1 (1) $M_1 \ddot{x}_1 = -k_1 x_1 - k_2(x_1 - x_2) - f_{v_3}(\dot{x}_1 - \dot{x}_2) - f_{v_1}\dot{x}_1 + f(t)$ — ① → 2.5 marks

$M_2 \ddot{x}_2 = -k_3 x_2 - k_2(x_2 - x_1) - f_{v_3}(\dot{x}_2 - \dot{x}_1) - f_{v_2}\dot{x}_2$ — ② → 2.5

Converting ① to Laplace

$$M_1 s^2 X_1(s) = -k_1 X_1(s) - k_2 X_1(s) + k_2 X_2(s) - f_{v_3} s X_1(s) + f_{v_3} s X_2(s) - f_{v_1} s X_1(s) + F(s)$$

$$F(s) = (s^2 M_1 + s(f_{v_3} + f_{v_1}) + k_1 + k_2) X_1(s) - (s f_{v_3} + k_2) X_2(s) \quad \text{— ③} \rightarrow 3$$

Converting ② to Laplace

$$M_2 s^2 X_2(s) = -k_3 X_2(s) - k_2 X_2(s) + k_2 X_1(s) - f_{v_3} s X_2(s) + f_{v_3} s X_1(s) - f_{v_2} s X_2(s)$$

$$(s^2 M_2 + s(f_{v_2} + f_{v_3}) + k_2 + k_3) X_2(s) = (k_2 + s f_{v_3}) X_1(s)$$

$$X_1(s) = \frac{(s^2 M_2 + s(f_{v_2} + f_{v_3}) + k_2 + k_3)}{(k_2 + s f_{v_3})} X_2(s) \quad \text{— ④} \rightarrow 3$$

Putting ④ into ③

$$F(s) = (s^2 M_1 + s(f_{v_3} + f_{v_1}) + k_1 + k_2)\frac{(s^2 M_2 + s(f_{v_2} + f_{v_3}) + k_2 + k_3)}{(k_2 + s f_{v_3})} X_2(s) - (s f_{v_3} + k_2) X_2(s)$$

$$G(s) = \frac{X_2(s)}{F(s)} = \frac{k_2 + s f_{v_3}}{(s^2 M_1 + s(f_{v_3} + f_{v_1}) + k_1 + k_2)(s^2 M_2 + s(f_{v_2} + f_{v_3}) + k_2 + k_3) - (k_2 + s f_{v_3})^2} \Rightarrow 4$$

(2) Let the state variables be z_1, z_2, z_3, z_4, u such that,

$$z_1(t) = y(t) = x_1(t) \qquad z_2(t) = x_2(t)$$
$$z_3(t) = \dot{x}_1(t) \qquad z_4(t) = \dot{x}_2(t) \qquad u(t) = f(t)$$

3

$$\dot{z}_1 = \dot{x}_1 = z_3 \qquad \dot{z}_2 = \dot{x}_2 = z_4$$

$$\dot{z}_3 = \ddot{x}_1 = \frac{-k_1 x_1 - k_2(x_1 - x_2) - b_{v_3}(\dot{x}_1 - \dot{x}_2) - b_{v_1}\dot{x}_1 + f(t)}{m_1}$$

$$= \frac{-k_1 z_1 - k_2 z_1 + k_2 z_2 - b_{v_3} z_3 + b_{v_3} z_4 - b_{v_1} z_3 + u}{m_1}$$

$$= -\frac{1}{m_1}(k_1 + k_2) z_1 + \frac{k_2}{m_1} z_2 - \frac{1}{m_1}(b_{v_1} + b_{v_3}) z_3 + \frac{b_{v_3}}{m_1} z_4 + \frac{u}{m_1}$$

3

$$\dot{z}_4 = \ddot{x}_2 = \frac{-k_3 x_2 - k_2(x_2 - x_1) - b_{v_3}(\dot{x}_2 - \dot{x}_1) - b_{v_2}\dot{x}_2}{m_2}$$

$$= \frac{-k_3 z_2 - k_2 z_2 + k_2 z_1 - b_{v_3} z_4 + b_{v_3} z_3 - b_{v_2} z_4}{m_2}$$

$$= \frac{k_2}{m_2} z_1 - \frac{(k_2 + k_3)}{m_2} z_2 + \frac{b_{v_3}}{m_2} z_3 - \frac{1}{m_2}(b_{v_2} + b_{v_3}) z_4$$

