From linear to bilinear, and beyond

Shu Kong

CS, ICS, UCI

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Outline

1. Bilinear CNN Models for Fine-grained Visual Recognition
2. At the very beginning of bi-linear idea
3. Bilinear SVM
4. Back to Bilinear CNN
5. Beyond bilinear
6. More?

Bilinear CNN Models for Fine-grained Visual Recognition

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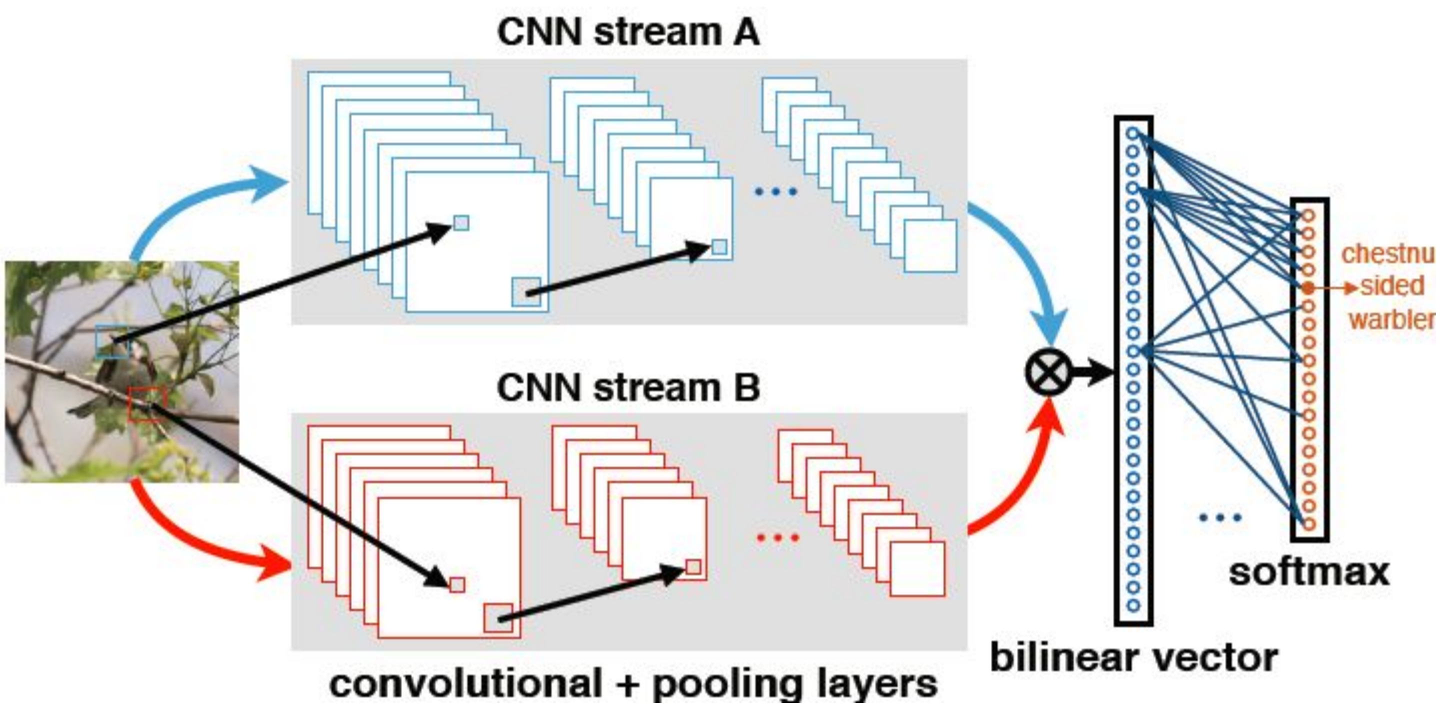

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2. It generalizes various orderless texture descriptors, such as Fisher Vector [1], VLAD [2], O2P [3].
3. It allows end-to-end training using image labels only, and achieves state-of-the-art performance on fine-grained classification.

