Assumptions:

1. Y is convex, closed and free disposal
2. F is convex, continuous and increasing
3. f is concave, continuous and (strictly) increasing

Table 1: Producer Duality

Problem	$\max_{y \in \mathbb{R}^L} p \cdot y$ s.t. $F(y) \leq 0$	$\min_{z \in \mathbb{R}_+^{L-1}} w \cdot z$ s.t. $f(z) \geq q$
FOC	$\dfrac{\frac{\partial F}{\partial y_k}}{\frac{\partial F}{\partial y_l}} = \dfrac{p_k}{p_l}$	$\dfrac{\frac{\partial f}{\partial z_k}}{\frac{\partial F}{\partial z_l}} = \dfrac{w_k}{w_l}$
Value Function	Profit function: $\pi(p)$	Cost function: $c(w, q)$
Properties	(a) $\pi(\lambda p) = \lambda \pi(p)$ (b) convex function of p	(a) $c(\lambda w, q) = \lambda c(w, q)$ (b) concave function of w (c) convex function of q (d) increasing in q
Solution	Supply correspondence: $y(p)$	Conditional factor demand correspondence: $z(w, q)$
Properties	(a) $y(\lambda p) = y(p)$ (b) Convex-valued (c) Single-valued, if Y strictly convex	(a) $z(\lambda w, q) = z(w, q)$ (b) Convex-valued (c) Single-valued, if f strictly concave
Comparative Statics	Hotelling's Identity $\dfrac{\partial \pi(p)}{\partial p_l} = y_l(p)$ $Dy(p) = D^2\pi(p)$ positive semidefinite	Shepard's Lemma $\dfrac{\partial c(w, q)}{\partial w_k} = z_k(w, q)$ $D_w z(w, q) = D_w^2 c(w, q)$ negative semidefinite

Caveat: If the production set Y exhibits non-decreasing return to scale, then either $\pi(p) = 0$ or $\pi(p) = +\infty$.

Table 2: Consumer Duality

Problem	$\max\limits_{x\geq 0} u(x)$ s.t. $p\cdot x\leq y$	$\min\limits_{x\geq 0} p\cdot x$ s.t. $u(x)\geq u$
FOC	$\dfrac{\partial u(x)}{\partial x_i}=\lambda p_i$	$\lambda\dfrac{\partial u(x)}{\partial x_i}=p_i$
Value Function	Indirect utility function: $u(p,y)$	Expenditure function: $e(p,u)$
Properties	(a) continuous (b) $v(\lambda p,\lambda y)=v(p,y)$ (c) strictly increasing in y (d) nonincreasing in p (e) quasiconvex in p	(a) continuous (b) $e(\lambda p,u)=\lambda e(p,u)$ (c) strictly increasing in v (d) nondecreasing in p (e) concave in p
Solution	Marshallian demand: $D(p,y)$	Hicksian demand: $h(p,u)$
Properties	(a) upper semicontinuous (b) $D(\lambda p,\lambda y)=D(p,y)$; (c) budget balanceness: $p\cdot x^*=y\ \forall x^*\in D(p,y)$ (d) convex-valued if $u(x)$ is quasiconcave; (e) single-valued and continuous if $u(x)$ is strictly quasiconcave. Roy's Identity $d_i(p,y)=-\dfrac{\frac{\partial v(p,y)}{\partial p_i}}{\frac{\partial v(p,y)}{\partial y}}$	(a) upper semicontinuous (b) $h(\lambda p,u)=h(p,u)$ (c) no excess utility: $u(x^h)=v\ \forall x^h\in h(p,u)$ (d) convex-valued if $u(x)$ is quasiconcave; (e) single-valued and continuous if $u(x)$ is strictly quasiconcave Shepard's Lemma $h_i(p,u)=\dfrac{\partial e(p,u)}{\partial p_i}$
Duality	$D(p,y)=h(p,v(p,y))$ $v(p,e(p,u))=u$	$h(p,y)=D(p,e(p,u))$ $e(p,v(p,y))=y$