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Masking Hybrid Homomorphic Encryption Schemes

Emira Salkić

13.02.2025

Introduction

Introduction to Homomorphic Encryption

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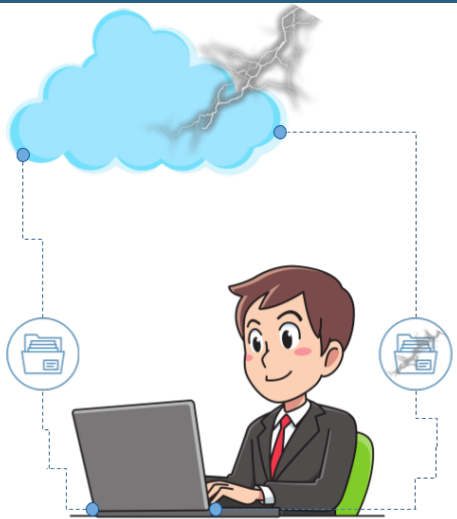
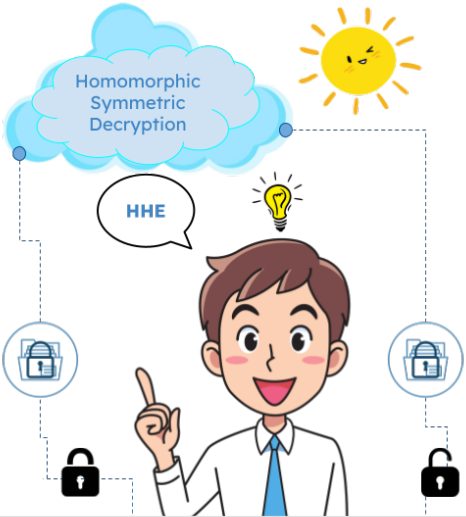


Homomorphic Encryption

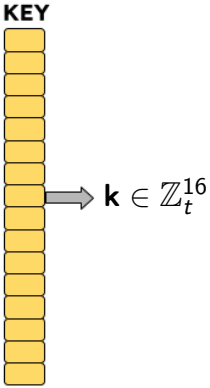
Homomorphic Encryption

Hybrid Homomorphic Encryption



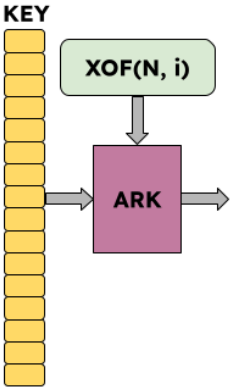
Hybrid Homomorphic Encryption

Hybrid Homomorphic Encryption: HERA



ASIACRYPT 2021: Transciphering Framework for Approximate Homomorphic Encryption, Jihoon Cho, Jincheol Ha, Seongkwang Kim, Byeonghak Lee, Joohee Lee, Jooyoung Lee, Dukjae Moon, and Hyojin Yoon

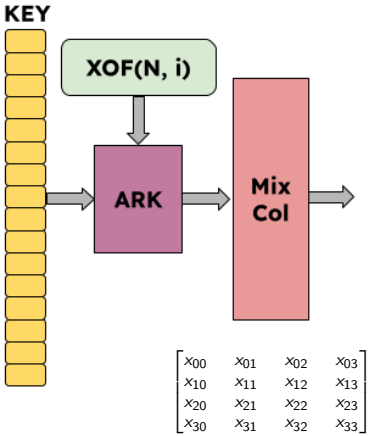
Hybrid Homomorphic Encryption: HERA



$\mathbf{rc} = (\mathbf{rc}_0, \ldots, \mathbf{rc}_r) \in (\mathbb{Z}_t^{16})^{r+1}$

$\mathrm{ARK}[\mathbf{k}, nc, i](\mathbf{x}) = \mathbf{x} + \mathbf{k} \bullet \mathbf{rc}_i$, *where* $\mathbf{x} \in \mathbb{Z}_t^{16}$

