Homework 7

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23. How many positive integers between 100 and 999 inclusive

a) are divisible by 7?
b) are odd?
c) have the same three decimal digits?
d) are not divisible by 4?
e) are divisible by 3 or 4?
f) are not divisible by either 3 or 4?
g) are divisible by 3 but not by 4?
h) are divisible by 3 and 4?

a) divisible by 7

- First, find the number of multiples of 7 between 100 and 999 inclusive.
- First multiple of 7 greater than or equal to 100 is 105.
- Last multiple of 7 less than or equal to 999 is 994.
- To find the count:
    - Count = $(994 - 105)/7 + 1 = 128$
- Hence, there are 128 positive integers between 100 and 999 that are divisible by 7.

b) odd

- Every other number in a consecutive sequence is odd.
- So, the numbers between 100 and 999 is half of the total numbers:
    - Count = $(999 - 100)/2 + 1 = 450$

- Hence, there are 450 odd positive integers between 100 and 999.

c) same three decimal digit

- 111, 222, 333, 444, 555, 666, 777, 888, 999
- Count = 9
- Hence, there are 9 positive integers with same three decimal digit.

d) not divisible by 4

- First, find the number of multiples of 4 between 100 and 999 inclusive.
- First multiple of 4 greater than or equal to 100 is 100.
- Last multiple of 4 less than or equal to 999 is 996.
- To find the total number of multiplies of 4 between the range:
  - total = $(996 - 100)/4 + 1 = 225.$
- To find the positive integers that are not divisible by 4:
  - Count = $999 - 100 - 225 + 1 = 675$
- Hence, there are 675 positive integers that are not divisible by 4.

e) are divisible by 3 or 4

- First, find the number of multiples of 4 between 100 and 999 inclusive.
- First multiple of 4 greater than or equal to 100 is 100.
- Last multiple of 4 less than or equal to 999 is 996.
- To find the total number of multiplies of 4 between the range:
  - total = $(996 - 100)/4 + 1 = 225.$

- Then, find the number of multiples of 3 between 100 and 999 inclusive.
- First multiple of 3 greater than or equal to 100 is 102.
- Last multiple of 3 less than or equal to 999 is 999.
- To find the total number of multiplies of 3 between the range:

- total = $(999 - 102)/3 + 1 = 300$.

- Lastly, find the number of multiples 3 and 4 between 100 and 999.
- First multiple of 3 and 4 greater than or equal to 100 is 108.
- Last multiple of 3 and 4 less than or equal to 999 is 996.
- To find the total number of multiplies of 3 and 4 between the range:
  - total = $(996 - 108)/12 + 1 = 75$.

- Hence, the count of numbers divisible by 3 or 4 = $300 + 225 - 75 = 450$

f) not divisible by either 3 or 4

- First, find the number of multiples of 4 between 100 and 999 inclusive.
- First multiple of 4 greater than or equal to 100 is 100.
- Last multiple of 4 less than or equal to 999 is 996.
- To find the total number of multiplies of 4 between the range:
  - total = $(996 - 100)/4 + 1 = 225$.

- Then, find the number of multiples of 3 between 100 and 999 inclusive.
- First multiple of 3 greater than or equal to 100 is 102.
- Last multiple of 3 less than or equal to 999 is 999.
- To find the total number of multiplies of 3 between the range:
  - total = $(999 - 102)/3 + 1 = 300$.

- Lastly, find the number of multiples 3 and 4 between 100 and 999.
- First multiple of 3 and 4 greater than or equal to 100 is 108.
- Last multiple of 3 and 4 less than or equal to 999 is 996.
- To find the total number of multiplies of 3 and 4 between the range:
  - total = $(996 - 108)/12 + 1 = 75$.

- Hence, the count of positive integers divisible by 3 or 4 = $300 + 225 - 75 = 450$
- To get the count of positive integers not divisible by either 3 or 4:
  - Count = $999 - 100 + 1 - 450 = 450.$
- Hence, the count of positive integers not divisible by either 3 or 4 is 450.

g) divisible by 3 but not by 4

- First, find the number of multiples of 3 between 100 and 999 inclusive.
- First multiple of 3 greater than or equal to 100 is 102.
- Last multiple of 3 less than or equal to 999 is 999.
- To find the total number of multiplies of 3 between the range:
  - total = $(999 - 102)/3 + 1 = 300.$

- Then, find the number of multiples 3 and 4 between 100 and 999.
- First multiple of 3 and 4 greater than or equal to 100 is 108.
- Last multiple of 3 and 4 less than or equal to 999 is 996.
- To find the total number of multiplies of 3 and 4 between the range:
  - total = $(996 - 108)/12 + 1 = 75$

- Hence, the count of positive integers divisible by 3 but not by 4:
  - Count = $300 - 75 = 225.$

h) divisible by 3 and 4

- First, find the number of multiples 3 and 4 between 100 and 999.
- First multiple of 3 and 4 greater than or equal to 100 is 108.
- Last multiple of 3 and 4 less than or equal to 999 is 996.
- To find the total number of multiplies of 3 and 4 between the range:

- Count = $(996 - 108)/12 + 1 = 75$

34. How many different functions are there from a set with 10 elements to sets with the following numbers of elements?

a) 2 **b)** 3 **c)** 4 **d)** 5

a) 2^{10}

b) 3^{10}

c) 4^{10}

d) 5^{10}

4. A bowl contains 10 red balls and 10 blue balls. A woman selects balls at random without looking at them.

a) How many balls must she select to be sure of having at least three balls of the same color?

b) How many balls must she select to be sure of having at least three blue balls?

a) She needs to select **5 balls** to be sure of having at least three balls of the same color.

