Machine Learning in Healthcare

# #C16 Independent component analysis

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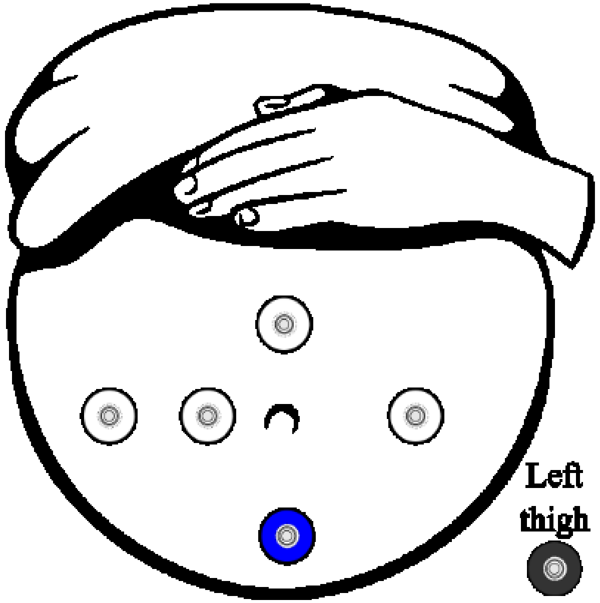
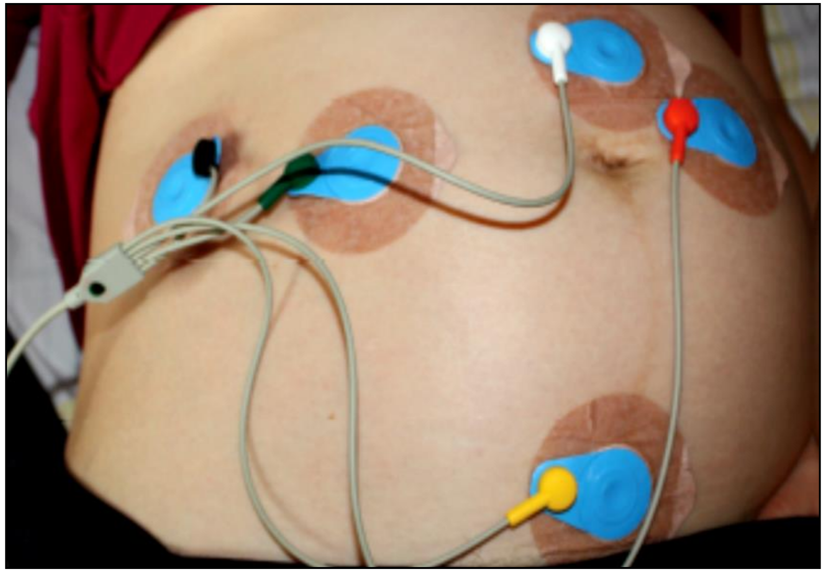
AIMLab.

# NI-FECG

## NI-FECG: opportunity

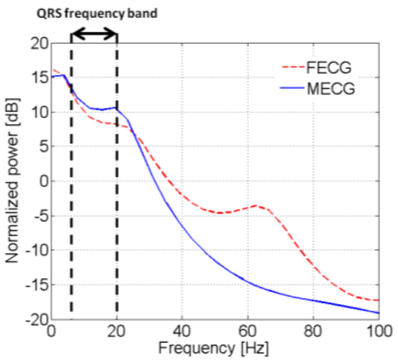
- Non-invasive,
- Information on conduction,
- Low-cost,
- Remote monitoring.

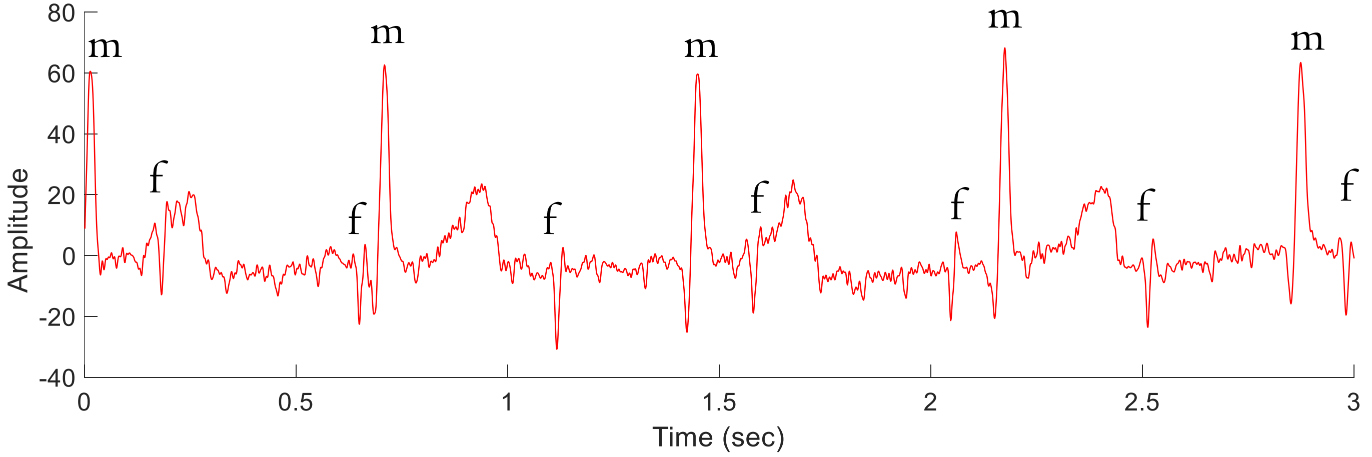


## NI-FECG: Challenges

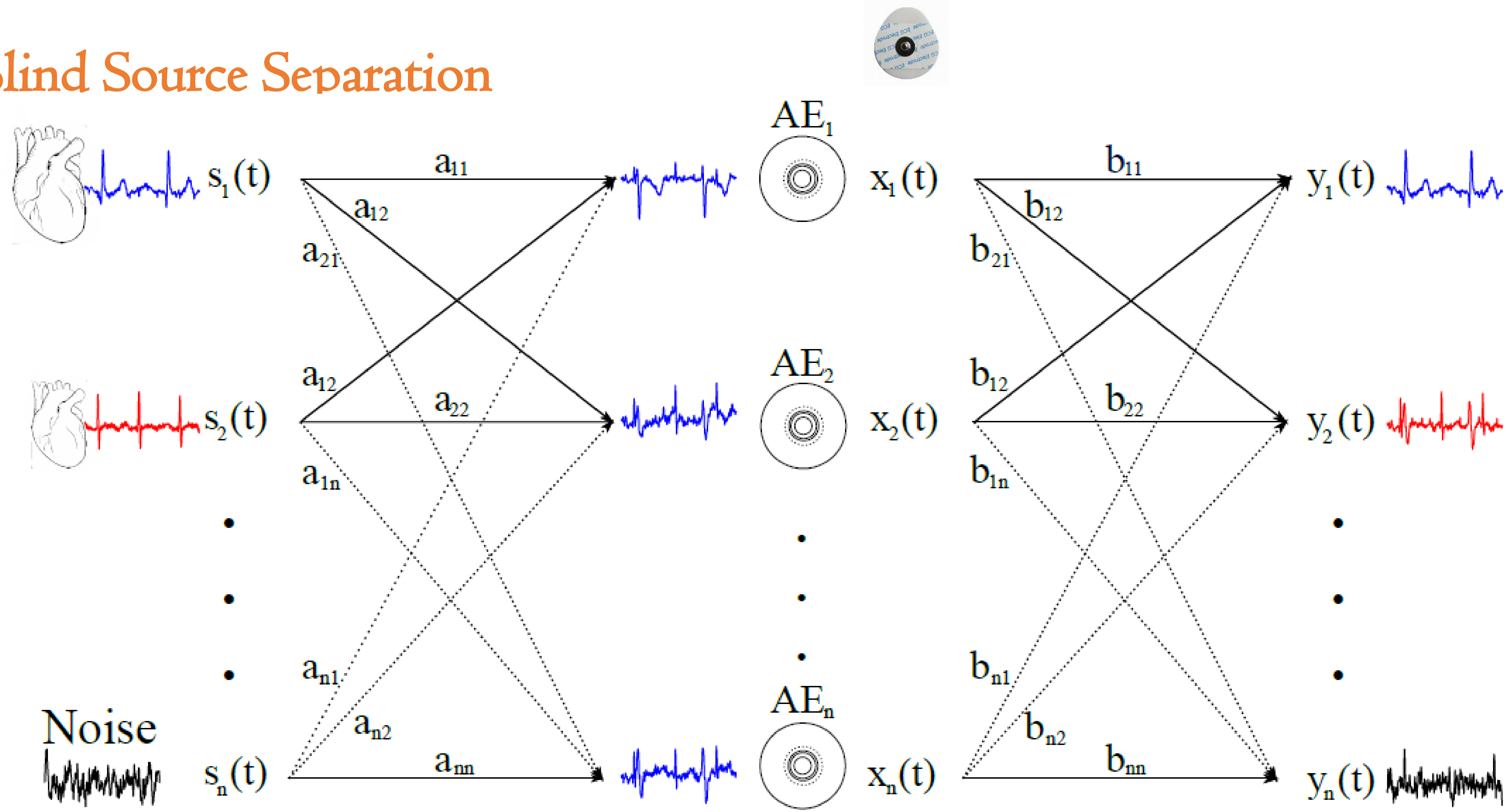
- Overlap in time and frequency,
- Non stationarities,
- Vernix caseosa.