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Cognitively…

Two streams

1. dorsal stream -- where pathway
2. ventral stream-- what pathway

Wikipediacally...

Bilinear as below

$$B(x, y) = x^{\mathrm{T}} A y = \sum_{i,j=1}^{n} a_{ij} x_i y_j.$$

function is linear w.r.t one variable when fixing the other

Mathematically...

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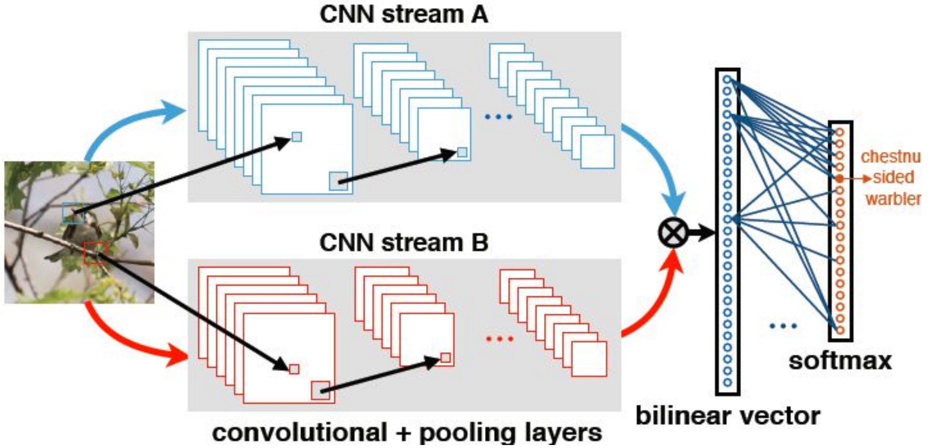

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By chain rule, we have

$$\frac{d\ell}{dA} = B\left(\frac{d\ell}{d\mathbf{x}}\right)^T, \quad \frac{d\ell}{dB} = A\left(\frac{d\ell}{d\mathbf{x}}\right)$$

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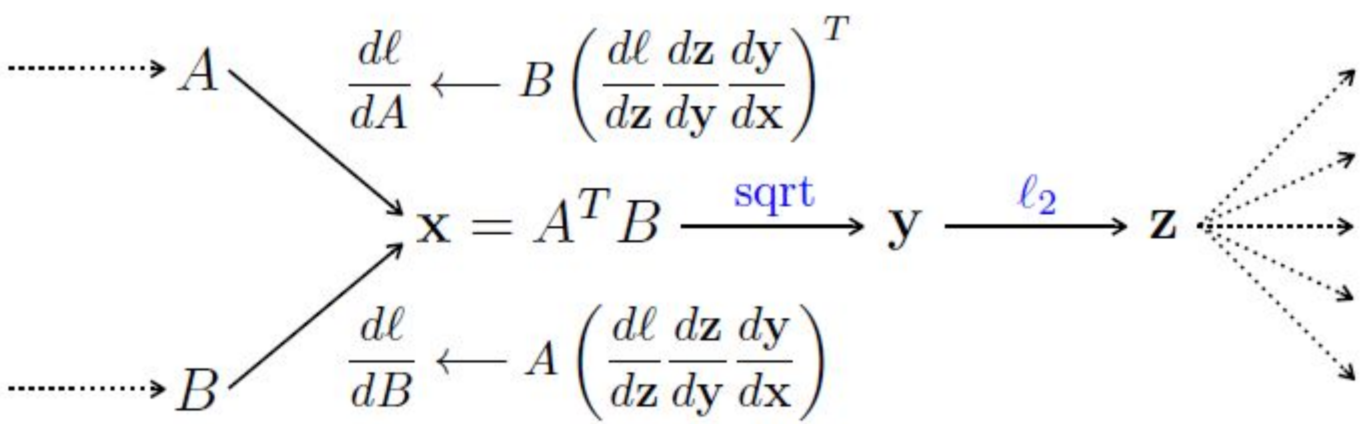

Experimentally...

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Bilinear pooling over the output of last convolutional layer

Finetune with softmax

Learn linear SVM as the classifier (Why?)

