Homework 1

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Problem 1.1.10

10. Let p and q be the propositions

p: I bought a lottery ticket this week.

q: I won the million dollar jackpot.

Express each of these propositions as an English sentence.

a) $\neg p$ **b)** $p \vee q$ **c)** $p \rightarrow q$

d) $p \wedge q$ **e)** $p \leftrightarrow q$ **f)** $\neg p \rightarrow \neg q$

g) $\neg p \wedge \neg q$ **h)** $\neg p \vee (p \wedge q)$

a) I didn't bought a lottery ticket this week.

b) I bought a lottery ticket this week or I won the million dollar jackpot.

c) If I bought a lottery ticket this week, then I won the million dollar jackpot.

d) I bought a lottery ticket this week and I won the million dollar jackpot.

e) I bought a lottery ticket this week if and only if I won the million dollar jackpot.

f) If I didn't bought a lottery ticket this week, then I didn't win the million dollar jackpot.

g) I didn't bought a lottery ticket this week and I didn't win the million dollar jackpot.

h) I didn't bought a lottery ticket this week or I bought a lottery ticket this week and I won the million dollar jackpot.

Problem 1.1.14

14. Let p, q, and r be the propositions

p: You have the flu.

q: You miss the final examination.

r: You pass the course.

Express each of these propositions as an English sentence.

a) $p \rightarrow q$ **b)** $\neg q \leftrightarrow r$

c) $q \rightarrow \neg r$ **d)** $p \vee q \vee r$

e) $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$ **f)** $(p \wedge q) \vee (\neg q \wedge r)$

a) If you have the flu, then you miss the final examination.

b) You don't miss the final examination if and only if you pass the course.

c) If you miss the final examination, then you fail the course.

d) You have the flu or you miss the final examination or you pass the course.

e) If you have the flu, then you fail the course or if you miss the final examination, then you fail the course.

f) You have the flu and you miss the final examination or you don’t miss the final examination and you pass the course.

Problem 1.1.29

29. State the converse, contrapositive, and inverse of each of these conditional statements.

a) If it snows today, I will ski tomorrow.

b) I come to class whenever there is going to be a quiz.

c) A positive integer is a prime only if it has no divisors other than 1 and itself.

a)

- Converse: If I ski tomorrow, then it has snowed today.
- Contrapositive: If I don’t ski tomorrow, then it doesn’t snows today.
- Inverse: If it doesn’t snows today, then I will not ski tomorrow.

b)

- Converse: If there is going to be a quiz, then I will come to class.
- Contrapositive: If there will be no quiz, then I may not come to class
- Inverse: If I don’t come to class, then there is no quiz.

c)

- Converse: If a positive integer has no divisors other than 1 itself
- Contrapositive: If a positive integer has divisors other than 1 and itself, then the it is not a prime number.
- Inverse: A positive integer is not a prime if it has divisors other than 1 and itself.

Problem 1.1.34

34. Construct a truth table for each of these compound propositions.

a) $p \to \neg p$ **b)** $p \leftrightarrow \neg p$

c) $p \oplus (p \vee q)$ **d)** $(p \wedge q) \to (p \vee q)$

e) $(q \to \neg p) \leftrightarrow (p \leftrightarrow q)$

f) $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

a)

p	¬p	p →¬p
T	F	F
F	T	T

b)

p	¬p	p ↔¬p

T	F	F
F	T	F

c)

p	q	(p∨q)	p⊕(p∨q)
T	T	T	F
T	F	T	F
F	T	T	T
F	F	F	F

d)

p	q	(p∧q)	(p∨q)	(p∧q) → (p∨q)
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

e)

p	q	¬p	(q → ¬p)	(p ↔q)	(q → ¬p) ↔(p ↔q)
T	T	F	F	T	F
T	F	F	T	F	F
F	T	T	T	F	F
F	F	T	T	T	T

f)

p	q	¬q	(p ↔ q)	(p ↔¬q)	(p ↔ q)⊕(p ↔¬q)
T	T	F	T	F	T
T	F	T	F	T	T
F	T	F	F	T	T
F	F	T	T	F	T

Problem 1.3.12

12. Show that each of these conditional statements is a tautology by using truth tables.

a) $[\neg p \wedge (p \vee q)] \rightarrow q$
b) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$
c) $[p \wedge (p \rightarrow q)] \rightarrow q$
d) $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$

a)

p	q	¬p	(p∨q)	[¬p∧(p∨q)]	[¬p∧(p∨q)] → q

T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

b)

p	q	r	$(p \to q)$	$(q \to r)$	$[(p \to q) \land (q \to r)]$	$(p \to r)$	$[(p \to q) \land (q$ - $r)] \to (p \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

c)

p	q	$(p \to q)$	$[p \land (p \to q)]$	$[p \land (p \to q)] \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

d)

p	q	r	$(p \lor q)$	$(p \to r)$	$(q \to r)$	$[(p \lor q) \land (p \to r) \land (q \to r)]$	$[(p \lor q) \land (p \to r) \land (q \to r)] \to$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	T	T	T	T	T
T	F	F	T	F	T	F	T
F	T	T	T	T	T	T	T
F	T	F	T	T	F	F	T
F	F	T	F	T	T	F	T
F	F	F	F	T	T	F	T