# Reminder

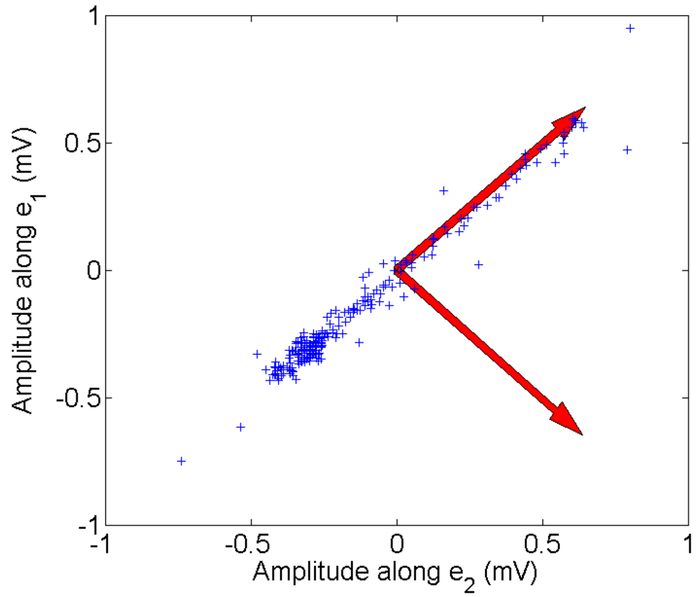
# Blind Source Separation



# Principal Component Analysis

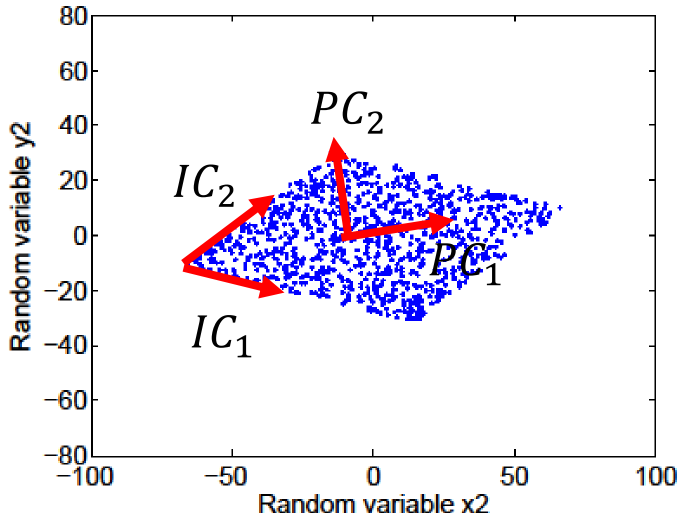
- Ideas we introduced here:
  - Expressing our dataset in a new basis may be a good idea!
  - PCA is a statistical procedure that uses an orthogonal transformation to convert a set of observations of possibly correlated variables into a set of values of linearly uncorrelated variables called principal components.

- Limitations with PCA:
  - Is maximal variance the right statistical criteria?
  - Limited to orthogonal basis. (Due to our criteria for independence which is second order.)



# Independent Component Analysis

- As in PCA, we want to find a new vector basis on which to project our observations in order to obtain a set of maximally independent source signals.
- Instead of using variance as our independence measure (i.e. decorrelation) as in PCA, we will look for <u>**statistical independence**</u> with ICA.




Invention of ICA: Herault, Jeanny, and Christian Jutten. "Space or time adaptive signal processing by neural network models." Neural networks for computing. Vol. 151. No. 1. AIP Publishing, 1986.

# Independent Component Analysis

# ICA

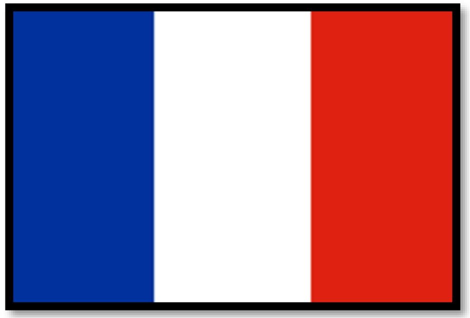
**Codename**

Herault and Jutten, 1986

**Special power**

Source separation

**Place of origin**



## ICA

Herault, Jeanny, and Christian Jutten. "Space or time adaptive signal processing by neural network models." Neural networks for computing. Vol. 151. No. 1. AIP Publishing, 1986.

# Independent Component Analysis

- Independent Component Analysis (ICA) consist of recovering unobserved signals or sources from several observed mixture by exploiting the assumption of **mutual independence** between the signals [Card1998].

- Two microphones recording two individuals: $x_1(t)$ and $x_2(t)$:
  - $\begin{cases} x_1(t) = a_{11}s_1 + a_{12}s_2 \\ x_2(t) = a_{21}s_1 + a_{22}s_2 \end{cases}$
  - More generally we write: $x = As$ and $s = Wx$
  - $A$ is commonly called the **mixing matrix**.
  - We assume that $x_1(t)$ and $x_2(t)$ are **linear** and **instantaneous mixtures**.

