Question 1: **(marks: 8+4)**

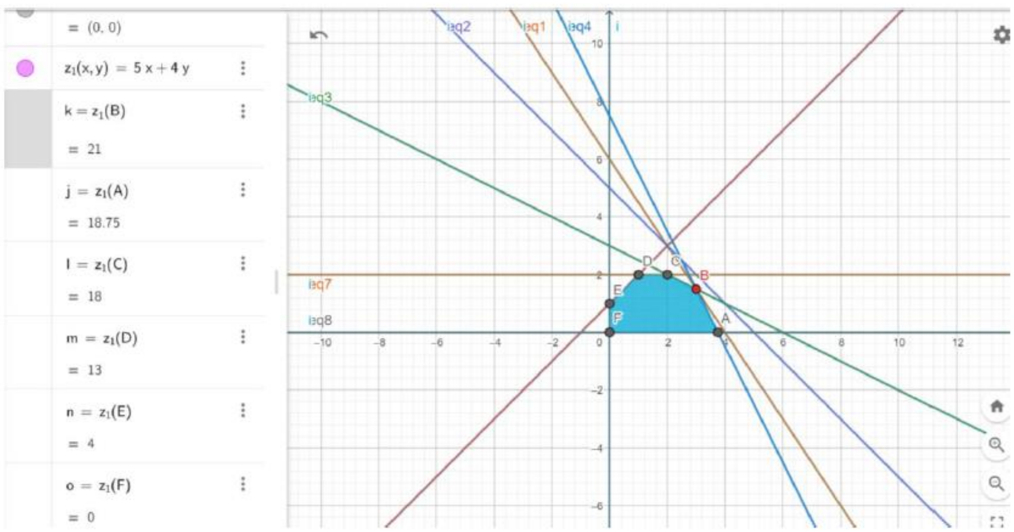
a. Solve the following linear programming model graphically.

$$\text{Max} \quad z = 5x + 4y$$

subject to

$$
\begin{aligned}
6x + 4y &\le 24 \\
6x + 3y &\le 22.5 \\
x + y &\le 5 \\
x + 2y &\le 6 \\
-x + y &\le 1 \\
y &\le 2 \\
x, y &\ge 0
\end{aligned}
$$

b. Identify the redundant constraints and show that their removal does not affect the solution space or the optimal solution.



First and third are redundant constraints.

Question 2: **(marks: 3+7)**

A compony produces 3 types of toys. The maximum production limit of the three types per month is 7 toys in total. Production time of a type 1, type 2 and type 3 toy is 2 hours, 5 hours and 3 hours respectively. The minimum work hours available in a month are 10 hours. The profit of a type 1, type 2 and type 3 toy is \$1, \$2 and \$3 respectively.

i. formulate a linear programming model for the given scenario.

ii. Use any appropriate technique to find the number of each type of toys to be produced to maximize the profit.

Solution

Objective Function:

Maximize: $Z = 1X_1 + 2X_2 + 3X_3$

Subject to:

$1X_1 + 1X_2 + 1X_3 \leq 7$

$2X_1 + 5X_2 + 3X_3 \geq 10$

$X_1, X_2, X_3 \geq 0$

Objective Function:

Maximize: $Z = 1X_1 + 2X_2 + 3X_3 + 0S_1 + 0S_2 - MA_1$

Subject to:

$1X_1 + 1X_2 + 1X_3 + 1S_1 + 0S_2 + 0A_1 = 7$

$2X_1 + 5X_2 + 3X_3 + 0S_1 - 1S_2 + 1A_1 = 10$

$X_1, X_2, X_3, S_1, S_2, A_1 \geq 0$

Initial Table

Table 1	C_j	1	2	3	0	0	-M	
C_b	Base	X_1	X_2	X_3	S_1	S_2	A_1	R
0	S_1	1	1	1	1	0	0	7
-M	A_1	2	5	3	0	-1	1	10
	Z	-2M-1	-5M-2	-3M-3	0	M	0	-10M

