For all homework problems where you are asked to give an algorithm, you must prove the correctness of your algorithm and establish the best upper bound that you can give for the running time. You should always write a clear informal description of your algorithm in English. You may also write pseudocode if you feel your informal explanation requires more precision and detail. As always, try to make your answers as clear and concise as possible.

1. Suppose we have an array A containing n numbers, some of which may be negative. We wish to find indices i and j so that

$$\sum_{k=i}^{j} A[k]$$

   is maximized. That is, you want a consecutive group of numbers with the maximum sum. Find an algorithm that runs in time $O(n)$, and also give the space used by your algorithm. Note: you should do an example or two; in particular, you should see that it is not the case that if you know the maximum consecutive sum of the first $n-1$ numbers, and the nth number, you can then find the maximum consecutive sum of the first n numbers.

2. A challenge that arises in databases is how to summarize data in easy-to-display formats, such as a histogram. A problem in this context is the minimal imbalance problem. Again suppose we have an array A containing n numbers, this time all positive, and another input k. Consider k indices $j_1, j_2, \ldots j_k$ that partition the array into $k+1$ subarrays $A[1, j_1], A[j_1 + 1, j_2], \ldots, A[j_k + 1, n]$. The weight $w(i)$ of the ith subarray is the sum of its entries. The *imbalance* of the partition is

$$\max_i \left| w(i) - \left(\sum_{\ell=1}^{n} A[\ell] \right) / (k+1) \right|.$$

   That is, the imbalance is the maximum deviation any partition has from the average size.

   Give an algorithm for determining the partition with the minimal imbalance given A, n, and k. (This corresponds to finding a histogram with k breaking points, giving $k+1$ bars, as close to equal as possible, in some sense.) Also give the space used by your algorithm.

   Explain how your algorithm would change if the imbalance was redefined to be

$$\sum_i \left| w(i) - \left(\sum_{\ell=1}^{n} A[\ell] \right) / (k+1) \right|.$$

3. Dynamic programming can also work well on tree structures. Let us call a subset S of vertices a *blanket* if every edge is adjacent to at least one vertex of S. Suppose that we wish to find the mininum sized blanket for a tree. Find an algorithm that finds a minimum-sized blanket of a tree and runs in time $O(|V|)$; also give the space used by your algorithm.

4. Suppose we want to print a paragraph neatly on a page. The paragraph consists of words of length $\ell_1, \ell_2, \ldots, \ell_n$. The maximum line length is M. (Assume $\ell_i \leq M$ always.) We define a measure of neatness as follows. The extra space on a line (using one space between words) containing words ℓ_i through ℓ_j is $M - j + i - \sum_{k=i}^{j} \ell_k$. The penalty is the sum over all lines **except the last** of the **cube** of the extra space at the end of the line. This has been proven to be an effective heuristic for neatness in practice. Find a dynamic programming algorithm to determine the neatest way to print a paragraph. Of course you should provide a recursive definition of the value of the optimal solution that motivates your algorithm. Also give the space used by your algorithm.

   For this problem, besides explaining/proving your algorithms as for other problems on the set, you should also code up your algorithm to print an optimal division of words into lines. The output should be the text split into lines appropriately, and the numerical value of the penalty. You can use any coding language you wish. You should assume that a *word* in this context is any contiguous sequence of characters not including blank spaces. You should not work with a partner on this coding problem.

   After coding your algorithm, determine the minimal penalty for the following review of the Season 1 Buffy DVD, apparently written by Ryan Crackell for the Apollo Guide, for the cases where $M = 40$ and $M = 72$. We will try to put the text of the review on the class page as well.

Buffy the Vampire Slayer fans are sure to get their fix with the DVD release of the show's first season. The three-disc collection includes all 12 episodes as well as many extras. There is a collection of interviews by the show's creator Joss Whedon in which he explains his inspiration for the show as well as comments on the various cast members. Much of the same material is covered in more depth with Whedon's commentary track for the show's first two episodes that make up the Buffy the Vampire Slayer pilot. The most interesting points of Whedon's commentary come from his explanation of the learning curve he encountered shifting from blockbuster films like Toy Story to a much lower-budget television series. The first disc also includes a short interview with David Boreanaz who plays the role of Angel. Other features include the script for the pilot episodes, a trailer, a large photo gallery of publicity shots and in-depth biographies of Whedon and several of the show's stars, including Sarah Michelle Gellar, Alyson Hannigan and Nicholas Brendon.