National University of Computer and Emerging Sciences

Lahore Campus

Operations Research (MT 4031) — **Final Exam**

Date: May 28, 2024

Course Instructor(s)

Dr. Misbah Irshad, Dr. Uzma Bashir, Dr. Shazia Javed, Dr. Ayesha Razzaq, Miss Aisha Rasheed, Mr. Yasir.

Total Time (Hrs.):	3
Total Marks:	70
Total Questions:	6

Roll No _____ Section _____ Student Signature _____

Do not write below this line.

i) Attempt all the questions neatly on the answer sheet.
ii) Solve all the parts of a question together in order.
iii) Don't use a red pen or lead pencil to solve the paper.

Question 1: [8+7]

a. For the following LP, find three alternative optimal basic solutions.

$x_3 = 10/3$

$x_3 = 3$
$x_1 = 1$

$$\text{Max } z = x_1 + 2x_2 + 3x_3$$

subject to

$$x_1 + 2x_2 + 3x_3 \le 10$$
$$x_1 + x_2 \le 5$$
$$x_1 \le 1$$
$$x_1, x_2, x_3 \ge 0.$$

$x_1 = 1$
$x_2 = 4$
x_3 1/3

b. Consider the following LP model:

$$\text{Maximize } z = 4x_1 + 14x_2$$

subject to

$$2x_1 + 7x_2 + x_3 = 21$$
$$7x_1 + 2x_2 + x_4 = 21$$
$$x_i \ge 0, \quad i = 1,2,3,4$$

Construct the entire simplex tableau associated with the following basic variables and check it for optimality and feasibility of the given basic solution,

Basic Variable = (x_2, x_4) and Inverse = $\begin{bmatrix} \frac{1}{7} & 0 \\ \frac{-2}{7} & 1 \end{bmatrix}$

0 0 2 0 42
3
15

Question 2: [7+8]

a. There are five registration counters in a university. Five persons are available for service. The details of the expected number of registered students is given below. How should the counters be assigned to persons to register the maximum number of students.

Counters	Persons				
	A	B	C	D	E
1	30	37	40	28	40
2	40	24	27	21	36
3	40	32	33	30	35
4	25	38	40	36	36
5	29	62	41	34	39

b. In the following transportation problem, the total demand exceeds the total supply. Suppose that the penalty costs per unit of unsatisfied demand are \$2, \$5, and \$3 for destinations 1, 2, and 3, respectively. Find the optimal minimum cost of the problem by using VAM for starting solution.

\$5	\$1	\$7	**10**
\$6	\$4	\$6	**80**
\$3	\$2	\$5	**15**
75	**20**	**50**	

Question 3: [8+4]

a. Solve the following integer linear programming problem using branch and bound algorithm by demonstrating the portioning graphically. Develop B & B tree as well.

$$\text{Maximize } z = 2x_1 + 9x_2$$

subject to

$$2x_1 + x_2 \leq 20$$

$$x_1 + 5x_2 \leq 24$$

$$x_1, x_2 \geq 0 \text{ and integers.}$$

b. A continuous optimal solution for an integer programming problem is given. Find a legitimate cut that will force the basic variable x_2 to take the integer value. Write the resulting solution as well.

Basic	x_1	x_2	x_3	x_4	x_5	x_6	sol
z	550/3	0	0	250/3	200/3	0	23000/3
x_2	1/3	1	0	1/3	-1/3	0	20/3
x_3	5/6	0	1	-1/6	2/3	0	50/3
x_6	-5/3	0	0	-2/3	-1/3	1	80/3

National University of Computer and Emerging Sciences

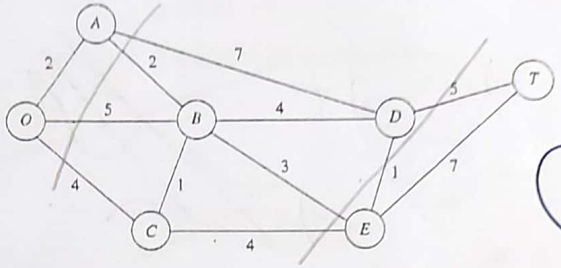
Lahore Campus

Question 4: [8+4]

a.

i. For the given network, enumerate all cuts that will terminate the flow between source node O and sink node T, hence identify the minimum cut.

ii. Find the maximal flow between source and sink nodes and verify that the maximal flow is equal to the minimum cut capacity.



11

b. A delivery company operates in a city with multiple distribution centers. The city has five distribution centers labeled A, B, C, D, and E. The distances between the distribution centers are as follows:

- Distance between A and B: 10 miles.
- Distance between A and C: 15 miles.
- Distance between B and C: 20 miles.
- Distance between B and D: 25 miles.
- Distance between C and D: 30 miles.
- Distance between C and E: 35 miles.
- Distance between D and E: 40 miles.

85

Construct the network and find a tree, a spanning tree and a minimal spanning tree by clearly differentiating the three.

Question 5: [4+4]

Suppose that the equation of a circle is $(x-5)^2 + (y+7)^2 = 49$.

a) Define the corresponding distributions f (x) and f (y), and then show how a sample point (x, y) is determined using the (0, 1) random pair (R1, R2).

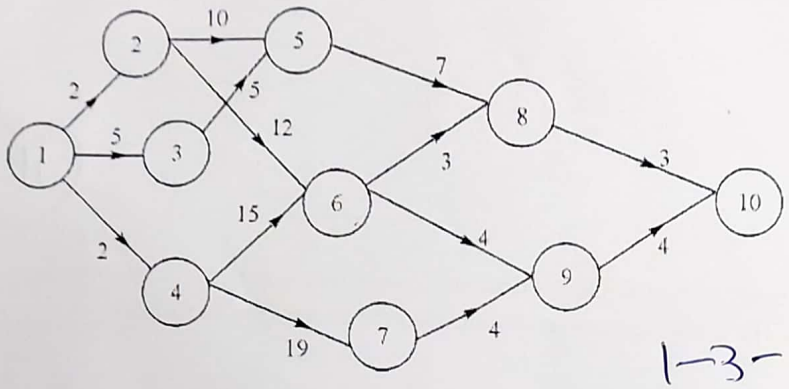
b) Estimate area of the circle for the following random pairs

R1: .0589 .3529 .5869 .7455 .7900 .6307

R2: .6733 .3646 .1281 .4871 .3698 .2346

Question 6: [8]

Mr. Ali, a sales manager, has decided to travel from city 1 to city 10. He wants to plan for a minimum distance program and visit maximum number of branch offices as possible on the route but no restrictions to visit all the offices. The route map of the various ways of reaching city 10 from city 1 is shown below. The number on the arrow indicates the distance in km. (× 100). **Using dynamic programing suggest a feasible minimum distance path plan to Mr. Ali.**



1-3-5-8-10

20