ARCH Models on S&P 500 Stock Returns

TIØ4317 Project

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Norwegian University of Science and Technology

Problem statement

Which forecasting model provides the most accurate out-of-sample volatility forecasts for stocks in the S&P 500, and does the best model vary across different market conditions?

Week	Date	Weekday	Time	Room	Teacher	Topic
Part I: Empirical finance						
2	10.01.2025	Friday	13:15-16:00	GL-RFB R9	Morten	Introduction
3	17.01.2025	Friday	13:15-16:00	GL-RFB R9	Maria	Statistical inference | Descriptive statistics | Cross-sectional data
4	24.01.2025	Friday	13:15-16:00	GL-RFB R9	Maria	Linear regression
5	31.01.2025	Friday	13:15-16:00	GL-RFB R9	Morten	Post estimation diagnostic tests
6	07.02.2025	Friday	13:15-16:00	GL-RFB R9	Morten	Introduction to time series data and models
7	14.02.2025	Friday	13:15-16:00	GL-RFB R9	Morten	Univariate time series models |Forecasting | Guest lecture: Hafslund
8	21.02.2025	Friday	13:15-16:00	GL-RFB R9	Morten	Volatility models
9	27.02.2025	Thursday	14:15-16:00	GL-RFB R10	#N/A	Guest lecture: NBIM
9	28.02.2025	Friday	13:15-16:00	GL-RFB R9	Morten	Multivariate time series models
10	07.03.2025	Friday	13:15-16:00	GL-RFB R9	#N/A	Winter holiday
11	14.03.2025	Friday	13:15-16:00	GL-RFB R9	Maria	Panel data | Q&A Part 1
Part II: Financial optimization and risk management						
12	21.03.2025	Friday	13:15-16:00	GL-RFB R9	Stein-Erik	Risk Measures I
13	28.03.2025	Friday	13:15-16:00	GL-RFB R9	Stein-Erik	Risk Measures II
14	04.04.2025	Friday	13:15-16:00	GL-RFB R9	Stein-Erik	Portfolio optimization
15	11.04.2025	Friday	13:15-16:00	GL-RFB R9	#N/A	Ind-øk trip break
16	18.04.2025	Friday	13:15-16:00	GL-RFB R9	#N/A	Easter break
Student project presentations						
19	08.05.2025	Thursday	08:00-19:00	F5	Morten	Presentations and peer-review
19	09.05.2025	Friday	08:00-15:00	F5	Morten	Presentations and peer-review

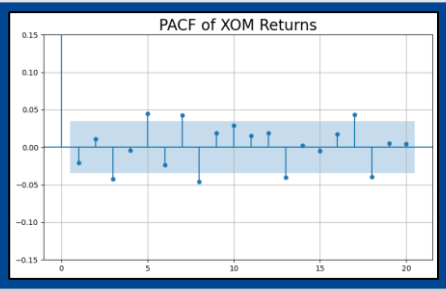
Week	Date	Weekday	Time	Room	Teacher	Topic
8	21.02.2025	Friday	13:15-16:00	GL-RFB R9	Morten	Volatility models

