Capacity of Additive White Gaussian Noise channel with BPSK Signaling

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We derive a closed form expression for the capacity of AWGN channel with M-ary signaling from source that follows the uniform distribution for the transmitted symbol.
We will use the following variables:
y: Received symbol
x: Transmitted symbol
n: Additive White Gaussian noise sample

Mathematically, we write AWGN channel as:

$$y = x + n$$

where n is sampled from the distribution $N(0, 1)$ i.e. the Gaussian distribution with zero mean and unit variance. From Information theory, we know that the capacity of the AWGN channel is given by:

$$C = H(y) - H(y|x)$$
$$= \int f(y) log f(y) dy + \int f(x, y) log f(y|x) dy$$

Assume that the M-ary signaling source X draws a symbol x with the probability p(x). Following this, we can write:

$$f(y) = f(y|x)p(x)$$

$$C = -\int \sum_x p(x) f(y|x) log f(y) dy + \int \sum_x f(x, y) log f(y|x) dy$$

$$C = -\int \sum_x p(x = a_k) f(y|x = a_k) log f(y) dy + \int \sum_x f(x = a_k, y) log f(y|x = a_k) dy$$

$$C = -\int \sum_x p(x = a_k) f(y|x = a_k) log f(y) dy + \int \sum_x p(x = a_k) f(y|x = a_k) log f(y|x = a_k) dy$$

$$C = -\int \sum_x p(x = a_k) f(y|x = a_k) log \frac{f(y|x = a_k)}{f(y)} dy$$

$$C = -\frac{1}{M} \int \sum_x f(y|x = a_k) log \frac{f(y|x = a_k)}{f(y)} dy$$