

# Probabilistic Caching Policy for Categorized Contents and Consecutive User Demands

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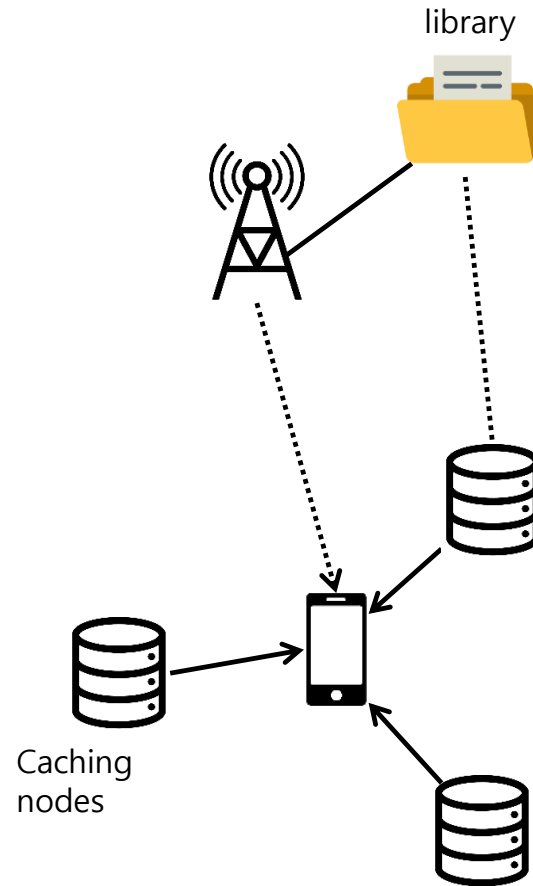
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# Outline

- Basics and motivation
- Content request model
- Network model
- Successful delivery rate for consecutive user demands
- Optimal probabilistic caching policy
- Numerical results

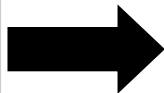
# Basics of Wireless Caching Network

- On-demand streaming service
  - A relatively **small number of popular contents** are requested at ultra high rates.
  - **Playback delay** is often the more important measure of goodness to the user than other usual performance metrics like **video quality**.
- **Wireless caching network** has considered as a promising technique.
- Wireless caching network
  - Bringing popular contents closer to users
  - Independent storages in caching nodes
- How to cache the content?