<u>Problem 1.4.8</u>

8. Translate these statements into English, where $R(x)$ is "x is a rabbit" and $H(x)$ is "x hops" and the domain consists of all animals.

a) $\forall x(R(x) \to H(x))$ **b)** $\forall x(R(x) \land H(x))$
c) $\exists x(R(x) \to H(x))$ **d)** $\exists x(R(x) \land H(x))$

a) For every animal x, if x is a rabbit, then x hops: All rabbits hop.

b) For every animal x, x is a rabbit and x hops: All animals that are rabbits also hop.

c) There exists an animal x such that if x is a rabbit, then x hops: There is a rabbit that hops.

d) There exists an animal x such that x is a rabbit and hops: There is an animal that is both a rabbit and it hops.

Problem 1.4.9

9. Let $P(x)$ be the statement "x can speak Russian" and let $Q(x)$ be the statement "x knows the computer language C++." Express each of these sentences in terms of $P(x)$, $Q(x)$, quantifiers, and logical connectives. The domain for quantifiers consists of all students at your school.

a) There is a student at your school who can speak Russian and who knows C++.

b) There is a student at your school who can speak Russian but who doesn't know C++.

c) Every student at your school either can speak Russian or knows C++.

d) No student at your school can speak Russian or knows C++.

a) $\exists x\ (P(x) \wedge Q(x))$

b) $\exists x\ (P(x) \wedge \neg Q(x))$

c) $\forall x\ (P(x) \vee Q(x))$

d) $\forall x\ \neg(P(x) \vee Q(x))$

Problem 1.4.36

36. Express the negation of each of these statements in terms of quantifiers without using the negation symbol.

a) $\forall x(-2 < x < 3)$

b) $\forall x(0 \leq x < 5)$

c) $\exists x(-4 \leq x \leq 1)$

d) $\exists x(-5 < x < -1)$

a) $\exists x\ (x \leq -2 \vee x \geq 3)$

b) $\exists x(x < 0 \vee x \geq 5)$

c) $\forall x\ (x < -4 \vee x > 1)$

d) $\forall x\ (x \leq -5 \vee x \geq -1)$

Problem 1.4.38

38. Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all real numbers.

a) $\forall x(x^2 \neq x)$ **b)** $\forall x(x^2 \neq 2)$
c) $\forall x(|x| > 0)$

a) Counterexample: x = 0, as 0 to the power of 2 is equal to 0.

b) Counterexample: x = square root of 2, as power of 2 to the square root of 2 will result in 2.

c) Counterexample: x = 0. The absolute value of 0 is not greater than 0.

Problem 1.5.28

28. Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.

a) $\forall x \exists y(x^2 = y)$ **b)** $\forall x \exists y(x = y^2)$
c) $\exists x \forall y(xy = 0)$ **d)** $\exists x \exists y(x + y \neq y + x)$

e) $\forall x(x \neq 0 \rightarrow \exists y(xy = 1))$
f) $\exists x \forall y(y \neq 0 \rightarrow xy = 1)$
g) $\forall x \exists y(x + y = 1)$
h) $\exists x \exists y(x + 2y = 2 \wedge 2x + 4y = 5)$
i) $\forall x \exists y(x + y = 2 \wedge 2x - y = 1)$
j) $\forall x \forall y \exists z(z = (x + y)/2)$

a) True. This statement is true for all real numbers. For any real number x power of 2, there is real number y that equals to it.

b) False. This statement is not valid for negative real number x as the square of any real number is always non negative.

c) True. If x is 0, then any real numbers multiplied by 0 will equal to 0.

d) False. The order of sum doesn’t affect the result. Hence x + y will always be equal to y + x.

e) True. For any non-zero x, y can be y = 1/x which results to 1 for xy = 1.

f) False. There is no a real number x that for every number y, excluding y = 0 results in xy = 1.

g) True. There exist a real number y that satisfies x + y = 1 as y = 1- x and then x + y = x + (1 - x) = 1.

h) False. There are no real numbers x and y that can satisfy both equation simultaneously. Example: if x = 0, y = 1 then the first equation is satisfiable but not the second equation.

i) False. For x = 1 which results in y = 1 results in both equation to be satisfiable simultaneously but when x is any other real numbers, it is impossible to satisfy both equation.

j) True. for any two real number x and y, there are real number z that is the average of x and y that can satisfy the equation.