4.6 INDEPENDENCE, CONDITIONAL PROBABILITY, AND THE MULTIPLICATION RULE

There is a probability rule that can be used to calculate the probability of the intersection of several events. However, this rule depends on the important statistical concept of **independent** or **dependent events.**

Definition Two events, A and B, are said to be **independent** if and only if the probability of event B is not influenced or changed by the occurrence of event A, or vice versa.

Colorblindness Suppose a researcher notes a person's gender and whether or not the person is colorblind to red and green. Does the probability that a person is colorblind change depending on whether the person is male or not? Define two events:

A: Person is a male

B: Person is colorblind

In this case, since colorblindness is a male sex-linked characteristic, the probability that a man is colorblind will be greater than the probability that a person chosen from the general population will be colorblind. The probability of event B, that a person is colorblind, depends on whether or not event A, that the person is a male, has occurred. We say that A and B are *dependent events.*

Tossing Dice On the other hand, consider tossing a single die two times, and define two events:

A: Observe a 2 on the first toss

B: Observe a 2 on the second toss

If the die is fair, the probability of event A is $P(A) = 1/6$. Consider the probability of event B. Regardless of whether event A has or has not occurred, the probability of observing a 2 on the second toss is still 1/6. We could write:

$P(B$ given that A occurred$) = 1/6$

$P(B$ given that A did not occur$) = 1/6$

Since the probability of event B is not changed by the occurrence of event A, we say that A and B are *independent events.*

The probability of an event A, given that the event B has occurred, is called the **conditional probability of A, given that B has occurred,** denoted by $P(A|B)$. The vertical bar is read "given" and the events appearing to the right of the bar are those that you know have occurred. We will use these probabilities to calculate the probability that *both A and B* occur when the experiment is performed.

THE GENERAL MULTIPLICATION RULE

The probability that *both A and B* occur when the experiment is performed is

$$P(A \cap B) = P(A)P(B|A)$$

or

$$P(A \cap B) = P(B)P(A|B)$$