Quiz 1

(MA6.102) Probability and Random Processes, Monsoon 2024

26 August, 2024

Max. Duration: 45 Minutes

Question 1 (5 Marks). Consider three events A, B, and C.

(a) Suppose A and B are disjoint events. What is the maximum possible number of elements in the smallest σ-field that contains A, B, and C?

(b) Suppose A and B are disjoint events, and B^c and C are also disjoint events. What is the maximum possible number of elements in the smallest σ-field that contains A, B, and C?

Question 2 (5 Marks). Suppose two events A and B satisfy the condition that the probability of A given B is equal to the probability of A given the complement of B, i.e., $P(A \mid B) = P(A \mid B^c)$. Does this condition imply that the events A and B are independent, given that $0 < P(B) < 1$? Provide a proof or a counterexample.

Question 3 (5 Marks). Let F_1 and F_2 be two cumulative distribution functions (CDFs) such that $F_1(x) < F_2(x)$, for all $x \in \mathbb{R}$. Assume that F_1 and F_2 are continuous and strictly increasing. Show that there exists random variables X_1 and X_2, with respective CDFs F_1 and F_2, defined on the same probability space (call Ω) such that $X_1 > X_2$, i.e., $X_1(\omega) > X_2(\omega)$, for all $\omega \in \Omega$.