EC5.201: Signal Processing **Mid-Semester Exam** **Monsoon 2024**

Marks: 40 **Date: 24th Sep 2024**

Instructions:

a) There are *6 questions* for a total of *40 marks*.
b) Mention any additional assumptions you make that is not given in the question.
c) Clearly show the steps used to arrive at the solutions.

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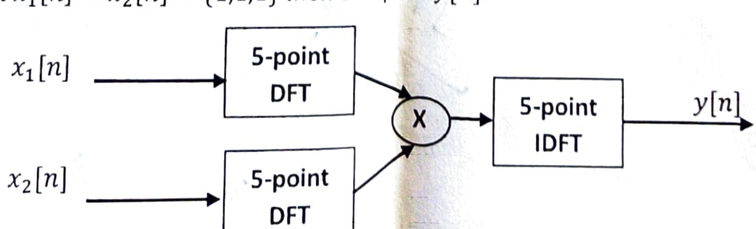
1. Let the $x_1[n] = \cos\left(\frac{\pi}{9}n\right)$, $x_2[n] = \cos\left(\frac{2\pi}{3}n\right)$ and $x[n] = x_1[n] + x_2[n]$ then **[2+3++3+2]**
   a. Compute DTFT of $x[n]$.
   b. A design engineer would like to extract $x_1[n]$ from $x[n]$ using a discrete LTI system, so an impulse response $h[n]$ is chosen with spectrum $H(\omega) = 1 \; \forall \; 0 \le \omega < \frac{\pi}{3}$ and $0 \; \forall \; \frac{\pi}{3} \le \omega < 2\pi$. Compute the impulse response $h[n]$.
   c. The same engineer mistakenly considers only real part of the $h[n]$ i.e., $h_R[n]$ in the discrete LTI system and obtain output $y[n]$. Is $y[n]$ is equal to $x_1[n]$? Justify your answer.
   d. Also, the engineer sends $h_{AM}[n] = e^{\frac{\pi}{4}n} h_R[n]$, which is a modulated impulse response of the $h_R[n]$ with carrier frequency of $\frac{\pi}{2}$, via a communication channel. At the receiving end, another engineer computes the output $y[n]$ of the discrete LTI system considering input as $x[n]$ and impulse response as $h_{AM}[n]$. Is $y[n]$ is equal to $x_1[n]$? If yes justify your answer else find $y[n]$

2. Let $X[k]$ and $H_1[k]$ are the sampled spectrums of $X(\omega)$ and $H_1(\omega)$ with N_x and N_h points respectively, where $X(\omega)$ and $H_1(\omega)$ are the DTFT of $x[n]$ and $h_R[n]$ defined in problem 1. Then **[1+1+2+2]**
   a. Compute the minimum value of N_x that can preserve the spectrums of both $x_1[n]$ and $x_2[n]$?
   b. $H_2[k]$ is N_h-point DFT of $h_R[n]$, then, is $H_2[k]$ equal to $H_1[k]$, Justify your answer?
   c. Is IDFT of $H_1[k]$ equal to $h_R[n]$, justify your answer.
   d. Compute the signal whose DFT is $X[k - N_x]$.

3. Answer the below questions **[4+2+2]**
   a. Let $X(\omega)$ and $X_1(\omega)$ are respective DTFTs of $x[n]$ and $x_1[n]$, where $x_1[n] = x[2n]$, then, express $X_1(\omega)$ in terms of $X(\omega)$. Also, verify the result by computing $X(\omega)$ and $X_1(\omega)$ for the case $x[n] = u[n] - u[n-k]$
   b. Let $x_1[n] = x_2[n] = \{1,1,1\}$ then compute $y[n]$.



   c. Let $x_3[n = \{3,2,1,0,1,2\}$ then clearly deduce the relation between $x_3[n]$ and $y[n]$ in 3.b.