Exam: Quiz-1

Marks: 20

Date: 30-Jan-2024

Time: 45 minutes

Instructions:

- Answering all the questions is compulsory.

- All steps should be justified in detail.

- **Any attempt at plagiarism will result in ZERO for the exam, apart from other academic consequences.**

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1. (5+3+2 = 10 marks) State and prove the conjugate symmetry property of the Fourier Transform for a real signal $x(t)$. Further, show that if the signal $x(t)$ is also an even signal (i.e., $x(t) = x(-t), \forall t$), then the Fourier transform is also real. What happens if $x(t)$ is real and odd? Investigate, write a formal statement, and prove the property.

2. (4+6=10 marks) Consider the block diagram for an AM signal generator shown below (please observe carefully that the first element in the block diagram is a *signal adder*, **not** a multiplier).
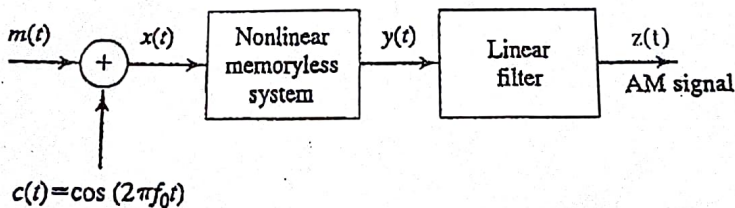
The operation of the non-linear device in the figure is given by the input-output relationship expression $y(t) = ax(t) + bx^2(t)$.

(a) Express $y(t)$, the output of the nonlinear device, in terms of the message $m(t)$ and the carrier $c(t) = cos(2\pi f_0 t)$ as shown.

(b) Suppose $m(t)$ is a baseband signal with bandwidth B. Assume that the carrier frequency is $f_0 \gg B$. Specify the characteristics of the linear filter if we wish to obtain an AM signal (with carrier *unsuppressed*) as the output $z(t)$. (That is, you should specify the type of filter, the centre frequency, and bandwidth of filter.) [Hint: Think about the spectrum of $y(t)$ and investigate which parts you want to keep, and which you want to filter out, to get $z(t)$.]