Experimentally...

Data augmentation -- left-right flipping

method	birds		birds + box		aircrafts		cars		FPS
	w/o ft	w/ ft	w/o ft	w/ ft	w/o ft	w/ ft	w/o ft	w/ ft	
FV-SIFT	18.8	-	22.4	-	61.0	-	59.2	-	10†
FC-CNN [M]	52.7	58.8	58.0	65.7	44.4	57.3	37.3	58.6	124
FC-CNN [D]	61.0	70.4	65.3	76.4	45.0	74.1	36.5	79.8	43
FV-CNN [M]	61.1	64.1	67.2	69.6	64.3	70.1	70.8	77.2	23
FV-CNN [D]	71.3	74.7	74.4	77.5	70.4	77.6	75.2	85.7	8
B-CNN [M,M]	72.0	78.1	74.2	80.4	72.7	77.9	77.8	86.5	87
B-CNN [D,M]	80.1	**84.1**	81.3	**85.1**	78.4	83.9	83.9	**91.3**	8
B-CNN [D,D]	80.1	84.0	80.1	84.8	76.8	**84.1**	82.9	90.6	10
Previous work	**84.1** [19], 82.0 [21] 73.9 [38], 75.7 [2]		82.8 [21], 73.5 [24] 73.0 [7], 76.4 [38]		72.5 [4], 80.7 [16]		**92.6** [21], 82.7 [16] 78.0 [4]		†on a cpu

method	birds mean per-class acc.		birds + box mean per-class acc.		aircrafts mean per-class acc.		cars fraction of correct	
	w/o ft	w/ ft	w/o ft	w/ ft	w/o ft	w/ ft	w/o ft	w/ ft
FV-SIFT	12.8	-	24.1	-	55.7	-	51.2	-
FC-CNN (M)	46.1	55.6	56.5	64.0	41.3	50.4	33.5	50.9
FC-CNN (D)	54.6	64.9	63.0	71.4	40.7	57.8	32.0	67.7
FV-CNN (M)	50.8	56.2	63.4	67.1	58.7	64.5	65.5	68.9
FV-CNN (D)	62.7	68.7	70.5	73.9	67.5	71.2	70.2	79.2
B-CNN (M,M)	66.6	72.5	70.7	77.2	67.9	73.5	73.9	82.3
B-CNN (D,M)	75.1	**80.9**	77.9	**81.9**	73.3	79.4	81.2	**88.2**
State-of-the-art	66.7 [7], 73.9 [35], 75.7 [2]		73.0 [7], 73.5 [21], 76.4 [35]		72.5 [4], **80.7** [16]		78.0 [4], 82.7 [16]	

Diagnostically...

B-CNN(M,D)

normalization	accuracy	mAP
square-root + ℓ_2	80.1	81.3
square-root only	79.4	77.9
ℓ_2 only	77.3	79.6
none	74.7	70.9

