EC5.101 – Network, Signals and Systems – Quiz 1

Date: 29th August, 2023
Exam duration: 45 minutes

Maximum marks: 24

Instructions:

a) There are *4 questions* for a total of *24 marks*.

b) Mention any additional assumptions you make that is not given in the question.

c) Clearly show the steps used to arrive at the solutions.

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1. [6 marks] Consider a periodic square wave given by

$$x(t) = \begin{cases} 3, & 0 < t \leq 5 \\ -3, & 5 < t < 10, \end{cases}$$

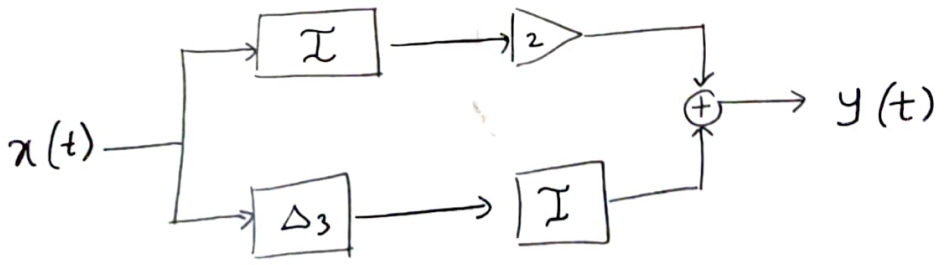
with period $T = 10$.

   (a) [4] Perform Fourier series (FS) analysis to find complex FS coefficients for this signal.

   (b) [1] As the signal $x(t)$ is real-valued, what symmetry property is expected for FS coefficients? Verify this for your answer in part (a).

   (c) [1] Do the absolute value of coefficients exhibit any symmetry?

2. [4 marks] A system block diagram is shown below which is made using integrator, scalar, and delay blocks. When the input to this system is the unit impulse signal, i.e., $x(t) = \delta(t)$, find and sketch the output signal $y(t)$.



3. [6 marks] Two periodic signals $x_1(t)$ and $x_2(t)$ have same period. Their Fourier series (FS) representations have M and N non-zero coefficients respectively. How many non-zero coefficients would the following signals have? FS

   (a) $x_1(t+5)$

   (b) $x_1(5-t)$

   (c) $x_1(t)+x_2(t)$

   (d) $x_1(t)+\frac{d}{dt}x_2(t)$

   Give the complete range in your answer and justify.

4. [8 marks] Tired of infinities, Shiva and Madhuri have decided to use a reduced set of basis signals to represent real-valued periodic signals. They are working with periodic signals with period 2π and each decide to keep only 3 signals in their basis set. Shiva decides to use $\{1, \sin(t), \cos(t)\}$ as the basis whereas Madhuri decides to use $\{1, \cos(t), \cos(2t)\}$ as the basis. Of course, not all signals can be represented using these reduced basis.

   (a) [6] For each of the following signals, identify who can correctly represent it using the basis signals at their disposal. Also, mention if neither of them can represent the signal. Justify your answers.

      i. $\sin^2(t)$

      ii. $\cos^2(t)+\sin(t)$

      iii. $\cos(t+1)$

   (b) [2] Give two examples each of signals $x_S(t)$ and $x_M(t)$ such that: $x_S(t)$ can be represented by Shiva; $x_M(t)$ can be represented by Madhuri; and, they are orthogonal, $\langle x_S(t), x_M(t)\rangle = 0$. Here orthogonality is defined over the time period $T = 2\pi$.