3

Converting to matrix form

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{1}{m_1}(k_1+k_2) & \frac{k_2}{m_1} & -\frac{(b_{v_1}+b_{v_3})}{m_1} & \frac{b_{v_3}}{m_1} \\ \frac{k_2}{m_2} & -\frac{(k_2+k_3)}{m_2} & \frac{b_{v_3}}{m_2} & -\frac{(b_{v_2}+b_{v_3})}{m_2} \end{bmatrix}}_{A} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_1} \\ 0 \end{bmatrix}}_{B} u$$

4

$$y = x_1 = z_1$$

2

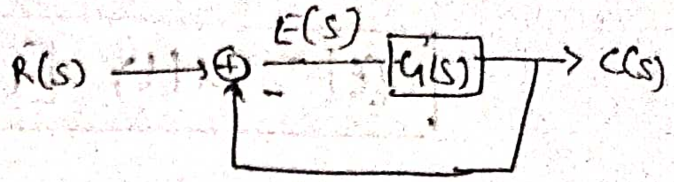
$$y = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}}_{C} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} + \underbrace{0}_{D} u$$

$$\dot{z} = Az + Bu$$

$$y = Cz + Du$$

(Approach) Midsem - q2

$$G(s) = \frac{w_n^2}{s^2 + 2\zeta w_n s}$$



$E(s) = R(s) - C(s)$

$C(s) = R(s) - E(s)$

$E(s) \cdot G(s) = C(s)$

$(R(s) - C(s)) \cdot G(s) = C(s)$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)}$$

$$\frac{C(s)}{R(s)} = \frac{w_n^2}{w_n^2 + 2\zeta w_n s + s^2}$$

$x(t) = u(t)$

$R(s) = \frac{1}{s}$, ROC: $Re(s) > 0$

i) underdamped

$0 < \zeta < 1$

$$C(s) = \frac{1}{s} \cdot \left[\frac{w_n^2}{w_n^2 + 2\zeta w_n s + s^2}\right]$$

$$= \frac{1}{s} - \frac{s + \zeta w_n}{(s + \zeta w_n)^2 + w_d^2} - \frac{\zeta w_n}{(s + \zeta w_n)^2 + w_d^2}$$

where $w_d = w_n\sqrt{1 - \zeta^2}$

natural damping freq.

inverse laplace

$$\mathcal{L}^{-1}\left[\frac{s + \zeta w_n}{(s + \zeta w_n)^2 + w_d^2}\right] = e^{-\zeta w_n t} \cos(w_d t) \cdot u(t)$$

$$\mathcal{L}^{-1}\left[\frac{w_d}{(s + \zeta w_n)^2 + w_d^2}\right] = e^{-\zeta w_n t} \sin(w_d t) \cdot u(t)$$

$$C(S) = \frac{1}{S} - \frac{S+\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2} - \left(\frac{\zeta\omega_n}{\omega_d}\right)\frac{\omega_d}{(S+\omega_n\zeta)^2+\omega_d^2}$$

$ROC_1 \cap ROC_2 \cap ROC_3 \leftarrow$ Final ROC: $Re(S) > 0$

$$c(t) = u(t) - e^{-\zeta\omega_n t}\cos(\omega_d t).u(t) - \frac{\zeta\omega_n}{\omega_d} e^{-\zeta\omega_n t}\sin(\omega_d t)\,u(t)$$

$$= u(t)\left[1 - e^{-\zeta\omega_n t}\cos(\omega_d t) - \frac{\zeta\omega_n}{\omega_d} e^{-\zeta\omega_n t}\sin(\omega_d t)\right]$$

$$= u(t)\left[1 - e^{-\zeta\omega_n t}\cos(\omega_d t) - \frac{\zeta}{\sqrt{1-\zeta^2}}\sin(\omega_d t)\right]$$

$$= u(t)\left[1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\left[\sqrt{1-\zeta^2}\cos(\omega_d t) + \zeta.\sin(\omega_d t)\right]\right]$$

$$\boxed{c(t) = u(t)\left[1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin\left(\omega_d t + \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right]}$$

ii) <u>critically damped</u>

$$\zeta = 1$$

$$C(S) = \frac{1}{S}\cdot\left[\frac{\omega_n^2}{(S+\omega_n)^2}\right]$$

$$= \frac{1}{S} - \frac{1}{S+\omega_n} - \frac{\omega_n}{(S+\omega_n)^2}$$

Final ROC: $Re(S) > 0$ $[Re(S)>0 \cap Re(S)>-\omega_n]$

$$\mathcal{L}^{-1}\left[\frac{1}{S+\omega_n}\right] = e^{-\omega_n t}.u(t)$$

$$\mathcal{L}^{-1}\left[\frac{1}{(S+\omega_n)^2}\right] = t.\omega_n e^{-\omega_n t}.u(t)$$

$$c(t) = u(t) - e^{-\omega_n t} \cdot u(t) - t \cdot e^{-\omega_n t} \cdot \omega_n u(t)$$

