

Sorting Algorithms & Convex Hull

Elias Moons (traduction par Guillaume Derval)

November 10, 2017

Table of Contents

Algorithmes de tri

- Tri par insertion

- Tri fusion (Merge sort)

- Quicksort

- Heapsort

Convex Hull

- Problem

- Graham Scan

Table of Contents

Algorithmes de tri

- Tri par insertion

- Tri fusion (Merge sort)

- Quicksort

- Heapsort

Convex Hull

- Problem

- Graham Scan

Table of Contents

Algorithmes de tri

Tri par insertion

Tri fusion (Merge sort)

Quicksort

Heapsort

Convex Hull

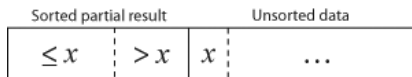
Problem

Graham Scan

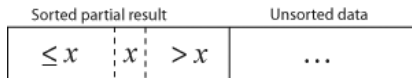
Tri par insertion

Idea

- ▶ A chaque itération, prendre un élément dans la partie non triée du tableau et le mettre au bon endroit dans la partie triée



- ▶ deviens



Tri par insertion

6	5	3	1	8	7	2	4
---	---	---	---	---	---	---	---

Tri par insertion

6	5	3	1	8	7	2	4
---	---	---	---	---	---	---	---

6	5	3	1	8	7	2	4
---	---	---	---	---	---	---	---

Tri par insertion

6	5	3	1	8	7	2	4
---	---	---	---	---	---	---	---

5	6	3	1	8	7	2	4
---	---	---	---	---	---	---	---

5	6	3	1	8	7	2	4
---	---	---	---	---	---	---	---

Tri par insertion

5	6	3	1	8	7	2	4
---	---	---	---	---	---	---	---

5	3	6	1	8	7	2	4
---	---	---	---	---	---	---	---

3	5	6	1	8	7	2	4
---	---	---	---	---	---	---	---

3	5	6	1	8	7	2	4
---	---	---	---	---	---	---	---

Tri par insertion

3	5	6	1	8	7	2	4
---	---	---	---	---	---	---	---

3	5	1	6	8	7	2	4
---	---	---	---	---	---	---	---

3	1	5	6	8	7	2	4
---	---	---	---	---	---	---	---

1	3	5	6	8	7	2	4
---	---	---	---	---	---	---	---

1	3	5	6	8	7	2	4
---	---	---	---	---	---	---	---

Tri par insertion

1	3	5	6	8	7	2	4
---	---	---	---	---	---	---	---

1	3	5	6	8	7	2	4
---	---	---	---	---	---	---	---

Tri par insertion

1	3	5	6	8	7	2	4
---	---	---	---	---	---	---	---

1	3	5	6	7	8	2	4
---	---	---	---	---	---	---	---

1	3	5	6	7	8	2	4
---	---	---	---	---	---	---	---

Tri par insertion

1	3	5	6	7	8	2	4
---	---	---	---	---	---	---	---

1	3	5	6	7	2	8	4
---	---	---	---	---	---	---	---

1	3	5	6	2	7	8	4
---	---	---	---	---	---	---	---

1	3	5	2	6	7	8	4
---	---	---	---	---	---	---	---

1	3	2	5	6	7	8	4
---	---	---	---	---	---	---	---

1	2	3	5	6	7	8	4
---	---	---	---	---	---	---	---

1	2	3	5	6	7	8	4
---	---	---	---	---	---	---	---

Tri par insertion

1	2	3	5	6	7	8	4
---	---	---	---	---	---	---	---

1	2	3	5	6	7	4	8
---	---	---	---	---	---	---	---

1	2	3	5	6	4	7	8
---	---	---	---	---	---	---	---

1	2	3	5	4	6	7	8
---	---	---	---	---	---	---	---

1	2	3	4	5	6	7	8
---	---	---	---	---	---	---	---

1	2	3	4	5	6	7	8
---	---	---	---	---	---	---	---

Tri par insertion

Complexité

Moyenne	Meilleure	Pire	Mémoire
$O(n^2)$	$O(n)$	$O(n^2)$	$O(1)$

Table of Contents

Algorithmes de tri

Tri par insertion

Tri fusion (Merge sort)

Quicksort

Heapsort

Convex Hull

Problem

Graham Scan

Merge Sort

Idée

- ▶ Division récursive du tableau en "moitiés", jusqu'à atteindre des tableaux de taille 1
- ▶ Fusion des sous-tableaux pour produire des sous-tableau plus grand et triés

Merge Sort

6	5	3	1	8	7	2	4
---	---	---	---	---	---	---	---

6	5	3	1
---	---	---	---

8	7	2	4
---	---	---	---

6	5
---	---

3	1
---	---

8	7	2	4
---	---	---	---

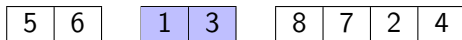
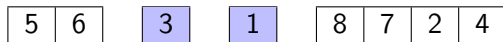
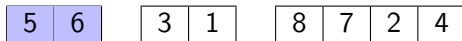
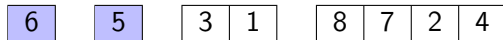
6

