

## EC5.101 – Network, Signals and Systems – Mid Exam

Date: 21st September, 2023

Maximum marks: 42

Exam duration: 90 minutes

Instructions:

- There are 6 questions for a total of 42 marks.
- Mention any additional assumptions you make that is not given in the question.
- Clearly show the steps used to arrive at the solutions.

1. [9 marks] Recall that the pulse signal is given by

$$p_{\Delta}(t) = \begin{cases} \frac{1}{\Delta}, & 0 < t \leq \Delta \\ 0, & \text{otherwise,} \end{cases}$$

Consider two pulse signals  $p_{\Delta_1}(t)$  and  $p_{\Delta_2}(t)$  for some  $\Delta_1 \neq \Delta_2$ .

- [1] Sketch the pulses  $p_{\Delta_1}(t)$  and  $p_{\Delta_2}(t)$ . If your roll number is odd consider  $\Delta_1 < \Delta_2$  and if it is even consider  $\Delta_1 > \Delta_2$ .
- [5] Find the convolution of these two pulse signals, i.e.,  $y(t) = p_{\Delta_1}(t) * p_{\Delta_2}(t)$ . Clearly plot and label this signal.
- [3] Analyse your answer as simultaneously both  $\Delta_1 \rightarrow 0$  and  $\Delta_2 \rightarrow 0$ . How can you use this to show that  $\delta(t) * \delta(t) = \delta(t)$ ?

2. [8 marks] Consider a system with input output relation given by

$$y(t) = \int_{t+1}^{\infty} x(\tau) d\tau$$

- [1] Is this system causal? Justify.
- [2] Is this system BIBO stable? Justify.
- [2] Find the impulse response of this system.
- [3] Analyse the system with regards to linearity and time-invariance.

3. [7 marks] Consider the signal given by

$$x(t) = \delta(t + 1) + u(t).$$

- (a) [1] Sketch this signal. ✓
- (b) [3] Find the Laplace transform expression and region of convergence (ROC) for this signal. ①
- (c) [3] Find another signal in time-domain which has the same Laplace transform expression as above. What is the corresponding ROC? ②

4. [5 marks] Consider the Laplace transform expression given by

$$X(s) = \frac{s + 2}{s(s + 1)(s - 2)}$$

- (a) [1] Sketch the pole-zero plot for this. ✓
- (b) [2] Indicate all the possible region of convergence (ROC) for this expression. ✓
- (c) [2] For each possible ROC, there is an associated time-domain signal. For each such ROC, identify the nature of the time-domain signal (i.e., right-sided, left-sided, or two-sided). You are not required to find the time-domain signals. ③

5. [8 marks] An LTI system has impulse response given by

$$h(t) = u(t) - u(t - 1).$$

Let  $x(t)$  and  $y(t)$  denote the input and output signals of this system.

- (a) [2] Comment on stability and causality for this system. Justify. ✓
  - (b) [2] How are  $x(t)$ ,  $y(t)$  and  $h(t)$  related for any LTI system? Using the impulse response given above, find the explicit relation between  $x(t)$  and  $y(t)$ .
  - (c) [3] Let the input  $x(t)$  to this system be a periodic signal with time period  $T$  and complex Fourier series (FS) coefficients given by  $a_k, k = 0, \pm 1 \dots \pm \infty$ . Show that the output signal  $y(t)$  is periodic and find its complex FS coefficients  $b_k$  in terms of the input signal coefficients  $a_k$  and  $T$ . ④
  - (d) [1] In above part, how is  $b_0$  related to  $a_0$ ?
6. [5 marks] Find the convolution of signals  $x_1(t)$  and  $x_2(t)$  given below. Give expression and sketch the answers.

- (a) [1]  $x_1(t) = \delta(t - 1)$  and  $x_2(t) = \sin(2\pi t)$
- (b) [2]  $x_1(t) = \delta(t) - \delta(t - 3)$  and  $x_2(t) = tu(t)$
- (c) [2]  $x_1(t) = \sum_{k=0}^{\infty} \delta(t - k)$  and  $x_2(t) = u(t)$