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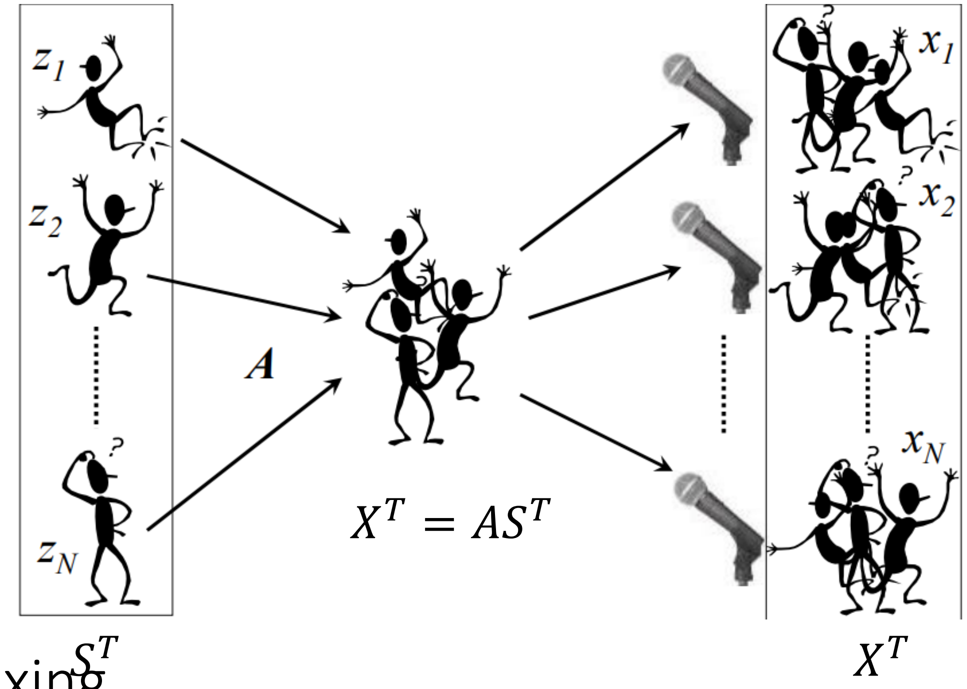
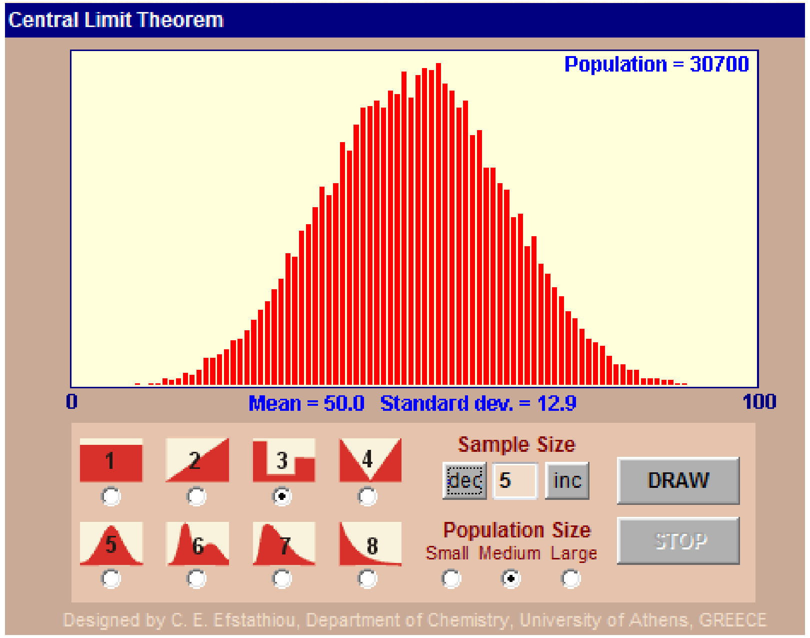
# The Cocktail Party Problem




- At each time instant:
  - $x(t) = As(t)$ and $s(t) = Wx(t)$
- For all recorded observations:
  - $X^T = AS^T$
    - $\hat{S}^T = WX^T$ with $W = \hat{A}^{-1}$
    - $A \in \mathbb{R}^{n \cdot n}$: linear square mixing.
    - $X \in \mathbb{R}^{m \cdot n}$: observations produced by the mixing.
    - $S \in \mathbb{R}^{m \cdot n}$: independent sources.
    - $n$ sources and observed signals.
    - $m$ observations (datapoint).
- We want to estimate $W = \hat{A}^{-1}$.
- Iterative process with some **cost function** which measures the statistical independence of the estimated sources at each iteration.

# Non-Gaussianity as Statistical Independence

- Reminder, **Central limit theorem**: any linear mixture of $N$ independent and identically distributed (i.i.d.) random variables is more Gaussian than the original variables.
- This is true, whatever the type of distributions of the individual random variables.
- Thus to demix our signal we will look for maximize non-Gaussianity. Non-Gaussianity is our marker of independence.




Example: http://www.chem.uoa.gr/applets/appletcentrallimit/appl_centrallimit2.html

# Higher Order Moments

- A moment is a specific quantitative measure of the shape of a probability distribution.
- $2^{nd}$ order is variance (we used it in PCA), $3^{rd}$ moment skewness and $4^{th}$ moment is kurtosis.

$$s(x) = \frac{1}{m}\sum_{i=1}^{m}\left[\frac{x^{(i)}-\mu}{\hat{\sigma}}\right]^3$$

Skewness

$$k(x) = \frac{1}{m}\sum_{i=1}^{m}\left[\frac{x^{(i)}-\mu}{\hat{\sigma}}\right]^4$$

Kurtosis

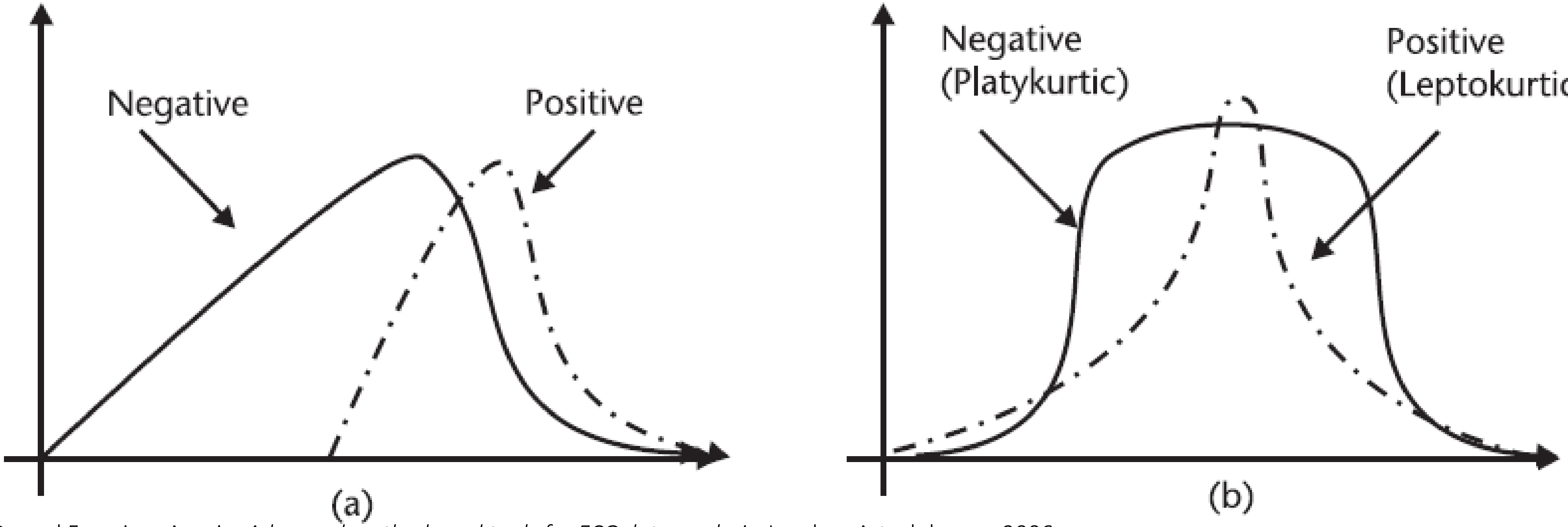



Image: Clifford, Gari D., and Francisco Azuaje. *Advanced methods and tools for ECG data analysis*. London: Artech house, 2006.

