

ARCH Models on S&P 500 Stock Returns

TIØ4317 Project

Group 1: Aidan Thomas Scholes Stautland, Aina Govil Lie,
Adrian Fagerland, Aleksander Fasting

Problem statement

Which forecasting model provides the most accurate out-of-sample volatility forecasts for stocks in the S&P 500, and does the best model vary across different market conditions?

Week	Date	Weekday	Time	Room	Teacher	Topic
Part I: Empirical finance						
2	10.01.2025	Friday	13:15-16:00	GL-RFB R9	Morten	Introduction
3	17.01.2025	Friday	13:15-16:00	GL-RFB R9	Maria	Statistical inference Descriptive statistics Cross-sectional data
4	24.01.2025	Friday	13:15-16:00	GL-RFB R9	Maria	Linear regression
5	31.01.2025	Friday	13:15-16:00	GL-RFB R9	Morten	Post estimation diagnostic tests
6	07.02.2025	Friday	13:15-16:00	GL-RFB R9	Morten	Introduction to time series data and models
7	14.02.2025	Friday	13:15-16:00	GL-RFB R9	Morten	Univariate time series models Forecasting Guest lecture: Hafslund
8	21.02.2025	Friday	13:15-16:00	GL-RFB R9	Morten	Volatility models
9	27.02.2025	Thursday	14:15-16:00	GL-RFB R10	#N/A	Guest lecture: NBIM
9	28.02.2025	Friday	13:15-16:00	GL-RFB R9	Morten	Multivariate time series models
10	07.03.2025	Friday	13:15-16:00	GL-RFB R9	#N/A	Winter holiday
11	14.03.2025	Friday	13:15-16:00	GL-RFB R9	Maria	Panel data Q&A Part 1
Part II: Financial optimization and risk management						
12	21.03.2025	Friday	13:15-16:00	GL-RFB R9	Stein-Erik	Risk Measures I
13	28.03.2025	Friday	13:15-16:00	GL-RFB R9	Stein-Erik	Risk Measures II
14	04.04.2025	Friday	13:15-16:00	GL-RFB R9	Stein-Erik	Portfolio optimization
15	11.04.2025	Friday	13:15-16:00	GL-RFB R9	#N/A	Ind-ek trip break
16	18.04.2025	Friday	13:15-16:00	GL-RFB R9	#N/A	Easter break
Student project presentations						
19	08.05.2025	Thursday	08:00-19:00	F5	Morten	Presentations and peer-review
19	09.05.2025	Friday	08:00-15:00	F5	Morten	Presentations and peer-review

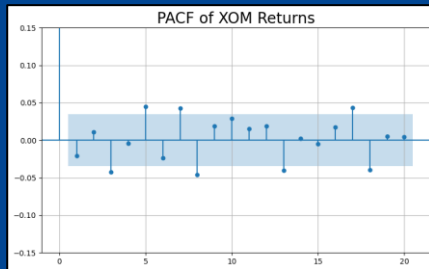
Week	Date	Weekday	Time	Room	Teacher	Topic
8	21.02.2025	Friday	13:15-16:00	GL-RFB R9	Morten	Volatility models

