1.1. Say we had a "black box," which takes two numbers as input and outputs their sum. See Figure 1.1(a). Say we had another box capable of multiplying two numbers together. See Figure 1.1(b). We can connect these boxes together to calculate p $\times$ (m + n). See Figure 1.1(c). Assume we have an unlimited number of these boxes. Show how to connect them together to calculate:

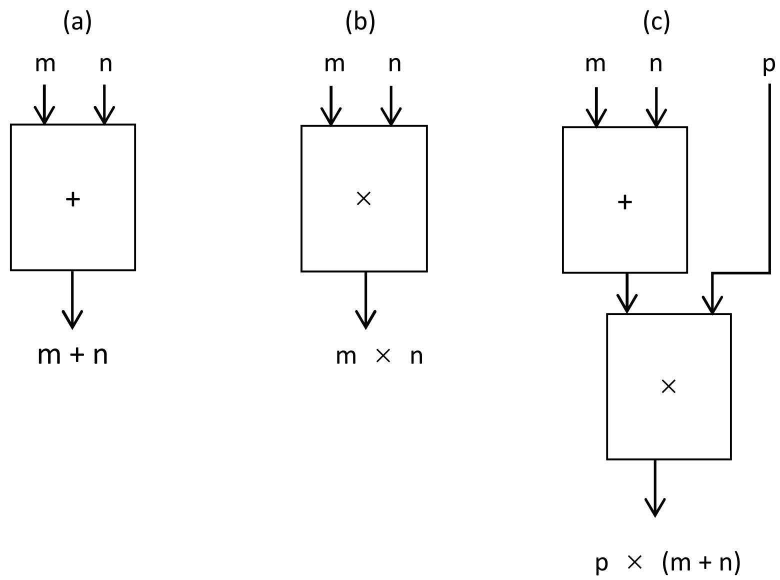
a. $ax + b$

b. The average of the four input numbers w, x, y, and z

c. $a^2 + 2ab + b^2$ (Can you do it with one add box and one multiply box?)

d. a^6

e. $ax^3 + bx^2 + cx + d$



1.2 Are items a through e in the following list algorithms? If not, what qualities required of algorithms do they lack?

a. Add the first row of the following matrix to another row whose first column contains a nonzero entry. (Reminder: Columns run vertically; rows run horizontally.)

$$\begin{pmatrix} 1 & 2 & 0 & 4 \\ 0 & 3 & 2 & 4 \\ 2 & 3 & 10 & 22 \\ 12 & 4 & 3 & 4 \end{pmatrix}$$

b. In order to show that there are as many prime numbers as there are natural numbers, match each prime number with a natural number in the following manner. Create pairs of prime and natural numbers by matching the first prime number with 1 (which is the first natural number) and the second prime number with 2, the third with 3, and so forth. If, in the end, it turns out that each prime number can be paired with each natural number, then it is shown that there are as many prime numbers as natural numbers.

c. Suppose you're given two vectors each with 20 elements and asked to perform the following operation. Take the first element of the first vector and multiply it by the first element of the second vector. Do the same to the second elements, and so forth. Add all the individual products together to derive the dot product.

d. Lynne and Calvin are trying to decided who will take the dog for a walk. Lynne suggests that they flip a coin and pulls a quarter out of her pocket. Calvin does not trust Lynne and suspects that the quarter may be weighted (meaning that it might favor a particular outcome when tossed) and suggests the following procedure to fairly determine who will walk the dog.
   1. Flip the quarter twice.
   2. If the outcome is heads on the first flip and tails on the second, then I will walk the dog.
   3. If the outcome is tails on the first flip, and heads on the second, then you will walk the dog.
   4. If both outcomes are tails or both outcomes are heads, then we flip twice again.

   Is Calvin's technique an algorithm?

e. Given a number, perform the following steps in order:
   1. Multiply it by four
   2. Add four
   3. Divide by two
   4. Subtract two
   5. Divide by two
   6. Subtract one
   7. At this point, add one to a counter to keep track of the fact that you performed steps 1 through 6. Then test the result you got when you subtracted one. If 0, write down the number of times you performed steps 1 through 6 and stop. If not 0, starting with the result of subtracting 1, perform the above 7 steps again.

1.3 a. Assume that there are about 400 students in your class. If every student is to be assigned a unique bit pattern, what is the minimum number of bits required to do this?

b. How many more students can be admitted to the class without requiring additional bits for each student's unique bit pattern?

1.4 a. What is the largest positive number one can represent in an 8-bit 2's complement code? Write your result in binary and decimal.

b. What is the greatest magnitude negative number one can represent in an 8-bit 2's complement code? Write your result in binary and decimal.

c. What is the largest positive number one can represent in n-bit 2's complement code?

d. What is the greatest magnitude negative number one can represent in n-bit 2's complement code?

1.5 Without changing their values, convert the following 2's complement binary numbers into 8-bit 2's complement numbers.

a. 1010
b. 011001
c. 1111111000
d. 01

1.6 Add the following 2's complement binary numbers. Also express the answer in decimal.

a. 01 + 1011
b. 11 + 01010101
c. 0101 + 110
d. 01 + 10

1.7 a. Describe what conditions indicate overflow has occurred when two 2's complement numbers are added.

b. Create two 16-bit 2's complement integers such that their sum causes an overflow.

c. Why does the sum of a negative 2's complement number and a positive 2's complement number never generate an overflow?

1.8 Compute the following:

a. NOT (1 0 1 1) OR N O T (l l O O)
b. NOT (1 0 0 0 AND (1100 OR 0 1 0 1))
c. NOT (NOT (1 1 0 1))
d. (0110 OR 0000) AND 1111

1.9 Perform the following logical operations. Express your answers in hexadecimal notation.

a. x5478 AND xFDEA
b. xABCD OR xl234
c. NOT((NOT(xDEFA)) AND (NOT(xFFFF)))
d. X00FF XOR x325C

1.10 We have represented numbers in base-2 (binary) and in base-16 (hex). We are now ready for unsigned base-4, which we will call quad numbers. A quad digit can be 0, 1, 2, or 3.

a. What is the maximum unsigned decimal value that one can represent with 3 quad digits?

b. What is the maximum unsigned decimal value that one can represent with n quad digits (Hint: your answer should be a function of n) ?

c. Add the two unsigned quad numbers: 023 and 221.

d. What is the quad representation of the decimal number 42?

e. What is the binary representation of the unsigned quad number 123.3?

f . Express the unsigned quad number 123.3 in IEEE floating point format.

g. Given a black box which takes m quad digits as input and produces one quad digit for output, what is the maximum number of unique functions this black box can implement?