H10.2 Writeup

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Problem

Let's say you have a bi-layered beam with one material having $E_1 = 100\ GPa, v_1 = 0.3, \rho_1 = 1000\ kg/m^3$ and the other material having $E_1 = 50\ GPa, v_1 = 0.5, \rho_1 = 3000\ kg/m^3$. The material is split equally in the y dimension between these two materials. It has the following dimensions: $L_x = 70\ cm, \mathrm{L_y} = 20\ \mathrm{cm\ and\ L_z} = 3\ \mathrm{m}.$

The material has pin supports at both ends.

a) Use the dynamic beam equation to find the 1st transverse resonant mode of the beam in the y-direction (report angular frequency and show a plot of mode shape)
b) Use FlexPDE to check your answer from part a), again reporting reporting the frequency found and showing the corresponding shape.

Solution

a)

Based on our results from the notes we know that:

$$v(z,t) = \hat{v}(z)\cos(\omega t + \phi) \rightarrow \omega^2 \hat{v}(z) = \frac{EI}{\mu}\hat{v}''''(z)$$

Therefore, the solution to the above differential equation is:

$$\hat{v}(z) = c_1\cos\beta z + c_2\sin\beta z + c_3\cosh\beta z + c_4\sinh\beta z \text{ where, } \omega = \beta^2\sqrt{\frac{EI}{\mu}}$$

Recall boundary conditions from unit 8, we are looking at the third row (pivot joint):

Type of boundary	$V = EI\dfrac{\partial^3 v}{\partial z^3}$	$M = EI\dfrac{\partial^2 v}{\partial z^2}$	$\theta = \dfrac{\partial v}{\partial z}$	v
Free	0	0	?	?
Clamped (Fixed)	?	?	0	0
Pivot joint (Pin)	?	0	?	0
Vertical Slider (Roller)	0	?	0	?

Therefore, the boundary conditions for this problem are:

At z = 0: $\hat{v}(0) = \hat{v}''(0) = 0$

At z = Lz: $\hat{v}(L_z) = \hat{v}''(L_z) = 0$

Now we solve in Maple:

```
#Solving DE
vh:=c1*cos(beta*z)+c2*sin(beta*z)+c3*cosh(beta*z)+c4*sinh(beta*z);
vhp:=diff(vh,z): vhpp:=diff(vhp,z): vhppp:=diff(vhpp,z):
c1:=solve(simplify(subs(z=0, vh)), c1);
simplify(subs(z=0, vhpp)); c3:=0:
vh;
c4:=solve(subs(z=Lz, vh), c4);
vh;
c2:=1: #Making sure all the constants are solved and accounted for
Lz:=3:
GenFn:=simplify(subs(z=Lz, vhp/beta));
```

$$vh := c1 \cos(\beta z) + c2 \sin(\beta z) + c3 \cosh(\beta z) + c4 \sinh(\beta z)$$

$$c1 := -c3$$

$$2\, c3\, \beta^2$$

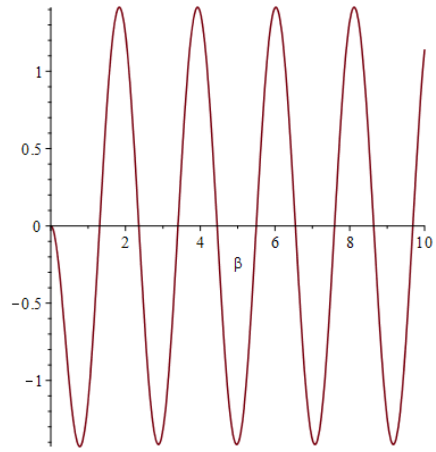
$$c2 \sin(\beta z) + c4 \sinh(\beta z)$$

$$c4 := -\frac{c2 \sin(\beta Lz)}{\sinh(\beta Lz)}$$

$$c2 \sin(\beta z) - \frac{c2 \sin(\beta Lz) \sinh(\beta z)}{\sinh(\beta Lz)}$$

$$GenFn := \frac{\cos(3\beta) \sinh(3\beta) - \sin(3\beta) \cosh(3\beta)}{\sinh(3\beta)}$$

```
#Finding transverse resonant modes
plot(GenFn, beta=0..10);
beta1:=fsolve(GenFn, beta=1..2);
beta2:=fsolve(GenFn, beta=2..3);
beta3:=fsolve(GenFn, beta=3..4);
vh1:=subs(beta=beta1, vh);
vh2:=subs(beta=beta2, vh);
vh3:=subs(beta=beta3, vh);
plot([vh1, vh2, vh3], z=0..Lz);
```



