355. Test the null hypothesis that $\sigma = .7$ against the alternative $\sigma > .7$. Use $\alpha = .05$.

10.52 Instrument Precision, continued Find a 90% confidence interval for the population variance in Exercise 10.51.

10.53 Drug Potency To properly treat patients, drugs prescribed by physicians must have a potency that is accurately defined. Consequently, not only must the distribution of potency values for shipments of a drug have a mean value as specified on the drug's container, but also the variation in potency must be small. Otherwise, pharmacists would be distributing drug prescriptions that could be harmfully potent or have a low potency and be ineffective. A drug manufacturer claims that his drug is marketed with a potency of $5 \pm .1$ milligram per cubic centimeter (mg/cc). A random sample of four containers gave potency readings equal to 4.94, 5.09, 5.03, and 4.90 mg/cc.

a. Do the data present sufficient evidence to indicate that the mean potency differs from 5 mg/cc?

b. Do the data present sufficient evidence to indicate that the variation in potency differs from the error limits specified by the manufacturer? [HINT: It is sometimes difficult to determine exactly what is meant by limits on potency as specified by a manufacturer. Since he implies that the potency values will fall into the interval $5 \pm .1$ mg/cc with very high probability—the implication is *always*—let us assume that the range .2; or (4.9 to 5.1), represents 6σ, as suggested by the Empirical Rule. Note that letting the range equal 6σ rather than 4σ places a stringent interpretation on the manufacturer's claim. We want the potency to fall into the interval $5 \pm .1$ with very high probability.]

10.54 Drug Potency, continued Refer to Exercise 10.53. Testing of 60 additional randomly selected containers of the drug gave a sample mean and variance equal to 5.04 and .0063 (for the total of $n = 64$ containers). Using a 95% confidence interval, estimate the variance of the manufacturer's potency measurements.

10.55 Hard Hats A manufacturer of hard safety hats for construction workers is concerned about the mean and the variation of the forces helmets transmit to wearers when subjected to a standard external force. The manufacturer desires the mean force transmitted by helmets to be 800 pounds (or less), well under the legal 1000-pound limit, and σ to be less than 40. A random sample of $n = 40$ helmets was tested, and the sample mean and variance were found to be equal to 825 pounds and 2350 pounds^2, respectively.

a. If $\mu = 800$ and $\sigma = 40$, is it likely that any helmet, subjected to the standard external force, will transmit a force to a wearer in excess of 1000 pounds? Explain.

b. Do the data provide sufficient evidence to indicate that when the helmets are subjected to the standard external force, the mean force transmitted by the helmets exceeds 800 pounds?

10.56 Hard Hats, continued Refer to Exercise 10.55. Do the data provide sufficient evidence to indicate that σ exceeds 40?

MY DATA EX1057 **10.57 Light Bulbs** A manufacturer of industrial light bulbs likes its bulbs to have a mean life that is acceptable to its customers and a variation in life that is relatively small. If some bulbs fail too early in their life, customers become annoyed and shift to competitive products. Large variations above the mean reduce replacement sales, and variation in general disrupts customers' replacement schedules. A sample of 20 bulbs tested produced the following lengths of life (in hours):

2100	2302	1951	2067	2415	1883	2101	2146	2278	2019
1924	2183	2077	2392	2286	2501	1946	2161	2253	1827

The manufacturer wishes to control the variability in length of life so that σ is less than 150 hours. Do the data provide sufficient evidence to indicate that the manufacturer is achieving this goal? Test using $\alpha = .01$.

10.7 COMPARING TWO POPULATION VARIANCES

Just as a single population variance is sometimes important to an experimenter, you might also need to compare two population variances. You might need to compare the precision of one measuring device with that of another, the stability of one manufacturing process with that of another, or even the variability in the grading procedure of one college professor with that of another.

One way to compare two population variances, σ_1^2 and σ_2^2, is to use the ratio of the sample variances, s_1^2/s_2^2. If s_1^2/s_2^2 is nearly equal to 1, you will find little evidence to indicate that σ_1^2 and σ_2^2 are unequal. On the other hand, a very large or very small value for s_1^2/s_2^2 provides evidence of a difference in the population variances. How large or small must s_1^2/s_2^2 be for sufficient evidence to exist to reject the following null hypothesis?