Using theory from “Volatility models” chapter in the curriculum

Data

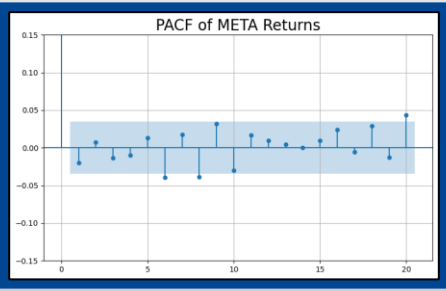
XOM
Oil & Gas

$\sigma^2 = 1.61$



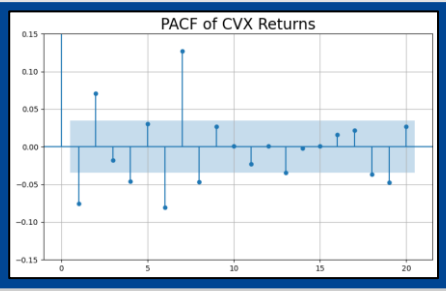
META
Technology

$\sigma^2 2.5$



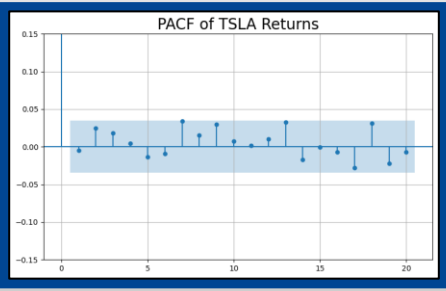
CVX
Oil & Gas

$\sigma^2 = 1.73$



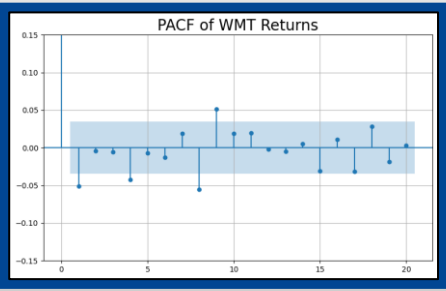
TSLA
Technology

$\sigma^2 = 3.58$



WMT
Retail

$\sigma^2 = 1.25$

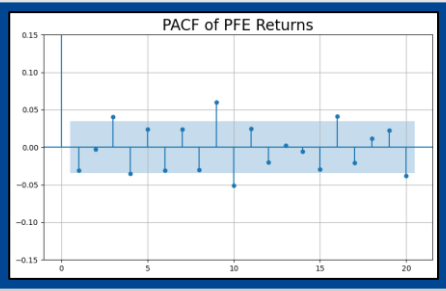


Selected 10 S&P 500 companies with different ACF and different volatility distributions

Data

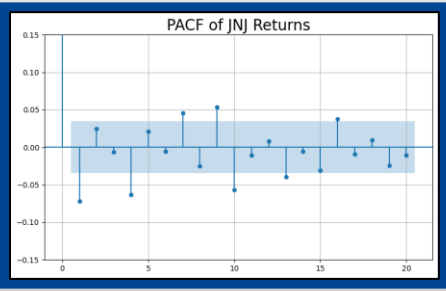
PFE
Pharmaceuticals

$\sigma^2 = 1.37$



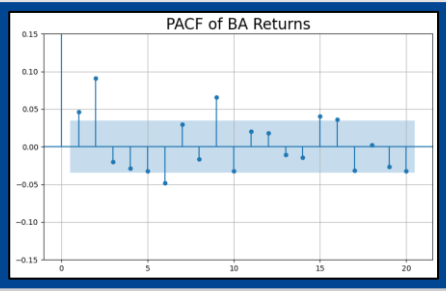
JNJ
Pharmaceuticals

$\sigma^2 = 1.08$



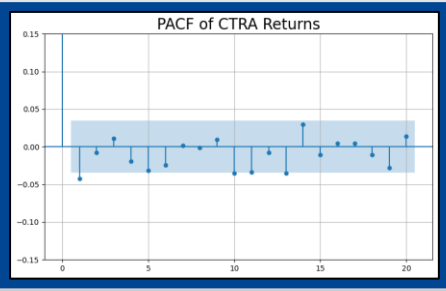
BA
Industrials

$\sigma^2 = 2.33$



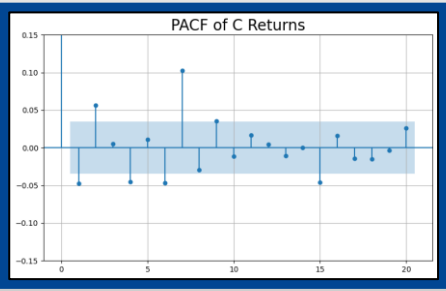
CTRA
Industrials

$\sigma^2 = 2.26$



C
Finance

$\sigma^2 = 1.98$



ARCH Models

ARCH (1)

- Models volatility as a function of past squared returns
- Captures volatility clustering

GARCH (1,1)

- Extends ARCH by adding lagged conditional variance terms
- Captures volatility persistence

GJR-GARCH (1,1)

