Suppose the experiment is repeated a large number of times—say, $n = 4{,}000{,}000$ times. Intuitively, you would expect to observe approximately 1 million zeros, 2 million ones, and 1 million twos. Then the average value of x would equal

$$\frac{\text{Sum of measurements}}{n} = \frac{1{,}000{,}000(0) + 2{,}000{,}000(1) + 1{,}000{,}000(2)}{4{,}000{,}000}$$

$$= \left(\frac{1}{4}\right)(0) + \left(\frac{1}{2}\right)(1) + \left(\frac{1}{4}\right)(2)$$

Note that the first term in this sum is $(0)p(0)$, the second is equal to $(1)p(1)$, and the third is $(2)p(2)$. The average value of x, then, is

$$\Sigma xp(x) = 0 + \frac{1}{2} + \frac{2}{4} = 1$$

This result provides some intuitive justification for the definition of the expected value of a discrete random variable x.

Definition Let x be a discrete random variable with probability distribution $p(x)$. The mean or **expected value of** $\boldsymbol{x}$ is given as

$$\mu = E(x) = \Sigma\, xp(x)$$

where the elements are summed over all values of the random variable x.

We could use a similar argument to justify the formulas for the **population variance** σ^2 and the **population standard deviation** σ. These numerical measures describe the spread or variability of the random variable using the "average" or "expected value" of the squared deviations of the x-values from their mean μ.

Definition Let x be a discrete random variable with probability distribution $p(x)$ and mean μ. The **variance of** $\boldsymbol{x}$ is

$$\sigma^2 = E[(x - \mu)^2] = \Sigma(x - \mu)^2 p(x)$$

where the summation is over all values of the random variable x.[†]

Definition The **standard deviation** $\boldsymbol{\sigma}$ **of a random variable** $\boldsymbol{x}$ is equal to the positive square root of its variance.

EXAMPLE 4.26

An electronics store sells a particular model of computer notebook. There are only four notebooks in stock, and the manager wonders what today's demand for this particular model will be. She learns from the marketing department that the probability distribution for x, the daily demand for the laptop, is as shown in the table. Find the

[†]It can be shown (proof omitted) that $\sigma^2 = \Sigma(x - \mu)^2 p(x) = \Sigma x^2 p(x) - \mu^2$. This result is analogous to the computing formula for the sum of squares of deviations given in Chapter 2.