Q1.

1. $m(a + bX) = a + b \times m(X)$

$m(a) + m(bX)$

$a + b \cdot m(X) = a + b \cdot m(X)$

2. $cov(X, a + bY) = b \times cov(X, Y)$

$cov(X, a) + cov(X, bY)$

$0 + b \cdot cov(X, Y) = b \cdot cov(X, Y)$

$b \cdot cov(X, Y) = b \cdot cov(X, Y)$

3. $cov(a + bX, a + bX) = b^2 cov(X, X)$

$cov(a, a) + cov(a, bX) + cov(bX, a) + cov(bX, bX)$

$0 + 0 + 0 + b^2 cov(X, X) = b^2 cov(X, X)$

$b^2 cov(X, X) = \sigma^2$

$\frac{1}{N} \sum_{i=1}^{N} (x_i - m(x))(x_i - m(x)) = \frac{1}{N} \sum_{i=1}^{N} (x_i - m(x))^2$

$\frac{1}{N} \sum_{i=1}^{N} (x_i - m(x))^2 = \frac{1}{N} \sum_{i=1}^{N} (x_i - m(x))^2$

4. A non-decreasing transformation of the median is the median of the transformed variable because the use of non-decreasing transformations means the order of the original function won't change so the median won't change. The statement applies to any quartile, the IQR, & the range.

5. Yes $m(g(X)) = g(m(X))$ when $g()$ is a non-decreasing function because it is simply a non-decreasing transformation of the median.