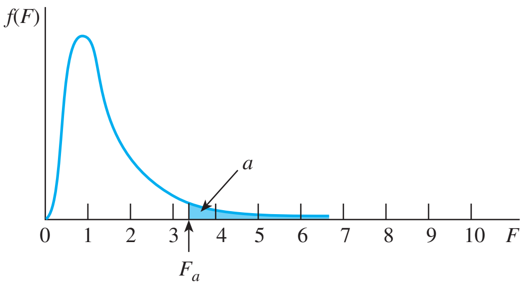
$$H_0 : \sigma_1^2 = \sigma_2^2$$

The answer to this question may be found by studying the distribution of s_1^2/s_2^2 in repeated sampling.

When independent random samples are drawn from two *normal* populations with *equal variances*—that is, $\sigma_1^2 = \sigma_2^2$—then s_1^2/s_2^2 has a probability distribution in repeated sampling that is known to statisticians as an ***F* distribution,** shown in Figure 10.20.

FIGURE 10.20

An F distribution with $df_1 = 10$ and $df_2 = 10$



ASSUMPTIONS FOR s_1^2/s_2^2 TO HAVE AN F DISTRIBUTION

- Random and independent samples are drawn from each of two normal populations.
- The variability of the measurements in the two populations is the same and can be measured by a common variance, σ^2; that is, $\sigma_1^2 = \sigma_2^2 = \sigma^2$.

It is not important for you to know the complex equation of the density function for F. For your purposes, you need only to use the well-tabulated critical values of F given in Table 6 in Appendix I.

Critical values of F and p-values for significance tests can also be found using the ***F* Probabilities** applet shown in Figure 10.21.

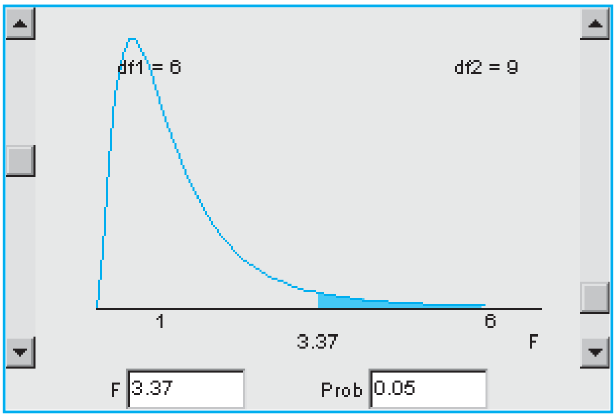
MY TIP

Testing two variances: $df_1 = n_1 - 1$ and $df_2 = n_2 - 1$

Like the χ^2 distribution, the shape of the F distribution is nonsymmetric and depends on the number of degrees of freedom associated with s_1^2 and s_2^2, represented as $df_1 = (n_1 - 1)$ and $df_2 = (n_2 - 1)$, respectively. This complicates the tabulation of critical values of the F distribution because a table is needed for each different combination of df_1, df_2, and a.

In Table 6 in Appendix I, critical values of F for right-tailed areas corresponding to $a = .100, .050, .025, .010$, and $.005$ are tabulated for various combinations of df_1 numerator degrees of freedom and df_2 denominator degrees of freedom. A portion of Table 6 is reproduced in Table 10.6. The numerator degrees of freedom df_1 are listed across the top margin, and the denominator degrees of freedom df_2 are listed along the side margin. The values of a are listed in the second column. For a fixed combination of df_1 and df_2, the appropriate critical values of F are found in the line indexed by the value of a required.

FIGURE 10.21
F Probabilities applet



EXAMPLE 10.13

Check your ability to use Table 6 in Appendix I by verifying the following statements:

1. The value of F with area .05 to its right for $df_1 = 6$ and $df_2 = 9$ is 3.37.
2. The value of F with area .05 to its right for $df_1 = 5$ and $df_2 = 10$ is 3.33.
3. The value of F with area .01 to its right for $df_1 = 6$ and $df_2 = 9$ is 5.80.