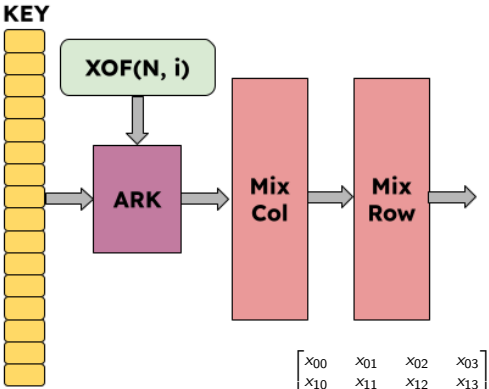
Hybrid Homomorphic Encryption: HERA



$$\begin{bmatrix} y_{0c} \\ y_{1c} \\ y_{2c} \\ y_{3c} \end{bmatrix} = \begin{bmatrix} 2 & 3 & 1 & 1 \\ 1 & 2 & 3 & 1 \\ 1 & 1 & 2 & 3 \\ 3 & 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} x_{0c} \\ x_{1c} \\ x_{2c} \\ x_{3c} \end{bmatrix}$$

$$\begin{bmatrix} x_{00} & x_{01} & x_{02} & x_{03} \\ x_{10} & x_{11} & x_{12} & x_{13} \\ x_{20} & x_{21} & x_{22} & x_{23} \\ x_{30} & x_{31} & x_{32} & x_{33} \end{bmatrix}$$

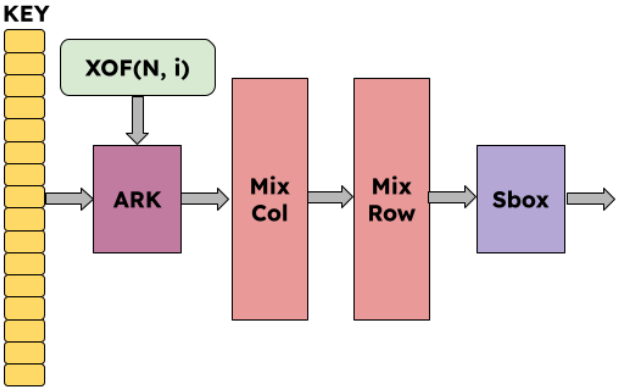
Hybrid Homomorphic Encryption: HERA



$$\begin{bmatrix} y_{c0} \\ y_{c1} \\ y_{c2} \\ y_{c3} \end{bmatrix} = \begin{bmatrix} 2 & 3 & 1 & 1 \\ 1 & 2 & 3 & 1 \\ 1 & 1 & 2 & 3 \\ 3 & 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} x_{c0} \\ x_{c1} \\ x_{c2} \\ x_{c3} \end{bmatrix}$$

$$\begin{bmatrix} x_{00} & x_{01} & x_{02} & x_{03} \\ x_{10} & x_{11} & x_{12} & x_{13} \\ x_{20} & x_{21} & x_{22} & x_{23} \\ x_{30} & x_{31} & x_{32} & x_{33} \end{bmatrix}$$