# Independent Component Analysis

- So we can use kurtosis for example as the cost function and proceed:
  - Initialize $W$.
  - Iterate to update $W$ with gradient descent to maximize kurtosis.
- Non-Gaussianity is one approximation, but sensitive to small changes in the distribution tail.
- Other measures of statistical independence may be used.

(Recall: Higher order moments $E(X^n) = \int_{-\infty}^{+\infty} x^n f(x)dx$. Mean, variance, skewness, kurtosis, hyperskewness, hyperflatness! )

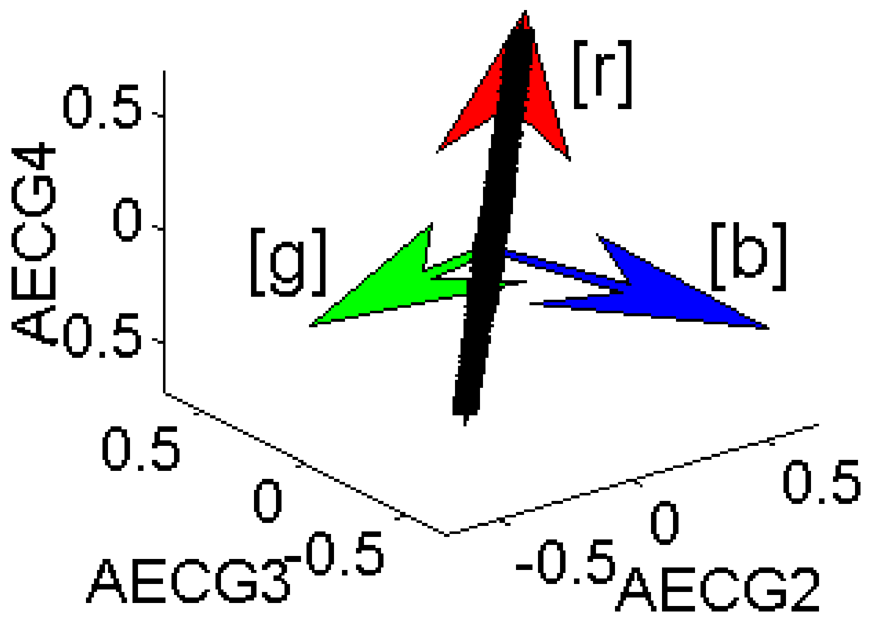
# Independent Component Analysis

- Looking for higher order statistics:
  - Kurtosis: $kurt(y) = E\{y^4\} - 3(E\{y^2\})^2$
  - Negentropy approximated: $J(y) \approx \frac{1}{12}E\{y^3\}^2 + \frac{1}{48}kurt(y)^2.$
  - Mutual information
  - Cummulants (JADE)
  - …
  - These are called **contrast function**. Their optimization allows to estimate the independent components.

(Recall: Higher order moments $E(X^n) = \int_{-\infty}^{+\infty} x^n f(x)dx$. Mean, variance, skewness, kurtosis, hyperskewness, hyperflatness! )

