9-101. In a random sample of 85 automobile engine crankshaft bearings, 10 have a surface finish roughness that exceeds the specifications. Do these data present strong evidence that the proportion of crankshaft bearings exhibiting excess surface roughness exceeds 0.10?
(a) State and test the appropriate hypotheses using $\alpha = 0.05$.
(b) If it is really the situation that $p = 0.15$, how likely is it that the test procedure in part (a) will not reject the null hypothesis?
(c) If $p = 0.15$, how large would the sample size have to be for us to have a probability of correctly rejecting the null hypothesis of 0.9?

9-102. A computer manufacturer ships laptop computers with the batteries fully charged so that customers can begin to use their purchases right out of the box. In its last model, 85% of customers received fully charged batteries. To simulate arrivals, the company shipped 100 new model laptops to various company sites around the country. Of the 105 laptops shipped, 96 of them arrived reading 100% charged. Do the data provide evidence that this model's rate is at least as high as the previous model? Test the hypothesis at $\alpha = 0.05$.

9-103. In a random sample of 500 handwritten zip code digits, 466 were read correctly by an optical character recognition (OCR) system operated by the U.S. Postal Service (USPS). USPS would like to know whether the rate is at least 90% correct. Do the data provide evidence that the rate is at least 90% at $\alpha = 0.05$?

9-104. Construct a 90% confidence interval for the proportion of handwritten zip codes that were read correctly using the data provided in Exercise 9-103. Does this confidence interval support the claim that at least 90% of the zip codes can be correctly read?

9-105. Construct a 95% lower confidence interval for the proportion of patients with kidney stones successfully removed in Exercise 9-95. Does this confidence interval support the claim that at least 78% of procedures are successful?

9-6 Summary Table of Inference Procedures for a Single Sample

The table in the end papers of this book (inside back cover) presents a summary of all the single-sample inference procedures from Chapters 8 and 9. The table contains the null hypothesis statement, the test statistic, the various alternative hypotheses and the criteria for rejecting H_0, and the formulas for constructing the $100(1-\alpha)\%$ two-sided confidence interval. It would also be helpful to refer to the roadmap table in Chapter 8 that provides guidance to match the problem type to the information inside the back cover.

9-7 Testing for Goodness of Fit

The hypothesis-testing procedures that we have discussed in previous sections are designed for problems in which the population or probability distribution is known and the hypotheses involve the parameters of the distribution. Another kind of hypothesis is often encountered: We do not know the underlying distribution of the population, and we wish to test the hypothesis that a particular distribution will be satisfactory as a population model. For example, we might wish to test the hypothesis that the population is normal.

We have previously discussed a very useful graphical technique for this problem called **probability plotting** and illustrated how it was applied in the case of a normal distribution. In this section, we describe a formal **goodness-of-fit test** procedure based on the chi-square distribution.

The test procedure requires a random sample of size n from the population whose probability distribution is unknown. These n observations are arranged in a frequency histogram, having k bins or class intervals. Let O_i be the observed frequency in the ith class interval. From the hypothesized probability distribution, we compute the expected frequency in the ith class interval, denoted E_i. The test statistic is

Goodness-of-Fit Test Statistic

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} \tag{9-47}$$

It can be shown that, if the population follows the hypothesized distribution, χ_0^2 has, approximately, a chi-square distribution with $k - p - 1$ degrees of freedom, when p represents the number of parameters of the hypothesized distribution estimated by sample statistics. This approximation improves as n increases. We should reject the null hypothesis that the population is the hypothesized distribution if the test statistic is too large. Therefore, the P-value would be the probability under the chi-square distribution with $k - p - 1$ degrees of freedom above the computed value of the test statistic χ_0^2 or $P = P\left(\chi_{k-p-1}^2 > \chi_0^2\right)$. For a fixed-level test, we would reject the hypothesis that the distribution of the population is the hypothesized distribution if the calculated value of the test statistic $\chi_0^2 > \chi_{\alpha,k-p-1}^2$.