Using theory from “Volatility models” chapter in the curriculum

Data

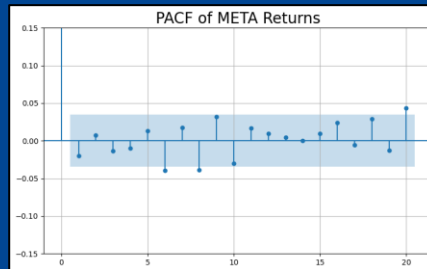
XOM
Oil & Gas

$$\sigma^2 = 1.61$$



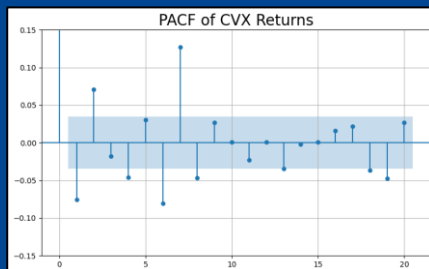
META
Technology

$$\sigma^2 2.5$$



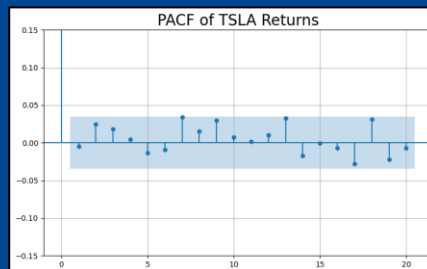
CVX
Oil & Gas

$$\sigma^2 = 1.73$$



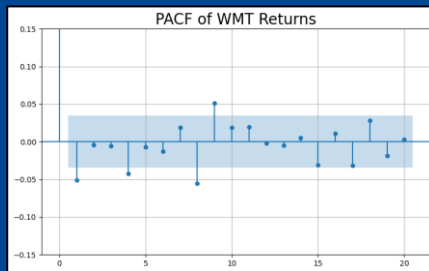
TSLA
Technology

$$\sigma^2 = 3.58$$



WMT
Retail

$$\sigma^2 = 1.25$$

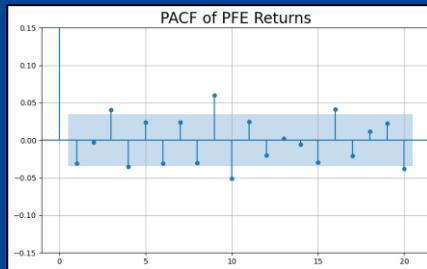


*Selected 10 S&P 500 companies
with different ACF and different
volatility distributions*

Data

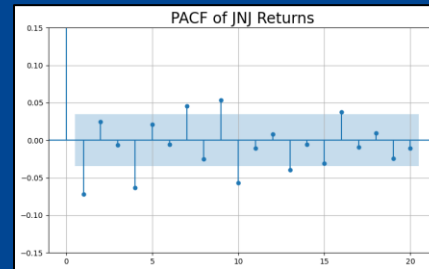
PFE
Pharmaceuticals

$$\sigma^2 = 1.37$$



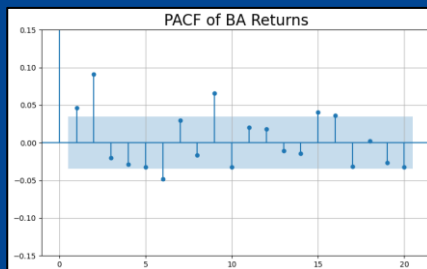
JNJ
Pharmaceuticals

$$\sigma^2 = 1.08$$



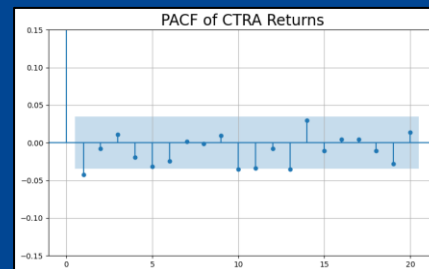
BA
Industrials

$$\sigma^2 = 2.33$$



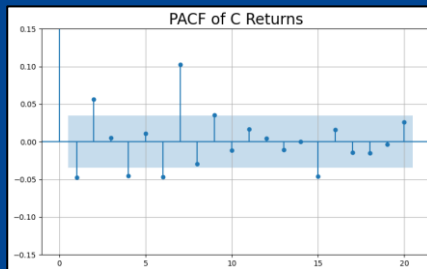
CTRA
Industrials

$$\sigma^2 = 2.26$$



C
Finance

$$\sigma^2 = 1.98$$



ARCH Models

ARCH (1)

- Models volatility as a function of past squared returns
- Captures volatility clustering

GARCH (1,1)

- Extends ARCH by adding lagged conditional variance terms
- Captures volatility persistence

GJR-GARCH (1,1)