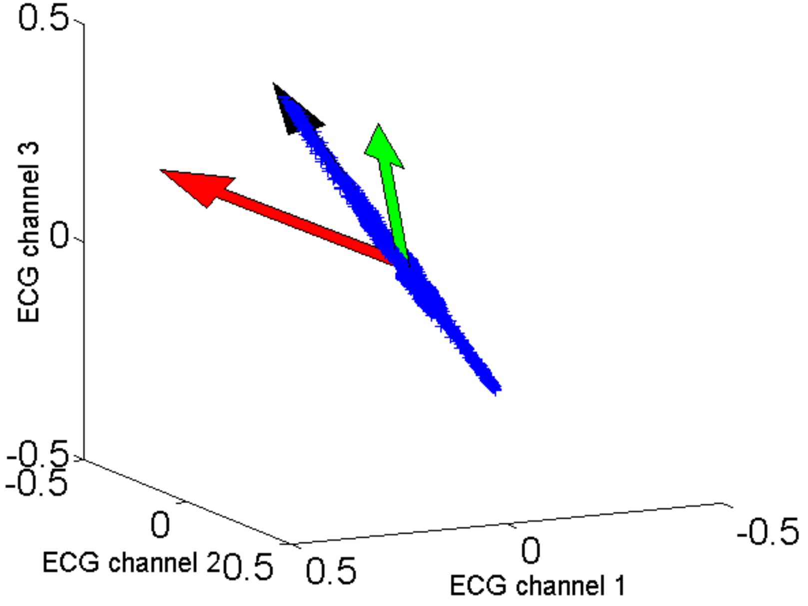
# Independent Component Analysis

PCA

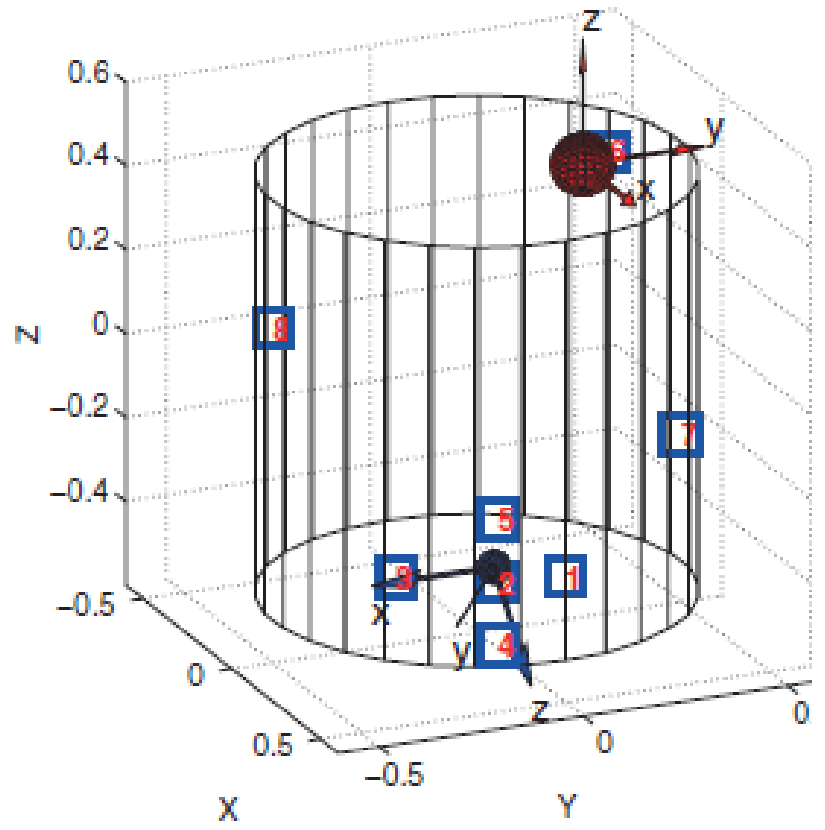


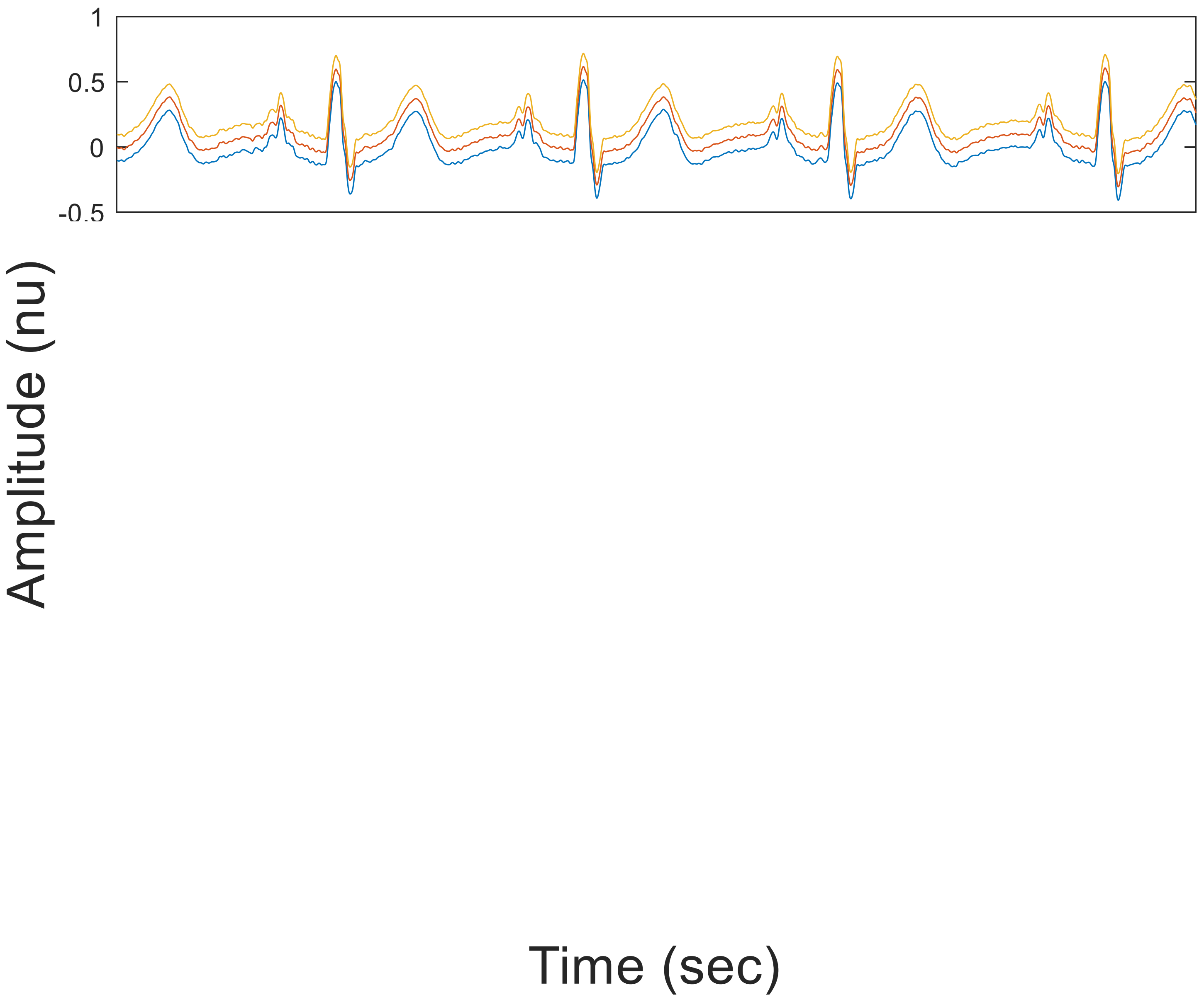

- Maximal covariance
- Closed form solution
- Constraint to orthogonal axis.

ICA




- Statistical independence
- No closed form solution
- Not constraint to orthogonal axis.


0.6
0.4
0.2
0
−0.2
−0.4
−0.5
0
0.5
Z
X
Y
−0.5
0
0
1
2
3
4
5
6
7
8


1
0.5
0
-0.5
Amplitude (nu)
Time (sec)

# Independent Component Analysis

- In summary:
  - ICA aims to find a linear representation of nongaussian data so that the components are statistically independent.
  - ICA exploits exploits the spatial diversity. Time structure are ignored. The mixture is assumed to be instantaneous.
  - ICA assumes, independence of the sources (and that the sources are not Gaussians). It does not assume orthogonality of the axis.
- Limitation:
  - Only works for linear mixture.
- Example good ICA implementations: JADE, FastICA.

