Information and Communication

IIIT-H

Spring-2024

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Exam: Mid-1
Marks: 50

Date: 29-Feb-2024
Time: 1 hr 30 minutes

Instructions:

- Answering all the questions is compulsory.
- All steps should be justified in detail.
- Clearly state the assumptions (*if any*) made that are not specified in the questions.

1. (16 marks)

   (a) (4 marks) A rectangular pulse $x(t) = rect(t/T)$ is passed through an ideal low-pass filter with bandwidth $1/T$. Express the output $y(t)$ in time domain or the frequency domain, whichever is easier. (Note: The bandwidth of a low-pass filter is the maximum frequency component it allows).

   (b) (5 marks) Suppose the output $y(t)$ is sampled at a rate of $4/T$ samples per second. show the fourier-transform of the sampled signal.

   (c) (3 marks) Let $u(t), v(t), w(t)$ be three time-signals. Is it true that the fourier-transform of the convolution $(u(t) \star v(t)) \star w(t)$ is the product of their individual transforms? Prove or disprove.

   (d) (4 marks) Instead of a single pulse, assume a pulse train, given by $x_1(t) = \sum_{k\in\mathbb{Z}} rect(\frac{t-4kT}{T})$. Obtain the fourier-transform of the output $z(t)$, when new $x_1(t)$ is passed through the LPF in part (a). (Hint: Try writing $x_1(t)$ as a convolution, and use Fourier Transform properties. Perhaps part (c) is useful too).

2. (10 marks) A signal given by $x(t) = \min(|\tan(2\pi 1000t)|, 5)$ is modulated using DSB-SC modulation, where the carrier signal is given by $A\cos(2\pi 10000t)$.

   (a) (3 marks) Sketch the modulated signal, if $A = 2$.

   (b) (2 marks) Also, sketch the spectrum (only magnitude part. You can use a justifiable approximate spectrum of $x(t)$. The exact spectrum of $x(t)$ need not be derived).

   (c) (5 marks) Show a technique to demodulate the signal to obtain $x(t)$ (you need to specify the parameters of the various blocks you may be using).

3. (8 marks) Consider a random experiment of rolling a dice with faces $\{1, 2, 3, 1, 2, 4\}$ such that any face is equally likely to show up (notice the numbers on the dice carefully!). Consider the events $A = \{1, 2\}$, $B = \{2, 4\}$, and $C = \{1, 2, 3, 4\}$.

   (a) (5 marks) Suppose a user is interested only in events A and B. Can you propose a way to simplify the associated event space? Write down the associated probability space. Specify the probabilities for all events in your event space.

(b) (3 marks) Suppose a user is interested only in event A. Can you propose a way to simplify the associated event space? Write down the associated probability space. Specify the probabilities for all events in your event space.

4. (5 marks) Find the mean of a binomial random variable with parameters n and p. (Hint: Try to find a relation between $\binom{n}{k}$ and $\binom{n-1}{k-1}$ to simplify the calculations.)

5. (5 marks) Let X be random variable with the support set $\mathcal{X} = \{1, 2, \ldots\}$. Then show that

$$\mathbb{E}(X) = \sum_{x=1}^{\infty} P_X(X \geq x)$$

6. (6 marks) A manufacturer produces light-bulbs that are packed into boxes of 100 bulbs each. Quality control observe that 0.5% of the light-bulbs produced are defective. What percentage of the boxes will contain:

(a) (3 marks) no defective light-bulbs?

(b) (3 marks) 2 or more defective light-bulbs?

Use only Poisson approximation to binomial distribution to solve this problem.