$$\beta 1 := 1.308867437$$

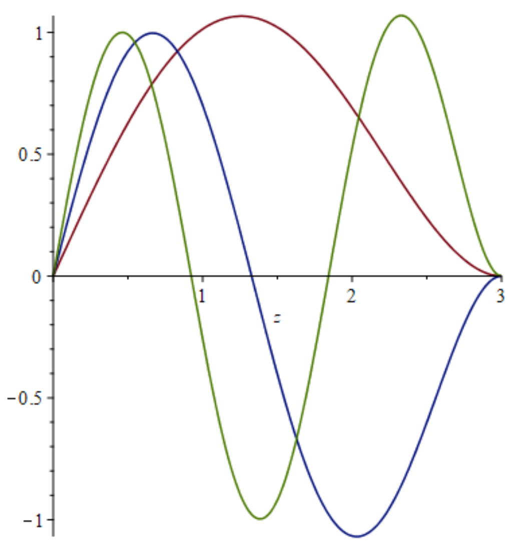
$$\beta 2 := 2.356194249$$

$$\beta 3 := 3.403392041$$

$$vh1 := \sin(1.308867437\, z) - \frac{\sin(3.926602311)\,\sinh(1.308867437\, z)}{\sinh(3.926602311)}$$

$$vh2 := \sin(2.356194249\, z) - \frac{\sin(7.068582747)\,\sinh(2.356194249\, z)}{\sinh(7.068582747)}$$

$$vh3 := \sin(3.403392041\, z) - \frac{\sin(10.21017612)\,\sinh(3.403392041\, z)}{\sinh(10.21017612)}$$



Note, for the above section of code we used the values of Beta we can see in the first plot to determine the range for solving for the first three solutions.

```
#Now we solve for frequency
Lx:=.7: Ly:=.2:
E:=piecewise(y>0, 100e9, y>Ly/2, 50e9): #Piecwise function for elasticity
EI:=int(E*Lx*(y)^3/12, y=0..Ly); #Flexural rigidity
rho:=piecewise(y>0, 1000, y>Ly/2, 3000):#Basically repeat work for E
mu:=int(rho, y=0..Ly)*Lx*Ly:
omega1:=beta1^2*sqrt(EI/mu);
omega2:=beta2^2*sqrt(EI/mu);
omega3:=beta3^2*sqrt(EI/mu);
```