- Incorporates asymmetry for negative returns
- Captures leverage effects

Approach



Models to be fitted

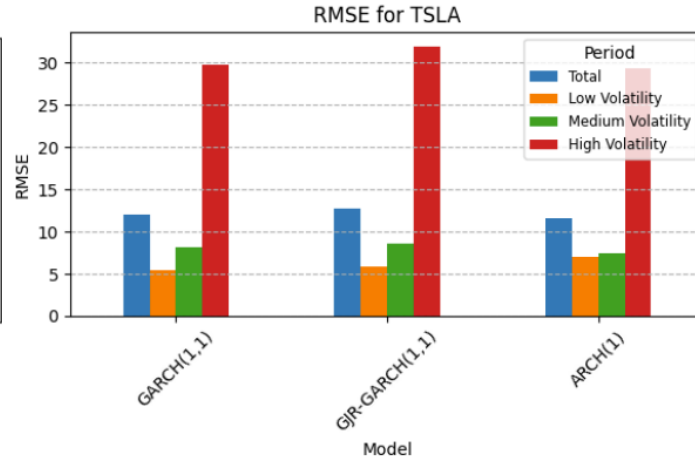
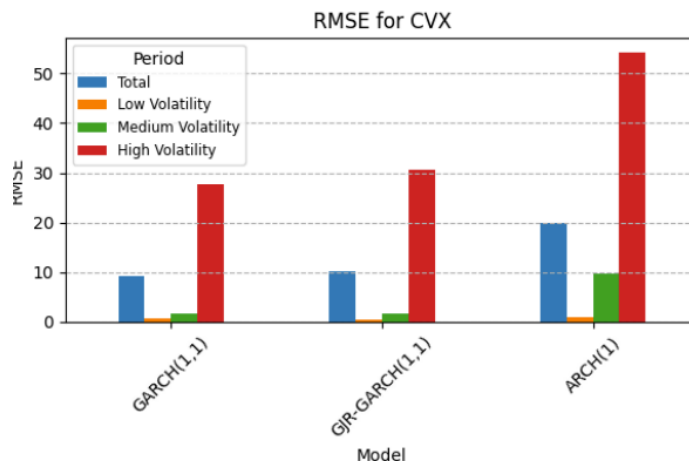
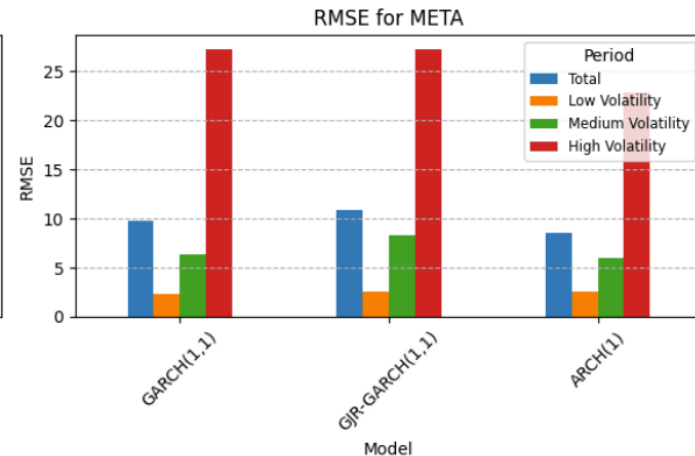
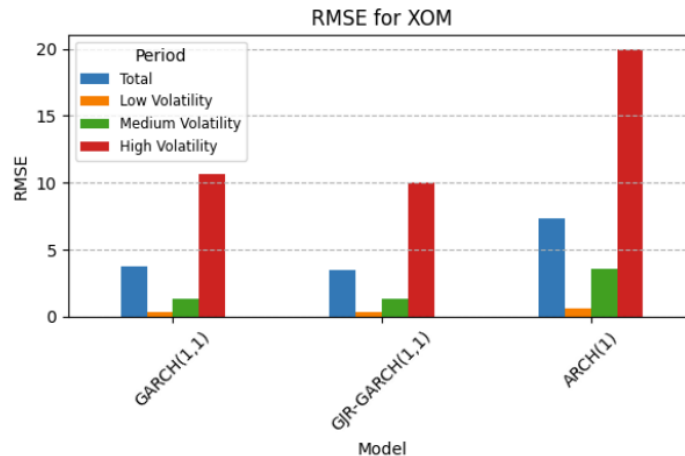
GARCH(1,1)

GJR-GARCH(1,1,1)

