

with their sum of squares. From the sum of squared deviations, a single measure called the **variance** is calculated. To distinguish between the variance of a *sample* and the variance of a *population*, we use the symbol s^2 for a sample variance and σ^2 (Greek lowercase sigma) for a population variance. *The variance will be relatively large for highly variable data and relatively small for less variable data.*

Definition The **variance of a population** of N measurements is the average of the squares of the deviations of the measurements about their mean μ . The population variance is denoted by σ^2 and is given by the formula

$$\sigma^2 = \frac{\sum (x_i - \mu)^2}{N}$$

Most often, you will not have all the population measurements available but will need to calculate the *variance of a sample* of n measurements.

Definition The **variance of a sample** of n measurements is the sum of the squared deviations of the measurements about their mean \bar{x} divided by $(n - 1)$. The sample variance is denoted by s^2 and is given by the formula

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{n - 1}$$

MY TIP

The variance and the standard deviation cannot be negative numbers.

For the set of $n = 5$ sample measurements presented in Table 2.1, the square of the deviation of each measurement is recorded in the third column. Adding, we obtain

$$\sum (x_i - \bar{x})^2 = 22.80$$

and the sample variance is

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{n - 1} = \frac{22.80}{4} = 5.70$$

The variance is measured in terms of the square of the original units of measurement. If the original measurements are in inches, the variance is expressed in square inches. Taking the square root of the variance, we obtain the **standard deviation**, which returns the measure of variability to the original units of measurement.

Definition The **standard deviation** of a set of measurements is equal to the positive square root of the variance.

NOTATION

n : number of measurements in the sample

s^2 : sample variance

$s = \sqrt{s^2}$: sample standard deviation

N : number of measurements in the population

σ^2 : population variance

$\sigma = \sqrt{\sigma^2}$: population standard deviation