

If the first selection *does not* result in a generic pill, then there are still 2 generic pills in the remaining 19, and the probability of a “success” (a generic pill) changes to

$$P(\text{generic on trial 2} | \text{no generic on trial 1}) = 2/19$$

Therefore the trials are dependent and the sampling does not represent a binomial experiment.

Think about the difference between these two examples. When the sample (the n identical trials) came from a large population, the probability of success p stayed about the same from trial to trial. When the population size N was small, the probability of success p changed quite dramatically from trial to trial, and the experiment *was not* binomial.

RULE OF THUMB

If the sample size is large relative to the population size—in particular, if $n/N \geq .05$ —then the resulting experiment is not binomial.

In Chapter 4, we tossed two fair coins and constructed the probability distribution for x , the number of heads—a binomial experiment with $n = 2$ and $p = .5$. The general binomial probability distribution is constructed in the same way, but the procedure gets complicated as n gets large. Fortunately, the probabilities $p(x)$ follow a general pattern. This allows us to use a single formula to find $p(x)$ for any given value of x .

THE BINOMIAL PROBABILITY DISTRIBUTION

A binomial experiment consists of n identical trials with probability of success p on each trial. The probability of k successes in n trials is

$$P(x = k) = C_k^n p^k q^{n-k} = \frac{n!}{k!(n-k)!} p^k q^{n-k}$$

for values of $k = 0, 1, 2, \dots, n$. The symbol C_k^n equals

$$\frac{n!}{k!(n-k)!}$$

where $n! = n(n-1)(n-2) \cdots (2)(1)$ and $0! \equiv 1$.

The general formulas for μ , σ^2 , and σ given in Chapter 4 can be used to derive the following simpler formulas for the binomial mean and standard deviation.

MEAN AND STANDARD DEVIATION FOR THE BINOMIAL RANDOM VARIABLE

The random variable x , the number of successes in n trials, has a probability distribution with this center and spread:

$$\begin{aligned} \text{Mean:} & \quad \mu = np \\ \text{Variance:} & \quad \sigma^2 = npq \\ \text{Standard deviation:} & \quad \sigma = \sqrt{npq} \end{aligned}$$