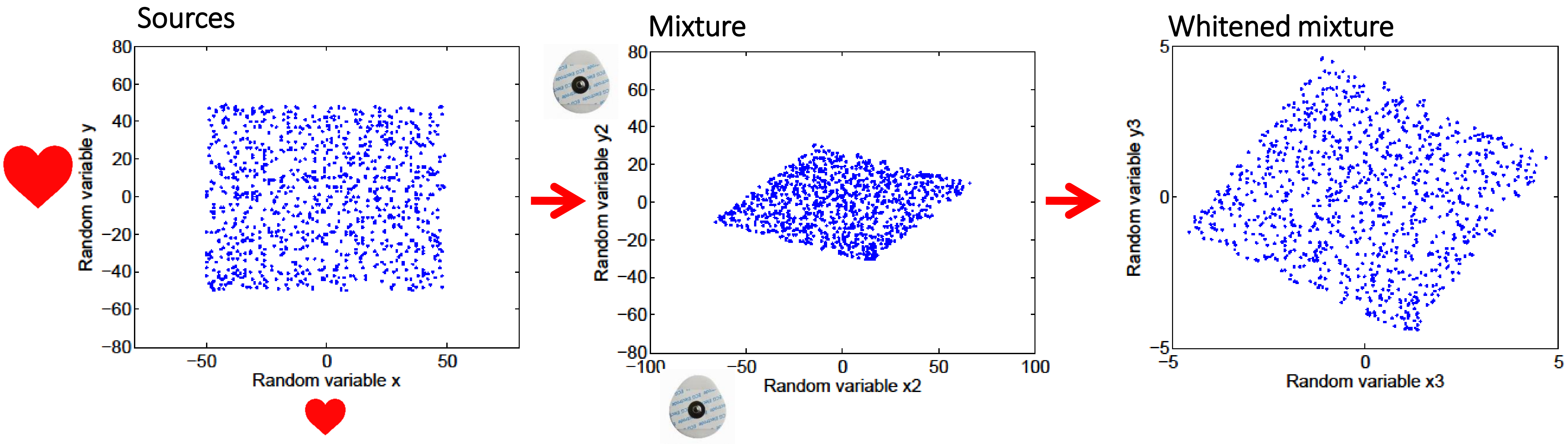
# Algorithm Evaluation

| CL | Method | $HRE$ | $RRE$ | $Se$ | $PPV$ | $F_1$ | $F_1\dagger$ |
|---|---|---|---|---|---|---|---|
| | | NU | NU | % | % | % | % |
| I | TS | 655.5 | 27.9 | 81.8 | 81.7 | 81.6 | 81.2 |
| I | $TS_c$ | 514.8 | 29.1 | 81.6 | 81.7 | 81.5 | 81.4 |
| I | $TS_m$ | 551.9 | 28.1 | 82.2 | 82.2 | 82.1 | 81.1 |
| I | $TS_{lp}$ | 902.0 | 46.2 | 82.1 | 81.9 | 81.8 | 78.5 |
| I | $TS_{pca}$ | 594.4 | 21.6 | 88.1 | 84.5 | 86.1 | 83.6 |
| I | $TS_{EKF}$ | 733.8 | 25.0 | 83.0 | 81.1 | 81.9 | 78.4 |
| II | ICA | 2852.1 | 39.3 | 69.1 | 60.0 | 63.7 | 61.7 |
| II | PCA | 3892.1 | 45.3 | 57.4 | 47.9 | 51.6 | 52.6 |
| III | TS-ICA | 272.7 | 17.1 | 93.0 | 91.1 | 92.0 | 91.3 |
| III | $TS_c$-ICA | 202.6 | 17.2 | 93.2 | 92.0 | 92.6 | 92.1 |
| III | $TS_m$-ICA | 251.9 | 18.4 | 91.7 | 90.8 | 91.2 | 92.3 |
| III | $TS_{lp}$-ICA | 399.1 | 37.9 | 88.4 | 88.7 | 88.4 | 85.2 |
| III | $TS_{pca}$-ICA | 153.2 | 16.9 | 93.8 | 92.2 | 93.0 | 92.4 |
| IV | ICA-$TS_{pca}$ | 396.9 | 27.1 | 90.1 | 88.8 | 89.2 | 89.4 |
| IV | ICA-$TS_{pca}$-ICA | 299.4 | 22.7 | 92.6 | 92.2 | 92.4 | 91.1 |
| | CONST-HR (143 bpm) | 172.2 | 8.9 | 23.2 | 23.1 | 23.0 | NA |
| | FUSE | 132.9 | 12.7 | 95.6 | 94.3 | 95.0 | 94.2 |
| | FUSE-SMOOTH | 19.1 | 6.3 | 95.9 | 96.0 | 96.0 | 95.2 |
| | FUSE-CHALL | 5.4 | 2.3 | NA | NA | NA | NA |