## Previous:

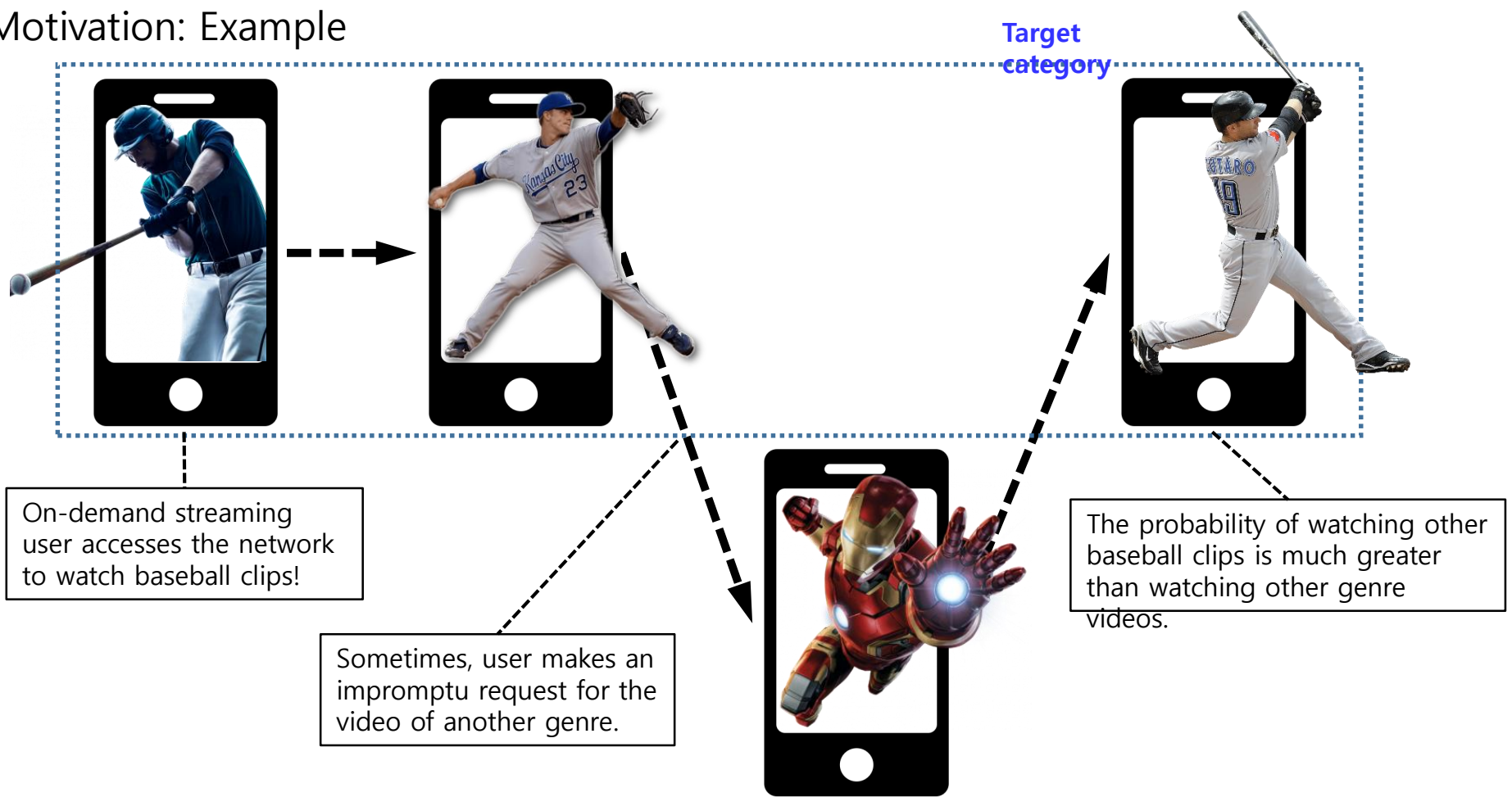
- Independent content requests
- Independent content popularity
- Caching policy optimized for one-shot request



