Real Analysis

Mid-Sem 2023

Time - 1.5 hours

Full marks 50

1.a) Prove that for each $n \geq 2$, $(n+1)! > 2^n$. induction

b) Prove that for all $n \in \mathbb{N}$, $(3+\sqrt{5})^n + (3-\sqrt{5})^n$ is an even integer. → JEE (4+6)

2.a) Prove that the set of natural numbers is not bounded from above.

b) Prove that there is an unique positive real number x, such that $x^2 = 2$ (5+5)

3.a) Prove that the union and intersection of finite number of open sets in $\mathbb{R}$ are open sets themselves.

b) Show that the set $\mathbb{N}$ has no limit points. (10+5)

4.a) Prove that $\lim_{n\to\infty} \frac{S_n}{t_n} = \frac{s}{t}$, given $\lim_{n\to\infty} S_n = s$ and $\lim_{n\to\infty} t_n = t$ with $t_n \neq 0 \ \forall n \in \mathbb{N}$ and $t \neq 0$.

b) Show whether the following sequence (x_n) with $x_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots.. + \frac{1}{n}$ is convergent or not.

c) Given $x \geq 1$, show that $\lim_{n\to\infty} \left(2x^{1/n} - 1\right)^n = x^2$ (5+5+5)