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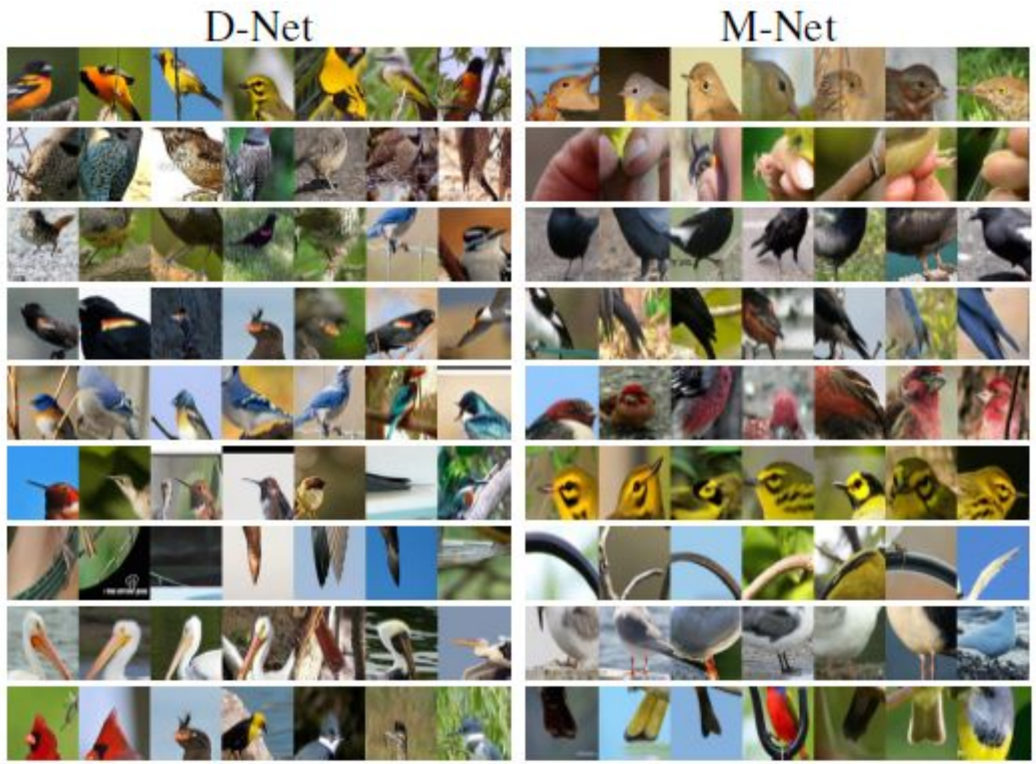

At the Very Beginning of Bilinear

Two real-world problems (content and style)

1. JB Tenenbaum, W. T. Freeman, Separating Style and Content, NIPS, 1997
2. W. T. Freeman, JB Tenenbaum, Learning bilinear models for two-factor problems in vision, CVPR, 1997

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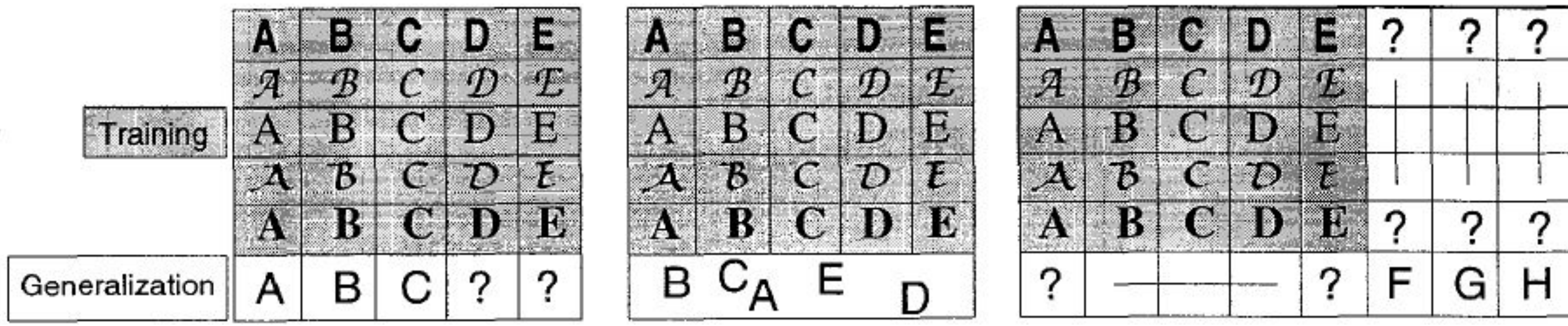
1. letters with different fonts