- Incorporates asymmetry for negative returns
- Captures leverage effects

Approach

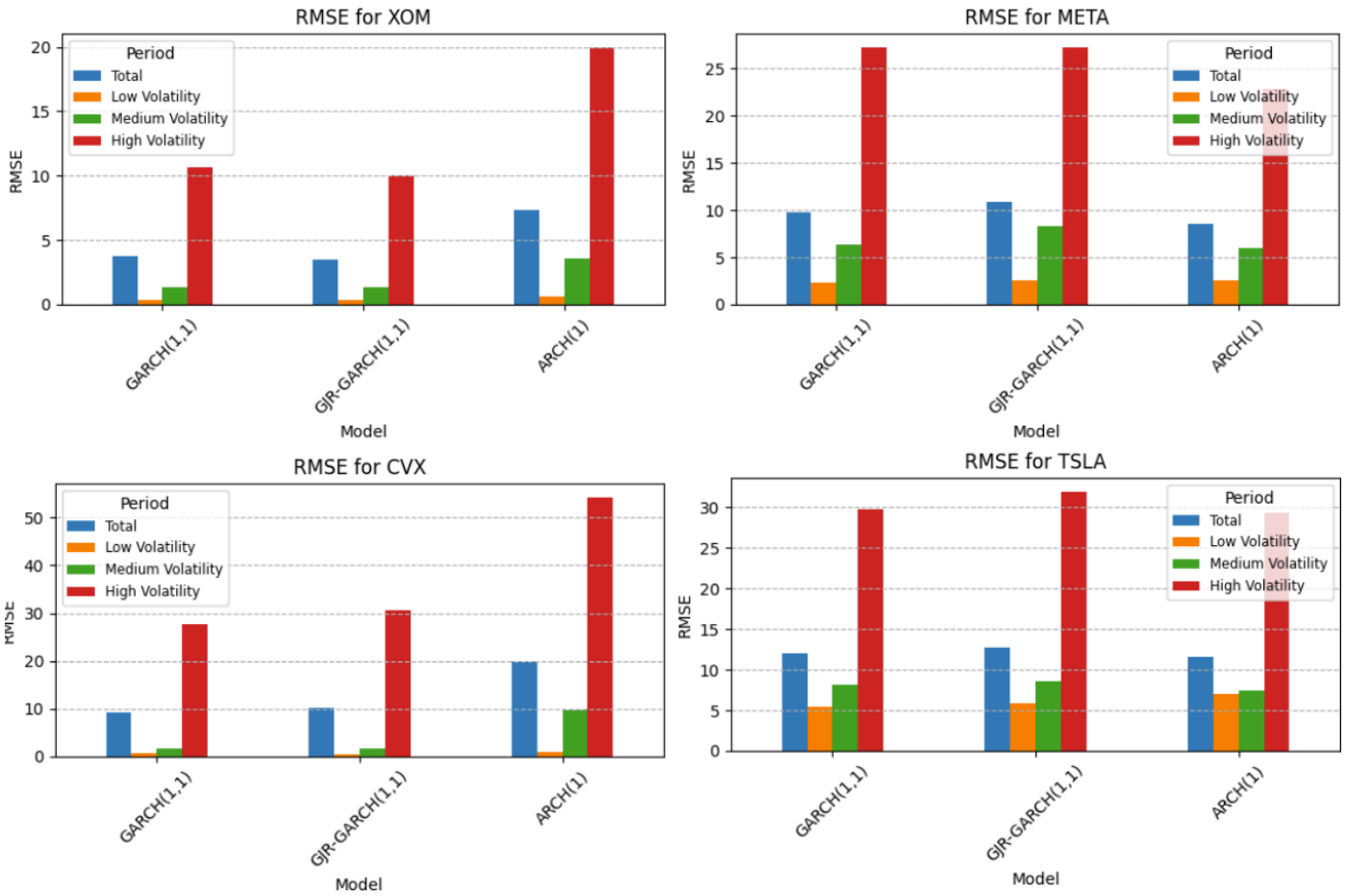
Select 5 S&P stocks → Fit ARCH models on absolute log returns → Evaluate fitness across models → Evaluate fitness across volatility levels → Cross-evaluate models across volatility levels