$$= u(t)\left[1 - e^{-\omega_n t} - t \cdot e^{-\omega_n t} \cdot \omega_n\right]$$

$$\boxed{c(t) = u(t)\left[1 - e^{-\omega_n t}(1 + t\omega_n)\right]}$$

Markings $6 + 17 + 17 = 40$

Feedback loop — 2 marks

Calculation for output / input — 2 marks

$R(s) = \frac{1}{s}$ + ROC — (1+1) marks

i) <u>underdamped</u> — 17 marks

- cond^n — 1 mark
- expression — 1 mark
- partial sum — 3 marks
- Inverse laplace — (3 + 3) marks
- Final ROCs — 2 marks
- Final ans/exp — 4 marks

ii) <u>critically damped</u> — 17 marks

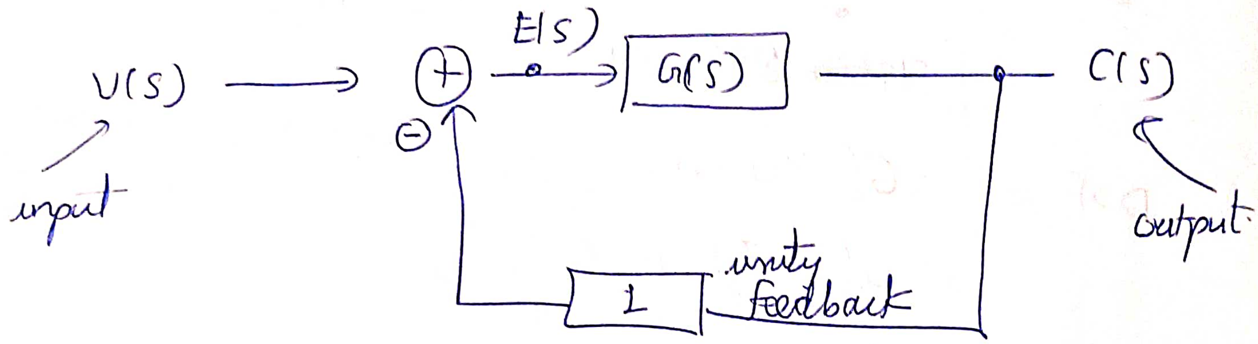
- cond^n — 1 mark
- exp — 1 mark
- partial sum — 3 marks
- Inverse laplace — (3 + 3) marks
- Final ROCs — 2 marks
- Final ans/exp — 4 marks

ST
Midsem

Paper key

(2) $G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s}$

ζ : damping Ratio
ω_n : natural frequency

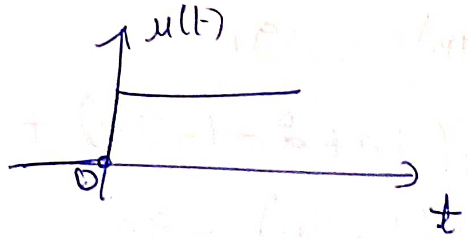


$E(s) = U(s) - C(s)$, $C(s) = E(s)\,G(s) = \left(U(s) - C(s)\right) G(s)$

$\Rightarrow \quad C(s)\left(1 + G(s)\right) = U(s)\,G(s)$

$\Rightarrow \quad \boxed{C(s) = U(s)\,\frac{G(s)}{1+G(s)}} \rightarrow (1)$

Input : unit step signal



Step (1): Calculate $U(s)$

$u(t) = u(t)$

$U(s) = LT\{u(t)\}$

$= \int_{-\infty}^{\infty} u(t)\, e^{-st}\, dt = \int_{0}^{\infty} (1)\, e^{-st}\, dt$

$= -\frac{1}{s}\left[e^{-st}\right]_0^{\infty}$, $(Re(s) > 0)$

$= -\frac{1}{s}\left[e^{-s(\infty)} - e^{-s(0)}\right]$

$\quad\;\; 0$

$= 1/s \quad (ROC : Re(s) > 0)$

$$\Rightarrow \quad U(s) = \frac{1}{s}, \quad Re(s) > 0$$

step ②: find $C(s) = \frac{U(s)\,G(s)}{1+G(s)}$ [from ①]

$$= \frac{U(s)}{1 + \frac{1}{G(s)}}$$

$$= \frac{(1/s)}{1 + \frac{1}{\omega_n^2}\cdot(s^2 + 2\zeta\omega_n s)}$$

$$= \frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

Roots of $s^2 + 2\zeta\omega_n s + \omega_n^2$: $\frac{-2\zeta\omega_n \pm \sqrt{4\zeta^2\omega_n^2 - 4\omega_n^2}}{2}$

step ③: Inverse Laplace Transform

$$= -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

(i) Underdamped System: $\zeta < 1$

$\Rightarrow$ roots: $-\zeta\omega_n \pm i\omega_n\sqrt{1-\zeta^2}$

: $(-\zeta + i\sqrt{1-\zeta^2})\omega_n$ and $(-\zeta - i\sqrt{1-\zeta^2})\omega_n$

