

IIT-H
EC5.102: Information and Communication
Summer-2024

Exam: Quiz-2

Marks: 20

Date: 2-Apr-2024

Time: 10:15 am to 11:00 am

Instructions:

- Answering all the questions is compulsory.
- All steps should be justified in detail.
- Clearly state the assumptions (if any) made that are not specified in the questions.

1. Let the joint distribution $P_{X,Y}$ of two random variables X, Y be described by the following matrix (the rows denote the values of X given by $\{x_1, x_2\}$, the columns denote the values of Y , given by $\{a, b, c\}$).

$$\begin{bmatrix} 0.3 & 0.1 & 0.3 \\ 0.15 & 0 & 0.15 \end{bmatrix}$$

Compute the following.

- (as) (2 points) the distribution P_X and $H(X)$ (remember that you need to tell which value of X will have which probability, and similarly for below questions.)
- (bs) (2 points) the conditional distribution $P_{Y|X}(\cdot|x_1)$
- (cs) (2 points) the distribution $P_{Y,X}$

2. Consider the following function of variables x and y .

$$f(x, y) = \begin{cases} 0.64e^{-0.8y} & \text{if } 0 < x < y \\ 0 & \text{otherwise} \end{cases}$$

- (a) (4 points) Is this a valid joint probability density function (PDF)? YES/NO? Justify your answer.
- (b) (3 points) If the answer to part (A) is YES, find $P_{XY}[1 < X < 2 \text{ and } 1 < Y < 2]$.
If the answer to part (A) is NO, justify how can you make $f(x, y)$ a valid PDF?
3. Answer the following questions related to "unique decodability" of a source code.
- (a) (2 points) Define extension of a source code.
- (b) (2 points) When does a source code is defined to be uniquely decodable? (Please note that in this question, you need to "define" unique decodability of a source code and not write intuitive understanding.)
- (c) (3 points) Give an example of a source code with at least four codewords such that the source code is uniquely decodable but not an instantaneous code.