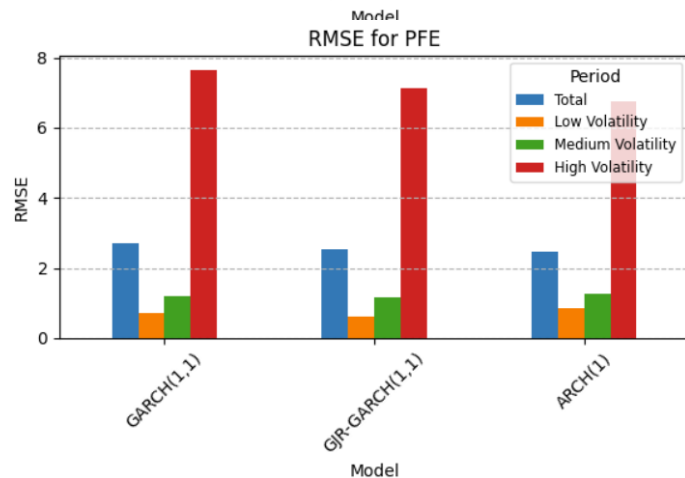
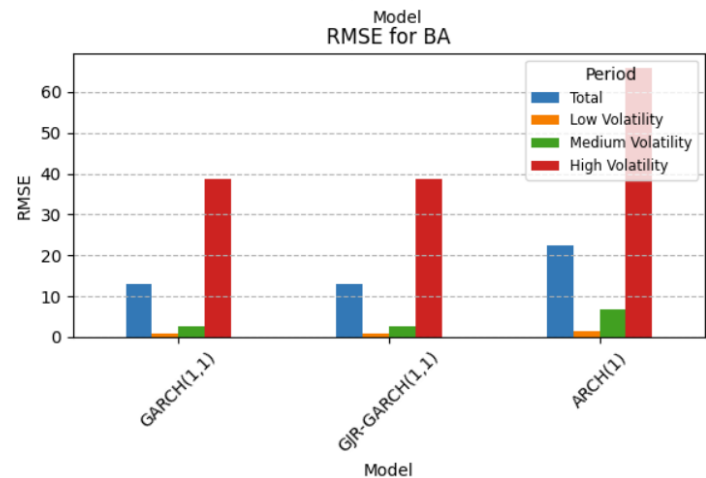
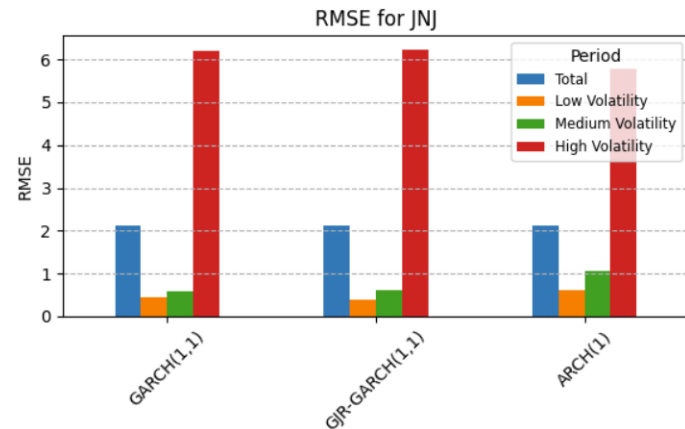
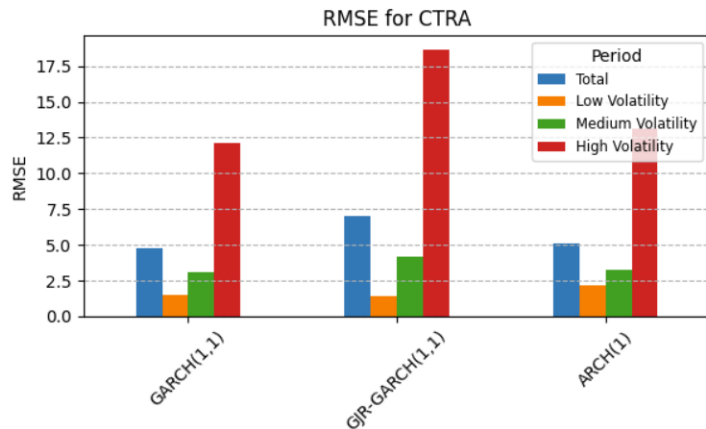
ARCH(1)

*The models are fitted on a rolling window of size 21 days.
When forecasting we use the volatilities in the window to predict the next-day volatility*