5

3	1
---	---

8	7	2	4
---	---	---	---

Merge Sort



Merge Sort

5	6	1	3	8	7	2	4
---	---	---	---	---	---	---	---

1	3	5	6	8	7	2	4
---	---	---	---	---	---	---	---

1	3	5	6	8	7	2	4
---	---	---	---	---	---	---	---

1	3	5	6	8	7	2	4
---	---	---	---	---	---	---	---

Merge Sort

1	3	5	6
---	---	---	---

8

7

2	4
---	---

1	3	5	6
---	---	---	---

7	8
---	---

2	4
---	---

1	3	5	6
---	---	---	---

7	8
---	---

2

4

1	3	5	6
---	---	---	---

7	8
---	---

2	4
---	---

Merge Sort

1	3	5	6
---	---	---	---

7	8
---	---

2	4
---	---

1	3	5	6
---	---	---	---

2	4	7	8
---	---	---	---

1	2	3	4	5	6	7	8
---	---	---	---	---	---	---	---

Merge Sort

Complexité

Moyenne	Meilleure	Pire	Mémoire
$O(n \log(n))$	$O(n \log(n))$	$O(n \log(n))$	$O(n)$

Table of Contents

Algorithmes de tri

Tri par insertion

Tri fusion (Merge sort)

Quicksort

Heapsort

Convex Hull

Problem

Graham Scan

Quicksort

Idée

- ▶ Choisir un élément dans le tableau (le pivot)
- ▶ Partitionnement: mettre les éléments du tableau plus grand que le pivot à droite de celui-ci (et donc les éléments plus petits à gauche)
- ▶ Appliquer récursivement à gauche et à droite

Quicksort

6	5	3	1	8	7	2	4
---	---	---	---	---	---	---	---

5	3	1	2	4
---	---	---	---	---

6

8	7
---	---

5	3	1	2	4
---	---	---	---	---

6

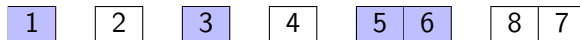
8	7
---	---

3	1	2	4
---	---	---	---

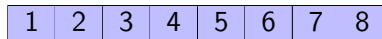
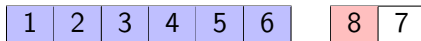
5	6
---	---

8	7
---	---

Quicksort



Quicksort



Quicksort

Complexité

Moyenne	Meilleure	Pire	Mémoire
$O(n \log(n))$	$O(n \log(n))$	$O(n^2)$	$O(1)$

Table of Contents

Algorithmes de tri

Tri par insertion

Tri fusion (Merge sort)

Quicksort

Heapsort

Convex Hull

Problem

Graham Scan

Tas (Heap)

Le tas est une structure de donnée qui a deux opérations:

- ▶ push: ajouter un élément au tas. $O(\log n)$.
- ▶ pop: retire l'élément le plus grand. $O(\log n)$.

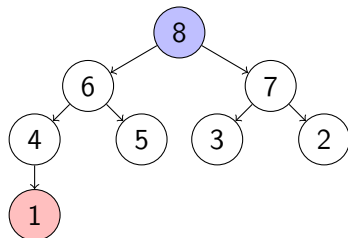
Tri par tas (Heap sort)

Idée

- ▶ Mettre tout les éléments dans un tas
- ▶ Retirer les éléments du tas, un à un