Enter the variable $\mathbf{X_2}$ and the variable $\mathbf{A_1}$ leaves the base. The pivot element is **5**

Iteration 1

Table 2	C_j	1	2	3	0	0	-M	
C_b	Base	X_1	X_2	X_3	S_1	S_2	A_1	R
0	S_1	3/5	0	2/5	1	1/5	-1/5	5
2	X_2	2/5	1	3/5	0	-1/5	1/5	2
	Z	-1/5	0	-9/5	0	-2/5	M+2/5	4

Enter the variable $\mathbf{X_3}$ and the variable $\mathbf{X_2}$ leaves the base. The pivot element is **3/5**

Iteration 2

Table 3	C_j	1	2	3	0	0	-M	
C_b	Base	X_1	X_2	X_3	S_1	S_2	A_1	R
0	S_1	1/3	-2/3	0	1	1/3	-1/3	11/3
3	X_3	2/3	5/3	1	0	-1/3	1/3	10/3
	Z	1	3	0	0	-1	M+1	10

Enter the variable $\mathbf{S_2}$ and the variable $\mathbf{S_1}$ leaves the base. The pivot element is **1/3**

Iteration 3

Table 4	C_j	1	2	3	0	0	-M	
C_b	Base	X_1	X_2	X_3	S_1	S_2	A_1	R
0	S_2	1	-2	0	3	1	-1	11
3	X_3	1	1	1	1	0	0	7
	Z	2	1	0	3	0	M	21

The optimal solution is Z = 21

$X_1= 0, X_2= 0, X_3= 7, S_1= 0, S_2= 11, A_1= 0$

Question 3: (marks: 8)

Determine dual price (the value of objective function) and the feasibility range of the variables from the given optimal tableau:

							Solution			
Basic	x_1	x_2	x_3	x_4	x_5	x_6	RHS	D_1	D_2	D_3
z	4	0	0	1	2	0	1350	1	2	0
x_2	$-\frac{1}{4}$	1	0	$\frac{1}{2}$	$-\frac{1}{4}$	0	100	$\frac{1}{2}$	$-\frac{1}{4}$	0
x_3	$\frac{3}{2}$	0	1	0	$\frac{1}{2}$	0	230	0	$\frac{1}{2}$	0
x_6	2	0	0	-2	1	1	20	-2	1	1

Solution

Dual prices: The value of the objective function can be written as

$$z = 1350 + \mathbf{1}D_1 + \mathbf{2}D_2 + \mathbf{0}D_3$$

Feasibility range: The current solution remains feasible if all the basic variables remain nonnegative—that is,

$$x_2 = 100 + \tfrac{1}{2}D_1 - \tfrac{1}{4}D_2 \geq 0$$
$$x_3 = 230 + \tfrac{1}{2}D_2 \geq 0$$
$$x_6 = 20 - 2D_1 + D_2 + D_3 \geq 0$$

The given conditions can produce the individual *feasibility ranges* associated with changing the resources *one at a time* (as defined in Section 3.6.1). For example, a change in operation 1 time only means that $D_2 = D_3 = 0$. The simultaneous conditions thus reduce to

$$\left.\begin{array}{l} x_2 = 100 + \frac{1}{2}D_1 \geq 0 \Rightarrow D_1 \geq -200 \\ x_3 = 230 > 0 \\ x_6 = 20 - 2D_1 \geq 0 \Rightarrow D_1 \leq 10 \end{array}\right\} \Rightarrow -200 \leq D_1 \leq 10$$

This means that the dual price for operation 1 is valid in the feasibility range $-200 \leq D_1 \leq 10$.

We can show in a similar manner that the feasibility ranges for operations 2 and 3 are $-20 \leq D_2 \leq 400$ and $-20 \leq D_3 \leq \infty$. respectively (verify!).