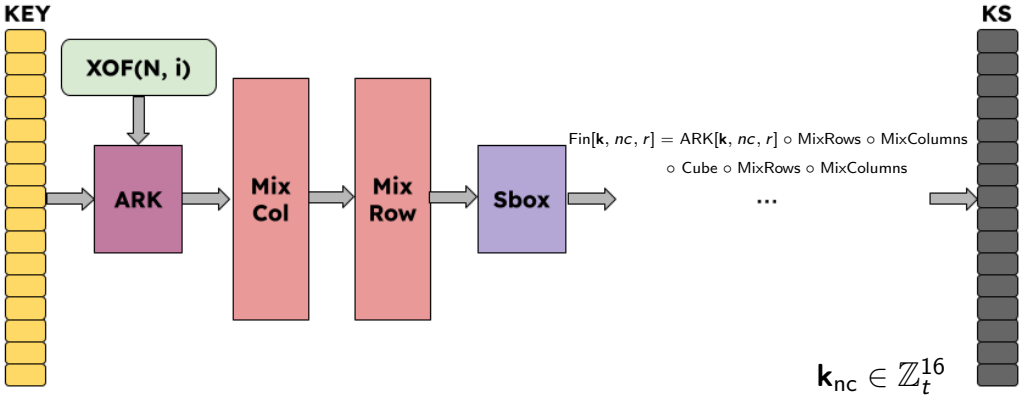
Hybrid Homomorphic Encryption: HERA



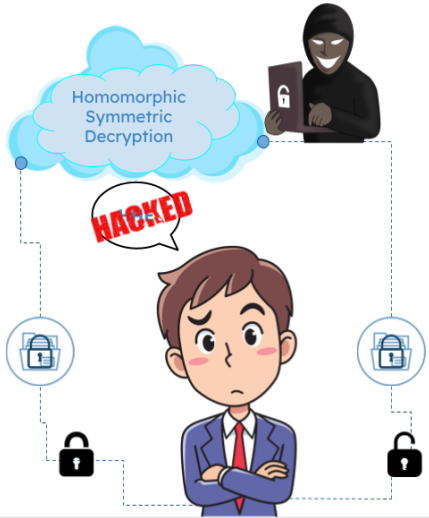
$$\text{Cube}(\mathbf{x}) = (x_0^3, \ldots, x_{15}^3)$$

$$\mathbf{x} = (x_0, \ldots, x_{15}) \in \mathbb{Z}_t^{16}$$

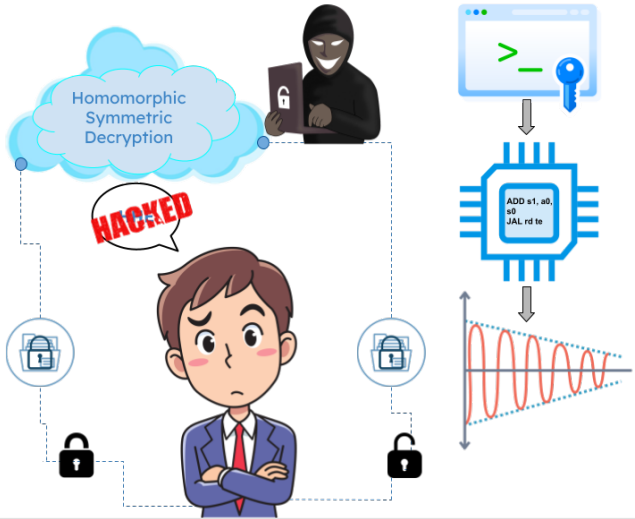
Hybrid Homomorphic Encryption: HERA



Are Implementations of Hybrid Homomorphic Schemes Secure?



Are Implementations of Hybrid Homomorphic Schemes Secure?



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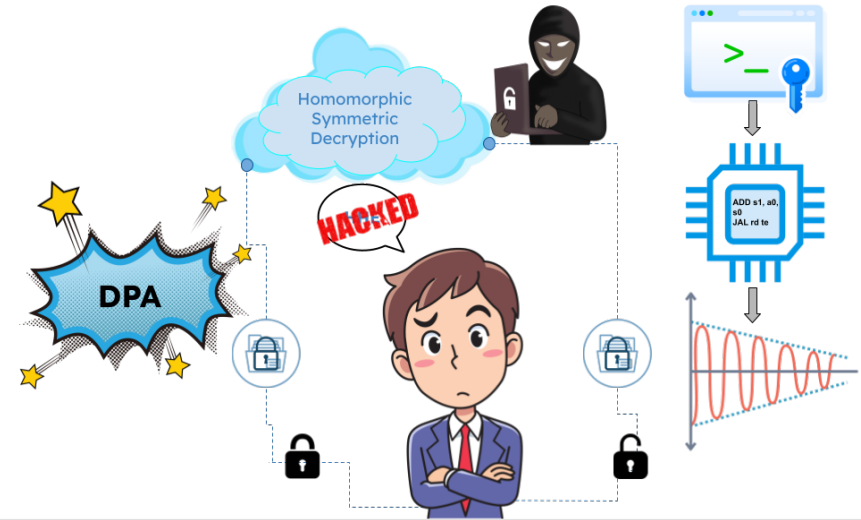
Are Implementations of Hybrid Homomorphic Schemes Secure?



