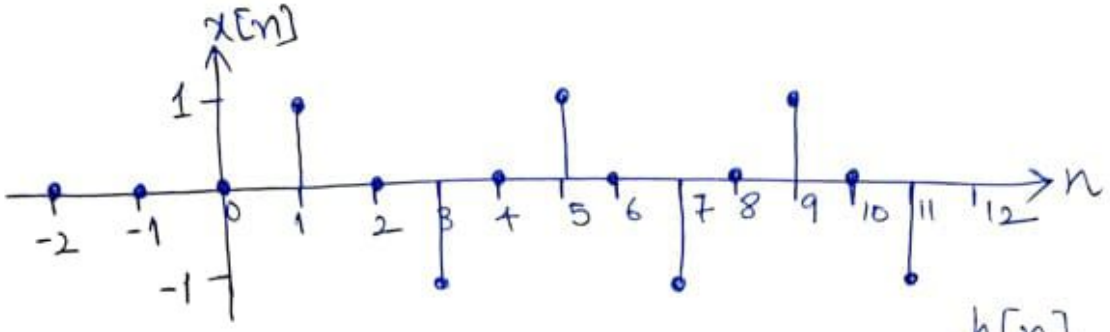
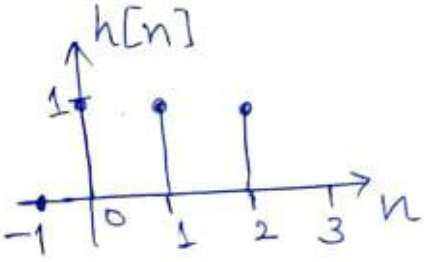
Q1. (a) $x[n] = \sin\left(\frac{\pi}{2}n\right) u[n]$

(2)

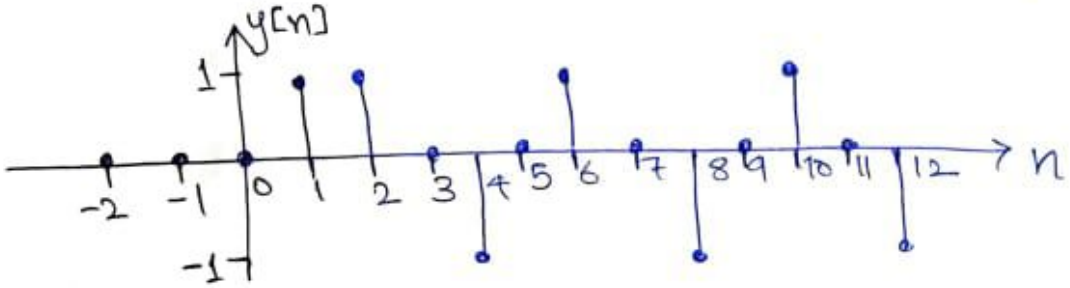


(b) $y[n] = x[n] * h[n]$

$= \sum_{K=-\infty}^{\infty} x[k]\, h[n-K]$



(2)



(c) $x'[n] = \sin\left(\frac{\pi}{2}\right)n$

$h[n] = u[n] - u[n-3]$

$= \delta[n] + \delta[n-1] + \delta[n-2]$.

$y'[n] = x'[n] * h[n]$

$= x'[n] + x'[n-1] + x'[n-2]$

$= \sin\left(\frac{\pi}{2}n\right) + \sin\left(\frac{\pi}{2}(n-1)\right) + \sin\left(\frac{\pi}{2}(n-2)\right)$

(0.5)

upon simplifying further ↓

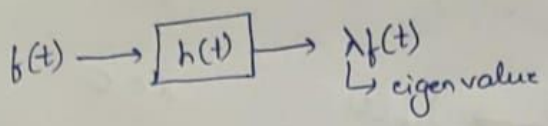
$= \sin\left(\frac{\pi}{2}n\right) + \sin\left(\frac{\pi}{2}n - \frac{\pi}{2}\right) + \sin\left(\frac{\pi}{2}n - \pi\right)$

$= \sin\left(\frac{\pi}{2}n\right) - \cos\left(\frac{\pi}{2}n\right) - \sin\left(\frac{\pi}{2}n\right)$

$= -\cos\left(\frac{\pi}{2}n\right)$.

