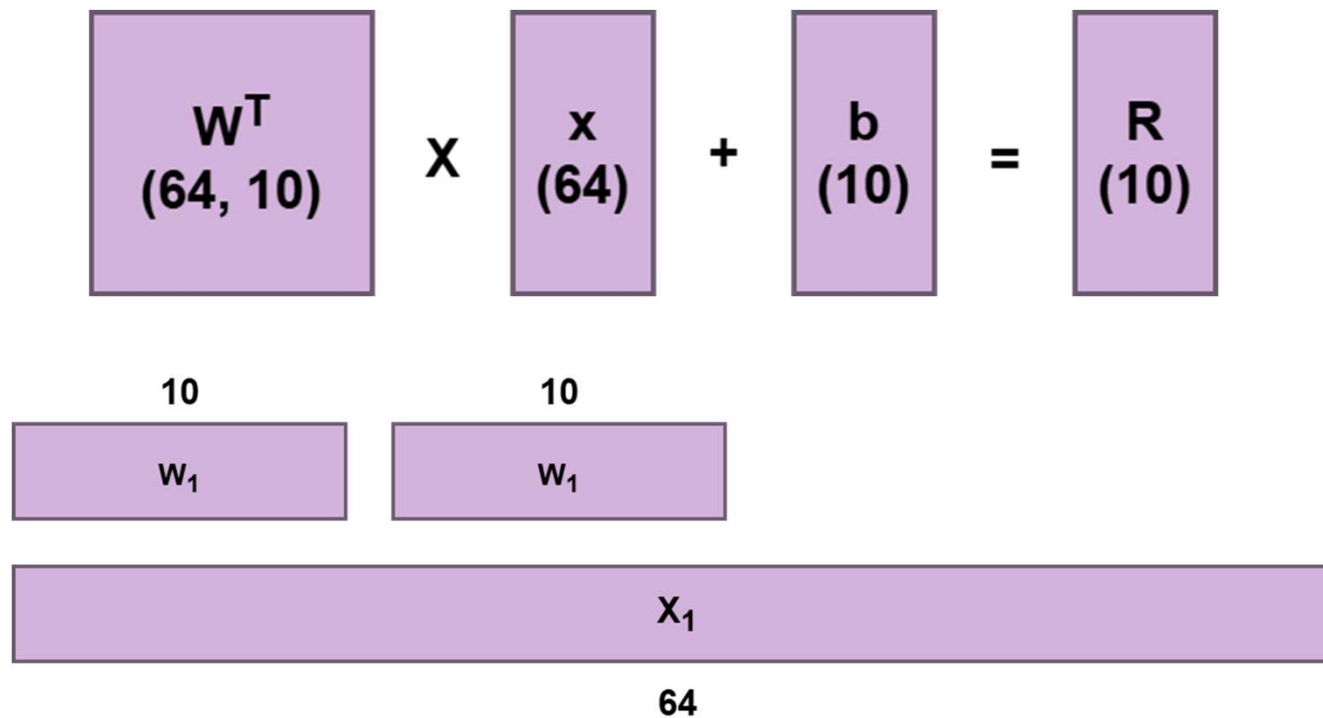
MLP - GELU

$$MLP(x) = \text{ApproxGELU}(x * W_1 + b_1) * W_2 + b_2$$

$$\begin{aligned} \text{ApproxGELU}(x) \\ = 0.5 * x (1 + \tanh(\sqrt{2/\pi}) * (x + 0.044715 * x^3)) \end{aligned}$$

$$\begin{aligned} \text{ApproxTanh}(x) \\ = \tanh(a) + (1 - \tanh^2(a)) * (x - a) - (\tanh(a) - \tanh^3(a)) \\ * (x - a)^2 \end{aligned}$$

Classifier



Result

- OpenFHE library
- Ring Dimension: 2^{16}
- Multipliative Depth: 25
- Accuracy compared to classifications: 43%
- Accuracy of original model: 46%
- Runtime: approx. 2 hours



Conclusion

- Privacy Preserving Vision Transformer using Homomorphic Evaluation
- Enable the use of Machine Learning in sensitive sectors
- Currently focused on inference but might be expanded to training in the future