Homework 5

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Question 1

1. **(20 points)** Recall that that the adjacency matrix A of a graph G is such that $A(i, j) = 1$ if nodes i and j are adjacent and $A(i, j) = 0$ otherwise. Let $\alpha_k(i, j)$ be the number of paths of length k between nodes i and j.For instance, the number of paths of length-1 between nodes i and j in a simple undirected graph is 1 if they are adjacent and zero otherwise. Show that one can determine $\alpha_2(i, j)$ by determining the (i, j)-th entry of A^2, i.e., the square of A.

- To determine the number of paths of length 2 between nodes i and j, we use matrix multiplication of the adjacency matrix A with itself, A^2.
- Each entry (i, j) in the resulting matrix A^2 will represent the number of paths of length 2 between nodes i and j.
- The (i, j)-th entry of A^2 , denoted as (A^2)(i, j) is calculated as follows:

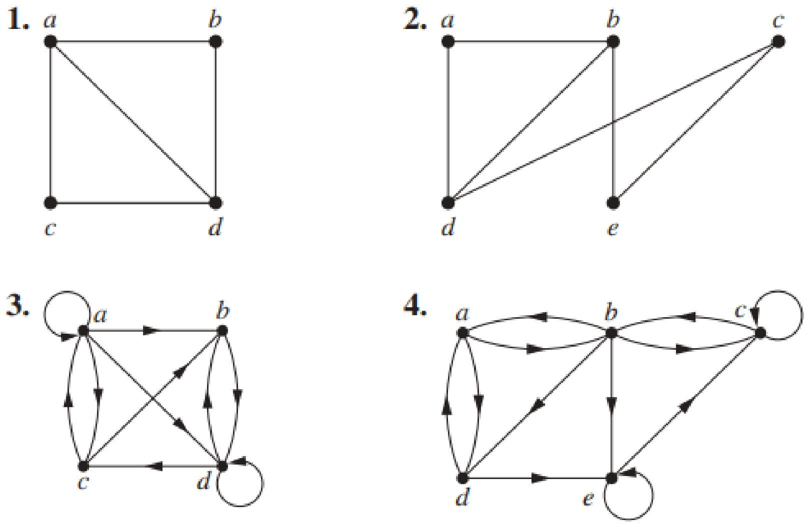
$$(A^2)(i, j) = \sum_{m=1}^{n} A(i, m) \times A(m, j)$$

where:

- A(i, m): The entry in the i-th row and m-th column of matrix A.
- A(m, j): The entry in the m-th row and j-th column of matrix A.
- n: Number of nodes in the graph

- If A(i, m) = 1 and A(m, j) = 1, it means that there is an edge from node i to node m and from node m to node j, forming a path of length 2 from node i to node j.
- If either A(i, m) = 0 or A(m, j) = 0, it means that there is no direct edge from node i to node m or from node m to node j. Hence, there is no path of length 2 from node i to node j through node m.
- Therefore, by calculating the (i, j)-th entry of A^2, we get $\alpha_2(i, j)$, the number of paths of length 2 between nodes i and j.

Question 2



7. Represent the graph in Exercise 3 with an adjacency matrix.

8. Represent the graph in Exercise 4 with an adjacency matrix.

7)

$$\tilde{A} = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

8)

$$\tilde{A} = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

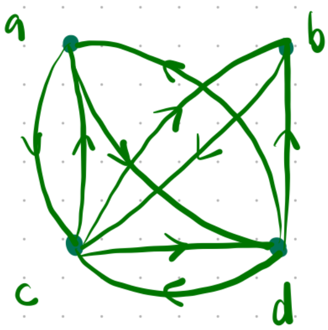
Question 3

In Exercises 10–12 draw a graph with the given adjacency matrix.

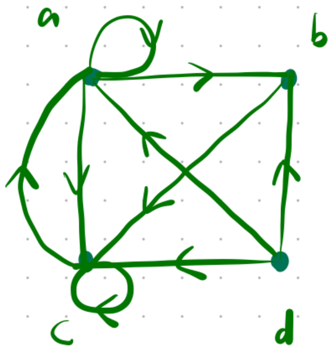
11. $\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

12. $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

11)



12)



Question 4

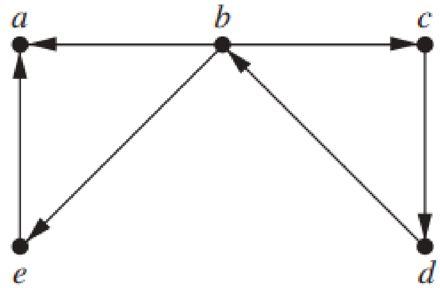
4. **(10 points)** What is the sum of the rows of the adjacency matrix for an undirected graph? For a directed graph?

- The sum of the rows of the adjacency matrix for both undirected and directed graphs provides information about the degree of each vertex in the graph.
  - Undirected graph: Each edge contributes to the degree of two vertices as there is no distinction between the start and end points. Hence, the sum of the rows of adjacency matrix for undirected graph will results in twice the degree of corresponding vertex.
  - Directed graph: The adjacency matrix can be asymmetric, as edges have a direction. Hence, the sum of each row in the adjacency matrix of directed graphs represents the out-degree of the corresponding vertex.

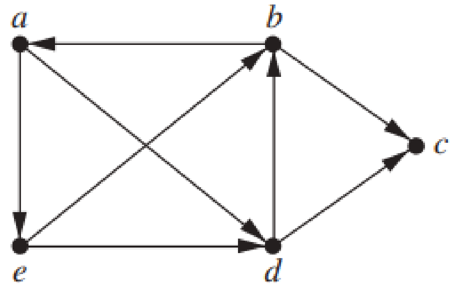
Question 5

11. Determine whether each of these graphs is strongly connected and if not, whether it is weakly connected.

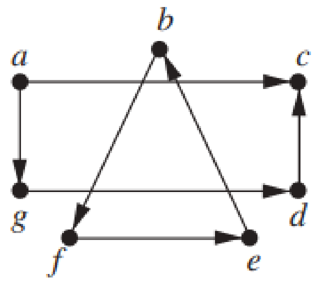
a)



b)



c)



a)

- This graph is not a strongly connected graph as it is not possible to reach every vertex from every other vertex in all cases. One of the evidence is that there is a path from vertex d to a but not from vertex a to d.
- However, this is a weakly connected graph as the underlying graph is connected.

b)

- This graph is not a strongly connected graph as it is not possible to reach every vertex from every other vertex in all cases. One of the evidence is that there is a path from vertex b to c but not from vertex c to b.
- However, this is a weakly connected graph as the underlying graph is connected.

c)

- This graph is not a strongly connected graph as it is not possible to reach every vertex from every other vertex in all cases. One of the evidence is that there is a path from vertex d to c but not from vertex c to d.
- This graph is also not a weakly connected graph as the underlying graph is not fully connected. One of the evidence is that there is no way that vertices a, c, g, d can reach vertices b, f, e.