One point to be noted in the application of this test procedure concerns the magnitude of the expected frequencies. If these expected frequencies are too small, the test statistic X_0^2 will not reflect the departure of observed from expected but only the small magnitude of the expected frequencies. There is no general agreement regarding the minimum value of expected frequencies, but values of 3, 4, and 5 are widely used as minimal. Some writers suggest that an expected frequency could be as small as 1 or 2 so long as most of them exceed 5. Should an expected frequency be too small, it can be combined with the expected frequency in an adjacent class interval. The corresponding observed frequencies would then also be combined, and k would be reduced by 1. Class intervals are not required to be of equal width.

We now give two examples of the test procedure.

Example 9-12 **Printed Circuit Board Defects-Poisson Distribution** The number of defects in printed circuit boards is hypothesized to follow a Poisson distribution. A random sample of $n = 60$ printed circuit boards has been collected, and the following number of defects observed.

Number of Defects	Observed Frequency
0	32
1	15
2	9
3	4

The mean of the assumed Poisson distribution in this example is unknown and must be estimated from the sample data. The estimate of the mean number of defects per board is the sample average, that is, $(32\cdot 0 + 15\cdot 1 + 9\cdot 2 + 4\cdot 3)/60 = 0\cdot 75$. From the Poisson distribution with parameter 0.75, we may compute p_i, the theoretical, hypothesized probability associated with the ith class interval. Because each class interval corresponds to a particular number of defects, we may find the p_i as follows:

$$p_1 = P(X = 0) = \frac{e^{-0.75}(0.75)^0}{0!} = 0.472$$

$$p_2 = P(X = 1) = \frac{e^{-0.75}(0.75)^1}{1!} = 0.354$$

$$p_3 = P(X = 2) = \frac{e^{-0.75}(0.75)^2}{2!} = 0.133$$

$$p_4 = P(X \geq 3) = 1 - (p_1 + p_2 + p_3) = 0.041$$

The expected frequencies are computed by multiplying the sample size $n = 60$ times the probabilities p_i. That is, $E_i = n_{pi}$. The expected frequencies follow:

Number of Defects	Probability	Expected Frequency
0	0.472	28.32
1	0.354	21.24
2	0.133	7.98
3 (or more)	0.041	2.46

Because the expected frequency in the last cell is less than 3, we combine the last two cells:

Number of Defects	Observed Frequency	Expected Frequency
0	32	28.32
1	15	21.24
2 (or more)	13	10.44

The seven-step hypothesis-testing procedure may now be applied, using $\alpha = 0.05$, as follows:

1. **Parameter of interest:** The variable of interest is the form of the distribution of defects in printed circuit boards.
2. **Null hypothesis:** H_0: The form of the distribution of defects is Poisson.
3. **Alternative hypothesis:** H_1: The form of the distribution of defects is not Poisson.
4. **Test statistic:** The test statistic is $\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$
5. **Reject H_0 if:** Because the mean of the Poisson distribution was estimated, the preceding chi-square statistic will have $k - p - 1 = 3 - 1 - 1 = 1$ degree of freedom. Consider whether the P-value is less than 0.05.
6. **Computations**:

$$\chi_0^2 = \frac{(32 - 28.32)^2}{28.32} + \frac{(15 - 21.24)^2}{21.24} + \frac{(13 - 10.44)^2}{10.44} = 2.94$$

7. **Conclusions:** We find from Appendix Table III that $\chi^2_{0.10,1} = 2.71$ and $\chi^2_{0.05,1} = 3.84$. Because $\chi_0^2 = 2.94$ lies between these values, we conclude that the P-value is between 0.05 and 0.10. Therefore, because the P-value exceeds 0.05, we are unable to reject the null hypothesis that the distribution of defects in printed circuit boards is Poisson. The exact P-value computed from software is 0.0864.

Example 9-13 **Power Supply Distribution-Continuous Distribution** A manufacturing engineer is testing a power supply used in a notebook computer and, using $\alpha = 0.05$, wishes to determine whether output voltage is adequately described by a normal distribution. Sample estimates of the mean and standard deviation of $\bar{x} = 5.04$ V and $s = 0.08$ V are obtained from a random sample of $n = 100$ units.

