① $X[K] = \sum_{n=0}^{N-1} x[n] W_N^{Kn}$

Splitting $x[n]$ into odd and even terms.

$= \sum_{m=0}^{\frac{N}{2}-1} x[2m] W_{N/2}^{Km} + \sum_{m=0}^{N/2-1} x[2m+1] W_N^{K} \cdot W_{N/2}^{Km}$

$K \in [0, N-1]$

$\Rightarrow X[K] = \sum_{m=0}^{\frac{N}{2}-1} \left[x[2m] + W_N^{K} x[2m+1]\right] W_{N/2}^{Kn}$

Replacing m by n

$\Rightarrow X[K] = \sum_{n=0}^{N/2-1} \left[x[2n] + W_N^{K} x[2n+1]\right] W_{N/2}^{Kn}$ — ①

$X[K+N/2] = \sum_{n=0}^{N/2-1} \left[x[2n] - W_N^{K} x[2n+1]\right] W_{N/2}^{Kn}$ — ②

② Marks

Now for the given faulty chip, the input locations 1, 3, 5 and 7 are permanently grounded.

$\therefore \quad x[2n+1] = 0 \quad \forall\, n \in [0, \frac{N}{2}-1]$

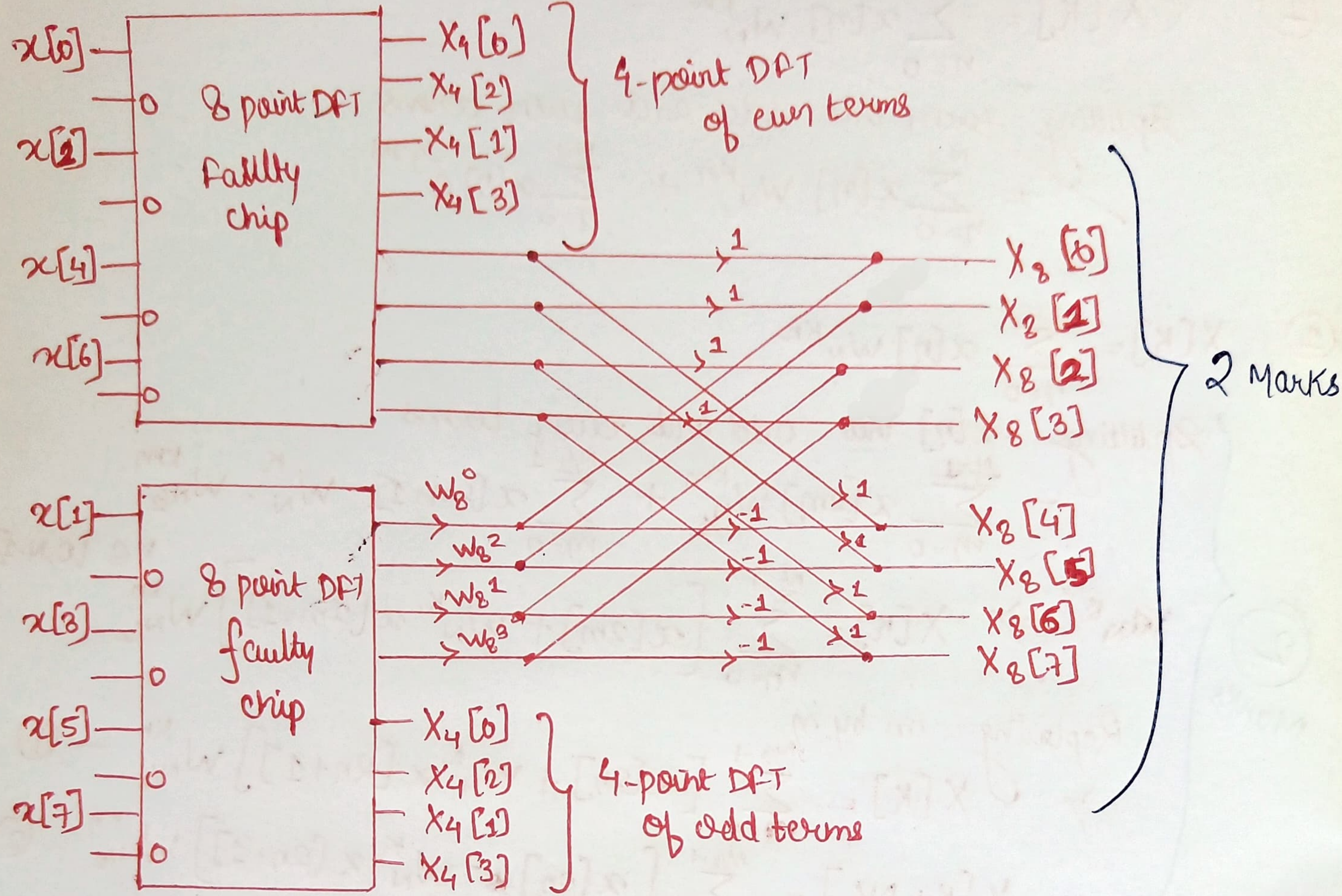
$\Rightarrow X[K] = X[K+N/2] = \sum_{n=0}^{\frac{N}{2}-1} x[2n] W_{N/2}^{Km}$ } 4-point DFT of input at 0, 2, 4, 6 locations.