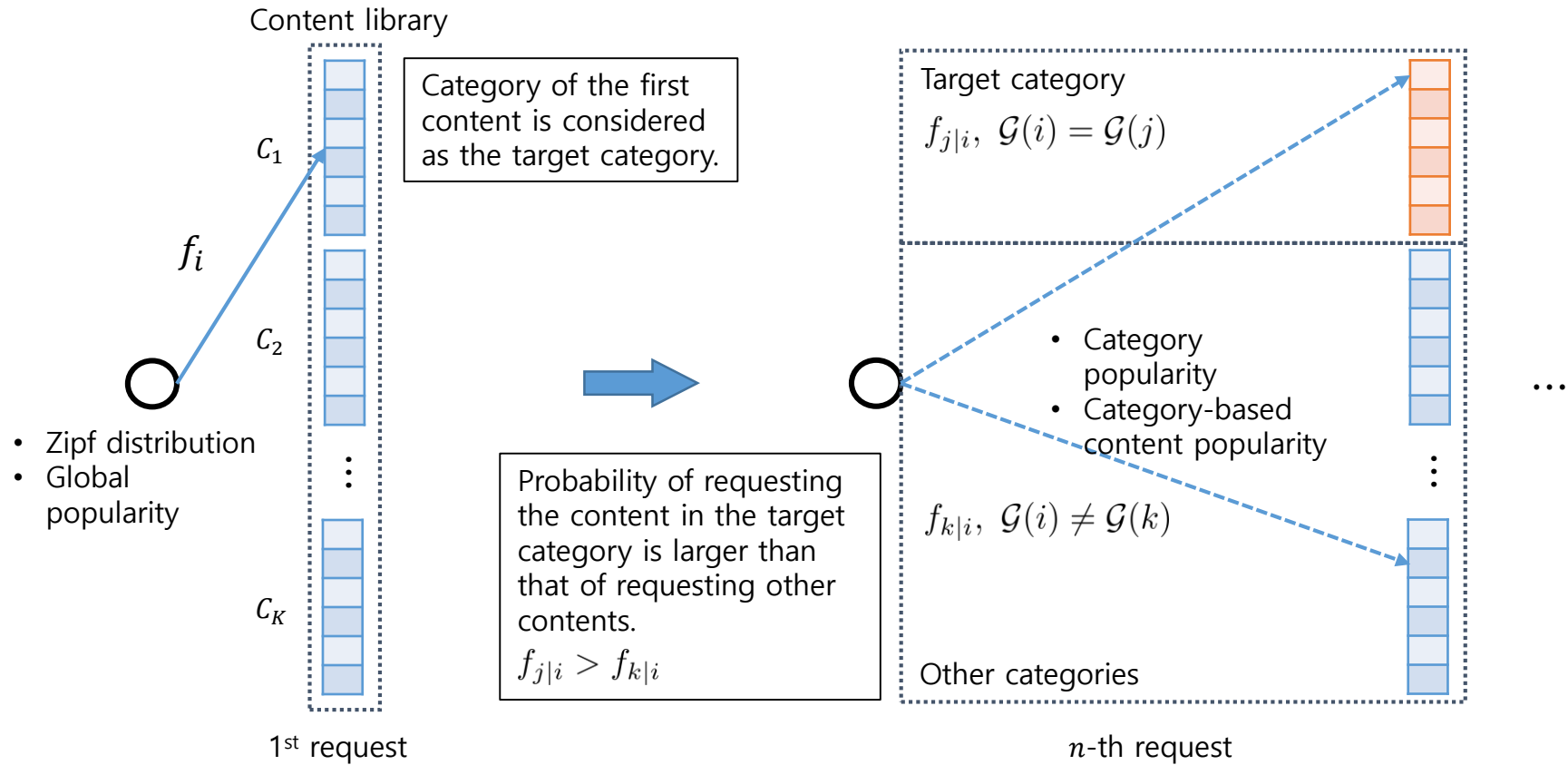
## Our paper:

- High correlation among multiple consecutive content requests
- Categorized contents
- Caching policy optimized for consecutive requests of categorized contents

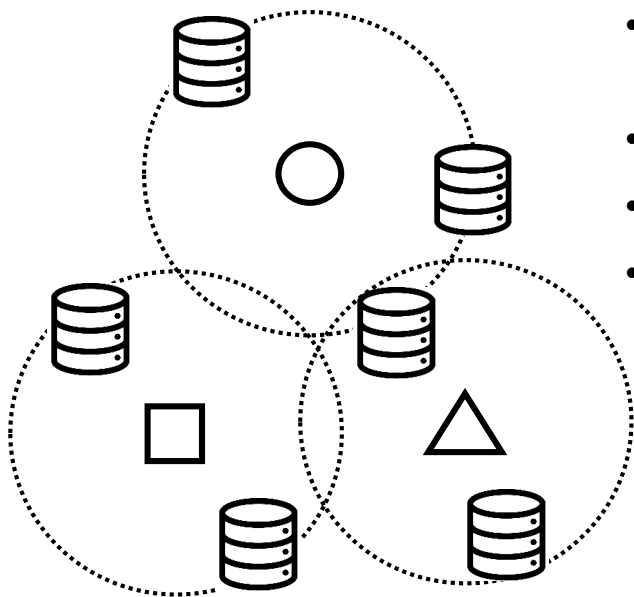
# Motivation: Example



# Content Request Model



# Network Model and Successful Delivery Rate



○ Type-1;  $L=1$

□ Type-2;  $L=2$

△ Type-3;  $L=3$

- Caching nodes are distributed by the independent Poisson point process with intensity of  $\lambda$ .
- The Rayleigh fading channel:  $h = \sqrt{D}g$  where  $D = 1/d^\alpha$
- A type- $l$  user watches consecutively  $l$  videos.
- The average successful delivery rate of type- $L$  user:

Data rate for delivering file  $i$  from the nearest helper which stores file  $i$ .

$$P_L^s = \sum_{i_1} \cdots \sum_{i_L} f_{i_1} f_{i_2|i_1} \cdots f_{i_L|i_1} \cdot \prod_{i=i_1}^{i_L} \Pr\{\tilde{R}_i \geq \rho_L\}$$

$$= \sum_{i_1} \cdots \sum_{i_l} f_{i_1} f_{i_2|i_1} \cdots f_{i_L|i_1} \cdot \prod_{i=i_1}^{i_L} (1 - e^{-Cp_i})$$

Popularity of consecutively consumed videos given file  $i_1$  requested at first.

No repeated request is assumed, i.e.,  $i_l \neq i_l, \forall l \in \{1, \dots, L-1\}$ .

# Optimization Problem Formulation

- Optimization problem to find probabilistic caching policy

$$\mathbf{p}^* = \arg \max_{p_i, i=1, \dots, F} \left[ \min \{P_1^s, \dots, P_L^s\} \right] \quad \text{s.t.} \quad \sum_{i=1}^N p_i \leq M$$
$$0 \leq p_i \leq 1$$

## Lemma 1.

$P_l^o > P_m^o$  for any  $l, m \in \{1, \dots, L\}$   
and  $l < m$



## Lemma 2.

The optimum vector  $\mathbf{p}^* = (p_1^*, \dots, p_F^*)^T$  satisfies  $\sum_{i=1}^F p_i^* = M$ .

$$\mathbf{p}^* = \arg \max_{p_i, i=1, \dots, F} P_L^s \quad \text{s.t.} \quad \sum_{i=1}^N p_i = M$$
$$0 \leq p_i \leq 1$$

- How to solve?



# Dual-Variable Subproblem

- Optimization problem with auxiliary variable
  - Given  $\{p_i\}_{i \neq n,m}$ , optimize  $p_m$  and  $p_n$ .
  - The average successful delivery rate of type- $L$  user:

The case in which the user requests both  $i_m$  and  $i_n$ .

The case in which the user requests  $i_n$  but  $i_m$ .

$$P_L^s = \overbrace{a_{m,n}(1 - e^{-Cp_m})(1 - e^{-Cp_n})}^{\text{The case in which the user requests both } i_m \text{ and } i_n.} + \overbrace{b_{m,n}(1 - e^{-Cp_m}) + d_{m,n}(1 - e^{-Cp_n})}^{\text{The case in which the user requests } i_n \text{ but } i_m.} + e_{m,n}$$

The case in which the user requests  $i_m$  but  $i_n$ .

The case in which the user does not request both files  $i_m$  and  $i_n$ .

- Dual-variable subproblem

$$\{p_m^*, p_n^*\} = \arg \min_{p_m, p_n} \mathcal{M}_{(p_m, p_n)} \quad \text{s.t.} \quad p_m + p_n = q_{m,n} = M - \sum_{i=1, i \neq m, n}^F p_i$$

$$\text{where } \mathcal{M}_{(p_m, p_n)} = x_{m,n}e^{-C \cdot p_m} + y_{m,n}e^{-C \cdot p_n} \quad 0 \leq p_m, p_n \leq 1$$

## Solution of Dual-Variable Subproblem

- According to arithmetic-geometric mean inequality,

$$x_{m,n}e^{-C \cdot p_m} + y_{m,n}e^{-C \cdot p_n} \geq 2\sqrt{x_{m,n}y_{m,n}e^{-C \cdot q_{m,n}}}$$