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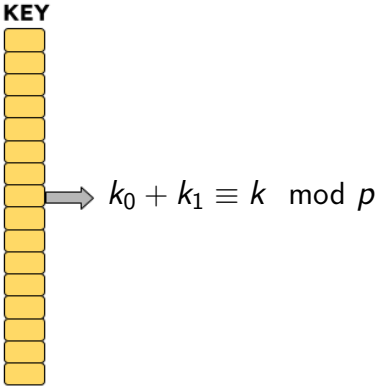
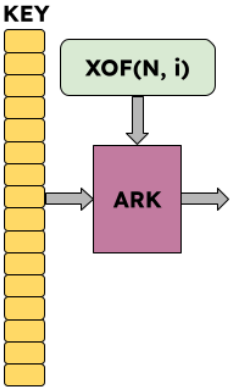
First-order arithmetic masking

First-order arithemtic masking of HERA



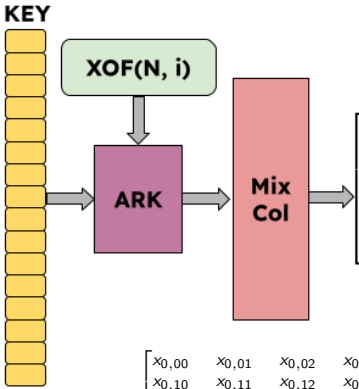
Hybrid Homomorphic Encryption: HERA



$\mathbf{rc} = (\mathbf{rc}_0, \ldots, \mathbf{rc}_r) \in (\mathbb{Z}_t^{16})^{r+1}$

$$\mathrm{ARK}[\mathbf{k}, nc, i] = \begin{cases} x_0 + (k_0 \bullet rc_i) \\ x_1 + (k_1 \bullet rc_i) \end{cases}$$

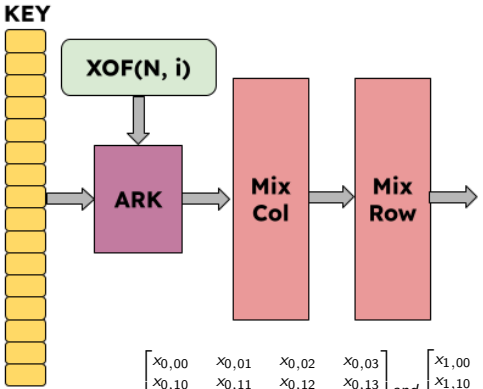
Hybrid Homomorphic Encryption: HERA



$$\begin{bmatrix} y_{m,0c} \\ y_{m,1c} \\ y_{m,2c} \\ y_{m,3c} \end{bmatrix} = \begin{bmatrix} 2 & 3 & 1 & 1 \\ 1 & 2 & 3 & 1 \\ 1 & 1 & 2 & 3 \\ 3 & 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} x_{m,0c} \\ x_{m,1c} \\ x_{m,2c} \\ x_{m,3c} \end{bmatrix}, \; m \in \{1, 0\}$$

$$\begin{bmatrix} x_{0,00} & x_{0,01} & x_{0,02} & x_{0,03} \\ x_{0,10} & x_{0,11} & x_{0,12} & x_{0,13} \\ x_{0,20} & x_{0,21} & x_{0,22} & x_{0,23} \\ x_{0,30} & x_{0,31} & x_{0,32} & x_{0,33} \end{bmatrix} \text{ and } \begin{bmatrix} x_{1,00} & x_{1,01} & x_{1,02} & x_{1,03} \\ x_{1,10} & x_{1,11} & x_{1,12} & x_{1,13} \\ x_{1,20} & x_{1,21} & x_{1,22} & x_{1,23} \\ x_{1,30} & x_{1,31} & x_{1,32} & x_{1,33} \end{bmatrix}$$

Hybrid Homomorphic Encryption: HERA



$$\begin{bmatrix} y_{m,c0} \\ y_{m,c1} \\ y_{m,c2} \\ y_{m,c3} \end{bmatrix} = \begin{bmatrix} 2 & 3 & 1 & 1 \\ 1 & 2 & 3 & 1 \\ 1 & 1 & 2 & 3 \\ 3 & 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} x_{m,c0} \\ x_{m,c1} \\ x_{m,c2} \\ x_{m,c3} \end{bmatrix}$$