: $\underbrace{-(\zeta - i\sqrt{1-\zeta^2})\omega_n}_{a}$ and $\underbrace{-(\zeta + i\sqrt{1-\zeta^2})\omega_n}_{b}$

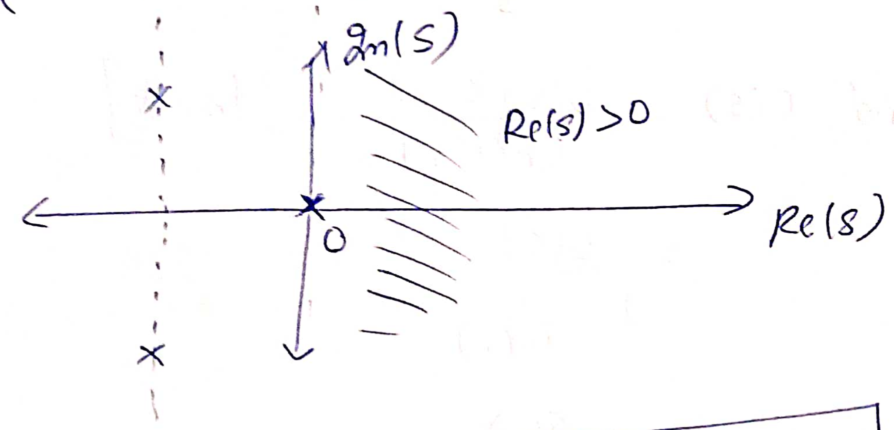
$$\Rightarrow \quad C(s) = \frac{\omega_n^2}{s\left(s + (\zeta - i\sqrt{1-\zeta^2})\omega_n\right)\left(s + (\zeta + i\sqrt{1-\zeta^2})\omega_n\right)}$$

$$= \frac{\omega_n^2}{s(s-a)(s-b)}$$

$$= \frac{\omega_n^2}{ab}\cdot\underbrace{\frac{1}{s}}_{Re(s)>0} + \frac{\omega_n^2}{a(a-b)}\cdot\underbrace{\frac{1}{s-a}}_{Re(s)>a} + \frac{\omega_n^2}{b(b-a)}\cdot\underbrace{\frac{1}{s-b}}_{Re(s)>b}$$

(Causal System)

$$\therefore\ a = -\omega_n\left(\zeta - i\sqrt{1-\zeta^2}\right) \qquad b = -\omega_n\left(\zeta + i\sqrt{1-\zeta^2}\right)$$



$$\boxed{\begin{array}{l} \therefore ILT\left(\frac{1}{s}\right),\ Re(s)>0 \Rightarrow u(t) \\ ILT\left(\frac{1}{s-a}\right),\ Re(s)>a \Rightarrow e^{at}u(t) \\ ILT\left(\frac{1}{s-b}\right),\ Re(s)>b \Rightarrow e^{bt}u(t) \end{array}}$$

$$ab = \omega_n^2\left(\zeta^2 + 1 - \zeta^2\right) = \omega_n^2$$

$$a-b = \omega_n\left(\cancel{\zeta} + i\sqrt{1-\zeta^2} - \cancel{\zeta} + i\sqrt{1-\zeta^2}\right) = 2\omega_n\sqrt{1-\zeta^2}\, i$$

$$C(s) = \frac{\omega_n^2}{ab}\cdot\frac{1}{s} + \frac{\omega_n^2}{a(a-b)}\cdot\frac{1}{s-a} + \frac{\omega_n^2}{b(b-a)}\cdot\frac{1}{s-b}$$

$$c(t) = \frac{\omega_n^2}{ab}\cdot u(t) + \frac{\omega_n^2}{a(a-b)}e^{at}u(t) + \frac{\omega_n^2}{b(b-a)}e^{bt}u(t)$$

$$= u(t) + \frac{\cancel{\omega_n^2}}{-\omega_n\left(\zeta - i\sqrt{1-\zeta^2}\right) 2\omega_n\sqrt{1-\zeta^2}\, i}\, e^{at}u(t) + \frac{\cancel{\omega_n^2}}{+\omega_n\left(\zeta + i\sqrt{1-\zeta^2}\right)\left(2\omega_n\right)\left(\sqrt{1-\zeta^2}\right) i}\, e^{bt}u(t)$$

$$\boxed{c(t) = u(t)\left[1 - \frac{1}{2\sqrt{1-\zeta^2}\left(\zeta - i\sqrt{1-\zeta^2}\right)i}e^{at} + \frac{1}{2\sqrt{1-\zeta^2}\left(\zeta + i\sqrt{1-\zeta^2}\right)i}e^{bt}\right]}$$

