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Problem 1.2

b. Is  $f(n) + g(n) = \theta(\max(f(n), g(n)))$  sometimes, always, or never true?

I believe that it is always true.

If  $f(n)$  is asymptotically larger than  $g(n)$ , then there is a function within  $\theta(f(n))$  which contains  $g(n)$ . The reverse is also true: whenever  $g(n)$  is asymptotically larger,  $\theta(g(n))$  will have a function within it that contains  $f(n)$ . This leaves only the case in which they are equal. In this case adding  $g(n)$  to  $f(n)$  is equivalent to  $2f(n)$ . That constants are contained should be obvious, so the last case is true too.