Models to be fitted

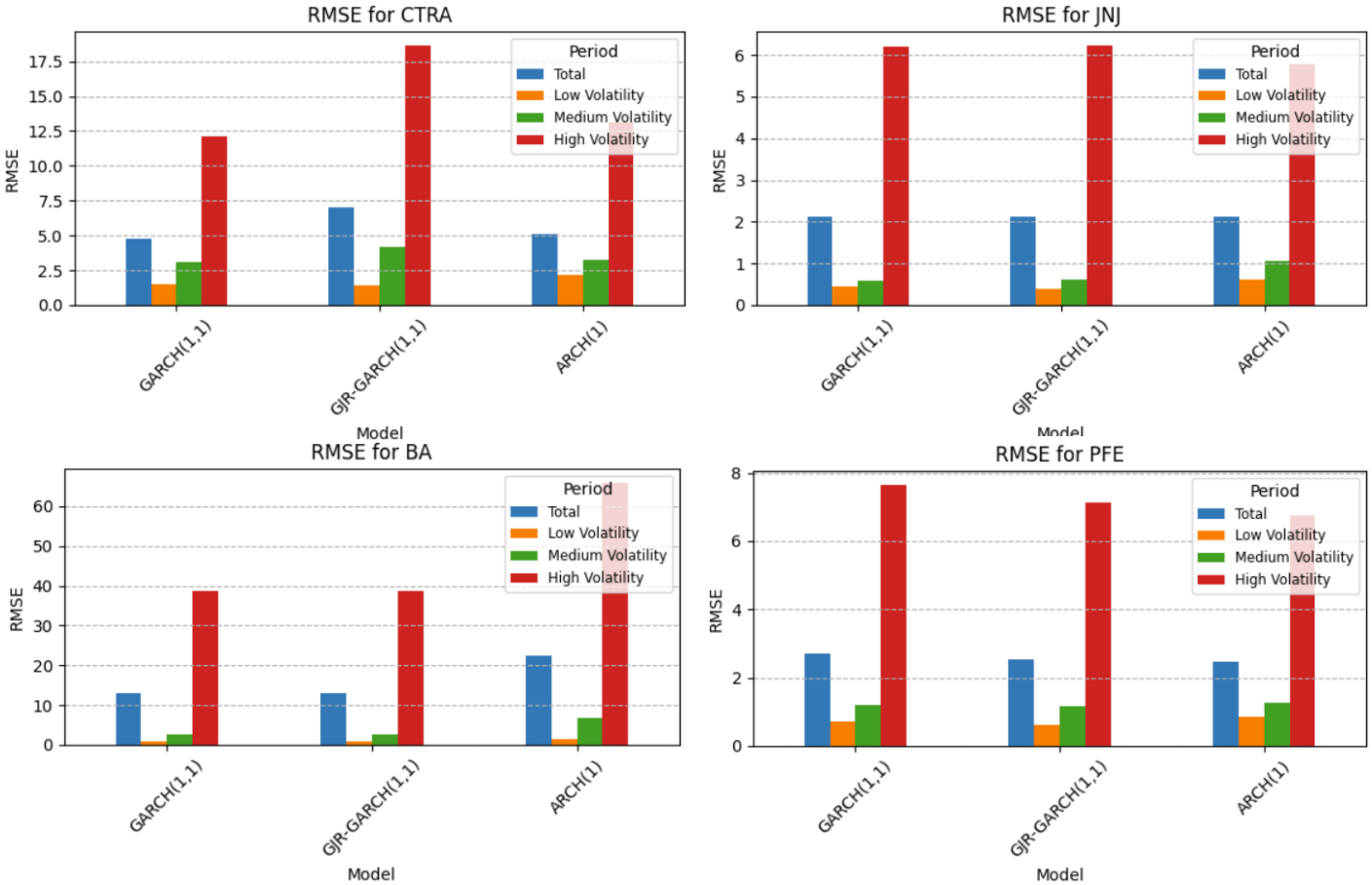
- GARCH(1,1)
- GJR-GARCH(1,1,1)
- ARCH(1)

The models are fitted on a rolling window of size 21 days.
When forecasting we use the volatilities in the window to predict the next-day volatility