(ii) Critically damped System:

$$a = b = -\omega_n \zeta$$

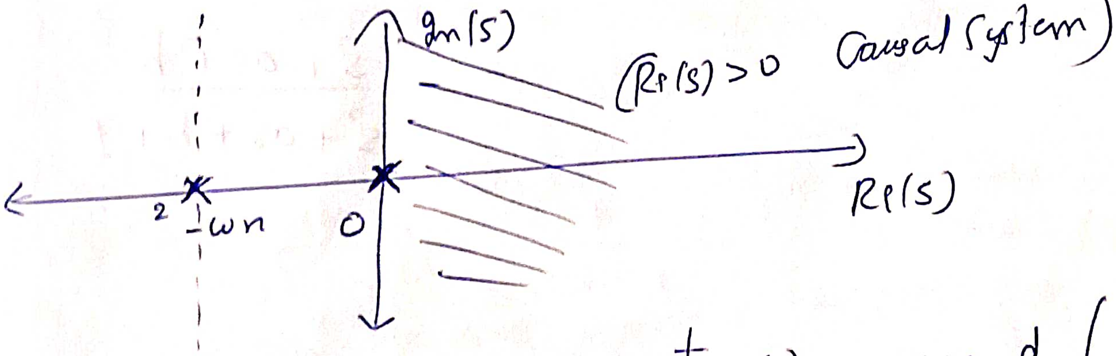
$$C(s) = \frac{\omega_n^2}{s(s^2 + 2\omega_n s + \omega_n^2)} = \frac{\omega_n^2}{s(s+\omega_n)^2}$$

$$= \frac{\omega_n^2((s+\omega_n) - (s))}{\cancel{(\omega_n)}\; s(s+\omega_n)^2}$$

$$= \frac{\omega_n}{s(s+\omega_n)} - \frac{\omega_n}{(s+\omega_n)^2}$$

$$= \frac{\omega_n((s+\omega_n) - (s))}{\cancel{\omega_n}\; s(s+\omega_n)} - \frac{\omega_n}{(s+\omega_n)^2}$$

$$= \frac{1}{s} - \frac{1}{s+\omega_n} - \frac{\omega_n}{(s+\omega_n)^2}$$



$$\Rightarrow c(t) = u(t) - e^{-\omega_n t}u(t) + \omega_n \frac{d}{ds}\left(\frac{1}{s+\omega_n}\right)$$

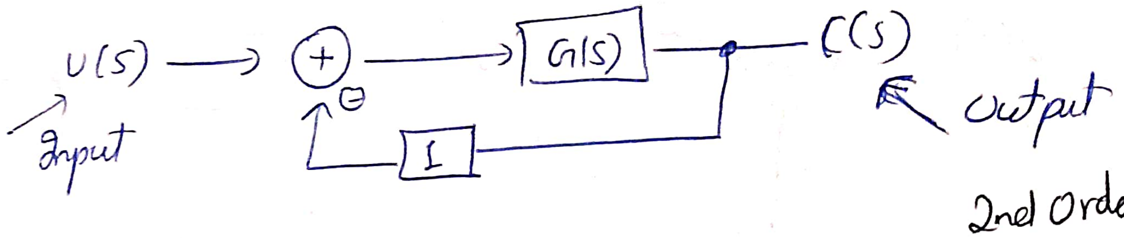
(Diff. in freq domain property) $\Longleftarrow$ $\updownarrow$ LT / ILS

$$-t \; ILS\left\{\frac{1}{s+\omega_n}\right\}$$

$$= u(t) - e^{-\omega_n t}u(t) + \omega_n(-t)e^{-\omega_n t}u(t)$$

$$\boxed{c(t) = u(t)\left[1 - e^{-\omega_n t}(1 + t\omega_n)\right]}$$

(3) $G(s) = \dfrac{1}{s^2 + as + b}$



2nd Order System

- $\dfrac{C(s)}{U(s)} = \dfrac{G(s)}{1+G(s)} = \dfrac{1}{1 + 1/G(s)} = \dfrac{1}{1 + s^2 + as + b} = \dfrac{1}{s^2 + as + (b+1)} \rightarrow (2)$

For $G(s) = \dfrac{1}{s^2+as+b}$ $\quad C(s)\left[s^2 + as + b + 1\right] = U(s)$