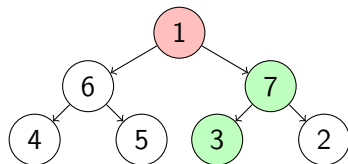
Heapsort

8	6	7	4	5	3	2	1
---	---	---	---	---	---	---	---



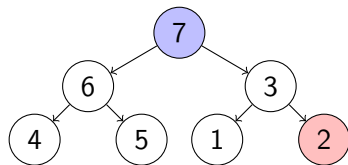
Heapsort

1	6	7	4	5	3	2	8
---	---	---	---	---	---	---	---



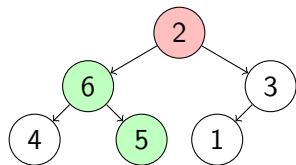
Heapsort

7	6	3	4	5	1	2	8
---	---	---	---	---	---	---	---



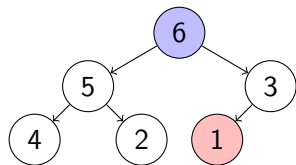
Heapsort

2	6	3	4	5	1	7	8
---	---	---	---	---	---	---	---



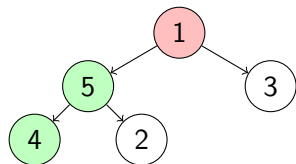
Heapsort

6	5	3	4	2	1	7	8
---	---	---	---	---	---	---	---



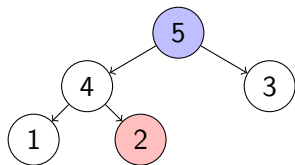
Heapsort

1	5	3	4	2	6	7	8
---	---	---	---	---	---	---	---



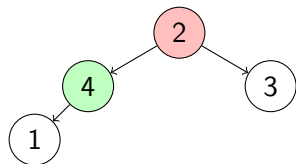
Heapsort

5	4	3	1	2	6	7	8
---	---	---	---	---	---	---	---



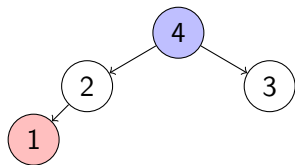
Heapsort

2	4	3	1	5	6	7	8
---	---	---	---	---	---	---	---

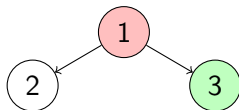
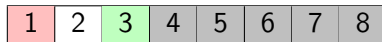


Heapsort

4	2	3	1	5	6	7	8
---	---	---	---	---	---	---	---

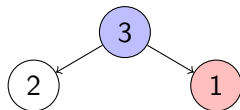


Heapsort

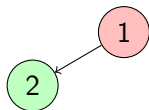
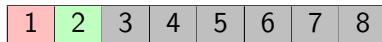


Heapsort

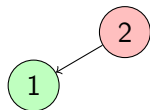
3	2	1	4	5	6	7	8
---	---	---	---	---	---	---	---



Heapsort



Heapsort



2	1	3	4	5	6	7	8
---	---	---	---	---	---	---	---

Heapsort

1

1	2	3	4	5	6	7	8
---	---	---	---	---	---	---	---

Heapsort

1	2	3	4	5	6	7	8
---	---	---	---	---	---	---	---

Heapsort

Complexité

Moyenne	Meilleure	Pire	Mémoire
$O(n \log(n))$	$O(n \log(n))$	$O(n \log(n))$	$O(1)$

Table of Contents

Algorithmes de tri

Tri par insertion

Tri fusion (Merge sort)

Quicksort

Heapsort

Convex Hull

Problem

Graham Scan

Table of Contents

Algorithmes de tri

Tri par insertion

Tri fusion (Merge sort)

Quicksort

Heapsort

Convex Hull

Problem

Graham Scan

Convex Hull

Given a set of points in the plane, compute the smallest convex polygon in the plane that contains all the points.

Table of Contents

Algorithmes de tri

Tri par insertion

Tri fusion (Merge sort)

Quicksort

Heapsort

Convex Hull

Problem

Graham Scan

Graham Scan

Method of computing the convex hull of a finite set of points in the plane with time complexity $O(n \log(n))$. The algorithm finds all vertices of the convex hull ordered along its boundary.

Graham Scan

Idea

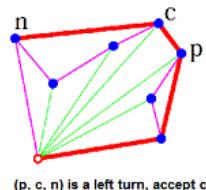
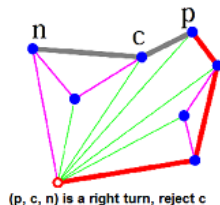
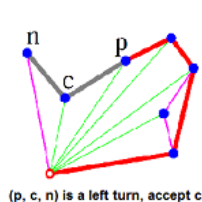
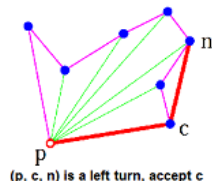
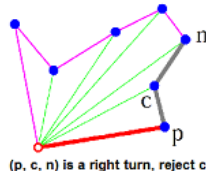
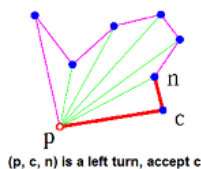
- ▶ find the point with the lowest y-coordinate. if there are multiple points with the lowest y-coordinate, the pick that one of them with the lowest x-coordinate. call this point P
- ▶ now sort all the points in increasing order of the angle they and point P make with the x-axis
- ▶ consider each point in the sorted array in sequence. For each point determine whether coming from the 2 previous points it makes a left of a right turn.
- ▶ if it makes a left turn, proceed with the next point
- ▶ if it makes a right turn, the second-to-last point is not part of the convex hull and should be removed from the convex hull, continue this removing for as long as the last 3 points make up a right turn

Graham Scan

p: previous

c: current

n:next



In the above algorithm and below code, a stack of points is used to store convex hull points. With reference to the code, p is next-to-top in stack, c is top of stack and n is points[i].

Graham Scan

Direction of the turn

To determine whether 3 points constitute a left or a right turn we do not have to compute the actual angles but we can use a cross product.

Consider the 3 points (x_1, y_1) , (x_2, y_2) and (x_3, y_3) , which we will call P_1 , P_2 and P_3 .

Now compute the z-component of the cross product of the vectors P_1P_2 and P_1P_3 . Which is given by the expression

$$(x_2 - x_1)(y_3 - y_1) - (y_2 - y_1)(x_3 - x_1).$$

If the result is 0, the points are collinear. If the result is positive, the 3 points constitute a left turn. If the result is negative, the 3 points constitute a right turn.