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Problem 1.2

a. Is $f(n) = O(f(n)^2)$ sometimes, never, or always true?

I believe that it is sometimes true. Whenever $f(n)$ outputs values in the range $[1, \infty)$ it is obvious that $f(n) \leq f(n)^2$. So a function like $f(n) = n$ shows that it is possible for this equation to be true. However, when $f(n)$ outputs values between $(0, 1)$ it is clear that $f(n)^2 < f(n)$. This means that functions with a constant numerator and a denominator which depends on n, like $f(n) = 10^9/n$ or $f(n) = 1/n$, are instances in which the equation is not true.

I actually came to a slightly different conclusion prior to looking at the answer sheet. For all cases in which $f(n) = O(f(n)^2)$ is not true, $f(n)$ will be below 1. As a consequence, it will be bounded by a constant. In these cases both $f(n)^2 = O(1)$ and $f(n) = O(1)$ will be true. After thinking and talking with friends about this for a few hours I decided that I was abusing the notation by claiming $f(n) = O(f(n)^2)$ in virtue of both being bounded by a constant, since plugging in the values shows this to be a lie.

I don't go to MIT (though I want to). I also don't have an instructor capable of giving me feedback. So if your reading this, I would really appreciate feedback.