These values are shaded in Table 10.6.

TABLE 10.6 Format of the *F* Table from Table 6 in Appendix I

		df_1					
df_2	a	1	2	3	4	5	6
1	.100	39.86	49.50	53.59	55.83	57.24	58.20
	.050	161.4	199.5	215.7	224.6	230.2	234.0
	.025	647.8	799.5	864.2	899.6	921.8	937.1
	.010	4052	4999.5	5403	5625	5764	5859
	.005	16211	20000	21615	22500	23056	23437
2	.100	8.53	9.00	9.16	9.24	9.29	9.33
	.050	18.51	19.00	19.16	19.25	19.30	19.33
	.025	38.51	39.00	39.17	39.25	39.30	39.33
	.010	98.50	99.00	99.17	99.25	99.30	99.33
	.005	198.5	199.0	199.2	199.2	199.3	199.3
3	.100	5.54	5.46	5.39	5.34	5.31	5.28
	.050	10.13	9.55	9.28	9.12	9.01	8.94
	.025	17.44	16.04	15.44	15.10	14.88	14.73
	.010	34.12	30.82	29.46	28.71	28.24	27.91
	.005	55.55	49.80	47.47	46.19	45.39	44.84
.	.			.			.
.	.			.			.
.	.			.			.
9	.100	3.36	3.01	2.81	2.69	2.61	2.55
	.050	5.12	4.26	3.86	3.63	3.48	3.37
	.025	7.21	5.71	5.08	4.72	4.48	4.32
	.010	10.56	8.02	6.99	6.42	6.06	5.80
	.005	13.61	10.11	8.72	7.96	7.47	7.13
10	.100	3.29	2.92	2.73	2.61	2.52	2.46
	.050	4.96	4.10	3.71	3.48	3.33	3.22
	.025	6.94	5.46	4.83	4.47	4.24	4.07
	.010	10.04	7.56	6.55	5.99	5.64	5.39
	.005	12.83	9.43	8.08	7.34	6.87	6.54

The statistical test of the null hypothesis

$$H_0 : \sigma_1^2 = \sigma_2^2$$

uses the test statistic

$$F = \frac{s_1^2}{s_2^2}$$

When the alternative hypothesis implies a one-tailed test—that is,

$$H_a : \sigma_1^2 > \sigma_2^2$$

you can find the right-tailed critical value for rejecting H_0 directly from Table 6 in Appendix I. However, when the alternative hypothesis requires a two-tailed test—that is,

$$H_0 : \sigma_1^2 \neq \sigma_2^2$$

the rejection region is divided between the upper and lower tails of the F distribution. These left-tailed critical values are *not given* in Table 6 for the following reason: You are free to decide which of the two populations you want to call "Population 1." If you always choose to call the population with the *larger* sample variance "Population 1," then the observed value of your test statistic will always be in the right tail of the F distribution. Even though half of the rejection region, the area $\alpha/2$ to its left, will be in the lower tail of the distribution, you will never need to use it! Remember these points, though, for a two-tailed test:

- The area in the right tail of the rejection region is only $\alpha/2$.
- The area to the right of the observed test statistic is only (p-value)/2.

The formal procedures for a test of hypothesis and a $(1 - \alpha)100\%$ confidence interval for two population variances are shown next.

TEST OF HYPOTHESIS CONCERNING THE EQUALITY OF TWO POPULATION VARIANCES

1. Null hypothesis: $H_0 : \sigma_1^2 = \sigma_2^2$
2. Alternative hypothesis:

One-Tailed Test	**Two-Tailed Test**
$H_a : \sigma_1^2 > \sigma_2^2$ (or $H_a : \sigma_1^2 < \sigma_2^2$)	$H_a : \sigma_1^2 \neq \sigma_2^2$