With equality if and only if  $x_{m,n}e^{-C \cdot p_m} = y_{m,n}e^{-C \cdot p_n} \iff$

$$\begin{aligned}\tilde{p}_m &= \frac{1}{2C} \log \frac{x_{m,n}}{y_{m,n}} + \frac{1}{2}q_{m,n} \\ \tilde{p}_n &= \frac{1}{2C} \log \frac{y_{m,n}}{x_{m,n}} + \frac{1}{2}q_{m,n}\end{aligned}$$

- Proposition 1.**

The optimal solution of the dual-variable subproblem is as follows:

$$\{p_m^*, p_n^*\} = \begin{cases} \{\tilde{p}_m, \tilde{p}_n\} & \text{if } 0 \leq \tilde{p}_m, \tilde{p}_n \leq 1 \\ \arg \min_{\{p_m, p_n\}} \{\mathcal{M}_{(0, q_{m,n})}, \mathcal{M}_{(q_{m,n}, 0)}\} & \text{elseif } q_{m,n} < 1 \\ \arg \min_{\{p_m, p_n\}} \{\mathcal{M}_{(1, q_{m,n}-1)}, \mathcal{M}_{(q_{m,n}-1, 1)}\} & \text{elseif } q_{m,n} \geq 1 \end{cases}$$

## Iterative Algorithm

- To find the optimum  $\mathbf{p}$ , iteration should be applied with respect to  $\forall (m, n) \in \{1, \dots, F\} \times \{1, \dots, F\}, m \neq n$
- Convergence
  - The updated objective sequence  $\{\mathcal{M}_{(p_m, p_n)}\}$  is nondecreasing.
  - The original objective function is upper bounded, i.e.  $P_L^s \leq 1$ .

# Numerical Results - Parameters

- Category structure

$\mathcal{G}_1$	$\mathcal{G}_2$	$\mathcal{G}_3$	$\mathcal{G}_4$	$\mathcal{G}_5$
1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

- Popularity model

- 1<sup>st</sup> content request (Zipf)

$$f_i = \frac{i^{-\gamma}}{\sum_{j=1}^F j^{-\gamma}}, \quad f_i > f_j \quad \forall i < j$$

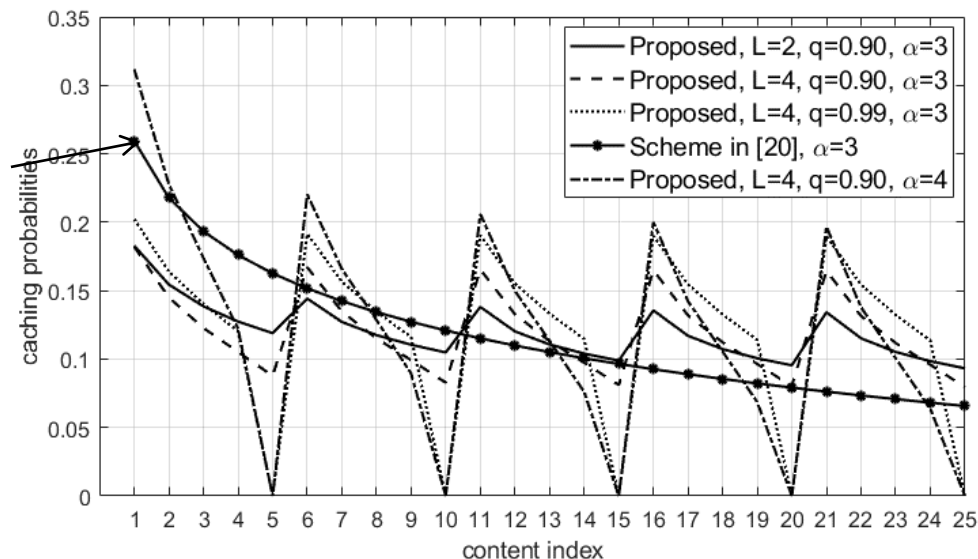
- Consecutive content requests (Uniform)

$$f_{j|i} = \begin{cases} \frac{q}{(N - \mathcal{N}(\mathcal{G}(i)))} & \text{if } \mathcal{G}(j) = \mathcal{G}(i) \\ \frac{(1-q)}{(F - N - \mathcal{N}(\bigcup_{n \neq i} \mathcal{G}(n)))} & \text{if } \mathcal{G}(j) \neq \mathcal{G}(i) \end{cases}$$