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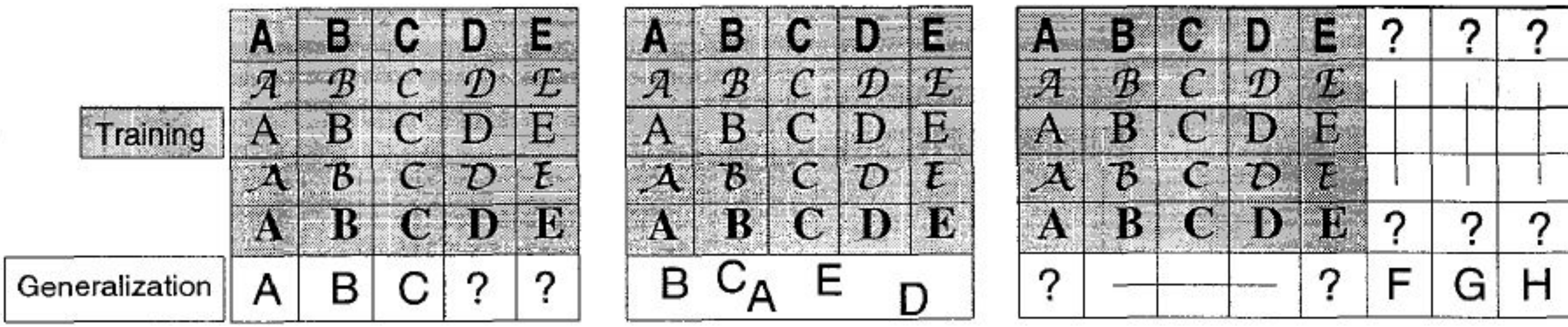
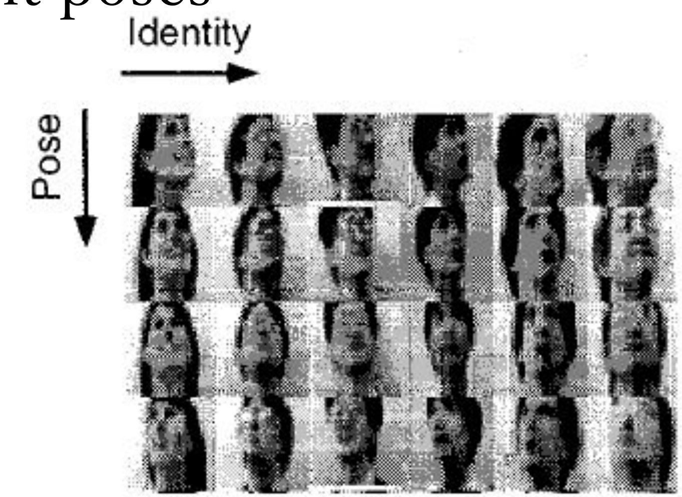


2. individual face images with different poses



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Original Bilinear Model

Observation $\mathbf{y}^{sc}$ can be characterized by content code $\mathbf{b}^c$ and style code $\mathbf{a}^s$, parameters W are independent of content and style but govern their interactions

Mathematically, the k-th element in observation $\mathbf{y}^{sc}$

$$y_k^{sc} = \mathbf{a}^{s^{\mathrm{T}}} \mathbf{W}_k \mathbf{b}^c = \sum_{ij} a_i^s b_j^c W_{ijk}$$

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Or…

$$\mathbf{y}^{sc} = \mathbf{A}^s \mathbf{b}^c \qquad A_{jk}^s \equiv \sum_i a_i^s W_{ijk}^s$$

$$\mathbf{y}^{sc} = \mathbf{B}^c \mathbf{a}^s$$

Training

By minimizing the least square fitting, the parameters can be learned iteratively.

$$\mathbf{y}^{sc} = \mathbf{A}^{s}\mathbf{b}^{c} \qquad A^{s}_{jk} \equiv \sum_{i} a^{s}_{i} W^{s}_{ijk}$$

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With learned codes, W can also be learned.

Bilinear SVM

Motivation -- image region can be naturally represented by a matrix, which is a 2D data, then why do people vectorize the matrix to train a linear SVM?