3. Test statistic:

One-Tailed Test	**Two-Tailed Test**
$F = \dfrac{s_1^2}{s_2^2}$	$F = \dfrac{s_1^2}{s_2^2}$

   where s_1^2 is the larger sample variance

4. Rejection region: Reject H_0 when

One-Tailed Test	**Two-Tailed Test**
$F > F_\alpha$	$F > F_{\alpha/2}$

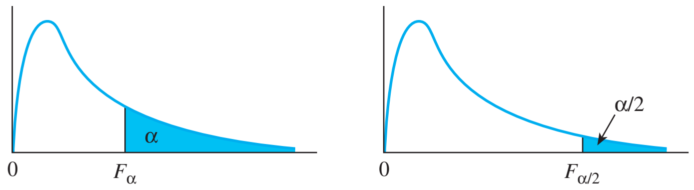
   or when p-value $< \alpha$

(continued)

TEST OF HYPOTHESIS CONCERNING THE EQUALITY OF TWO POPULATION VARIANCES

(continued)

The critical values of F_α and $F_{\alpha/2}$ are based on $df_1 = (n_1 - 1)$ and $df_2 = (n_2 - 1)$. These tabulated values, for $a = .100, .050, .025, .010$, and $.005$, can be found using Table 6 in Appendix I, or the ***F* Probabilities** applet.



Assumptions: The samples are randomly and independently selected from normally distributed populations.

CONFIDENCE INTERVAL FOR σ_1^2/σ_2^2

$$\left(\frac{s_1^2}{s_2^2}\right)\frac{1}{F_{df_1,df_2}} < \frac{\sigma_1^2}{\sigma_2^2} < \left(\frac{s_1^2}{s_2^2}\right)F_{df_2,df_1}$$

where $df_1 = (n_1 - 1)$ and $df_2 = (n_2 - 1)$. F_{df_1,df_2} is the tabulated critical value of F corresponding to df_1 and df_2 degrees of freedom in the numerator and denominator of F, respectively, with area $\alpha/2$ to its right.

Assumptions: The samples are randomly and independently selected from normally distributed populations.

EXAMPLE 10.14

An experimenter is concerned that the variability of responses using two different experimental procedures may not be the same. Before conducting his research, he conducts a prestudy with random samples of 10 and 8 responses and gets $s_1^2 = 7.14$ and $s_2^2 = 3.21$, respectively. Do the sample variances present sufficient evidence to indicate that the population variances are unequal?

Solution Assume that the populations have probability distributions that are reasonably mound-shaped and hence satisfy, for all practical purposes, the assumption that the populations are normal. You wish to test these hypotheses:

$$H_0 : \sigma_1^2 = \sigma_2^2 \quad \text{versus} \quad H_a : \sigma_1^2 \neq \sigma_2^2$$

Using Table 6 in Appendix I for $\alpha/2 = .025$, you can reject H_0 when $F > 4.82$ with $\alpha = .05$. The calculated value of the test statistic is

$$F = \frac{s_1^2}{s_2^2} = \frac{7.14}{3.21} = 2.22$$

Because the test statistic does not fall into the rejection region, you cannot reject $H_0 : \sigma_1^2 = \sigma_2^2$. Thus, there is insufficient evidence to indicate a difference in the population variances.

EXAMPLE 10.15

Refer to Example 10.14 and find a 90% confidence interval for σ_1^2/σ_2^2.

Solution The 90% confidence interval for σ_1^2/σ_2^2 is

$$\left(\frac{s_1^2}{s_2^2}\right)\frac{1}{F_{df_1,df_2}} < \frac{\sigma_1^2}{\sigma_2^2} < \left(\frac{s_1^2}{s_2^2}\right)F_{df_2,df_1}$$

where

$$s_1^2 = 7.14 \qquad s_2^2 = 3.21$$
$$df_1 = (n_1 - 1) = 9 \qquad df_2 = (n_2 - 1) = 7$$
$$F_{9,7} = 3.68 \qquad F_{7,9} = 3.29$$

Substituting these values into the formula for the confidence interval, you get

$$\left(\frac{7.14}{3.21}\right)\frac{1}{3.68} < \frac{\sigma_1^2}{\sigma_2^2} < \left(\frac{7.14}{3.21}\right)3.29 \quad \text{or} \quad .60 < \frac{\sigma_1^2}{\sigma_2^2} < 7.32$$

The calculated interval estimate .60 to 7.32 includes 1.0, the value hypothesized in H_0. This indicates that it is quite possible that $\sigma_1^2 = \sigma_2^2$ and therefore agrees with the test conclusions. Do not reject $H_0 : \sigma_1^2 = \sigma_2^2$.

