5.1 INTRODUCTION

Examples of *discrete random variables* can be found in a variety of everyday situations and across most academic disciplines. However, there are three discrete probability distributions that serve as *models* for a large number of these applications. In this chapter we study the binomial, the Poisson, and the hypergeometric probability distributions and discuss their usefulness in different physical situations.

5.2 THE BINOMIAL PROBABILITY DISTRIBUTION

A coin-tossing experiment is a simple example of an important discrete random variable called the **binomial random variable.** Many practical experiments result in data similar to the head or tail outcomes of the coin toss. For example, consider the political polls used to predict voter preferences in elections. Each sampled voter can be compared to a coin because the voter may be in favor of our candidate—a "head"—or not—a "tail." In most cases, the proportion of voters who favor our candidate does not equal 1/2; that is, the coin is not fair. In fact, the proportion of voters who favor our candidate is exactly what the poll is designed to measure!

Here are some other situations that are similar to the coin-tossing experiment:

- A sociologist is interested in the proportion of elementary school teachers who are men.
- A soft-drink marketer is interested in the proportion of cola drinkers who prefer her brand.
- A geneticist is interested in the proportion of the population who possess a gene linked to Alzheimer's disease.

Each sampled person is analogous to tossing a coin, but the probability of a "head" is not necessarily equal to 1/2. Although these situations have different practical objectives, they all exhibit the common characteristics of the **binomial experiment.**

Definition A **binomial experiment** is one that has these five characteristics:

1. The experiment consists of n identical trials.
2. Each trial results in one of two outcomes. For lack of a better name, the one outcome is called a success, S, and the other a failure, F.
3. The probability of success on a single trial is equal to p and remains the same from trial to trial. The probability of failure is equal to $(1 - p) = q$.
4. The trials are independent.
5. We are interested in x, the number of successes observed during the n trials, for $x = 0, 1, 2, \dots, n$.

EXAMPLE 5.1

Suppose there are approximately 1,000,000 adults in a county and an unknown proportion p favor term limits for politicians. A sample of 1000 adults will be chosen in such a way that every one of the 1,000,000 adults has an equal chance of being selected, and each adult is asked whether he or she favors term limits. (The ultimate objective of this survey is to estimate the unknown proportion p, a problem that we will discuss in Chapter 8.) Is this a binomial experiment?