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Generalize predictor from vector to matrix, and consider low-rank constraint to reduce the degrees of freedom in the matrix W

$$f_w(x) = w^T x$$

$$f_W(X) = \mathrm{Tr}(W^T X)$$

$$f_{W_y,W_x}(X) = \mathrm{Tr}(W_x W_y^T X) = \mathrm{Tr}(W_y^T X W_x) \qquad W = W_y W_x^T$$

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from linear SVM to bilinear SVM

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Focusing on W_y

$$\min_{\tilde{W}_y} L(\tilde{W}_y, W_x) = \frac{1}{2}\operatorname{Tr}(\tilde{W}_y^T \tilde{W}_y) + C\sum_n \max(0, 1 - y_n \operatorname{Tr}(\tilde{W}_y^T \tilde{X}_n))$$

where $\quad \tilde{W}_y = W_y A^{\frac{1}{2}} \quad$ and $\quad \tilde{X}_n = X_n W_x A^{-\frac{1}{2}} \quad$ and $\quad A = W_x^T W_x.$

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Is bilinear CNN easy to be extended to multilinear? Two streams/nets work better than a single net? How about three nets? Tensor product?

Can we see it as a new way to fuse features from two sources?

Beyond Bilinear -- Compact

Compact Bilinear Pooling -- reducing the dimensionality of outer-product features

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Recall bilinear pooling resulting into a c^2 length vector

$$B(\mathcal{X}) = \sum_{s \in \mathcal{S}} x_s x_s^T \qquad \mathcal{X} = (x_1, \ldots, x_{|\mathcal{S}|}, x_s \in \mathbb{R}^c)$$

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linear kernel

$$\begin{aligned} \langle B(\mathcal{X}), B(\mathcal{Y}) \rangle &= \langle \sum_{s \in \mathcal{S}} x_s x_s^T, \sum_{u \in \mathcal{U}} y_u y_u^T \rangle \\ &= \sum_{s \in \mathcal{S}} \sum_{u \in \mathcal{U}} \langle x_s x_s^T, y_u y_u^T \rangle \\ &= \sum_{s \in \mathcal{S}} \sum_{u \in \mathcal{U}} \langle x_s, y_u \rangle^2 \end{aligned}$$

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If a low-dimension feature $\phi(x) \in R^d$ can be found ($d << c^2$) to approximate $\langle \phi(x), \phi(y) \rangle \approx \langle x, y \rangle^2$

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If a low-dimension feature $\phi(x) \in R^d$ can be found ($d << c^2$) to approximate $\langle \phi(x), \phi(y) \rangle \approx \langle x, y \rangle^2$ (second-order pooling approx.)

then...

$$\begin{aligned} \langle B(\mathcal{X}), B(\mathcal{Y}) \rangle &= \langle \sum_{s \in \mathcal{S}} x_s x_s^T, \sum_{u \in \mathcal{U}} y_u y_u^T \rangle \\ &= \sum_{s \in \mathcal{S}} \sum_{u \in \mathcal{U}} \langle x_s x_s^T, y_u y_u^T \rangle \\ &= \sum_{s \in \mathcal{S}} \sum_{u \in \mathcal{U}} \langle x_s, y_u \rangle^2 \end{aligned}$$

$$\Longrightarrow$$

$$\begin{aligned} \langle B(\mathcal{X}), B(\mathcal{Y}) \rangle &= \sum_{s \in \mathcal{S}} \sum_{u \in \mathcal{U}} \langle x_s, y_u \rangle^2 \\ &\approx \sum_{s \in \mathcal{S}} \sum_{u \in \mathcal{U}} \langle \phi(x), \phi(y) \rangle \\ &= \langle \sum_{s \in \mathcal{S}} \phi(x_s), \sum_{u \in \mathcal{U}} \phi(y_u) \rangle \\ &\equiv \langle C(\mathcal{X}), C(\mathcal{Y}) \rangle, \end{aligned}$$

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Beyond Bilinear -- Compact

Then the compact bilinear pooling is

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1. P. Kar and H. Karnick. Random feature maps for dot product kernels, AISTATS 2012
2. N. Pham and R. Pagh. Fast and scalable polynomial kernels via explicit feature maps, SIGKDD 2013

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Two approximations are presented:

1. Random Maclaurin
2. Tensor Sketch

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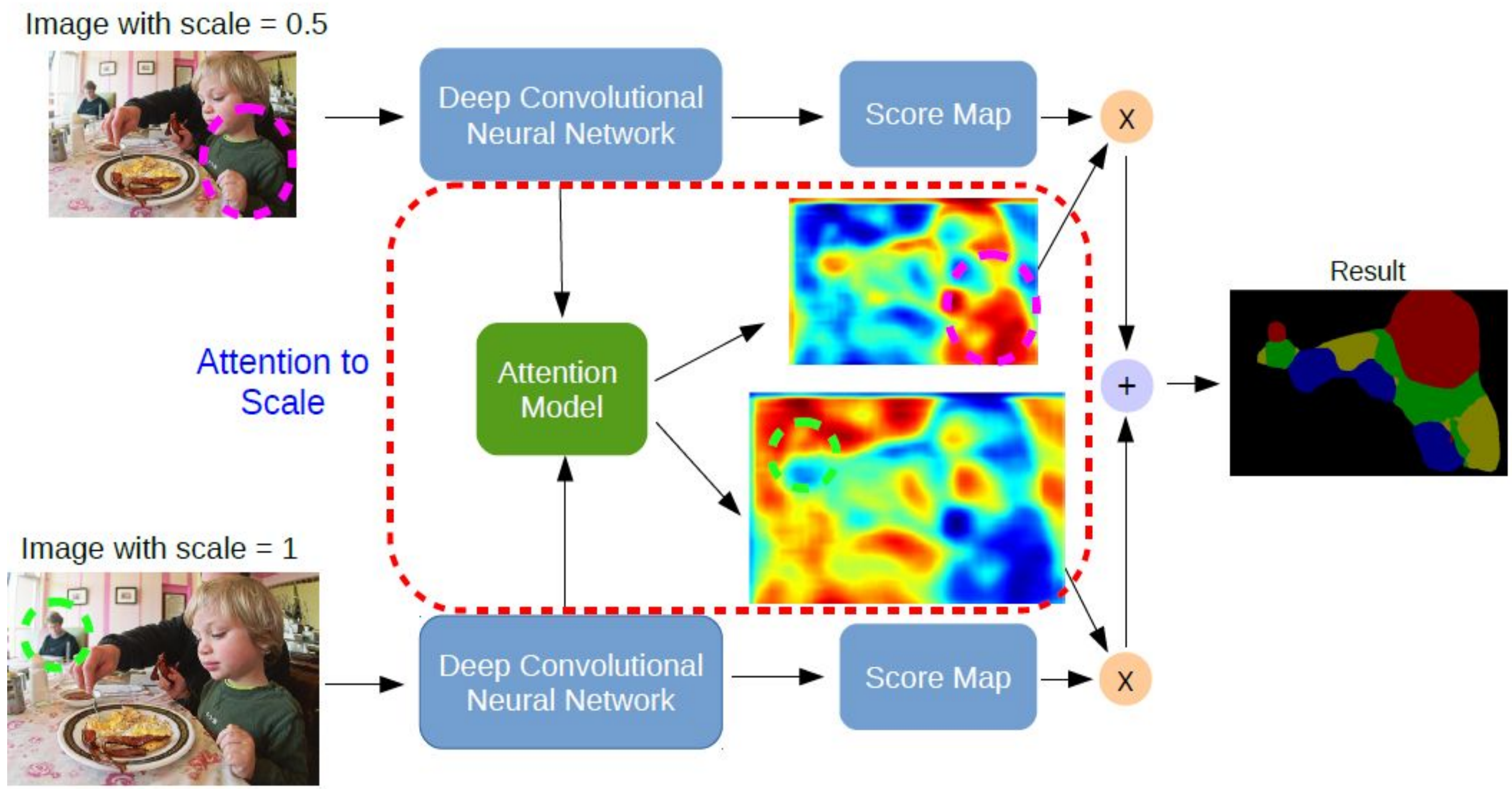
Beyond Bilinear -- Multiplicative Pooling

Motivation -- consider multiple scales, and softly weight features from different input scales when predicting the semantic label of a pixel.