The *MINITAB* command **Stat → Basic Statistics → 2 Variances** allows you to enter either raw data or summary statistics to perform the F-test for the equality of variances and calculates confidence intervals for the two individual standard deviations (which we have not discussed). The relevant printout, containing the F statistic and its p-value, is shaded in Figure 10.22.

FIGURE 10.22
MINITAB output for Example 10.14

Test for Equal Variances

```
95% Bonferroni confidence intervals for standard deviations
Sample    N     Lower     StDev     Upper
     1   10   1.74787   2.67208   5.38064
     2    8   1.12088   1.79165   4.10374

F-Test (Normal Distribution)
Test statistic = 2.22, p-value = 0.304
```

EXAMPLE 10.16

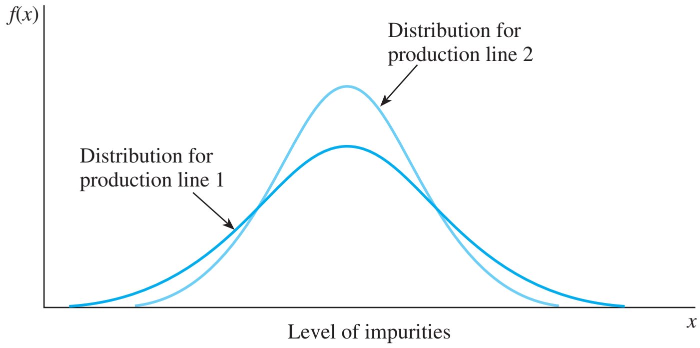
The variability in the amount of impurities present in a batch of chemical used for a particular process depends on the length of time the process is in operation. A manufacturer using two production lines, 1 and 2, has made a slight adjustment to line 2, hoping to reduce the variability as well as the average amount of impurities in the chemical. Samples of $n_1 = 25$ and $n_2 = 25$ measurements from the two batches yield these means and variances:

$$\bar{x}_1 = 3.2 \qquad s_1^2 = 1.04$$
$$\bar{x}_2 = 3.0 \qquad s_2^2 = .51$$

Do the data present sufficient evidence to indicate that the process variability is less for line 2?

Solution The experimenter believes that the average levels of impurities are the same for the two production lines but that her adjustment may have decreased the variability of the levels for line 2, as illustrated in Figure 10.23. This adjustment would be good for the company because it would decrease the probability of producing shipments of the chemical with unacceptably high levels of impurities.

FIGURE 10.23
Distributions of impurity measurements for two production lines



To test for a decrease in variability, the test of hypothesis is

$$H_0 : \sigma_1^2 = \sigma_2^2 \quad \text{versus} \quad H_a : \sigma_1^2 > \sigma_2^2$$

and the observed value of the test statistic is

$$F = \frac{s_1^2}{s_2^2} = \frac{1.04}{.51} = 2.04$$

Using the p-value approach, you can bound the one-tailed p-value using Table 6 in Appendix I with $df_1 = df_2 = (25 - 1) = 24$. The observed value of F falls between $F_{.050} = 1.98$ and $F_{.025} = 2.27$, so that $.025 < p\text{-value} < .05$. The results are judged significant at the 5% level, and H_0 is rejected. You can conclude that the variability of line 2 is less than that of line 1.

The F-test for the difference in two population variances completes the battery of tests you have learned in this chapter for making inferences about population parameters under these conditions:

- The sample sizes are small.
- The sample or samples are drawn from normal populations.

You will find that the F and χ^2 distributions, as well as the Student's t distribution, are very important in other applications in the chapters that follow. They will be used for different estimators designed to answer different types of inferential questions, but the basic techniques for making inferences remain the same.

In the next section, we review the assumptions required for all of these inference tools, and discuss options that are available when the assumptions do not seem to be reasonably correct.

