Topology

Aiken Ji

April 12, 2025

Contents

Conventions

$\mathbb{F}$ denotes either $\mathbb{R}$ or $\mathbb{C}$.
$\mathbb{N}$ denotes the set $\{1, 2, 3, \ldots\}$ of natural numbers (excluding 0).
Inner products are taken to be linear in the first argument and conjugate linear in the second.
The Einstein summation convention is used for tensors unless otherwise specified.

1 Topological Spaces

Definition 1.1. Topological Spaces

Let X be a non-empty set and τ is a collection of subsets U of X. If

1. $\varnothing, X \in \tau$
2. $\bigcup_\alpha U_\alpha \in \tau$
3. $\bigcap_i U_i \in \tau$

Then we call τ the **topology** on X, and the pair (X, τ) is called a **topological space**.

Definition 1.2. Discrete and Indiscrete Topology

Let X be any non-empty set and τ be the collection of subsets of X.

1. If $\tau = \mathcal{P}(X)$, then τ is called the **discrete topology**, and X is called a **discrete space**.
2. If $\tau = \{\varnothing, X\}$, then τ is called the **indiscrete topology**, and X is called a **indiscrete space**.