NOTES ON MECHANICS

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THE EQUATIONS OF MOTION

The position of a particle in three-dimensional Euclidean space is defined by a vector $\boldsymbol{r} \in \mathbb{R}^3$. We denote the derivative of $\boldsymbol{r}$ with respect to time by $\dot{\boldsymbol{r}}$, referred to as the *velocity of the particle*, and the second derivative $\ddot{\boldsymbol{r}}$ is its *acceleration*.

1.1 GENERALISED COORDINATES

In general, the number of independent quantities which are necessary to uniquely define the position of a system is the number of *degrees of freedom of the system*. Any $n \in \mathbb{Z}_{>0}$ quantities $q_1, \dots, q_n$, each of which is in some subset of $\mathbb{R}$, which completely define the position of a system with n degrees of freedom are referred to as *generalised coordinates of the system*, and the derivatives $\dot{q}_1, \dots, \dot{q}_n$ are its *generalised velocities*. Generalised coordinates span the *configuration space of the system*. We denote generalised coordinates by an n-dimensional vector $\boldsymbol{q}$.

In principle, if all the coordinates $\boldsymbol{q}$ and velocities $\dot{\boldsymbol{q}}$ of a system are simultaneously specified for some instant, then accelerations $\ddot{\boldsymbol{q}}$ for that instant are uniquely determined. The relations between the coordinates, velocities, and accelerations are the *equations of motions of the system*, which are second-order differential equations for the function $\boldsymbol{q}(t)$ with respect to time $t \in \mathbb{R}$. Solving for $\boldsymbol{q}(t)$ makes possible the determination of the motion of the system.

1.2 THE STATIONARY-ACTION PRINCIPLE

The most general formulation of the law governing the motion of mechanical systems is the *stationary-action principle* or the *principle of least action*, according to which every mechanical system is characterised by a definite function $L(\boldsymbol{q}(t), \dot{\boldsymbol{q}}(t), t)$, referred to as the *Lagrangian*, and the motion of the system is such that a certain condition is satisfied.

Let the system occupy, at the instants t_0 and t_1, positions defined by two sets of values of the coordinates, $\boldsymbol{q}_0$ and $\boldsymbol{q}_1$. Then the condition is that the system moves between these positions in such a way that

$$S[\boldsymbol{q}] = \int_{t_0}^{t_1} L(\boldsymbol{q}(t), \dot{\boldsymbol{q}}(t), t)\,\mathrm{d}t$$

is stationary. The functional $S[\boldsymbol{q}]$ is referred to as the *action*.