Started on	Friday, 20 October 2023, 8:56 AM
State	Finished
Completed on	Friday, 20 October 2023, 9:26 AM
Time taken	30 mins 9 secs
Grade	**9.00** out of 10.00 (**90**%)

Question 1

Correct

Mark 1.00 out of 1.00

Find $\sum_{n=0}^{\infty} \frac{1}{(n+1)(n+2)}$,

Answer: 1 ✔

The correct answer is: 1

Question 2

Incorrect

Mark 0.00 out of 1.00

Cardinality of power set of natural numbers is

- ☐ Uncountable
- ☑ Countable ✖
- ☐ Can't determine

The correct answer is:
Uncountable

Question 3

Correct

Mark 1.00 out of 1.00

Let A and B be bounded non-empty sets. Which of the following statements would be equivalent to saying that $\sup(A) = \inf(B)$

- [ ] For every $a \in A$ there exists a $b \in B$ such that $a + \epsilon < b$.
- [x] There exists a real number L such that $a \leq L \leq b$ for all $a \in A$ and $b \in B$. ✖
- [x] For every $a \in A$ and every $b \in B$, we have $a \leq b$. Also, for every $\epsilon > 0$, there exists $a \in A$ and $b \in B$ such that $b - a < \epsilon$ ✔
- [ ] There exists $a \in A$ and $b \in B$ such that $a \leq b$. However, for any $\epsilon > 0$, there does not exist $a \in A$ and $b \in B$ for which $a + \epsilon \leq b$.

The correct answer is:
For every $a \in A$ and every $b \in B$, we have $a \leq b$. Also, for every $\epsilon > 0$, there exists $a \in A$ and $b \in B$ such that $b - a < \epsilon$

Question 4

Correct

Mark 1.00 out of 1.00

Let A and B be bounded non-empty sets such that $\inf(A) \leq \sup(B)$. Which of the following statements must be true?

- [ ] For every $a \in A$ there exists a $b \in B$ such that $a \leq b$
- [ ] There exists $b \in B$ such that $a \leq b$ for all $a \in A$.
- [x] For every $\epsilon > 0$ there exists $a \in A$ and $b \in B$ such that $a < b + \epsilon$. ✔
- [ ] There exists $a \in A$ such that $a \leq b$ for all $b \in B$.

The correct answer is:
For every $\epsilon > 0$ there exists $a \in A$ and $b \in B$ such that $a < b + \epsilon$.

Question 5

Correct

Mark 1.00 out of 1.00

Find $\lim (x^2 e^{-nx})$ for $x \in \mathbb{R}, x \geq 0$.

Answer: 0 ✔

The correct answer is: 0

Question 6

Correct

Mark 1.00 out of 1.00

Find $\sum_{n=0}^{\infty} \frac{1}{(\alpha+n)(\alpha+n+1)}$

Assume that the value of alpha is 2.

Answer: 0.50 ✔

The correct answer is: 0.5

Question 7

Correct

Mark 1.00 out of 1.00

Let f : [a, b] → R be continuous on [a, b]. Choose the false statement.

- [ ] If f(a) > 0 > f(b) then there exists c ∈ (a, b) such that f(c) = 0.
- [ ] If f(a) < 0 < f(b) then there exists c ∈ (a, b) such that f(c) ≠ 0.
- [ ] If f(a) < 0 < f(b) then there exists c ∈ (a, b) such that f(c) = 0.
- [x] If a < 0 < b then there exists c ∈ (a, b) such that f(c) = 0. ✔

The correct answer is:
If a < 0 < b then there exists c ∈ (a, b) such that f(c) = 0.

Question 8

Correct

Mark 1.00 out of 1.00

Which of the following statements is correct?

- $\sum_{n=1}^{\infty} \cos n$ is divergent and the series $\sum_{n=1}^{\infty} (\cos n)/n^2$ is divergent.
- $\sum_{n=1}^{\infty} \cos n$ is divergent and the series $\sum_{n=1}^{\infty} (\cos n)/n^2$ is convergent. ✔
- $\sum_{n=1}^{\infty} \cos n$ is convergent and the series $\sum_{n=1}^{\infty} (\cos n)/n^2$ is divergent.
- $\sum_{n=1}^{\infty} \cos n$ is convergent and the series $\sum_{n=1}^{\infty} (\cos n)/n^2$ is convergent.

The correct answer is:

$\sum_{n=1}^{\infty} \cos n$ is divergent and the series $\sum_{n=1}^{\infty} (\cos n)/n^2$ is convergent.

Question 9

Correct

Mark 1.00 out of 1.00

Choose a function that is continuous at 0.

- [x] \($f(x) = \sin x$ \)
- [x] \($f(x) = 10x$ \)
- [x] \($f(x) = |x|$ \)
- [ ] \($f(x) = \frac{1}{x}$ \)

The correct answers are:
\(

$$f(x) = \sin x$$

\)

,
\(

$$f(x) = |x|$$

\)

,
\(

$$f(x) = 10x$$

\)

Question 10

Correct

Mark 1.00 out of 1.00

Which one of the following function is continuous at $x = 3$?

- $f(x) = \begin{cases} 4 & x = 3 \\ 8 - x & x \neq 3 \end{cases}$
- $f(x) = \dfrac{1}{x^3 - 27} \quad x \neq 3$
- $f(x) = \begin{cases} 2 & x = 3 \\ x - 1 & x > 3 \\ \frac{x+3}{3} & x < 3 \end{cases}$ ✔
- $f(x) = \begin{cases} x + 3 & x \leq 3 \\ x - 4 & x > 3 \end{cases}$

The correct answer is: $f(x) = \begin{cases} 2 & x = 3 \\ x - 1 & x > 3 \\ \frac{x+3}{3} & x < 3 \end{cases}$