- Maximal number of types:  $L = 4$
- Cache size:  $M = 3$
- Probability of requesting the preferred category:  $q = 0.9$
- The intensity of caching node distribution:  $\lambda = 0.2$

# Numerical Results – Caching Probabilities

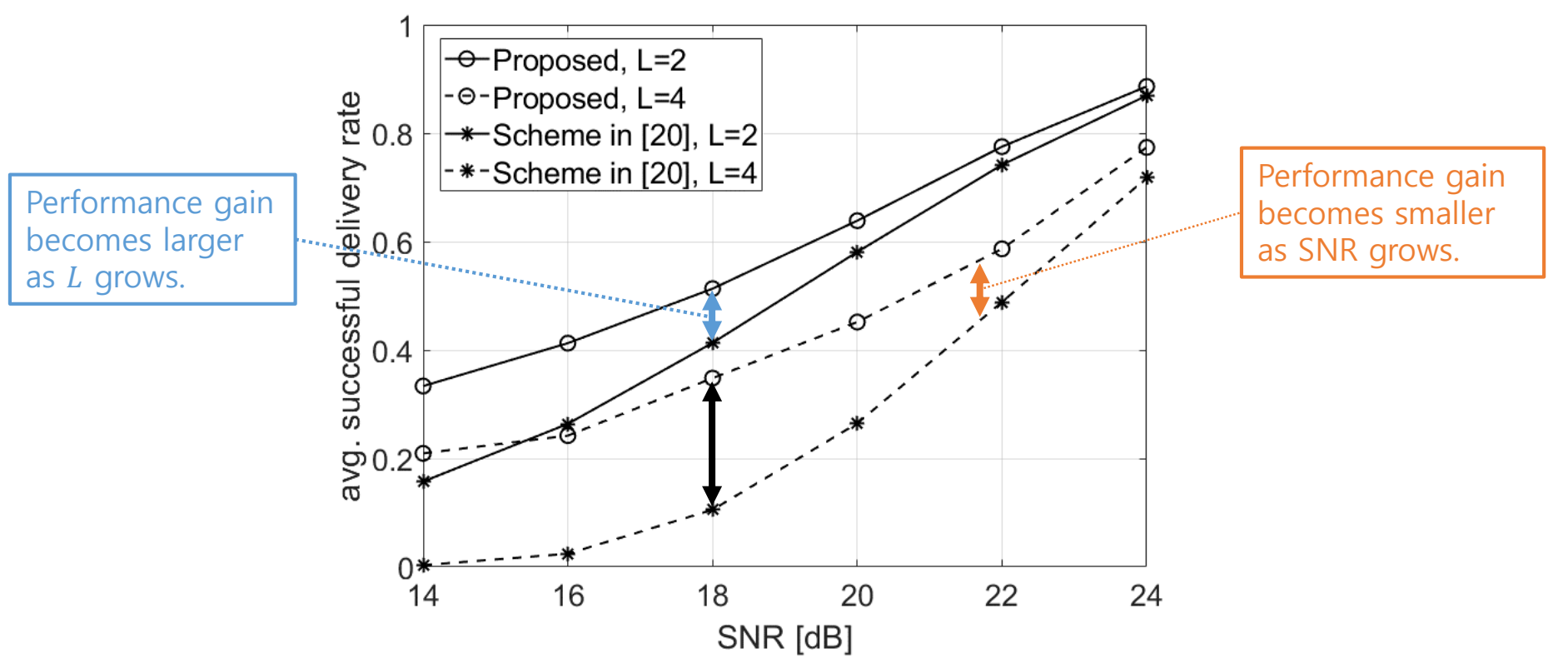
Optimized for  
one-shot request



$L$ : consecutive content request  
number  
 $q$ : probability of requesting contents  
in the target category  
 $\alpha$ : pathloss exponent