A common practice in constructing the class intervals for the frequency distribution used in the chi-square goodness-of-fit test is to choose the cell boundaries so that the expected frequencies $E_i = np_i$ are equal for all cells. To use this method, we want to choose the cell boundaries $a_0, a_1, \ldots, a_k$ for the k cells so that all the probabilities

$$p_i = P(a_{i-1} \le X \le a_i) = \int_{a_{i-1}}^{a_i} f(x)\,dx$$

are equal. Suppose that we decide to use $k = 8$ cells. For the standard normal distribution, the intervals that divide the scale into eight equally likely segments are (0, 0.32), (0.32, 0.675), (0.675, 1.15), (1.15, ∞), and their four "mirror image" intervals on the other side of zero. For each interval $p_i = 1/8 = 0.125$, so the expected cell frequencies are $E_i = n_{pi} = 100(0.125) = 12.5$. The complete table of observed and expected frequencies is as follows:

Class Interval	Observed Frequency o_i	Expected Frequency E_i
$x < 4.948$	12	12.5
$4.948 \le x < 4.986$	14	12.5
$4.986 \le x < 5.014$	12	12.5
$5.014 \le x < 5.040$	13	12.5
$5.040 \le x < 5.066$	12	12.5
$5.066 \le x < 5.094$	11	12.5
$5.094 \le x < 5.132$	12	12.5
$5.132 \le x$	14	12.5
Totals	100	100

The boundary of the first class interval is $\bar{x} - 1.15s = 4.948$. The second class interval is $\left[\bar{x} - 1.15s, \bar{x} - 0.675s\right]$ and so forth. We may apply the seven-step hypothesis-testing procedure to this problem.

1. **Parameter of interest:** The variable of interest is the form of the distribution of power supply voltage.

2. **Null hypothesis:** H_0: The form of the distribution is normal.

3. **Alternative hypothesis:** H_1: The form of the distribution is nonnormal.

4. **Test statistic:** The test statistic is

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

5. **Reject H_0 if:** Because two parameters in the normal distribution have been estimated, the preceding chi-square statistic will have $k - p - 1 = 8 - 2 - 1 = 5$ degrees of freedom. We will use a fixed significance level test with $\alpha = 0.05$. Therefore, we will reject H_0 if $\chi_0^2 > \chi_{0.05,5}^2 = 11.07$.

6. **Computations:**

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(o_i - E_i)^2}{E_i} = \frac{(12 - 12.5)^2}{12.5} + \frac{(14 - 12.5)^2}{12.5} + \ldots + \frac{(14 - 12.5)^2}{12.5} = 0.64$$

7. **Conclusions:** Because $\chi_0^2 = 0.64 < \chi_{0.05,5}^2 = 11.07$, we are unable to reject H_0, and no strong evidence indicates that output voltage is not normally distributed. The P-value for the chi-square statistic $\chi_0^2 = 0.64$ is $P = 0.9861$.

EXERCISES FOR SECTION 9-7

⊕ Problem available in *WileyPLUS* at instructor's discretion.
⊕ **Go Tutorial** Tutoring problem available in *WileyPLUS* at instructor's discretion.

9-106. ⊕ Consider the following frequency table of observations on the random variable X.

Values	0	1	2	3	4
Observed frequency	24	30	31	11	4

(a) Based on these 100 observations, is a Poisson distribution with a mean of 1.2 an appropriate model? Perform a goodness-of-fit procedure with $\alpha = 0.05$.
(b) Calculate the P-value for this test.

9-107. ⊕ Let X denote the number of flaws observed on a large coil of galvanized steel. Of 75 coils inspected, the following data were observed for the values of X:

Values	1	2	3	4	5	6	7	8
Observed frequency	1	11	8	13	11	12	10	9

(a) Does the assumption of the Poisson distribution seem appropriate as a probability model for these data? Use $\alpha = 0.01$.
(b) Calculate the P-value for this test.

9-108. ⊕ The number of calls arriving at a switchboard from noon to 1:00 P.M. during the business days Monday through Friday is monitored for six weeks (i.e., 30 days). Let X be defined as the number of calls during that one-hour period. The relative frequency of calls was recorded and reported as

Value	5	6	8	9	10
Relative frequency	0.067	0.067	0.100	0.133	0.200
Value	11	12	13	14	15
Relative frequency	0.133	0.133	0.067	0.033	0.067

(a) Does the assumption of a Poisson distribution seem appropriate as a probability model for this data? Use $\alpha = 0.05$.
(b) Calculate the P-value for this test.