# H8.2 Writeup

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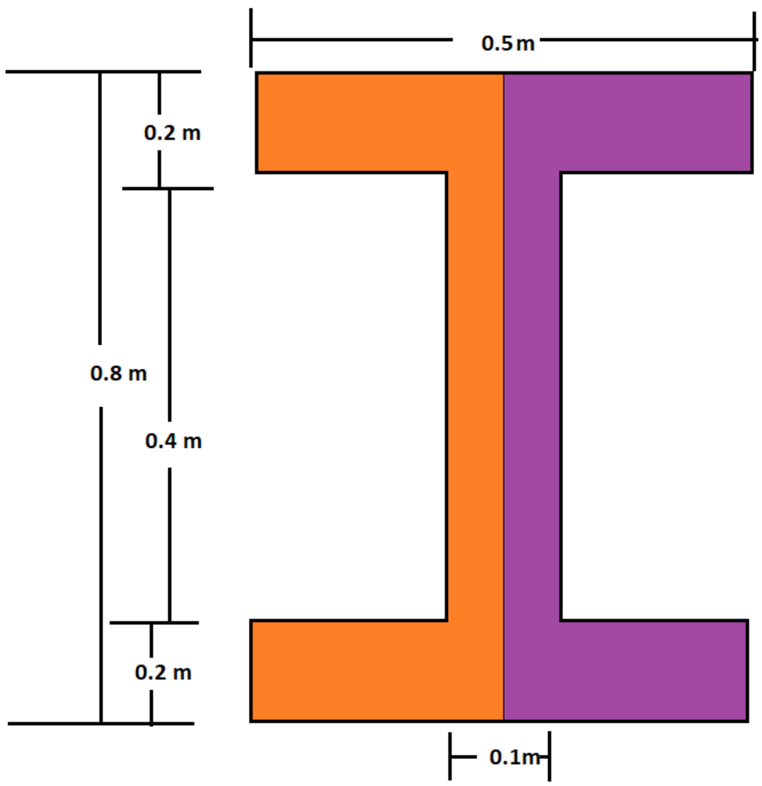
## Problem

You are designing a bi-layered I beam for fun. You have two materials to work with and need to make sure the beam is made of 50% by volume of each material. They are steel, with E = 210 GPA, v = 0.4 and $\rho = 7600$ kg/$m^3$ and "*Softened Daniellium*", with E = 5 GPA, v = 0.2, $\rho = 990\ kg/m^3$. You decide to have the material structured like this in the x-y plane:




Where orange is steel and the purple is softened daniellium. The shape extends for z=10 m in the +z direction.

a) Solve for the displacement of the centroid using Maple and verify in FlexPDE (use a magnified grid plot to ensure we can see the deformations). Assume that the force of gravity acts in the -y direction. Also assume that the surface is fixed at z=0 and free at z=10 m (cantilever setup).
b) Determine the displacement of the centroid if a force of 10kN acts on the z=10m in the -y direction. Use Maple or FlexPDE.

## Solution

a) Set x=0 to be at the centre of the shape and y=0 to be at the bottom of the shape. This makes the x<0 to be steel and x>0 to be the softened daniellium. Use these conditions to form piecewise functions for the various properties of the beam. Since we have piecewise functions that have conditions in both x and y, we must use a double integral for mu.

```
restart;
#Defining Material properties
Lz:= 10:
E:=piecewise(x<0, 210e9, x>0, 5e9):
nu:=piecewise(x<0, 0.4, x>0, 0.2):
rho:=piecewise(x<0, 7600, x>0, 990):
width:=piecewise(y<.0.2, 0.5, y<0.6, 0.1, y<0.8, 0.5);
mu:=int(int(width*rho, y=0..0.8),x=-0.25..0.25);
```

$$width := \begin{cases} 0.5 & y < 0. \\ 0.1 & y < 0.6 \\ 0.5 & y < 0.8 \end{cases}$$

$$\mu := 343.6000000$$

Use these properties to calculate the flexural rigidity and the weighted centroid:

```
Ybar:=int(width*E*y,y=0..0.8)/int(width*E,y=0..0.8); #AE weighted centroid of beam
EI:=int(int(width*E*(y-Ybar)^2, x=-0.25..0.25),y=0..0.8); #Flexural rigidity
```

$$Ybar := 0.5500000000$$

$$EI := 4.371666667\ 10^{8}$$

Since we know that the z=0 face is fixed and the z=10 m face is free then we have to set up the differential equations accordingly:

| Type of boundary | $V = EI\frac{\partial^3 v}{\partial x^3}$ | $M = EI\frac{\partial^2 v}{\partial x^2}$ | $\theta = \frac{\partial v}{\partial x}$ | $v$ |
|---|---|---|---|---|
| Free | 0 | 0 | ? | ? |
| Clamped (Fixed) | ? | ? | 0 | 0 |