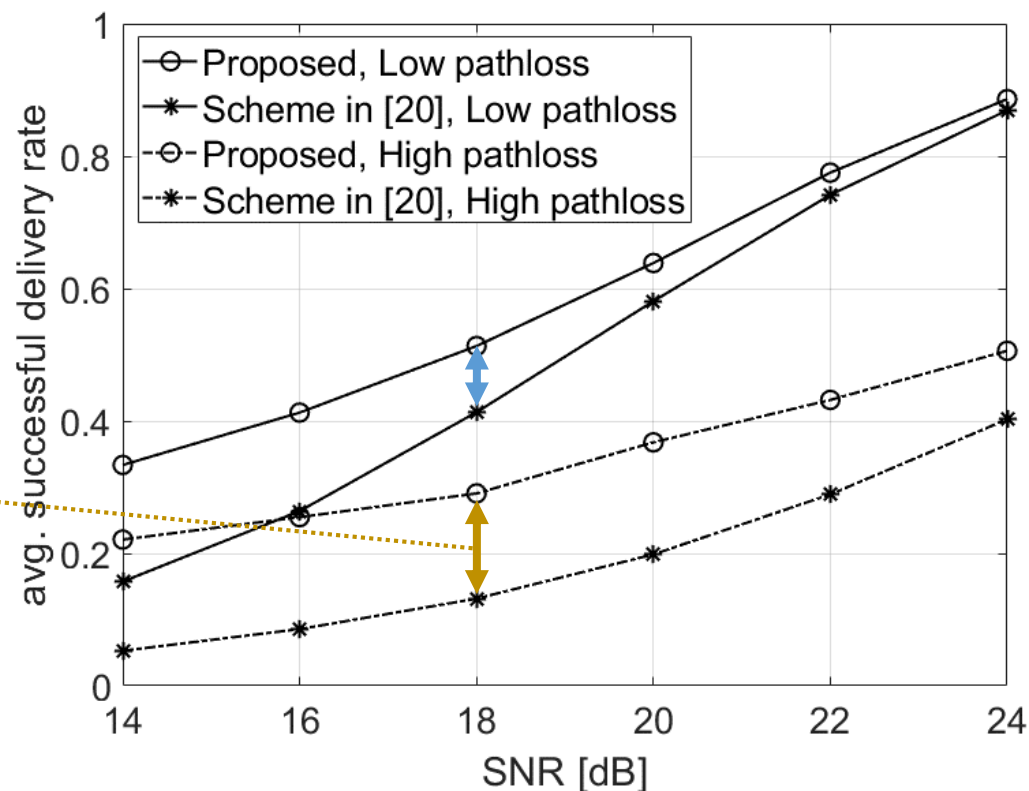
- Every peak point represents the most popular file in each category  
→ Their differences result from global content popularity
- As  $q$  or  $L$  or  $\alpha$  grows, every peak becomes sharper.  
→ Category-based content popularity becomes stronger than global content popularity.

# Numerical Results – Avg. Successful Delivery Rates (1)



➔ The proposed scheme outperforms the caching policy optimized for one-shot requests