$$EI := 2.333333333\ 10^{6}$$

$$\omega 1 := 494.5391788$$

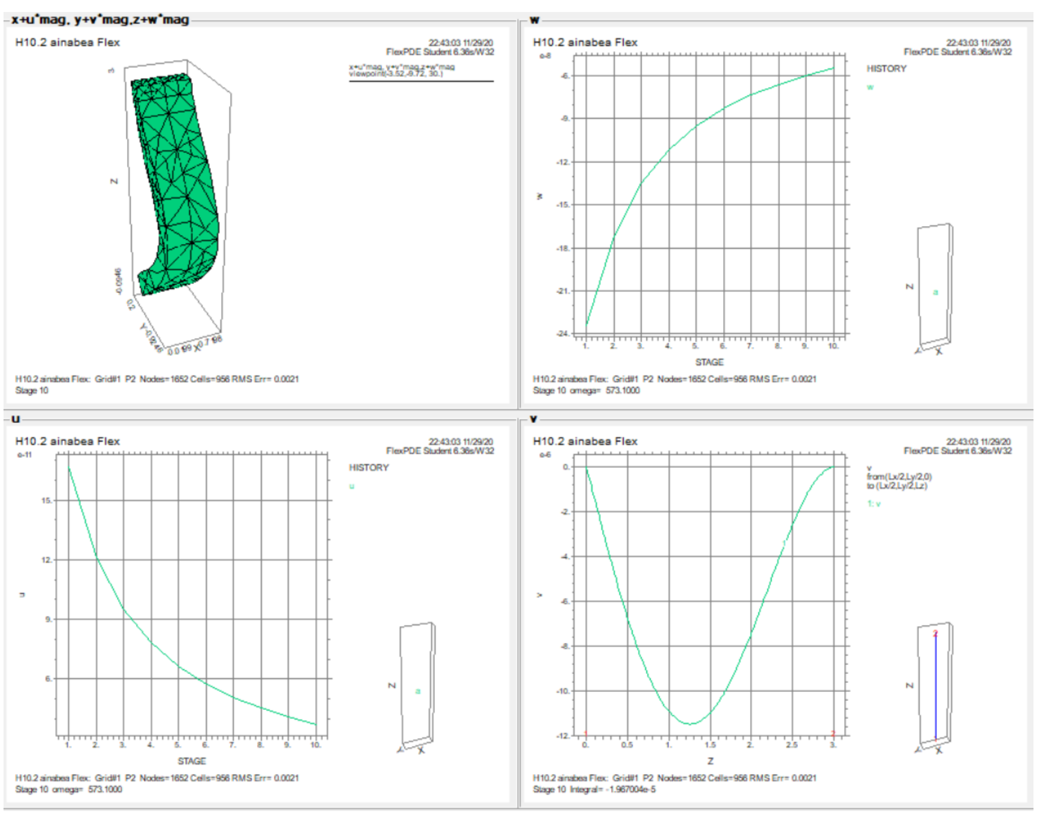
$$\omega 2 := 1602.623698$$

$$\omega 3 := 3343.746422$$

Therefore, according to Maple, the first resonant frequency is 494.5 rad/s

b)

See appendix for the FlexCode:



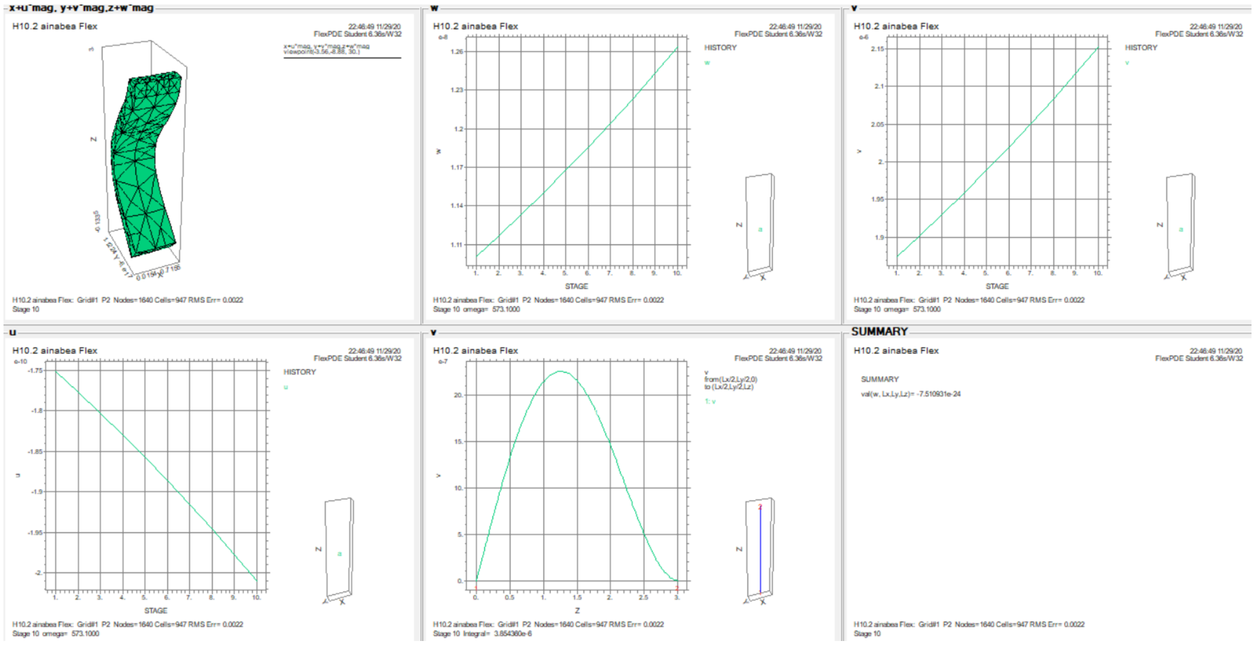
According to FlexPDE the result is: 572.1+.1*stage rad/s which is a bit off from our Maple calculations but close enough.