The blue circles the conditions for the free end and pink circles the conditions for the fixed end. In Maple this is what it looks like:

```
#Now solve for shear and moment and plot them
M:=EI*diff(vFlexure,z,z); #Taken from last table in notes,notice how it is only based
on vFlexure
V:=diff(M,z);
plot([V,M],z=0..Lz, legend = ["Shear", "Moment"]);
```

$$v(z) = -\frac{64253069}{200000000000000}z^4 + \frac{64253069}{5000000000000}z^3 - \frac{192759207}{1000000000000}z^2$$

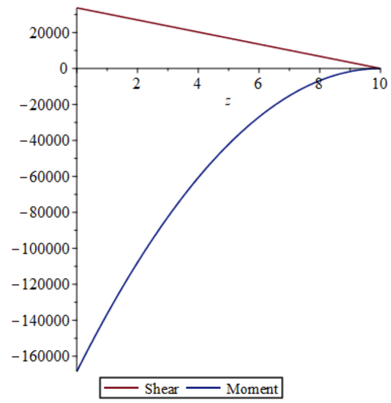
$$vFlexure := -3.212653450\ 10^{-7}\ z^4 + 0.00001285061380\ z^3 - 0.0001927592070\ z^2$$

Now we plot the shear and moment:

```
#Now solve for shear and moment and plot them
M:=EI*diff(vFlexure,z,z); #Taken from last table in notes,notice how it is only based
on vFlexure
V:=diff(M,z);
plot([V,M],z=0..Lz, legend = ["Shear", "Moment"]);
```

$$M := -1685.358000\, z^2 + 33707.16000\, z - 168535.8000$$

$$V := -3370.716000\, z + 33707.16000$$




Note how both plots converge to zero at the free end as they should.

Now we account for shear and calculate the displacement:

```
#Now plot displacement in y (should be negative)
plot([vFlexure,vShear,vTotal],z=0..Lz);

#Calculate the displacement at the centroid specifically
vShearCent:=int(gammazy,z=0..Lz/2);
vFlexureCent:=subs(z=Lz/2, vFlexure);
vTotalCent:=vShearCent+vFlexureCent;
```