Behar et al. Physiol. Meas. 2014a

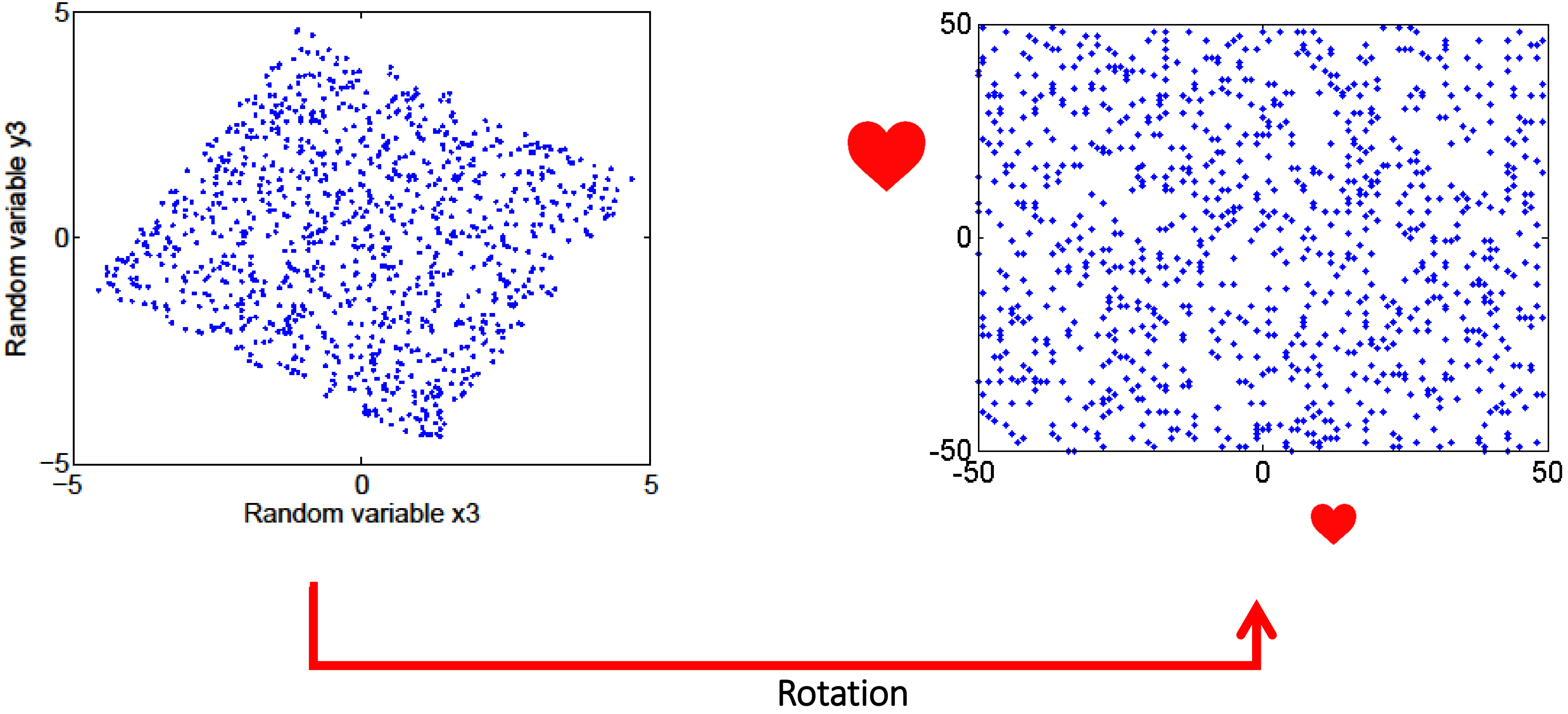
# Extra insights on ICA

# ICA: whitening the data



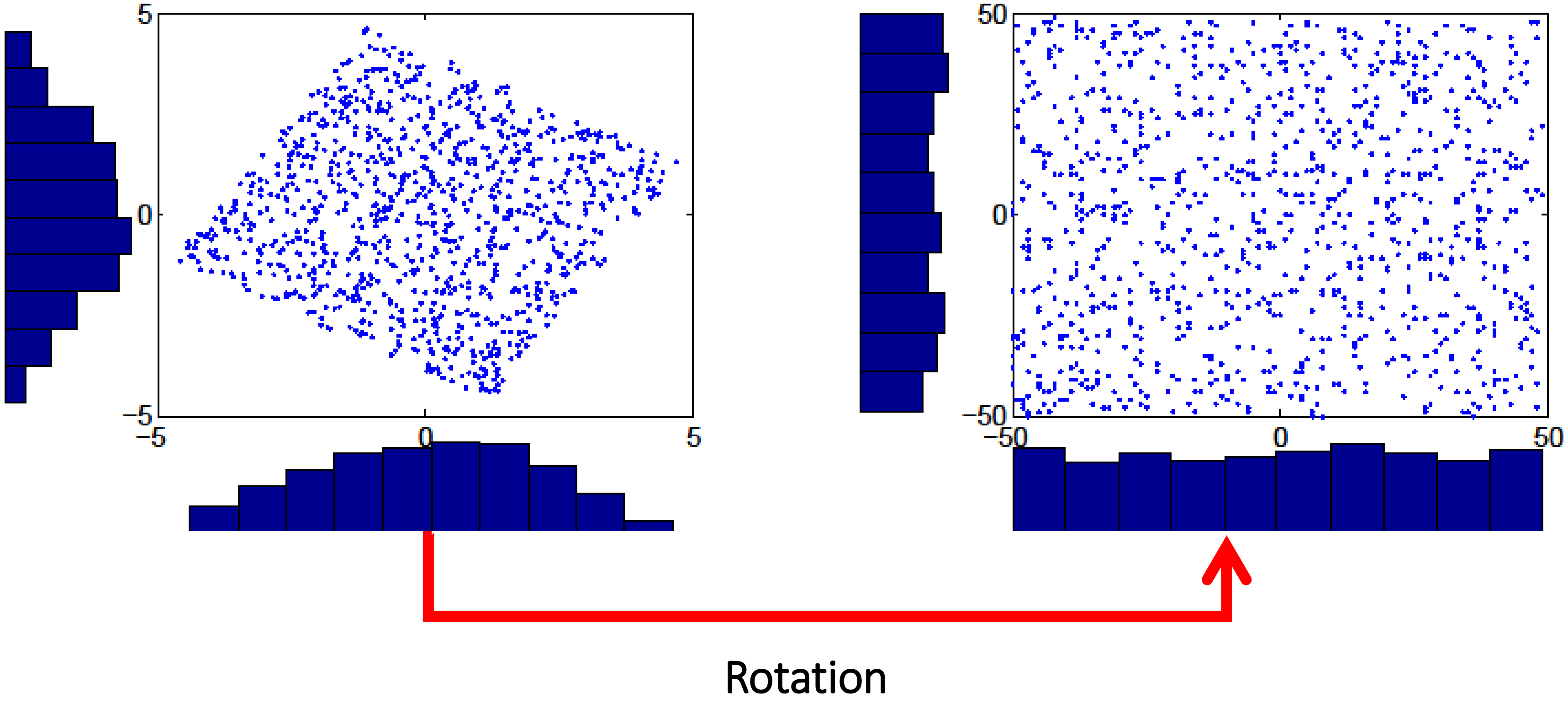

- **Whitening** removes any correlation in the data.
- The geometric interpretation is that whitening restores the initial shape of the data then ICA `only’ has to rotate.

# ICA: whitening the data

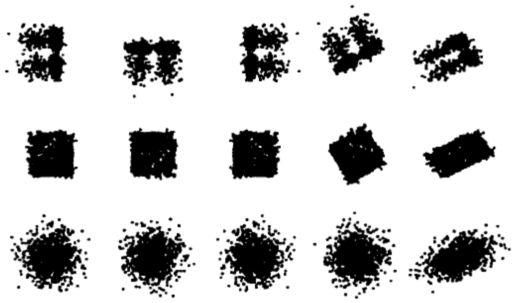


# ICA: whitening the data

- Central limit theorem: any linear mixture of 2 i.i.d. random variables is more Gaussian than the original variables.



# ICA: whitening the data



- This examples also highlight that BSS exploits spatial diversity.
- Time structure are ignored and the information contained in the data is represented exhaustively by the sample distribution of the observed random variables.

# ICA: whitening the data

- Whitening procedure: we want to transform the matrix $X$ linearly so that we obtain a new observation matrix $X_{new}$ which is white i.e. its components are **uncorrelated** and with **unit variance** i.e. $E\{X^T X\} = I$ i.e. we want the covariance matrix to be the unity matrix.
- How can we produce this transform?
- Covariance matrix can be diagonalized (we saw that in PCA)
  - $C = PDP^T$ where $P$ is orthogonal.
- So an idea is that we can diagonalize first but then we need to normalize somehow to make the covariance matrix of unit variance.
  - $X_w = D^{1/2} P^T X$
  - We can show that whitening is like transforming by $D^{1/2} P^T$
    - $X_w X_w^T = D^{1/2} P^T X X^T P D^{-1/2} = D^{1/2} P^T C P D^{-1/2} = I$

# ICA: whitening the data

- So we have:
  - $X_w = D^{-1/2}P^T X = D^{-1/2}P^T AS = A_{new}S$
  - $A_{new}$ is orthogonal:
    - $X_w X_w^T = I = A_{new}S(A_{new}S\ )^T = A_{new}SS^T A_{new}^T$
    - We assume unit variance of the sources we look for,
    - $A_{new}SS^T A_{new}^T = A_{new}A_{new}^T = I$
  - $A$ is an $N \cdot N$ matrix with $N^2$ degree of freedom.
  - $A_{new}$ is an $N \cdot N$ matrix with $N \cdot (N-1)/2$ degree of freedom.
  - So by making **the whitening transformation we reduced by half the complexity** of the problem.
  - This is the purpose of whitening.

# ICA algorithm

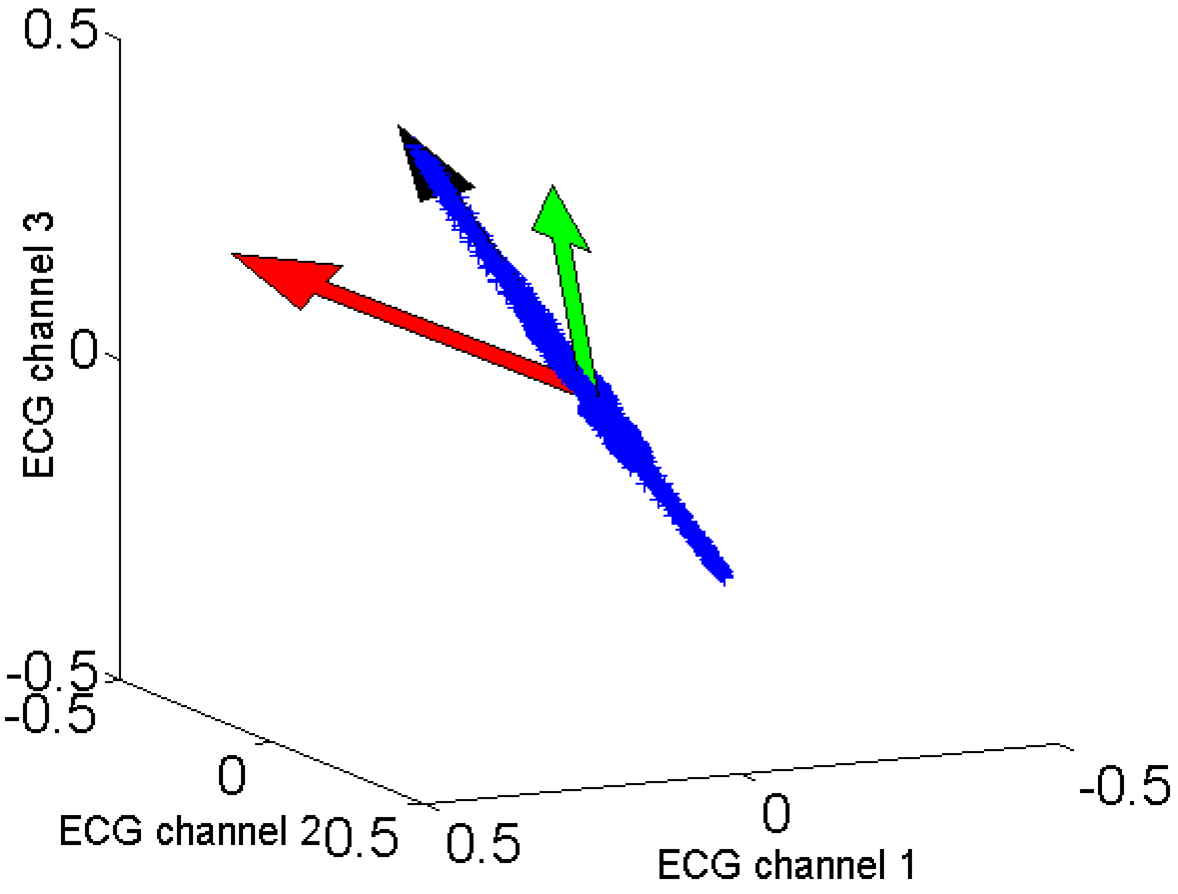
- Typical steps:
  - Centering: subtract the mean of the signal.
  - Whitening: uncorrelated the data. This means to treat all dimensions equally and simplify the ICA problem.
  - Dimensionality reduction (optional): remove PCA components with the least variance.
  - Iterative algorithm: find the independent components

# Independent Component Analysis

- Joint Approximation Diagonalization of Eigen-matrices (JADE)
  - Cardoso, Jean-François, and Antoine Souloumiac. "Blind beamforming for non-Gaussian signals." IEE proceedings F (radar and signal processing). Vol. 140. No. 6. IET Digital Library, 1993.
  - Cardoso, Jean-François. "High-order contrasts for independent component analysis." Neural computation 11.1 (1999): 157-192.
- FastICA
  - Hyvarinen, Aapo. "Fast and robust fixed-point algorithms for independent component analysis." IEEE transactions on Neural Networks 10.3 (1999): 626-634.
- ICA has been widely used in biosignals processing, in particular for ECG and EEG analysis.