$a \neq 0,\ b \neq 0,\ \Rightarrow$ type 0 $\quad$ In time domain

$a = 0,\ b \neq 0,\ \Rightarrow$ type 0

$a \neq 0,\ b = 0,\ $ type 1 $\quad \dfrac{d^2y}{dt^2} + a\dfrac{dy}{dt} + (b+1)y = u(t) + C_0$

$a = 0,\ b = 0,$ $\quad$ Input $\quad$ constant

type 2

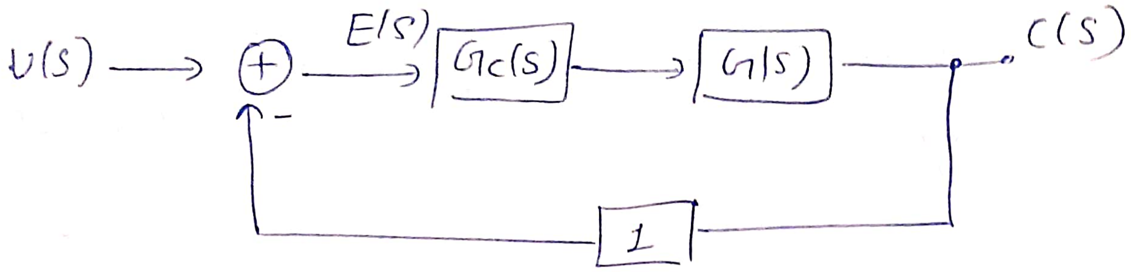
No of poles at origin $(s=0)$ $\quad \therefore$ Type of System: 2nd Order System (Type 0)

(No Integral term)

- $E(s) = \dfrac{U(s)}{1+G(s)} = \dfrac{(1/s)}{1 + \dfrac{1}{s^2+as+b}} = \dfrac{s^2+as+b}{s(s^2+as+b+1)}$

steady state error response $= \lim_{s\to 0} sE(s) = \dfrac{b}{b+1} \rightarrow (1)$

- Adding a controller:



Error response $E'(s) = \dfrac{U(s)}{1 + G_c(s)\,G(s)}$

Let controlling input be $r(t)$

- Only proportional Control added: $G_c(s) = K_p$

$$r(t) = k_p\, e(t)$$
$$R(s) = K_p\, E(s)$$

$$E'(s) = \frac{\overset{LT\{u(t)\}}{(1/s)}}{1 + K_p \cdot \frac{1}{s^2+as+b}} = \frac{1}{s}\,\frac{s^2+as+b}{s^2+as+b+K_p}$$

steady state error response: $\lim_{t\to\infty} e(t) = \lim_{s\to 0} s\,E'(s)$

can be reduced

$$= \lim_{s\to 0} \frac{s^2+as+b}{s^2+as+b+K_p}$$

$$= \frac{b}{b+K_p} \rightarrow \begin{cases} < \frac{b}{b+1} & K_p > 1 \\ > \frac{b}{b+1} & 0 < K_p < 1 \end{cases}$$

Transient Response:

$$\frac{C(s)}{U(s)} = \frac{G_c(s)G(s)}{1+G_c(s)G(s)} = \frac{K_p \cdot \frac{1}{s^2+as+b}}{1 + K_p \cdot \frac{1}{s^2+as+b}} = \frac{K_p}{s^2+as+(b+K_p)}$$

from (2): without controller: $\dfrac{C(s)}{U(s)} = \dfrac{1}{s^2+as+(b+1)}$

Proportional term increased by $K_p - 1$

⇒ more change in output when small error in input

⇒ Improves settling time when ↑K

⇒ Faster Response

⇒ Imposes Oscillations/Overshoots

- proportional + derivative Control

$$a(t) = k_p\, e(t) + k_d \frac{d\, e(t)}{dt}$$

$$R(s) = k_p E(s) + k_d\, s\, E(s)$$

$$= \cancel{k_p}\, E(s)\,(k_p + k_d s)$$

$$G_c(s) = k_p + k_d s$$

$$E'(s) = \frac{U(s)}{1 + G_c(s)\, G(s)} = \frac{(1/s)}{1 + (k_p + k_d s)\cdot \frac{1}{s^2+as+b}}$$

$$= \frac{1}{s} \cdot \frac{s^2 + as + b}{s^2 + (k_d + a)s + (b + k_p)}$$

steady state error response: can be reduced

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} s\, E'(s)$$

$$= \lim_{s\to 0} \frac{s^2 + as + b}{s^2 + (k_d + a)s + (b + k_p)}$$

$$= \frac{b}{b + k_p} \rightarrow \begin{cases} < \frac{b}{b+1}, & k_p > 1 \\ > \frac{b}{b+1} & 0 < k_p < 1 \end{cases}$$