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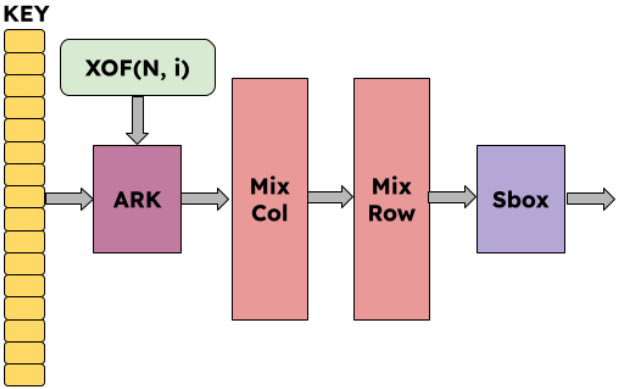
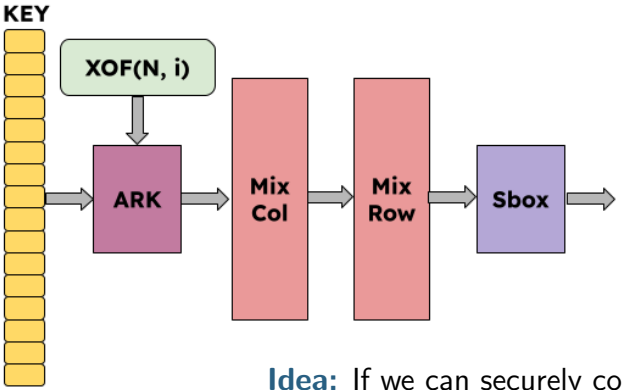
Hybrid Homomorphic Encryption: HERA



$$\text{Cube}(\mathbf{x}) = x^3$$

$$x_0 + x_1 \equiv x \mod p$$

Hybrid Homomorphic Encryption: HERA

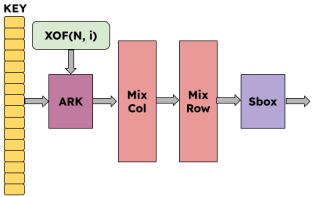


$$\text{Cube}(\mathbf{x}) = x^3$$

$$x_0 + x_1 \equiv x \mod p$$

Idea: If we can securely compute x^2,
then extending this approach to x^3 is straightforward.

Hybrid Homomorphic Encryption: HERA



$$\text{Cube}(\mathbf{x}) = x^3$$

$$x_0 + x_1 \equiv x \mod p$$

$$\text{Cube}(x) = \begin{cases} r = \text{rand}() \mod p \\ x_0' = (x_0 + r) \mod p \\ x_1' = (x_1 + p - r) \mod p \end{cases}$$

Note:

$$x_0' + x_1' \equiv x \mod p$$

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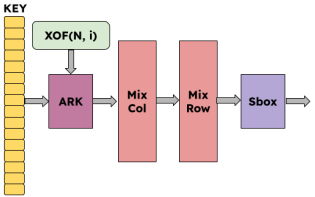
Hybrid Homomorphic Encryption: HERA



$$\text{Cube}(\mathbf{x}) = x^3$$

$$x_0 + x_1 \equiv x \mod p$$

$$\text{Cube}(x) = \begin{cases} r = \text{rand}() \mod p \\ x_0' = (x_0 + r) \mod p \\ x_1' = (x_1 + p - r) \mod p \\ \mathbf{f}(x_0, x_1, x_0', x_1') \\ \mathbf{f}(x_0'', x_1'', x_0', x_1') \end{cases}$$

Note:

$x_0' + x_1' \equiv x \mod p$

Note:

f(...) executes secure multiplication

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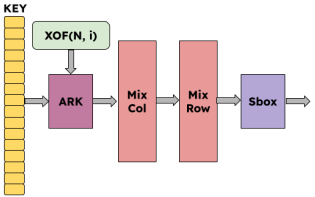
Hybrid Homomorphic Encryption: HERA



$$\text{Cube}(\mathbf{x}) = x^3$$

$$x_0 + x_1 \equiv x \mod p$$

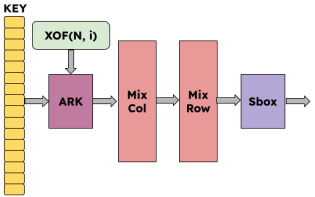
Note:

First call to f(...): $(x_0 + x_1)^2 \equiv x^2 \mod p$

Second call to f(...): $(x_0 + x_1)^3 \equiv x^3 \mod p$

$$\text{Cube}(x) = \begin{cases} r = \text{rand}() \mod p \\ x_0' = (x_0 + r) \mod p \\ x_1' = (x_1 + p - r) \mod p \\ \mathbf{f}(x_0, x_1, x_0', x_1') \qquad \Rightarrow (x_0'' + x_1'')^2 \equiv x^2 \mod p \\ \mathbf{f}(x_0'', x_1'', x_0', x_1') \end{cases}$$

Hybrid Homomorphic Encryption: HERA



$$\text{Cube}(\mathbf{x}) = x^3$$
$$x_0 + x_1 \equiv x \mod p$$

$$\text{Cube}(x) = \begin{cases} r = \text{rand}() \mod p \\ x_0' = (x_0 + r) \mod p \\ x_1' = (x_1 + p - r) \mod p \\ \mathbf{f}(x_0, x_1, x_0', x_1') \\ \mathbf{f}(x_0'', x_1'', x_0', x_1') \end{cases} \quad \mathbf{f}(\ldots) = \begin{cases} c_0 = \text{rand}() \mod p \\ c_1 = (c_0 + (x_0 \cdot x_1') + (x_1 \cdot x_0')) \mod \\ x_0'' = ((x_0 \cdot x_0') + p - c_0) \mod p \\ x_1'' = ((x_1 \cdot x_1') + c_1) \mod p \end{cases}$$

Results

Results

Metric	Not Masked	Masked
Encryption Time (μs)	84	123
Decryption Time (μs)	66	106
CPUs Utilized	0.617	0.785
Task Clock (ms)	3.24	3.45
Instructions Executed	6,044,834	6,114,087

Table: Comparison of Masked vs. Non-Masked Implementations

Conclusion

Conclusion and Future Scope

- First-order masking protects against first-order attacks.
- Implementation is easily extendable to higher-order masking to protect against higher-order attacks.
- Taking hybrid homomorphic encryption one step closer to real-world secure cloud computing.

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Extending HERA to Higher-Order Security

Key Idea: Increase the number of shares to improve security.

Current Two-Share Representation

$$x_0 + x_1 \equiv x \mod p$$

Generalized N-Share Representation

$$x_0 + x_1 + x_2 + \ldots + x_{N-1} \equiv x \mod p$$

Note:

- Each share is randomized but maintains modular sum.
- Security improves, but computational cost increases.

Formal proof: AddRoundKey

Algorithm 4 HERA AddRoundKey Function (Masked)

Require: Masked state arrays S_0, S_1, masked key arrays K_0, K_1, length $n = HERA\_K$.
Ensure: Updated masked state arrays S_0, S_1.

1: **for** $i = 0$ **to** $n - 1$ **do**
2: $r_i \leftarrow \text{UniformRandom}(\mathbb{Z}_p)$
3: $m_{0,i} \leftarrow (K_{0,i} \cdot r_i) \mod p$
4: $m_{1,i} \leftarrow (K_{1,i} \cdot r_i) \mod p$
5: $S'_{0,i} \leftarrow (S_{0,i} + m_{0,i}) \mod p$
6: $S'_{1,i} \leftarrow (S_{1,i} + m_{1,i}) \mod p$
7: **end for**

Formal proof: MixColumns

Algorithm 7 HERA MixColumns Function (Masked)

Require: Masked state shares $S_0 = \{c_{i,j,0}\}$, $S_1 = \{c_{i,j,1}\}$

Ensure: Updated masked state shares $S'_0 = \{c'_{i,j,0}\}$, $S'_1 = \{c'_{i,j,1}\}$