## Quiz: axis

- Does ICA looks for orthogonal axes?




ICA **<u>DOES NOT</u>** require orthogonal axis

# Quiz: whitening the data

- Does ICA preserves Scaling?

$$\begin{cases} x_1(t) = a_{11}s_1 + a_{12}s_2 \\ x_2(t) = a_{21}s_1 + a_{22}s_2 \end{cases}$$

Both a and s are unknowns so any scalar multiplying one of the source s can be cancelled by dividing by the corresponding a by the same value.

ICA **DOES NOT** preserve scaling

# Quiz: whitening the data

- Are uncorrelated and statistical independent variables the same thing?

Independant variables

$$p(y_1, y_2) = p_1(y_1)p_2(y_2)$$

Uncorrelated variables

$$E\{y_1 y_2\} - E\{y_1\}E\{y_2\} = 0$$

<u>Independence implies uncorrelated but not the opposite.</u>

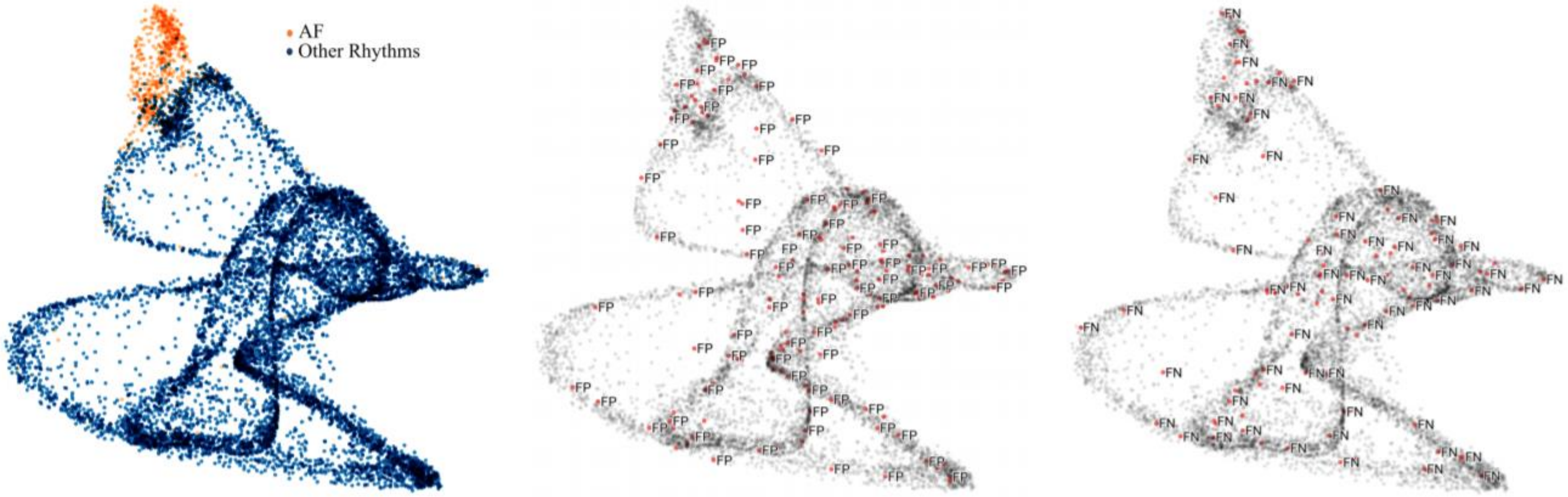
# Beyond ICA

# Beyond PCA and ICA

- PCA and ICA are commonly used algorithms in ML.
- However, often non-linear relationships between features exist and both standard PCA and ICA will not capture that.
- Dimensionality reduction is a field of research on its own.
- These are advanced techniques to tackle the limitation of PCA and ICA.
- t-Distributed Stochastic Neighbor Embedding (t-SNE) is a probabilistic **non-linear technique** which is well suited for the visualization of high-dimensional datasets.
  - Application: e.g. image processing and genomic.
  - t-SNE is computationally expensive.
  - t-SNE is non-linear whereas PCA and ICA are linear.
- Uniform manifold approximation and projection UMAP…

Maaten, Laurens van der, and Geoffrey Hinton. "Visualizing data using t-SNE." Journal of machine learning research 9.Nov (2008): 2579-2605.
https://lvdmaaten.github.io/tsne/
McInnes, Leland, John Healy, and James Melville. "Umap: Uniform manifold approximation and projection for dimension reduction." arXiv preprint arXiv:1802.03426 (2018).
https://umap-learn.readthedocs.io/en/latest/

# t-SNE




(a) Visualization of data belonging to *AF* and *Other Rhythm* class

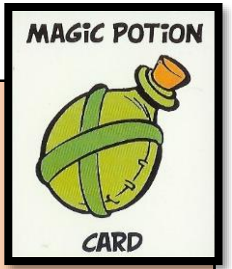
(b) The False Positive (FP) cases highlighted

(c) The False Negative (FN) cases highlighted

**Figure 5: Visualization of the features extracted from the deep learning pipeline and the time series covariates by performing t-SNE [37] based clustering. Clustering was performed on the testing dataset**

Shashikumar, Supreeth P., et al. "Detection of paroxysmal atrial fibrillation using attention-based bidirectional recurrent neural networks." Proceedings of the 24th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining. ACM, 2018.

# Take Home




- ICA look for **statistical independence**. It assumes a linear, instantaneous mixture of statistically independent sources (and not Gaussian).
- ICA exploits **spatial diversity**. Time structure are ignored.
- It looks to **maximize non-Gaussianity** of the sources it estimates.
- ICA is non-parametric.
- To solve the ICA problem we use gradient descent on a cost function that we call the **contrast function**.

# References


[1] Gari D. Clifford course note: http://www.mit.edu/~gari/teaching/6.555/SLIDES/BSShandouts.pdf

[2] Joachim Behar. Course note, ML in Healthcare course.

[3] Independent Component Analysis: Algorithms and Applications. Aapo Hyvärinen and Erkki Oja. URL: http://mlsp.cs.cmu.edu/courses/fall2012/lectures/ICA_Hyvarinen.pdf

[4] ICA for dummies. Online tutorial by Arnaud Delorme. URL: http://sccn.ucsd.edu/~arno/indexica.html

[5] Clifford, Gari D., and Francisco Azuaje. Advanced methods and tools for ECG data analysis. London: Artech house, 2006.

[6] Aapo Hyvarinen. tutorial on whitening. URL: http://cis.legacy.ics.tkk.fi/aapo/papers/IJCNN99_tutorialweb/node26.html

[7] Cardoso, J-F. "Blind signal separation: statistical principles." Proceedings of the IEEE 86.10 (1998): 2009-2025.

[8] Course notes: https://www.stat.cmu.edu/~cshalizi/uADA/12/lectures/ch18.pdf

[9] Course notes: http://cis.legacy.ics.tkk.fi/aapo/papers/NCS99web/node33.html

[10] Arnaud Delorme research page: http://arnauddelorme.com/ica_for_dummies/