(2)

1) Eigen Signals - Signals when passed through an LTI system, output a scaled version of input signal



— 1M

$\lambda \in C$

Note - Any Definition demonstrating knowledge of Eigen Signal will be given full marks

a) $h(n) = \delta(n-4)$

$y(n) = h(n) * x(n)$

— 2.5M

$y(n) = x(n-4)$

$\Rightarrow x(n-4) = \lambda x(n)$

Note if → Any periodic signal example with time period = 2, 4 with reasoning will fetch full marks (2.5M)

Ex:- $\cos\left(\frac{\pi n}{2}\right)$

Note that in the context of the question sum of complex expo are accepted

→ Trivial examples like DC signal or any complex exponentials will be given (1M)

Ex: $x(n) = k \in C$

or $x(n) = a^n$ { note that $R \subseteq C$ }

b) $h(n) = \delta(n-2) + \delta(n-4)$ — 2.5M

$y(n) = x(n-2) + x(n-4) = \lambda x(n)$

→ Any periodic signal example with time period = 2 with reasoning will fetch full marks — 2.5M Ex: $\cos(\pi n)$

Quiz 1

Solution

[3] Shiva : $y[n] = \frac{1}{2} y[n-1] + x[n]$

Madhuri : $y[n] = \frac{5}{6} y[n-1] - \frac{1}{6} y[n-2] + x[n] - \frac{1}{3} x[n-1]$

(a) → 3 Marks

Shiva : $Y(z) = \frac{z^{-1}}{2} Y(z) + X(z)$ — 0.5 Mark for steps

$\Rightarrow Y(z)\left(1 - \frac{z^{-1}}{2}\right) = X(z)$

$\rightarrow \boxed{H(z) = \frac{1}{1 - z^{-1}/2}}$ — 0.5 for answer

Madhuri : $Y(z) = \frac{5}{6} z^{-1} Y(z) - \frac{z^{-2}}{6} Y(z) + X(z) - \frac{z^{-1}}{3} X(z)$

1 Mark for steps

$\Rightarrow Y(z)\left[1 - \frac{5}{6} z^{-1} + \frac{z^{-2}}{6}\right] = X(z)\left[1 - \frac{z^{-1}}{3}\right]$

$\Rightarrow H(z) = \frac{\left(1 - z^{-1}/3\right)}{\left(1 - \frac{5}{6} z^{-1} + \frac{z^{-2}}{6}\right)}$

$= \frac{\left(1 - z^{-1}/3\right)}{\left(1 - z^{-1}/2\right)\left(1 - z^{-1}/2\right)}$

$\boxed{H(z) = \frac{1}{1 - z^{-1}/2}}$ — 0.5 for answer

0.5 marks for final inference

Since both $H(z)$ are same & both systems are causal $\Rightarrow$ same ROC

$\Rightarrow$ Shiv & Madhuri are working with same systems

(b) → 2 Marks

$H(z) = \frac{1}{1 - z^{-1}/2}$ → 1 Mark for identifying correct ROC

Since system causal, ROC : $\boxed{|z| > \frac{1}{2}}$

$\Rightarrow h[n] = \mathcal{Z}^{-1}\{H(z)\}$

using standard z transform pair

$\downarrow$

$\alpha^n u[n] \xrightarrow{z} \frac{1}{1 - \alpha z^{-1}}$; $|z| > |\alpha|$

1 Mark if all is mentioned.

we get, $\alpha = \frac{1}{2}$ & $\boxed{h[n] = \left(\frac{1}{2}\right)^n u[n]}$

↳ impulse response of system

(c) → 2 Marks

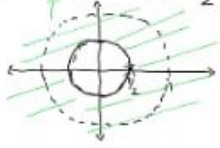
↳ only Yes / No → 0.5 Marks

Time domain → Bounded Input stability

Freq. domain → ROC : $|z| > \frac{1}{2}$

Any of these shown then full marks.

$\Rightarrow$



$\Rightarrow$ ROC includes unit circle

$\Rightarrow$ system is stable.