

In Table 2.3, we choose a few numerical values for k and compute $[1 - (1/k^2)]$.

TABLE 2.3 Illustrative Values of $[1 - (1/k^2)]$

k	$1 - (1/k^2)$
1	$1 - 1 = 0$
2	$1 - 1/4 = 3/4$
3	$1 - 1/9 = 8/9$

From the calculations in Table 2.3, the theorem states:

- At least none of the measurements lie in the interval $\mu - \sigma$ to $\mu + \sigma$.
- At least $3/4$ of the measurements lie in the interval $\mu - 2\sigma$ to $\mu + 2\sigma$.
- At least $8/9$ of the measurements lie in the interval $\mu - 3\sigma$ to $\mu + 3\sigma$.

Although the first statement is not at all helpful, the other two values of k provide valuable information about the proportion of measurements that fall in certain intervals. The values $k = 2$ and $k = 3$ are not the only values of k you can use; for example, the proportion of measurements that fall within $k = 2.5$ standard deviations of the mean is at least $1 - [1/(2.5)^2] = .84$.

EXAMPLE

2.6

The mean and variance of a sample of $n = 25$ measurements are 75 and 100, respectively. Use Tchebysheff's Theorem to describe the distribution of measurements.

Solution You are given $\bar{x} = 75$ and $s^2 = 100$. The standard deviation is $s = \sqrt{100} = 10$. The distribution of measurements is centered about $\bar{x} = 75$, and Tchebysheff's Theorem states:

- At least $3/4$ of the 25 measurements lie in the interval $\bar{x} \pm 2s = 75 \pm 2(10)$ —that is, 55 to 95.
- At least $8/9$ of the measurements lie in the interval $\bar{x} \pm 3s = 75 \pm 3(10)$ —that is, 45 to 105.

Since Tchebysheff's Theorem applies to *any* distribution, it is very conservative. This is why we emphasize “at least $1 - (1/k^2)$ ” in this theorem.

Another rule for describing the variability of a data set does not work for *all* data sets, but it does work very well for data that “pile up” in the familiar mound shape shown in Figure 2.11. The closer your data distribution is to the mound-shaped curve in Figure 2.11, the more accurate the rule will be. Since mound-shaped data distributions occur quite frequently in nature, the rule can often be used in practical applications. For this reason, we call it the **Empirical Rule**.

FIGURE 2.11

Mound-shaped distribution