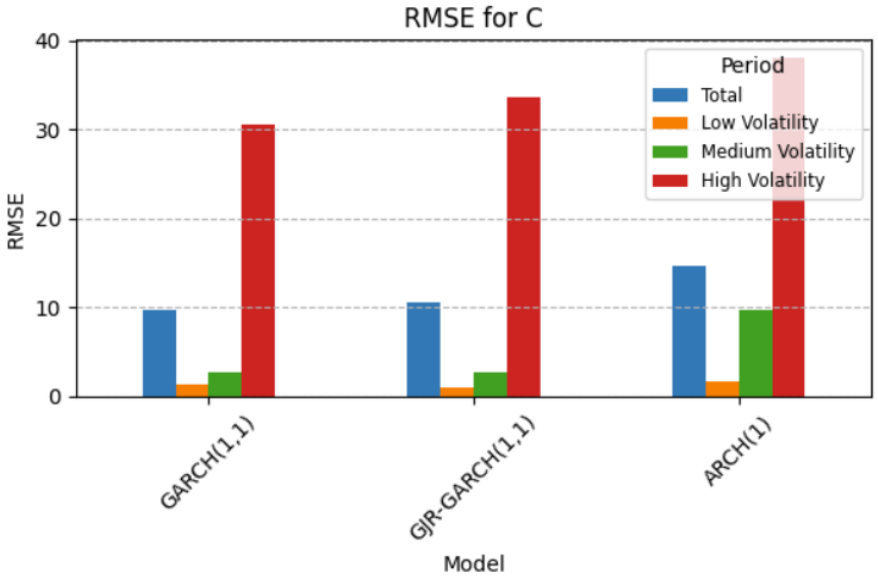
Results

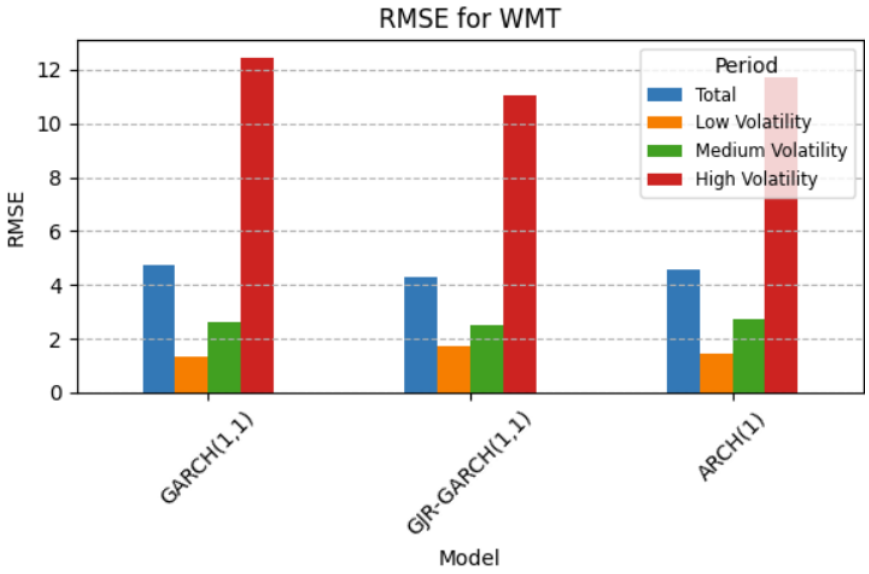

RMSE for XOM
RMSE for META
RMSE for CVX
RMSE for TSLA
Period
Total
Low Volatility
Medium Volatility
High Volatility
RMSE
Model
GARCH(1,1)
GJR-GARCH(1,1)
ARCH(1)

