Homework Eight, for Thu 11/19

CSE 101

Prepare a PDF file in which your solution to each of the following problems (1–7) begins on a fresh page. Upload the file to Gradescope, using your campus email address as login. The deadline is 11pm on Thursday.

These problems cover the following skills and concepts:

- Dynamic programming, given the subproblem.
- Dynamic programming, not given the subproblem.

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1. *Maximum-sum contiguous subsequence.* Textbook problem 6.1. Use the following subproblem definition: for each $1 \leq j \leq n$,

   $$S(j) = \text{maximum sum of any contiguous subsequence ending exactly at position } j.$$

   Note that a subsequence can have length zero, in which case the sum is also zero. Thus no $S(j)$ should ever be negative. The final answer we want is $\max_j S(j)$.

2. *A long journey.* Textbook problem 6.2. Use the following subproblem definition: for $1 \leq j \leq n$,

   $$P(j) = \text{total penalty to get to milepost } a_j, \text{ assuming you stop at that hotel.}$$

3. *Palindromes.* Textbook problem 6.7. For this one, a natural subproblem is

   $$T(i, j) = \text{length of longest palindromic subsequence of } x[i \ldots j].$$

4. *Change-making.* Textbook problem 6.17. Use the following subproblem: for $0 \leq s \leq v$,

   $$T(s) = \texttt{true} \text{ if it is possible to make change for } s \text{ using coins of the given denominations.}$$

5. *Number of paths in a DAG.* Given a directed *acyclic* graph $G = (V, E)$, and two nodes $s, t \in V$, we want to find the *number of distinct paths* from s to t. Show how to do this in linear time using dynamic programming. Hint: this is a variant of a problem from class, shortest paths in DAGs.

6. *Vertex cover in trees.* Textbook problem 6.21.

7. *A variant on change-making.* Textbook problem 6.19. Hint: use a two-dimensional table.