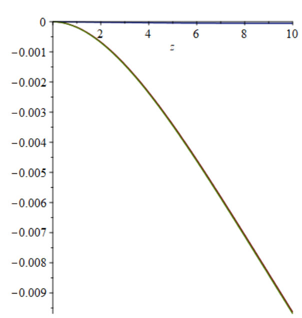
$$A := 0.1600000000$$

$$GCombined := 1.927083333\ 10^{10}$$

$$tauzy := 21066.97500\, z - 210669.7500$$

$$gammazy := 1.093205189\ 10^{-6}\, z - 0.00001093205189$$

$$vShear := 5.466025945\ 10^{-7}\, z^2 - 0.00001093205189\, z$$



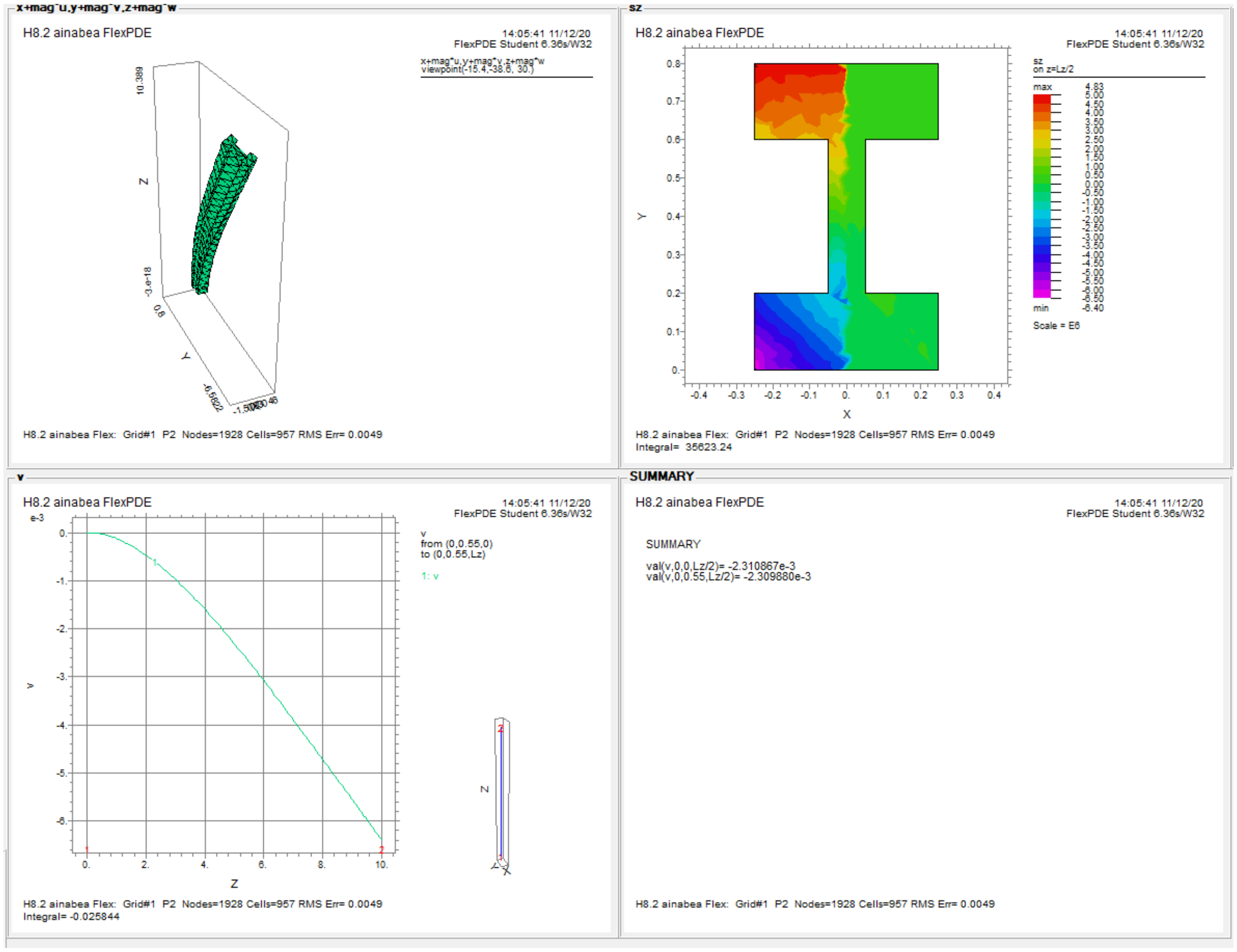

$$vShearCent := -0.00004099519459$$

$$vFlexureCent := -0.003413444291$$

$$vTotalCent := -0.003454439486$$

It appears that all the displacements follow the same trend. Notice how it does not go to zero. Because the end is free to move it will not stay in the same position it started in.

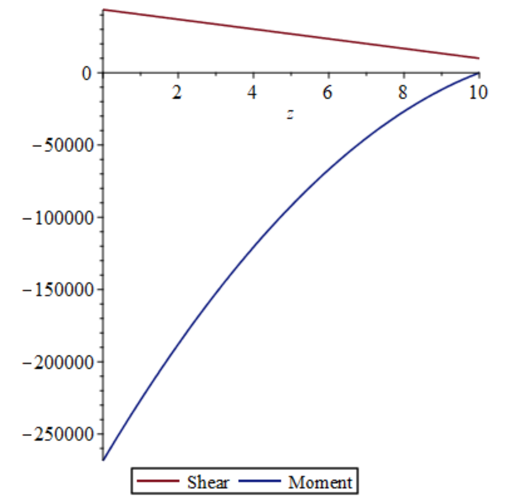
Verify results in FlexPDE:

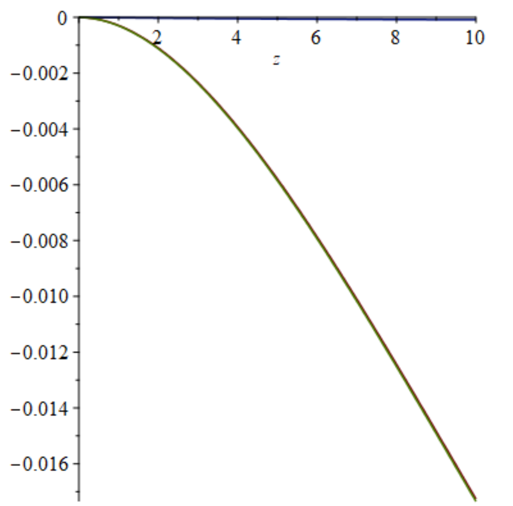



The results are similar in FlexPDE and Maple. I would prefer them to be closer, but this may be due to the rounding error in Flex.

b)

The only thing that changes for the maple code is the boundary condition for the third derivative at Lz. It is now equal to the absolute value of the new load/EI. (See attached load). Maple results:




$$vShearCent := -0.00005721141084$$

$$vFlexureCent := -0.005796212113$$

$$vTotalCent := -0.005853423524$$

The value of the displacement increased in the negative y direction which makes sense because we added more load there.

## Reflection

Solving this problem proved more difficult than I expected because I had width which varied in the y direction and every other material property which varied in the x direction. In order to get a lot of the values that involved width and another material I had to do a double integral. This problem helped me to figure out how to incorporate double integrals in Maple and better understand equations.

And since this beam was a cantilever, I had to modify the FlexPDE boundary conditions for z=Lz. At first, I didn't, and I was confused why all my plots looked off. But I eventually figured out where the boundary conditions were.

I think the hardest part of this problem was figuring it out conceptually and then applying the math was easier after that.

# Appendix A: FlexPDE Code

```
TITLE 'H8.2 ainabea FlexPDE'
COORDINATES cartesian3
VARIABLES       { system variables }
            u
            v
            w
SELECT       { method controls }
ngrid=20
DEFINITIONS    { parameter definitions }
mag=1e3

grav=9.81

Lz = 10

!piecewise functions
E=  if x<0 then 210e9 else 5e9
nu= if x<0 then 0.4 else 0.2
rho= if x<0 then 7600 else 990

G=E/(2*(1+nu))

C11 =E*(1-nu)/((1+nu)*(1-2*nu))
C22 = C11
C33 = C11

C12 = E*nu/((1+nu)*(1-2*nu))
C13 = C12
C21 = C12
C23 = C12
C31 = C12
C32 = C12

!!Strain
!Axial Strain
ex=dx(u)
ey=dy(v)
ez=dz(w)
!Engineering Shear Strain
gxy=dx(v)+dy(u)
gyz=dy(w)+dz(v)
gxz=dz(u)+dx(w)
```

```
!Stress via Hooke's Laws
!Axial Stress
sx = C11*ex + C12*ey + C13*ez
sy = C21*ex + C22*ey + C23*ez
sz = C31*ex + C32*ey + C33*ez
!Shear Stress
syz= G*gyz
sxz= G*gxz
sxy= G*gxy

EQUATIONS	{ PDE's, one for each variable }
!Fnet = 0
u:		dx(sx)+dy(sxy)+dz(sxz)=0
v:		dx(sxy)+dy(sy)+dz(syz)-rho*grav=0
w:	dx(sxz)+dy(syz)+dz(sz)=0

EXTRUSION
surface 'bottom' z=0
surface 'top' z=Lz

BOUNDARIES	{ The domain definition }
surface 'bottom'
	value(u)=0
	value(v)=0
	value(w)=0
surface 'top' !remember to say boundary conditions!
	!value(u)=0
	!value(v)=0
	!value(w)=0

  REGION 1	{ For each material region }
   START(-0.25,0)
	load(u)=0
	load(v)=0
	load(w)=0
	LINE TO(0.25,0)TO(0.25,0.2)TO(0.05,0.2)TO(0.05,0.6)TO(0.25,0.6)TO(0.25,0.8)TO(-0.25,0.8)TO(-
0.25,0.6)TO(-0.05,0.6)TO(-0.05,0.2)TO(-0.25,0.2)TO CLOSE
PLOTS
	grid(x+mag*u,y+mag*v,z+mag*w)
	contour(sz) painted on z=Lz/2
	contour(ez) painted on z=Lz/2
	elevation(v) from (0,0.55,0) to (0,0.55,Lz)
SUMMARY
	report val(v,0,0,Lz/2)
```

```
    report val(v,0,0.55,Lz/2)
END
```