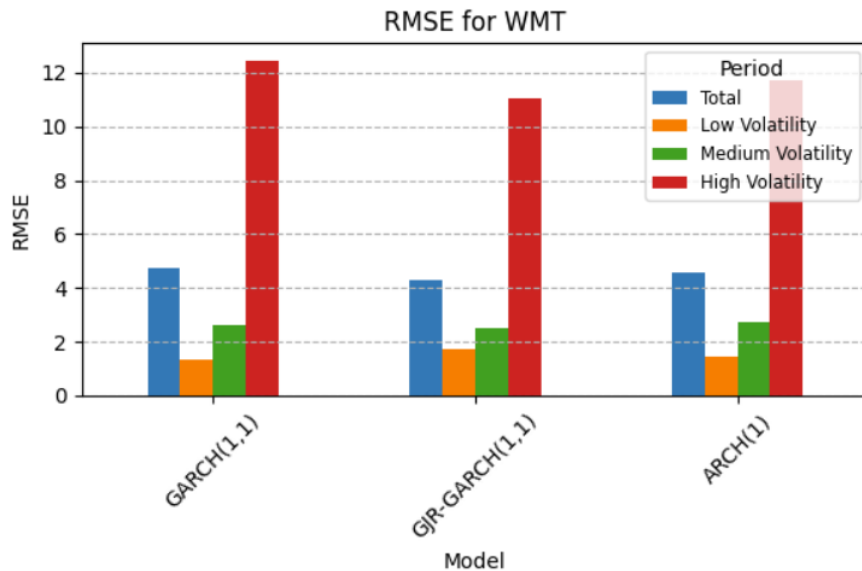
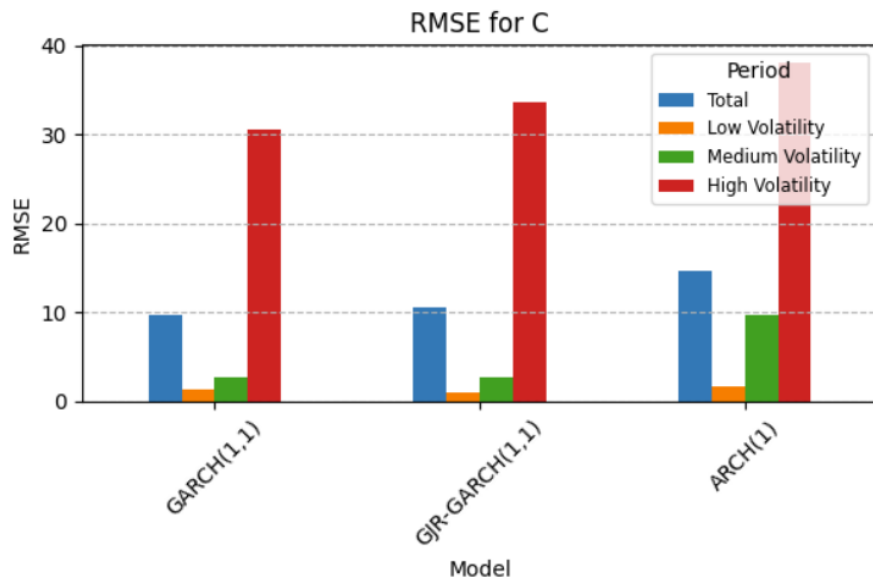
Results