Results

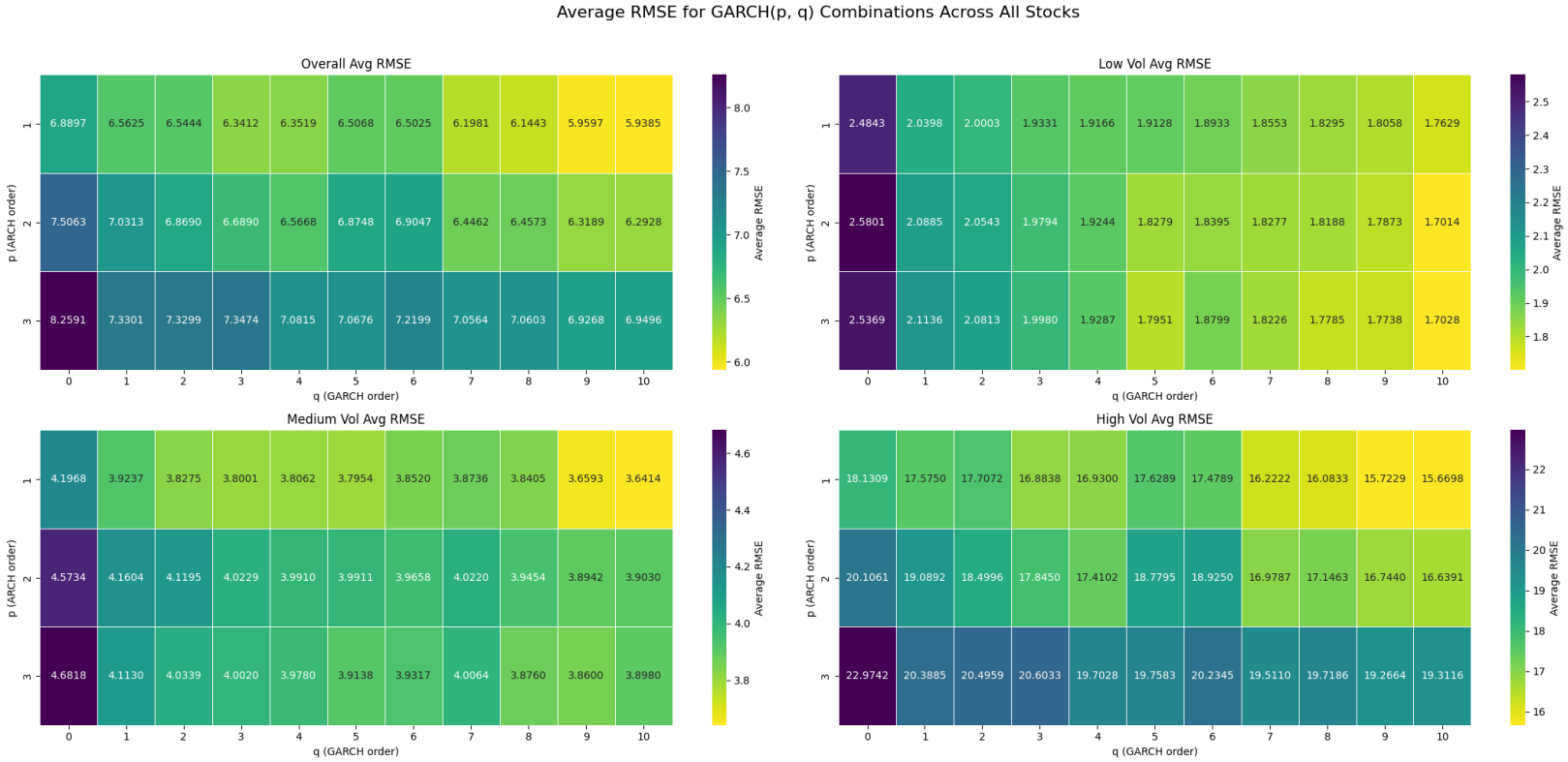

RMSE for CTRA
RMSE for JNJ
RMSE for BA
RMSE for PFE
Period
Total
Low Volatility
Medium Volatility
High Volatility
RMSE
Model
GARCH(1,1)
GJR-GARCH(1,1)
ARCH(1)

Results

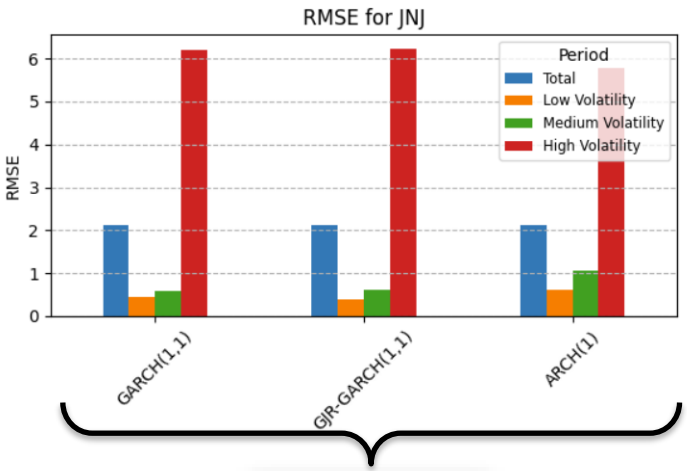

RMSE for C
RMSE
Model
GARCH(1,1)
GJR-GARCH(1,1)
ARCH(1)
Period
Total
Low Volatility
Medium Volatility
High Volatility
0
10
20
30
40