10.7 EXERCISES

BASIC TECHNIQUES

10.58 Independent random samples from two normal populations produced the variances listed here:

Sample Size	Sample Variance
16	55.7
20	31.4

a. Do the data provide sufficient evidence to indicate that σ_1^2 differs from σ_2^2? Test using $\alpha = .05$.

b. Find the approximate p-value for the test and interpret its value.

10.59 Refer to Exercise 10.58 and find a 95% confidence interval for σ_1^2/σ_2^2.

10.60 Independent random samples from two normal populations produced the given variances:

Sample Size	Sample Variance
13	18.3
13	7.9

a. Do the data provide sufficient evidence to indicate that $\sigma_1^2 > \sigma_2^2$? Test using $\alpha = .05$.

b. Find the approximate p-value for the test and interpret its value.

APPLICATIONS

10.61 SAT Scores The SAT subject tests in chemistry and physics[11] for two groups of 15 students each electing to take these tests are given below.

Chemistry	Physics
$\bar{x} = 629$	$\bar{x} = 643$
$s = 110$	$s = 107$
$n = 15$	$n = 15$

To use the two-sample t-test with a pooled estimate of σ^2, you must assume that the two population variances are equal. Test this assumption using the F-test for equality of variances. What is the approximate p-value for the test?

10.62 Product Quality The stability of measurements on a manufactured product is important in maintaining product quality. In fact, it is sometimes better to have small variation in the measured value of some important characteristic of a product and have the process mean be slightly off target than to suffer wide variation with a mean value that perfectly fits requirements. The latter situation may produce a higher percentage of defective products than the former. A manufacturer of light bulbs suspected that one of her production lines was producing bulbs with a wide variation in length of life. To test this theory, she compared the lengths of life for $n = 50$ bulbs randomly sampled from the suspect line and $n = 50$ from a line that seemed to be "in control." The sample means and variances for the two samples were as follows:

"Suspect Line"	Line "in Control"
$\bar{x}_1 = 1520$	$\bar{x}_2 = 1476$
$s_1^2 = 92{,}000$	$s_2^2 = 37{,}000$

a. Do the data provide sufficient evidence to indicate that bulbs produced by the "suspect line" have a larger variance in length of life than those produced by the line that is assumed to be in control? Test using $\alpha = .05$.

b. Find the approximate p-value for the test and interpret its value.

10.63 Construct a 90% confidence interval for the variance ratio in Exercise 10.62.

10.64 Tuna III In Exercise 10.25 and dataset EX1025, you conducted a test to detect a difference in the average prices of light tuna in water versus light tuna in oil.

a. What assumption had to be made concerning the population variances so that the test would be valid?

b. Do the data present sufficient evidence to indicate that the variances violate the assumption in part a? Test using $\alpha = .05$.

10.65 Runners and Cyclists III Refer to Exercise 10.26. Susan Beckham and colleagues conducted an experiment involving 10 healthy runners and 10 healthy cyclists to determine if there are significant differences in pressure measurements within the anterior muscle compartment for runners and cyclists.[7] The data—compartment pressure, in millimeters of mercury (Hg)—are reproduced here:

	Runners		Cyclists	
Condition	Mean	Standard Deviation	Mean	Standard Deviation
Rest	14.5	3.92	11.1	3.98
80% maximal O_2 consumption	12.2	3.49	11.5	4.95
Maximal O_2 consumption	19.1	16.9	12.2	4.47

For each of the three variables measured in this experiment, test to see whether there is a significant difference in the variances for runners versus cyclists. Find the approximate p-values for each of these tests. Will a two-sample t-test with a pooled estimate of σ^2 be appropriate for all three of these variables? Explain.

10.66 Impurities A pharmaceutical manufacturer purchases a particular material from two different suppliers. The mean level of impurities in the raw material is approximately the same for both suppliers, but the manufacturer is concerned about the variability of the impurities from shipment to shipment. If the level of impurities tends to vary excessively for one source of supply, it could affect the quality of the pharmaceutical product. To compare the variation in percentage impurities for the two suppliers, the manufacturer selects 10 shipments from each of the two suppliers and measures the percentage of impurities in the raw material for each shipment. The sample means and variances are shown in the table.