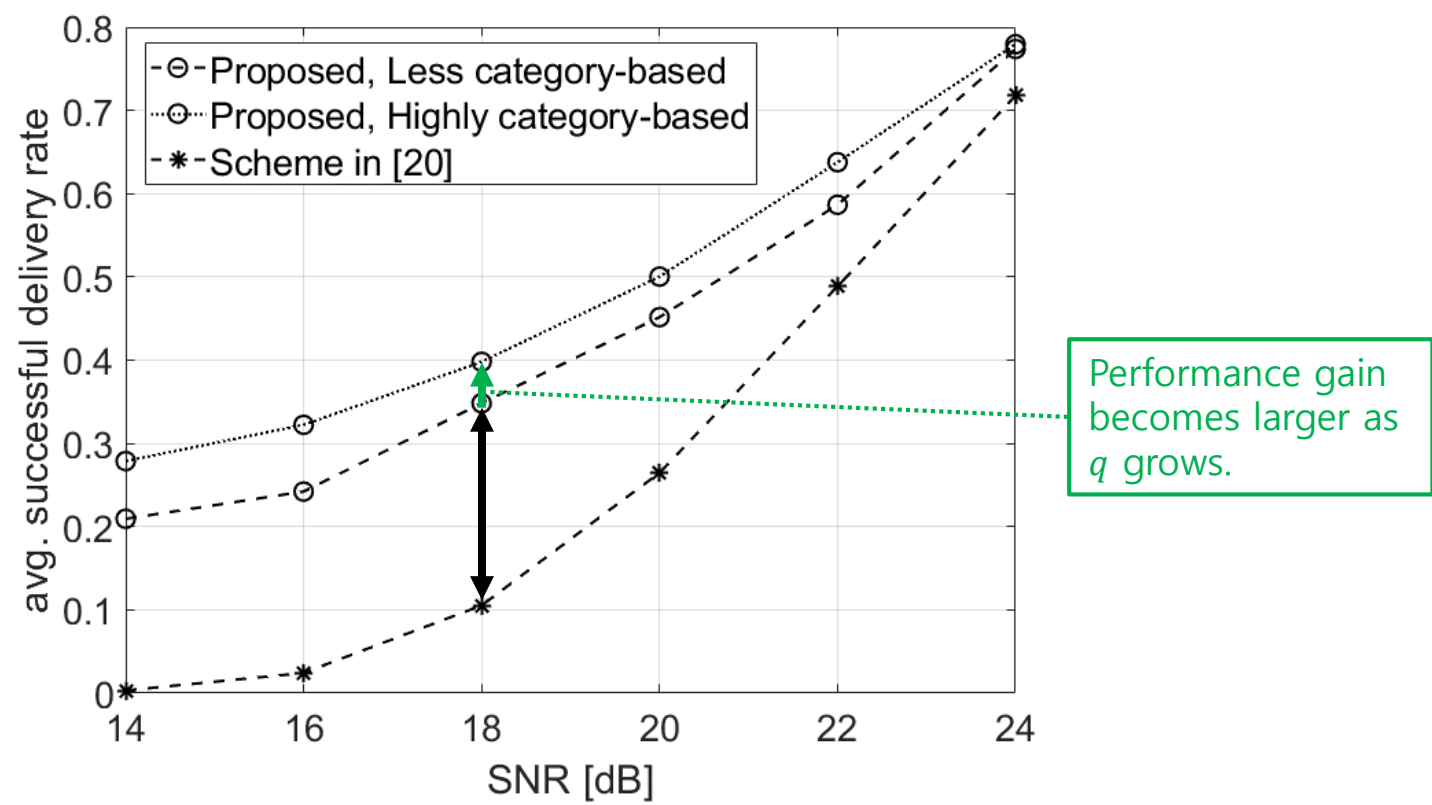
## Numerical Results – Avg. Successful Delivery Rates (2)



Performance gain becomes larger as  $\alpha$  grows.

➔ The proposed scheme outperforms the caching policy optimized for one-shot requests

# Numerical Results – Avg. Successful Delivery Rates (3)



➔ The proposed scheme outperforms the caching policy optimized for one-shot requests



## Conclusion

- The probabilistic caching policy that maximizes the minimum average successful delivery rates among all users is proposed when users request different numbers of categorized contents.
- The proposed scheme captures the essential characteristics of video delivery:
  - Contents in the same category have higher relevance.
  - Different users demand different content consumption.
- The iterative algorithm optimizing the dual-variable subproblems can solve the original optimization problem with the guarantee of convergence.
- The impacts of categorized contents and consecutive user demands on the caching policy are clearly shown by numerical results.