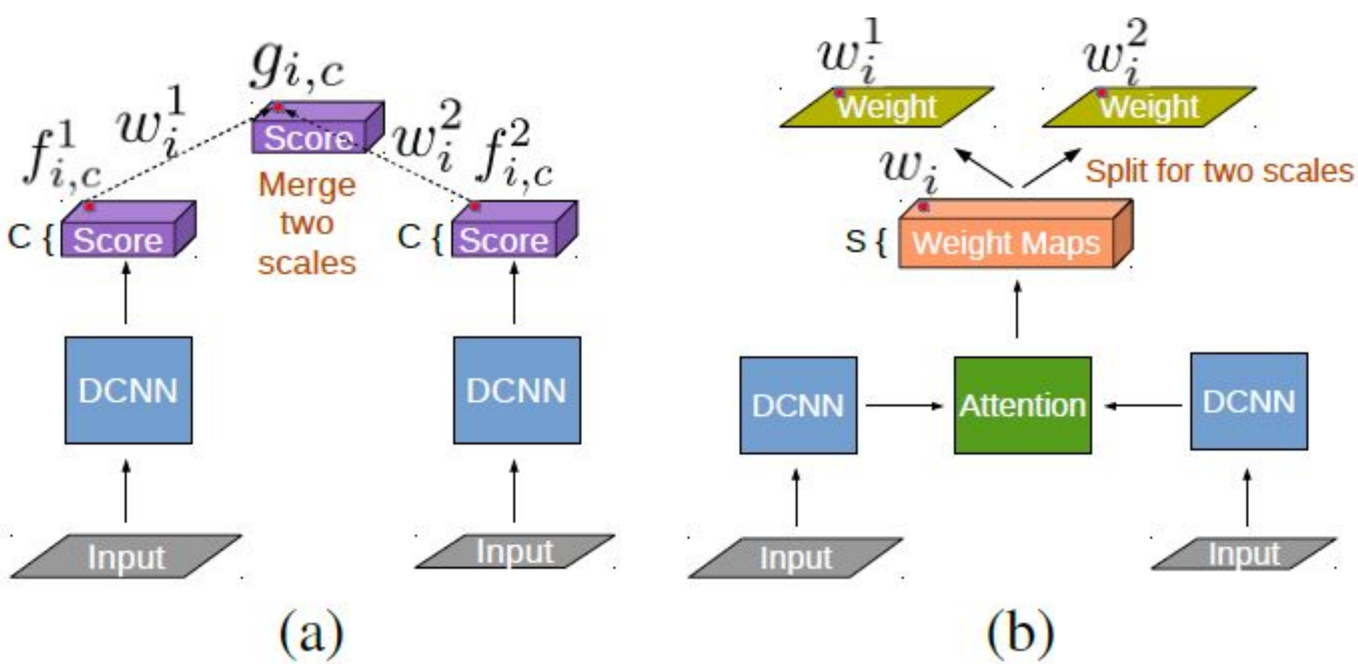
1. L-C Chen, Y Yang, J. Wang, W. Xu, A. Yuille, Attention to Scale: Scale-aware Semantic Image Segmentation, arxiv, 2015

Beyond Bilinear -- Multiplicative Pooling

Look into each pixel:

$f_{i,c}^{s}$ "i" index the spatial location of this pixel, at scale "s", for class-"c"

$$g_{i,c} = \sum_{s=1}^{S} w_i^s \cdot f_{i,c}^s \qquad w_i^s = \frac{\exp(h_i^s)}{\sum_{t=1}^{S} \exp(h_i^t)}$$



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More to think

A trend is emerging -- bilinear-, multilinear- and multiplicative-operation allow models to produce instance-adaptive features and weights in specific problems.

More to think

A trend is emerging -- bilinear-, multilinear- and multiplicative-operation allow models to produce instance-adaptive features and weights in specific problems.

Some typical problems -

1. detection and recognition
2. style, content, and photo aesthetics
3. human pose, human size, human detection
4. face viewpoint, keypoint, face detection
5. ......

Conclusion

1. Bilinear CNN Model
2. At the very beginning of bi-linear idea
3. Bilinear SVM
4. Back to Bilinear CNN
5. Beyond bilinear
6. More!

Thanks

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