RMSE for WMT
RMSE
Model
GARCH(1,1)
GJR-GARCH(1,1)
ARCH(1)
Period
Total
Low Volatility
Medium Volatility
High Volatility
0
2
4
6
8
10
12

Results – GARCH parameters



Analysis – RMSE value



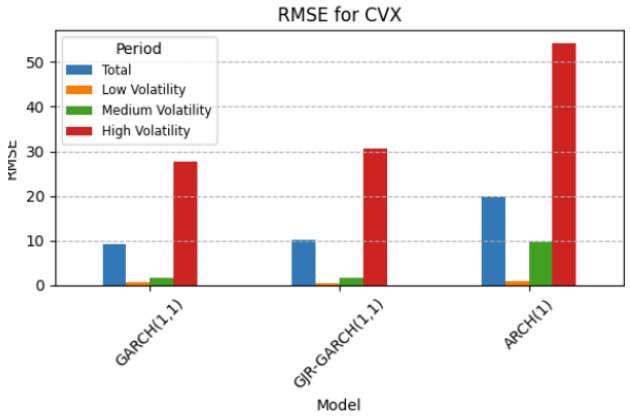
$$\sigma = 1.08$$

$$\mathrm{corr}(\sigma, RMSE) = 0.76$$

Analysis – ARCH performance

4 stocks have this property:

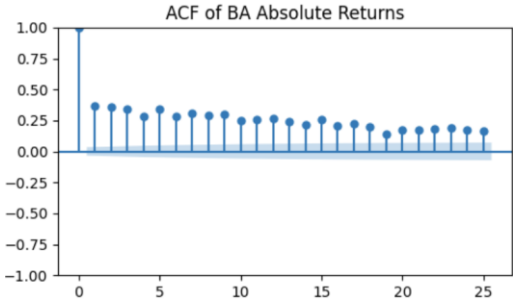
- XOM, CVX, C, BA

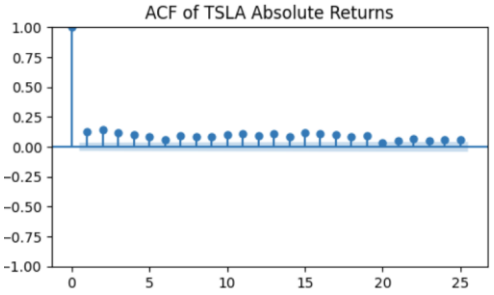


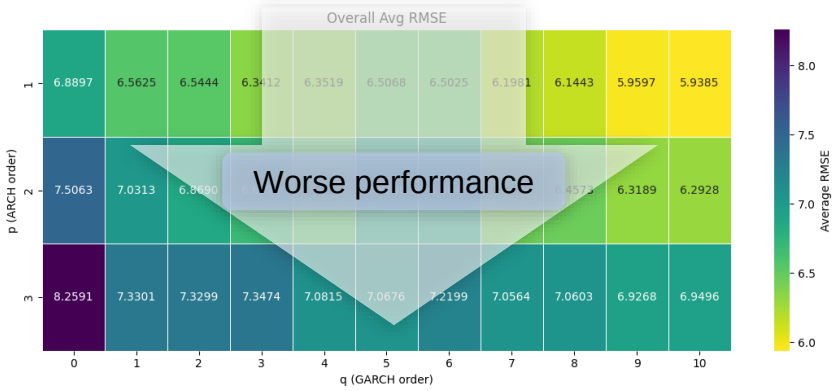
Interpretation:
ACF shape and *industry*

Correlation

Causation?





Analysis – GARCH parameters

Insight:
Shocks do not persist over multiple lags, volatility does