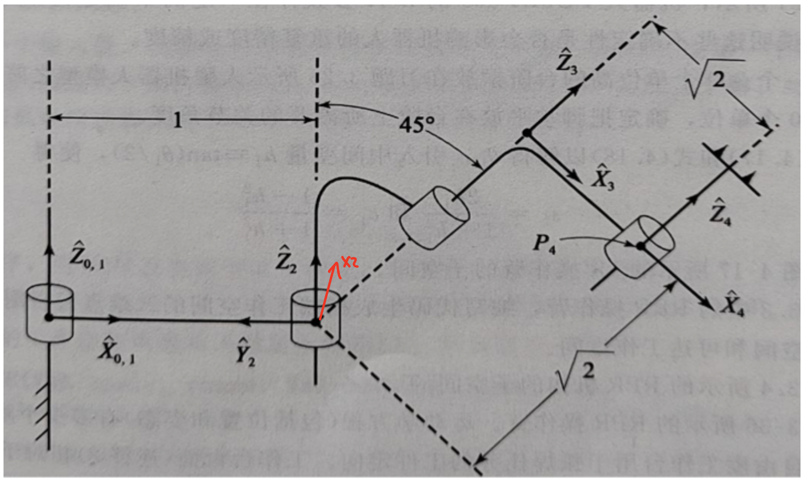
第4次作业.　范睿诗　2014076



解：①由图坐标系得DH参数表：

	α_{i-1}	a_{i-1}	d_i	θ_i
1	0°	0	0	θ_1
2	0°	1	0	θ_2
3	45°	0	$\sqrt{2}$	θ_3
4	0°	$\sqrt{2}$	0	θ_4

② 正运动学：

$${}^0_1T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^1_2T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 1 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^2_3T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & 0 \\ \frac{\sqrt{2}}{2}s\theta_3 & \frac{\sqrt{2}}{2}c\theta_3 & -\frac{\sqrt{2}}{2} & -1 \\ \frac{\sqrt{2}}{2}s\theta_3 & \frac{\sqrt{2}}{2}c\theta_3 & \frac{\sqrt{2}}{2} & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^3_4T = \begin{bmatrix} c\theta_4 & -s\theta_4 & 0 & \sqrt{2} \\ s\theta_4 & c\theta_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0_4T = {}^0_1T \cdot {}^1_2T \cdot {}^2_3T \cdot {}^3_4T = \begin{bmatrix} C_{12} & -S_{12} & 0 & C_1 \\ S_{12} & C_{12} & 0 & S_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_3 & -S_3 & 0 & 0 \\ \frac{\sqrt{2}}{2}S_3 & \frac{\sqrt{2}}{2}C_3 & -\frac{\sqrt{2}}{2} & -1 \\ \frac{\sqrt{2}}{2}S_3 & \frac{\sqrt{2}}{2}C_3 & \frac{\sqrt{2}}{2} & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_4 & -S_4 & 0 & \sqrt{2} \\ S_4 & C_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_{12} & -S_{12} & 0 & C_1 \\ S_{12} & C_{12} & 0 & S_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_{34} & -S_{34} & 0 & \sqrt{2}C_3 \\ \frac{\sqrt{2}}{2}S_{34} & \frac{\sqrt{2}}{2}C_{34} & -\frac{\sqrt{2}}{2} & S_3-1 \\ \frac{\sqrt{2}}{2}S_{34} & \frac{\sqrt{2}}{2}C_{34} & \frac{\sqrt{2}}{2} & S_3+1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C_{12}C_{34}-\frac{\sqrt{2}}{2}S_{12}S_{34} & -(C_{12}S_{34}+\frac{\sqrt{2}}{2}S_{12}C_{34}) & \boxed{\frac{\sqrt{2}}{2}S_{12}} & \boxed{\sqrt{2}C_{12}C_3-S_{12}S_3+S_{12}+C_1} \\ S_{12}C_{34}+\frac{\sqrt{2}}{2}C_{12}S_{34} & -S_{12}S_{34}+\frac{\sqrt{2}}{2}C_{12}C_{34} & \boxed{-\frac{\sqrt{2}}{2}C_{12}} & \boxed{\sqrt{2}S_{12}C_3+C_{12}S_3-C_{12}+S_1} \\ \boxed{\frac{\sqrt{2}}{2}S_{34}} & \boxed{\frac{\sqrt{2}}{2}C_{34}} & \frac{\sqrt{2}}{2} & \boxed{S_3+1} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

③ 逆运动学：已知 ${}^0_4T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & 1.1 \\ r_{21} & r_{22} & r_{23} & 1.5 \\ r_{31} & r_{32} & r_{33} & 1.707 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ 求解 θ_3. θ_4. θ_1. θ_2

(所有关节范围为$(-\pi,\pi)$)

1° 由 $S_3+1=1.707$　∴ $\sin\theta_3 = 0.707 = \frac{\sqrt{2}}{2}$，$\cos\theta_3 = \pm\sqrt{1-\sin^2\theta_3} = \pm\frac{\sqrt{2}}{2}$

∴ $\theta_3 = \arctan(s_3, c_3) = \arctan(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$ 或 $\arctan(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$　∴ $\theta_3 = \frac{\pi}{4}$ 或 $\frac{3}{4}\pi$

2° 由 $\frac{\sqrt{2}}{2}S_{34}=r_{31}$，$\frac{\sqrt{2}}{2}C_{34}=r_{32}$． $\therefore \theta_3+\theta_4=\operatorname{atan2}(r_{31},r_{32})$ $\theta_4=\operatorname{atan2}(r_{31},r_{32})-\theta_3$

3° 由 $\begin{cases}\sqrt{2}C_{12}C_3-S_{12}S_3+S_{12}+C_1=1.1\\ \sqrt{2}S_{12}C_3+C_{12}S_3-C_{12}+S_1=1.5\end{cases}$ 代入 $\frac{\sqrt{2}}{2}S_{12}=r_{13}$，$-\frac{\sqrt{2}}{2}C_{12}=r_{23}$

即 $S_{12}=\sqrt{2}r_{13}$，$C_{12}=-\sqrt{2}r_{23}$ 得：

$S_1=1.5-2r_{13}S_3-\sqrt{2}r_{23}+\sqrt{2}r_{23}S_3$，$C_1=1.1+2r_{23}C_3+\sqrt{2}r_{13}S_3-\sqrt{2}r_{13}$

$\therefore \theta_1=\operatorname{atan2}(S_1,C_1)$

4°. 又由 $S_{12}=\sqrt{2}r_{13}$，$C_{12}=-\sqrt{2}r_{23}$ 得 $\theta_1+\theta_2=\operatorname{atan2}(r_{13},-r_{23})$

$\therefore \theta_2=\operatorname{atan2}(r_{13},-r_{23})-\theta_1$

综上 θ_3为双解．θ_4．θ_1．θ_2 均为单解．

不会出现自由度丢失．没有奇异解