Reflection

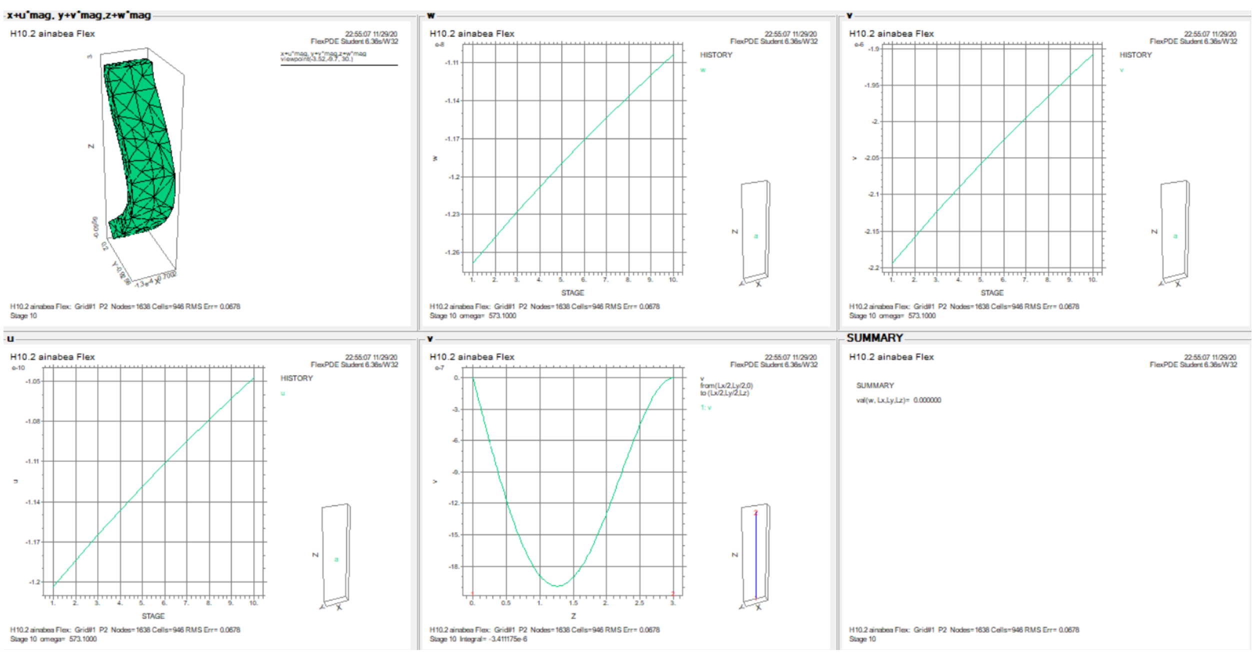
Applying the same method, I used in H8.2 for a bi-layered beam was straightforward for the Maple code. The only difference was the formula for flexural rigidity.

Also, I was curious as to why the Maple code did not require the Poisson's ratio. But the Flex code did. I guessed that maybe the Poisson's ratio does not have much affect on the resonant frequency. So, I ran a simulation with various values for the Poisson's ratio of the beam. You can see the results below. The frequency does not change, much if at all. Therefore, the Poisson's ratio does not have much effect.

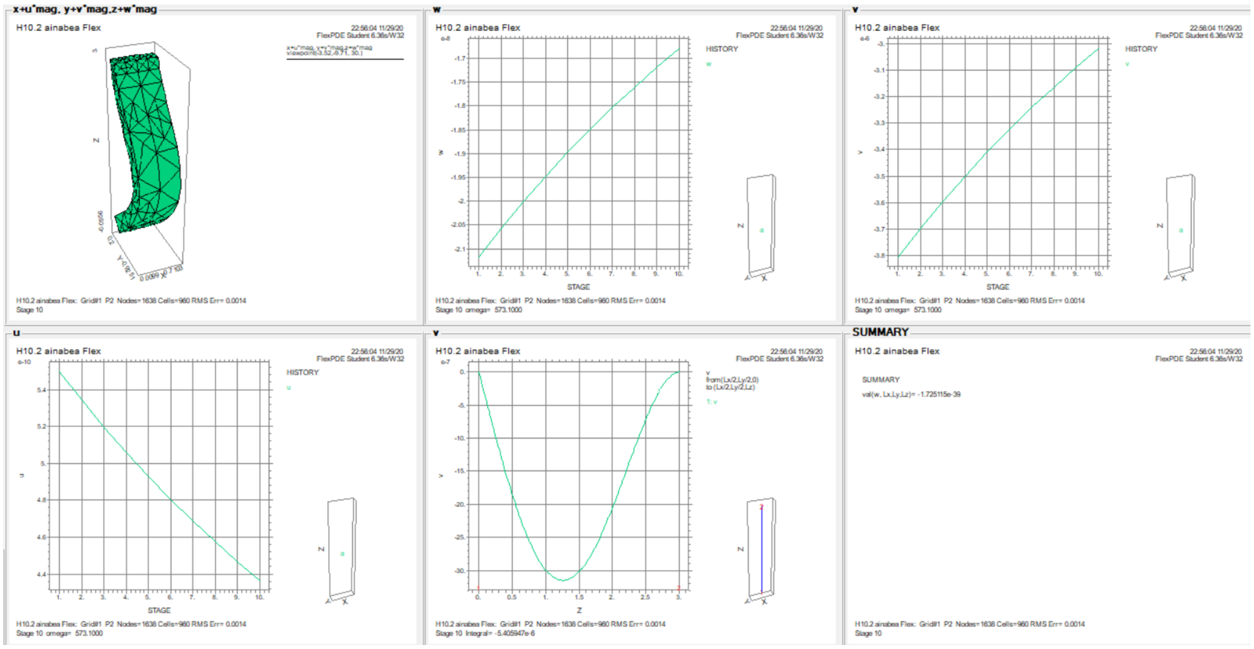
0.4:


x+u*mag, y+v*mag,z+w*mag
w
v
u
v
SUMMARY
H10.2 ainabea Flex

0:


x+u*mag, y+v*mag,z+w*mag
w
v
u
v
SUMMARY
H10.2 ainabea Flex

0.2:


x+u*mag, y+v*mag,z+w*mag
H10.2 ainabea Flex
w
HISTORY
STAGE
v
u
SUMMARY
val(w, Lx,Ly,Lz)= -1.725115e-39

Appendix A: FlexPDE Code

```
TITLE 'H10.2 ainabea Flex'     { the problem identification }
COORDINATES cartesian3  { coordinate system, 1D,2D,3D, etc }
VARIABLES       { system variables }
u
v
w
SELECT        { method controls }
!spectral_colors
!ngrid=15
stages = 10

DEFINITIONS    { parameter definitions }
mag = .3*globalmax(magnitude(x,y,z))/globalmax(magnitude(u,v,w))

Lx = .7
Ly = .2
Lz = 3

omega=572.1+.1*stage !determined through trial error

E = if(y<Ly/2) then 100e9 else 50e9
nu = if(y<Ly/2) then 0.3 else 0.5
rho = if(y<Ly/2) then 1000 else 3000
G = E/(2*(1+nu))

ex = dx(u)
ey = dy(v)
ez = dz(w)

gyz = dz(v) + dy(w)
gxz = dz(u) + dx(w)
gxy = dy(u) + dx(v)

C11 = E/((1+nu)*(1-2*nu))*(1-nu)
C12 = E/((1+nu)*(1-2*nu))*nu
C13 =C12
C21 =C12
C22 = C11
C23 = C12
C31 = C12
C32 = C12
C33 = C11
```

```
sx = C11*ex + C12*ey+C13*ez
sy = C21*ex + C22*ey+C23*ez
sz = C31*ex + C32*ey+C33*ez

syz = G*gyz
sxz = G*gxz
sxy = G*gxy

phi = atan2(y,x)
xp = x+u
yp = y+v
thetatest = atan2(yp,xp)-phi
theta = if(thetatest<-pi) then thetatest+2*pi else if(thetatest > +pi) then thetatest-2*pi else thetatest

! INITIAL VALUES
EQUATIONS       { PDE's, one for each variable }
u: dx(sx) + dy(sxy) + dz(sxz) = -rho*omega^2*u
v: dx(sxy) + dy(sy) + dz(syz) =  -rho*omega^2*v
w: dx(sxz) + dy(syz) + dz(sz) =  -rho*omega^2*w

EXTRUSION
surface 'bottom' z = 0
surface 'top' z = Lz

BOUNDARIES      { The domain definition }
surface 'bottom'
value(u) = 0
value(v) = 0
load(w) = 0
surface 'top'
value(u) = 0
value(v) = 0
value(w) = 0

!load(w) = 735294.1176*y+73529.41176


  REGION 1      { For each material region }
    START (0,0) line to (Lx,0)
line to (Lx,Ly)
load(v) = 5  line to (0,Ly)
load(v) = 0          line to close
            !start(Lx,0) arc(center=0,0) angle=360
```

```
! TIME 0 TO 1    { if time dependent }
MONITORS        { show progress }
PLOTS           { save result displays }
            grid(x+u*mag, y+v*mag,z+w*mag)
history(w) at (Lx/2,Ly/2, Lz/2) report(omega)
history(v) at (Lx/2,Ly/2, Lz/2) report(omega)
history(u) at (Lx/2,Ly/2, Lz/2) report(omega)
elevation(v) from(Lx/2,Ly/2,0) to (Lx/2,Ly/2,Lz)

SUMMARY
report val(w, Lx,Ly,Lz)

END
```