1: **for** $i = 0$ **to** 3 **do**

2: $\quad sum_0 \leftarrow (c_{i,0,0} + c_{i,1,0} + c_{i,2,0} + c_{i,3,0}) \mod p.$

3: $\quad sum_1 \leftarrow (c_{i,0,1} + c_{i,1,1} + c_{i,2,1} + c_{i,3,1}) \mod p.$

4: $\quad$ **for** $j = 0$ **to** 3 **do**

5: $\quad\quad c'_{i,j,0} \leftarrow (sum_0 + c_{i,j,0} + c_{i,(j+1 \mod 4),0} + c_{i,(j+2 \mod 4),0}) \mod p.$

6: $\quad\quad c'_{i,j,1} \leftarrow (sum_1 + c_{i,j,1} + c_{i,(j+1 \mod 4),1} + c_{i,(j+2 \mod 4),1}) \mod p.$

7: $\quad$ **end for**

8: **end for**

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Formal proof: MixRows

Algorithm 8 HERA MixRows Function (Masked)

Require: Masked state shares $S_0 = \{r_{i,j,0}\}$, $S_1 = \{r_{i,j,1}\}$
Ensure: Updated masked state shares $S'_0 = \{r'_{i,j,0}\}$, $S'_1 = \{r'_{i,j,1}\}$

for $j = 0$ **to** 3 **do**
  $sum_0 \leftarrow (r_{0,j,0} + r_{1,j,0} + r_{2,j,0} + r_{3,j,0}) \mod p.$
  $sum_1 \leftarrow (r_{0,j,1} + r_{1,j,1} + r_{2,j,1} + r_{3,j,1}) \mod p.$
  for $i = 0$ **to** 3 **do**
    $r'_{i,j,0} \leftarrow (sum_0 + r_{i,j,0} + r_{(i+1 \mod 4),j,0} + r_{(i+2 \mod 4),j,0}) \mod p.$
    $r'_{i,j,1} \leftarrow (sum_1 + r_{i,j,1} + r_{(i+1 \mod 4),j,1} + r_{(i+2 \mod 4),j,1}) \mod p.$
  end for
end for

Formal proof: S-box

Algorithm 6 HERA S-Box Cube Function (Masked)

Require: Masked state shares $s_0 = \{s_{0,0}, s_{0,1}, \ldots, s_{0,n}\}$, $s_1 = \{s_{1,0}, s_{1,1}, \ldots, s_{1,n}\}$

Ensure: Updated masked state shares s'_0, s'_1, satisfying $s' = (s^3) \mod p$.

1: **for** $i = 0$ **to** $n - 1$ **do**
2: Generate random value $t_1 \in \mathbb{Z}_p$.
3: $t_2 \leftarrow (s_{0,i} + t_1) \mod p$.
4: $t_3 \leftarrow (s_{1,i} + p - t_1) \mod p$.
5: SECURE_MUL$(s_{0,i}, s_{1,i}, t_2, t_3)$, resulting in intermediate masked shares $(z_{0,i}, z_{1,i})$
6: SECURE_MUL$(z_{0,i}, z_{1,i}, t_2, t_3)$, resulting in updated masked shares $(s'_{0,i}, s'_{1,i})$
7: $s_{0,i} \leftarrow s'_{0,i}$
8: $s_{1,i} \leftarrow s'_{1,i}$
9: **end for**

Formal proof: Masked Gadget

Algorithm 5 HERA Secure Multiplication (Masked Gadget)

Require: Masked inputs (x_0, x_1) and (y_0, y_1)

Ensure: Masked outputs (z_0, z_1), where $z = (x \cdot y) \mod p$

1: Generate random value $r_{0,1} \in \mathbb{Z}_p$

2: $r_{1,0} \leftarrow \big((x_0 \cdot y_1) + (x_1 \cdot y_0)\big) \mod p$

3: $z_0 \leftarrow \big((x_0 \cdot y_0) + p - r_{0,1}\big) \mod p$

4: $z_1 \leftarrow \big((x_1 \cdot y_1) + r_{1,0}\big) \mod p$