- It is possible that she may selected two red balls and two blue balls.
- When she picks another ball, it is ensured that she will have either three red balls or three blue balls.

b) She needs to select **13 balls** to be sure of having at least three blue balls.

- It is possible that she may selected ten red balls in her first ten tries.
- If that so, the next three balls should be confirmed to be blue balls.

18. How many numbers must be selected from the set $\{1, 3, 5, 7, 9, 11, 13, 15\}$ to guarantee that at least one pair of these numbers add up to 16?

- The pair of sum that results in 16 are:
  - (15, 1)
  - (13, 3)
  - (11, 5)
  - (9, 7)
- The worst case scenario is that we picked a number from each pair that sums to 16.
- Hence, we need at least 5 numbers to guarantee that at least one pair of those numbers add up to 16.

5. **(10 points)** A computer programming team has 15 members.

   (a) How many ways can a group of seven be chosen to work on a project?

   (b) Suppose nine team members are SE students and six are CPRE students.

      i. How many groups of seven can be chosen that contain four SE and three CPRE students?

      ii. How many groups of seven can be chosen that contain at least one SE student?

      iii. How many groups of seven can be chosen that contain at most four CPRE students?

Equation for combination:

$$C(n, r) = \frac{n!}{r!(n-r)!}$$

with $0 \le r \le n$, r-combinations of a set with n elements

a) n = 15, r = 7

$$C(15,7) = \frac{15!}{7!(15-7)!} = 6,435 \text{ ways}$$

b) i.

- SE: n = 9, r = 4

$$C(9,4) = \frac{9!}{4!(9-4)!} = 126 \text{ ways}$$

- CPRE: n = 6, r = 3

$$C(6,3) = \frac{6!}{3!(6-3)!} = 20 \text{ ways}$$

- Total = $126 \times 20 = 2,520$ ways.

ii.

- The total ways of a group of 7 can be chosen, n = 15, r = 7:

$$C(15,7) = \frac{15!}{7!(15-7)!} = 6,435 \text{ ways}$$

- The total ways of a group can be chosen with all CPRE students, n = 6, r = 7

$$C(6,7) = \frac{6!}{7!(6-7)!} = 0 \text{ ways}$$
$$\text{Since n} < \text{r}$$

- Hence, the number of ways that a group of seven that contain a least one SE student can be chosen is 6435 ways.

iii.

<u>0 CPRE students:</u>

$$C(9,7) = \frac{9!}{7!(9-7)!} = 36 \text{ ways}$$

1 CPRE students:

$$C(9,6) = \frac{9!}{6!(9-6)!} = 84 \times 6 = 504 \text{ ways}$$

2 CPRE students:

- SE students

$$C(9,5) = \frac{9!}{5!(9-5)!} = 126 \text{ ways}$$

- CPRE students

$$C(6,2) = \frac{6!}{2!(6-2)!} = 15 \text{ ways}$$

- Total = $126 \times 15 = 1890$ ways.

3 CPRE students:

- SE students

$$C(9,4) = \frac{9!}{4!(9-4)!} = 126 \text{ ways}$$

- CPRE students

$$C(6,3) = \frac{6!}{3!(6-3)!} = 20 \text{ ways}$$

- Total = $126 \times 20 = 2520$ ways.

4 CPRE students:

- SE students

$$C(9,3) = \frac{9!}{3!(9-3)!} = 84 \text{ ways}$$

- CPRE students

$$C(6,4) = \frac{6!}{4!(6-4)!} = 15 \text{ ways}$$

- Total = $84 \times 15 = 1260$ ways.

- Total number of ways where at most 4 out of 7 group members are CPRE:

$1260 + 2520 + 1890 + 504 + 36 = 6,210$ ways.

6. **(10 points)** If there are 4 colors of jellybeans and you are trying to fill up a jar that holds 100 beans, how many different color combinations exist (assuming no restrictions on the distributions of the colors)?

$$C(100 + 4 - 1, 4 - 1) = \frac{103!}{3!(103-3)!}$$

$$\frac{103 \times 102 \times 101}{3 \times 2 \times 1} = 176,851$$

7. **(10 points)** In how many ways can you place 2 identical rooks on an 8×8 chessboard such that they will not be able to capture each other (i.e., they do not share the same row or column).

$$C(64, 2) = \frac{64!}{2!(64-2)!} = 2016 \text{ ways}$$

- Assume that the first rook is placed at (1, 1).
- The other rook can the be placed at other 49 squares without being attack.
- If the first rook is placed at (1, 2), the other rook can still be placed at another 49 squares without being attack.
- Hence, every time that a rook is placed at one of the 64 squares, the other rook can safely be placed in 49 squares.
- To find the total number of safe placements, we sum up the number of safe placements for each possible placement of the first rook.
- Since there are 64 possible placement s for the first rook, the total number of safe placements: $64 \times 49 = 3136$.
- However, we have counted each safe placement twice (once for each rook).

- So, we need to divide by 2 to avoid double counting =
    - Correct number of ways to place 2 identical rooks on an 8×8 chessboard = $3136 \div 2 = 1568$.