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Instructions:

- a) There are 9 questions for a total of 60 marks.
  - b) Clearly show the steps used to arrive at the solutions.
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1. 10 Marks [1+2+3+4]

A causal FIR filter with impulse response  $h[n] = \{1, 0, -1\}$  is to be implemented.

- a. Find and plot the magnitude spectrum of this filter.
- b. Show that the filter has linear phase and plot it. Hint: use trigonometric identities.
- c. If the input to this filter is the signal  $x[n] = 1 + \sin\left(\frac{\pi n}{6}\right) + \sin\left(\frac{\pi n}{2}\right) + \cos(\pi n)$ , find the output of this filter using frequency domain analysis. Give exact analytical expression.
- d. Due to a mistake, the filter  $h_1[n] = h[n - 1]$  gets implemented.
  - i. Show that  $h_1[n]$  is also a linear phase filter.
  - ii. How are its magnitude and phase spectrum related to that of  $h[n]$ ?
  - iii. If the above signal  $x[n]$  is given as input to this filter, what is the output?

2. 4 Marks [2+2]

A non-zero signal  $x[n]$  is passed through an ideal low pass filter (cutoff frequency  $\omega_1$ ) to give output  $y[n]$ . The signal  $y[n]$  is then passed through an ideal high pass filter (cutoff frequency  $\omega_2$ ) to give output  $z[n]$ . If it is observed that  $z[n] = x[n]$ , then answer the following:

- a. What must be the relation between  $\omega_1$  &  $\omega_2$  for this to happen? Justify.
- b. The Fourier transform of input signal  $x[n]$  is necessarily zero in which frequency regions?

3. [6 Marks]

An FIR system with transfer function  $H(z)$  is known to be causal, stable, and has linear phase characteristics. What can you say about the causality and stability of the following systems? If they are causal & stable, comment on their linear phase nature with reasons.

- a.  $\frac{1}{H(z)}$
- b.  $H(z^{-1})$
- c.  $H(-z)$
- d.  $-H(z)$

4. 10 Marks [3+3+4]

Shiva and Madhuri are designing digital IIR filters using different methods. Shiva is using the method of derivative approximation with  $T = 1$ , i.e.,  $s = 1 - z^{-1}$ . Madhuri is using a modified version of the bilinear transformation given by  $s = \frac{z}{z+1}$ .

- For both, find how does the imaginary axis, i.e.,  $s = j\omega$ , maps in the  $z$ -plane.
- For both, show that a stable and causal filter in continuous-time gives a stable and causal filter in the discrete-time.
- If they both start with the same analog low pass filter with transfer function  $H(s) = \frac{1}{s+2}$ , find the corresponding digital filter, the pole-zero plots, and the nature of the filter (low pass, high pass, etc.) obtained by each of them.

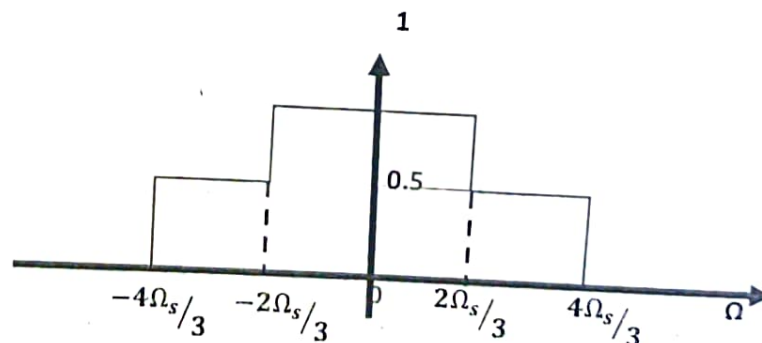
5. 10 Marks [3+2+3+2]

Let  $x[n] = \{1, 1, 1, 1, 1, 1, 1, 1\}$ , then do the following and show the necessary calculations.

- Draw the magnitude spectrum ( $|X(e^{j\omega})|$ ) of  $x[n]$
- Compute  $\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega$
- Compute  $y(n)$  whose DTFT is  $X_R(e^{j\omega})e^{j2\omega}$ , where  $X_R(e^{j\omega})$  is the real part of the spectrum  $X(e^{j\omega})$ .
- Compute the 8-point DFT of  $x[n]$ .

6. [3 Marks]

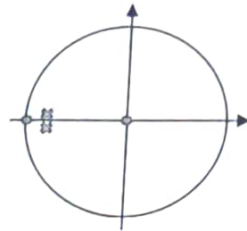
Let's consider  $H(j\Omega)$  is a CTFT of  $h(t)$  with a magnitude spectrum as shown below with zero phase spectrum then



- Plot the amplitude spectrum of DTFT  $H(e^{j\omega})$  of  $h[n]$ , where  $h[n]$  is the discrete time signal obtained from  $h(t)$  using sampling frequency  $F_s$  ( $\Omega_s = 2\pi F_s$ ).

7. [5 Marks]

Let the pole-zero plot of a system is shown below then compute and precisely plot the magnitude and phase spectrum of the system. Explain necessary calculations and assumptions.



8. [5 Marks]

A signal  $x[n] = \{x[0], x[1], x[2], x[3], x[4], x[5], x[6], x[7]\}$  of length 8 i.e., there are 8 non-zero consecutive values and the remaining values are zero. For this signal, it is required to obtain following four DFT values  $X[1], X[3], X[5], X[7]$  using FFT chip. Unfortunately, there is no 8-point FFT chip, only one 4-point FFT chip is available. Let's assume that the 4-point chip takes any arbitrary signal  $y[n] = \{y[0], y[1], y[2], y[3]\}$  as the input in the normal sequence order, compute 4-point DFT and produce the output in normal sequence order  $Y[k] = \{Y[0], Y[1], Y[2], Y[3]\}$ .

- a. Using this 4-point FFT chip, can the required DFT values ( $X[1], X[3], X[5], X[7]$ ) computed for any  $x[n]$ ? if yes derive the necessary equations and draw the complete connections else argue the reasons by deriving the necessary equations.

9. [7 Marks]

Let  $x_1[n] = x_2[n] = \{1, 1, 1\}$  then compute  $y[n]$ . Please note that the 5-point FFT chip takes any arbitrary signal as the input in the normal sequence order, compute a 5-point DFT and produce the output in normal sequence order.