Transient Response :

$$\frac{C(s)}{U(s)} = \frac{(k_p + k_d s)\, G(s)}{1 + (k_p + k_d)\, G(s)} = \frac{k_p + k_d s}{s^2 + (a + k_d)s + (b + k_p)}$$

without controller: $\frac{C(s)}{U(s)} = \frac{1}{s^2 + as + (b+1)}$

⇒ improves Settling time when ↑ k_p

⇒ damping coefficient increased

⇒ ~~Output~~ System's opposition to rapid changed decreased

⇒ Overshoot decreases

⇒ System settles more smoothly

- proportional + Integral Control:

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau)\, d\tau$$

$$R(s) = K_p E(s) + \frac{K_i}{s} E(s) = \left(K_p + \frac{K_i}{s}\right) E(s)$$

$$G_c(s) = \left(K_p + \frac{K_i}{s}\right)$$

$$E'(s) = \frac{U(s)}{1 + G_c(s) G(s)} = \frac{(1/s)}{1 + \left(K_p + \frac{K_i}{s}\right)\frac{1}{s^2 + as + b}}$$

$$= \frac{1}{s} \cdot \frac{s(s^2 + as + b)}{s^3 + as^2 + bs + K_p s + K_i}$$

$$= \frac{s^2 + as + b}{s^3 + as^2 + (b + K_p)s + K_i}$$

steady state error response $= \lim_{s \to 0} sE'(s) = \cancel{\frac{b(0)}{K_i} = 0}$

can be brought close to 0

$$= \lim_{s \to 0} \frac{s(s^2 + as + b)}{s^3 + as^2 + (b + K_p)s + K_i}$$

$$= 0 \quad \left(< \frac{b}{b + L}\right)$$

Transient Response:

$$\frac{C(s)}{U(s)} = \frac{G_c(s) G(s)}{1 + G_c(s) G(s)} = \frac{\left(K_p + \frac{K_i}{s}\right)\frac{1}{s^2 + as + b}}{1 + \left(K_p + \frac{K_i}{s}\right)\frac{1}{s^2 + as + b}} = \frac{K_p s + K_i}{s^3 + (a + K_p)s^2 + (b + K_i)s}$$

⇒ System's Order increased

⇒ one extra pole added

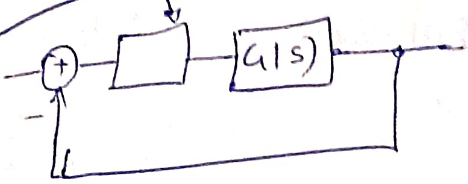
Increase in K_p (proportional term) leads towards decrease in settling time, in this process, as K_i accumulates error, it may lead to overshoot. Since there is no much effect of derivative constant, the settling time gets increased although zero steady state error can be achieved.

leading to sluggish Response but steady state error achieved to 0.

PID Control

$$r(t) = K_p e(t) + K_i \int_0^t e(\tau)\, d\tau + K_d \frac{d}{dt} e(t)$$

$$R(s) = \left(K_p + \frac{1}{s} K_i + K_d s\right) E(s)$$



$$G_c(s) = K_p + \frac{K_i}{s} + K_d s$$

$$E'(s) = \frac{U(s)}{1 + G_c(s) G(s)} = \frac{(1/s)}{1 + \left(K_p + \frac{K_i}{s} + K_d s\right) \cdot \frac{1}{s^2 + as + b}}$$

$$= \frac{1}{s} \cdot \frac{s^3 + as^2 + bs}{s^3 + as^2 + bs + K_d s^2 + K_p s + K_i}$$

$$= \frac{1}{s} \cdot \frac{s^3 + as^2 + bs}{s^3 + (a + K_d) s^2 + (b + K_p) s + K_i}$$

steady state error response : $= \lim_{s \to 0} s E'(s) = \lim_{s \to 0} \frac{s^3 + as^2 + bs}{s^3 + (a + K_d) s^2 + (b + K_d) s + K_i}$

can be brough close to 0

$$= 0 \quad \left(< \frac{b}{b+1} \right)$$

Transient Response :

$$\frac{C(s)}{U(s)} = \frac{G_c(s) G(s)}{1 + G_c(s) G(s)}$$

$$= \frac{\left(K_p + K_d s + \frac{K_i}{s}\right) \frac{1}{s^2 + as + b}}{1 + \left(K_p + K_d s + \frac{K_i}{s}\right) \cdot \frac{1}{s^2 + as + b}} = \frac{K_p s + K_d s^2 + K_i}{s^3 + (a + K_d) s^2 + (b + K_p) s + K_i}$$

effects of K_p : Improves settling time, fast response, Imposes Oscillations

~~K_i : Accumulates error~~

K_d : reduces overshoot and resists sudden changes in output

K_i : Eliminates steady state error. by accumulating error and increasing systems order by 1 (1 pole addition)

→ System order increased but balanced by derivative Controller

Why PID ? → K_p imposes oscillations (P controller), Using PD, reduces oscillations but brings about steady state error. Using PI, ~~we~~ bring steady state error to 0 but makes response sluggish. PID overcomes.