To compute the 8 point DFT, we only require a 4 point DFT of the even signal $x[2m]$ and odd signal $x[2m+1]$.

$\therefore$ Input 1, 3, 5, 7 are grounded of faulty chip, we can connect $x[0], x[2], x[4], x[6]$ to 1 faulty chip and $x[1], x[3], x[5], x[7]$ to the other chip in the location 0, 2, 4, 6 of the faulty chip.

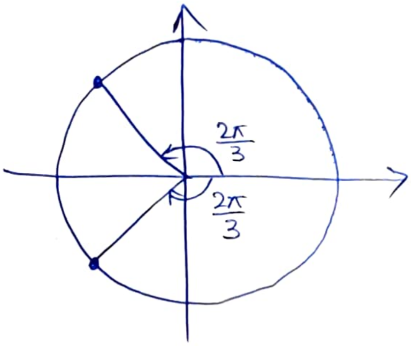
$\Rightarrow$ we will obtain 4 point DFTs of the even and odd sequence separately using the two faulty chip. and then we can combine them using eqn ① & ②

① Mark



(2) marks are given for the diagram or for providing the correct explanation and equation

Q3.



Zeroes : $e^{j2\pi/3}$, $e^{-j2\pi/3}$

Poles : NONE.

1

$$H(z) = k\left(z - e^{j2\pi/3}\right)\left(z - e^{-j2\pi/3}\right)$$
$$= k\left[z^2 - \left(e^{j2\pi/3} + e^{-j2\pi/3}\right)z + e^{j2\pi/3}e^{-j2\pi/3}\right]$$
$$= k\left[z^2 - 2\cos\left(\frac{2\pi}{3}\right)z + 1\right]$$
$$= k\left[z^2 + z + 1\right].$$

2

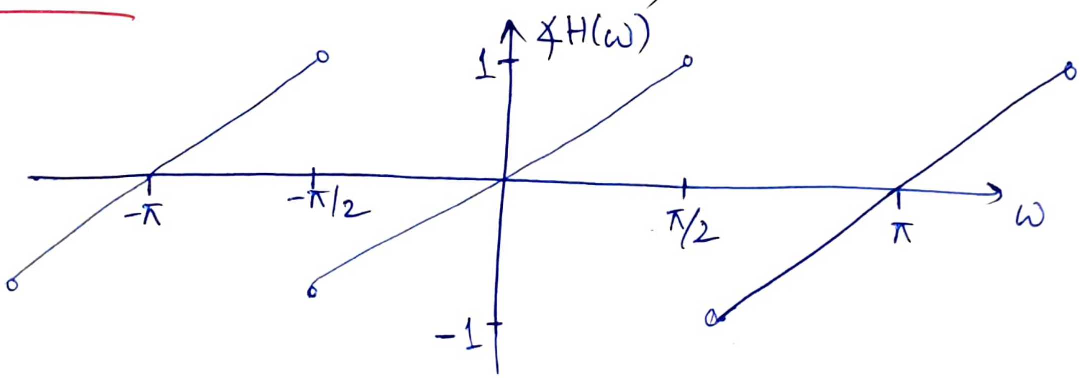
$$\Rightarrow H(\omega) = k\left[e^{2j\omega} + e^{j\omega} + 1\right]$$
$$= k\left[(\cos 2\omega + j\sin 2\omega) + (\cos\omega + j\sin\omega) + 1\right]$$
$$= k\left[(\cos 2\omega + \cos\omega + 1) + j(\sin 2\omega + \sin\omega)\right]$$

$$\measuredangle H(\omega) = \tan^{-1}\left(\frac{\sin 2\omega + \sin\omega}{\cos 2\omega + \cos\omega + 1}\right)$$

3

$$= \tan^{-1}\left(\frac{2\sin\omega\cos\omega + \sin\omega}{2\cos^2\omega - \cancel{1} + \cos\omega + \cancel{1}}\right)$$
$$= \tan^{-1}\left(\frac{\sin\omega\,\cancel{(2\cos\omega + 1)}}{\cos\omega\,\cancel{(2\cos\omega + 1)}}\right)$$
$$= \tan^{-1}(\tan\omega)$$

4



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