Roll No. ....

OLE-3007

B. Tech. 1st Semester (ME)
Examination – April, 2021

MATH - I (CALCULUS AND MATRICES)

Paper : BSC-MATH-101-G

Time : Three Hours] ***[Maximum Marks : 75***

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** questions in total by selecting ***one*** from each Unit. Question No. **1** is ***compulsory***.

1. (a) State mean value theorem.

(b) Give relation between Beta and Gamma function.

(c) Test the convergence of $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \sin \frac{1}{n}$.

(d) If $u = e^{xyz}$, find the value of $\frac{\partial^3 u}{\partial x \, \partial y \, \partial z}$.

(e) Define rank of matrix.

(f) Define orthogonal matrix.

UNIT – I

2. (a) Evaluate $\lim\limits_{x \to 0} \dfrac{xe^x - \log(1+x)}{x^2}$.

(b) Find the Maclaurin's theorem with Lagrange's form of remainder for $f(x) = \cos x$.

3. (a) Find the evolute of the curve $x = a\cos^3\theta, y = a\sin^3\theta$.

(b) Find the volume formed by the revolution of loop of the curve $y^2(a+x) = x^2(3a-x)$, about the x-axis.

UNIT – II

4. (a) Test the Convergence of the series

$$\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\infty.$$

(b) Expand $\log_e x$ in powers of $(x-1)$.

5. Expand $f(x) = x$ as half range sine and cosine series in $0 < x < 2$.

UNIT – III

6. (a) Find the points on the surface $z^2 = xy + 1$ nearest to the origin.

(b) If $z = \tan(y + ax) - (y - ax)^{\frac{3}{2}}$, show that $\frac{\partial^2 z}{\partial x^2} = a^2 \frac{\partial^2 z}{\partial y^2}$.

7. (a) Find a unit vector normal to the surface $xy^3z^2 = 4$ at the point (–1, –1, 2).

(b) Find the value of a if the vector $(ax^2y + yz)i + (xy^2 - xz^2)j + (2xyz - 2x^2y^2)k$ has zero divergence.

UNIT – IV

8. (a) Reduce the following matrix into its normal form and hence find its rank :

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

(b) Test for consistency and solve $2x - 3y + 7z = 5$, $3x + y - 3z = 13$, $2x + 19y - 47z = 32$

9. Find the eigen values, eigen vectors and verify Cayley-Hamilton theorem of the matrix :

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$