Q3

Final Rubrik:

$(1+2+2) = 5$

- Order - $\boxed{1\text{ mark}}$
  Type - $\boxed{2\text{ marks}}$
  Why Type - $\boxed{2\text{ marks}}$ (no poles at origin)

(Type 0 ($a\neq 0, b\neq 0$))

$(1+1+1+1+1) = 5$

- - P controller ($K_p > 0$)

  Error response $E(s)$
  calculation → $\boxed{1\text{ mark}}$
  final ans → $\left(\frac{b}{b+K_p}\right)$, $\boxed{1\text{ mark}}$
  comment on error → $\boxed{1\text{ mark}}$ ($K_p \propto \frac{1}{\text{error}}$, s-s error non zero)

  K_p → faster response $\boxed{1\text{ mark}}$
  Oscillations / Overshoots occur $\boxed{1\text{ mark}}$

$(1+1+1+1+1)$

- PD control

  error $E(s)$
  calculation → $\boxed{1\text{ mark}}$
  final ans → $\left(\frac{b}{b+K_p}\right)$ $\boxed{1\text{ mark}}$
  comment on error → $\boxed{1\text{ mark}}$ (non zero error, $K_p \propto \frac{1}{\text{error}}$, no change in error compared to P controller)

  Transient response comments:
  K_p → faster response $\boxed{1\text{ mark}}$
  K_d → oppose sudden change to response / reduces overshoot / decrease in oscillations / decreases response rate compared to P control $\boxed{1\text{ mark}}$

$(1+1+1+1+1)$

- PI control:

  error $E(s)$
  calculation → $\boxed{1\text{ mark}}$
  final ans → (0) $\boxed{1\text{ mark}}$
  comment on error → $\boxed{1\text{ mark}}$ (integrator accumulates the error.)
  Transient response comments: (As one extra pole added, order of system increased by 1, sluggish response / slower response) $\boxed{1\text{ mark}}$
  Can impose overshoots / oscillations $\boxed{1\text{ mark}}$

PID control (10 Marks)

Error $E(s)$ calculate → $\boxed{2 \text{ marks}}$

final ans → (0) $\boxed{1 \text{ mark}}$

comment on error → (accumulates error) $\boxed{1 \text{ mark} + 1 \text{ mark}}$

and effect of Ki

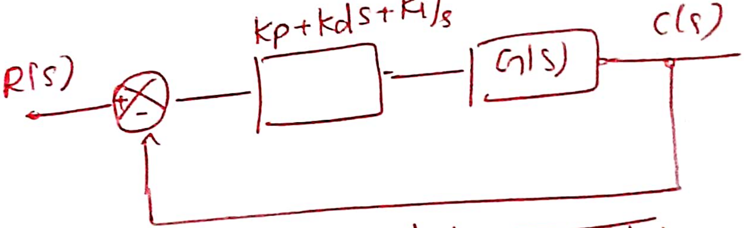
Transient response comments → effect of kp (Faster response / ~~improves oscillations~~) $\boxed{1 \text{ mark}}$

→ effect of Kd (reduces oscillations / oppose sudden changes in response / causes damping) $\boxed{1 \text{ mark}}$

→ effect of Ki

Why PID over PD and PI → (Oscillation caused due to kp can be reduced by kd, ~~So~~ steady state error in PD, P is reduced by Ki, a pole added by but balanced by kd) $\boxed{1 \text{ mark}}$

Block diagram for PID $\boxed{1 \text{ mark}}$



~~To~~ Transfer function $\frac{C(s)}{R(s)}$: To be calculated $\boxed{1 \text{ mark}}$

$2+1+1+1+1+1+1+1 = 10$ Marks

× × × ×

→ IN YOUR ANSWER SCRIPTS, WE DID EXPECT YOU TO WRITE ABOUT SETTLING TIME BUT IF YOU HAVE WRITTEN WE CONSIDERED THAT AS AN ~~stp~~ ALTERNATIVE.

→ FOR ERROR ANALYSIS ,
FOR P, PD Control , WE DID NOT EXPECT ANALYSIS FOR both kp<1 and kp>1. Any ANALYSIS for kp>0 WORKS.