Results

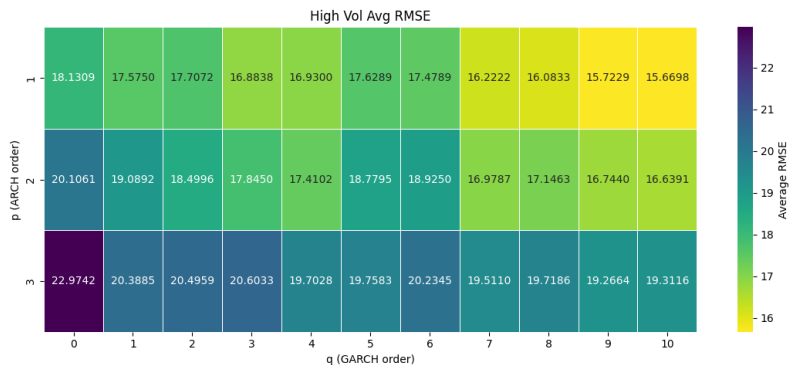
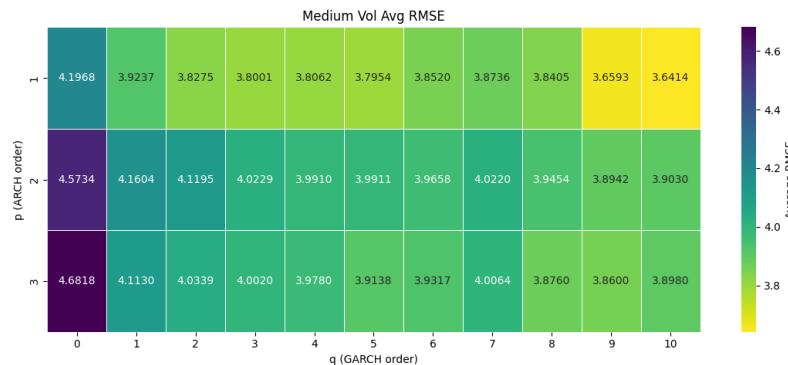
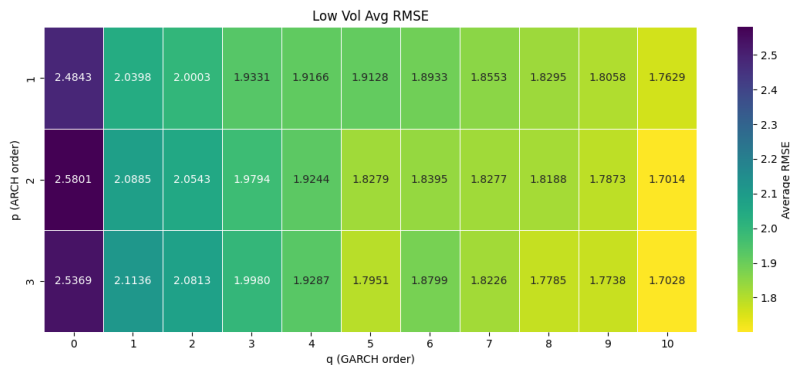
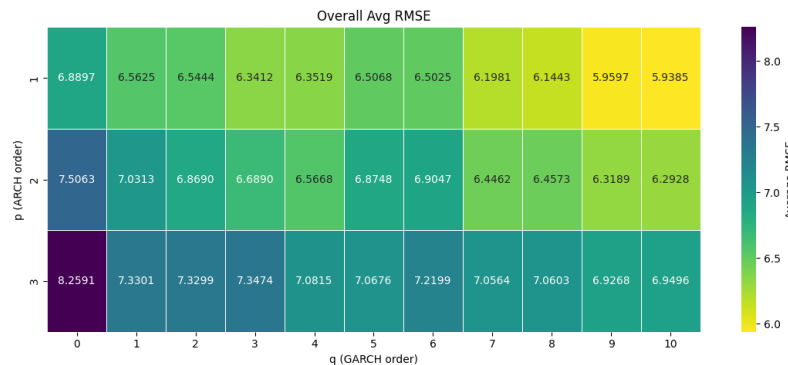


Results

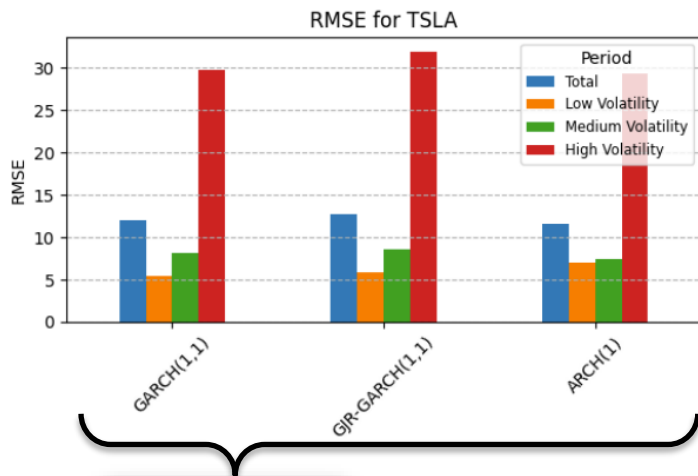


Results – GARCH parameters

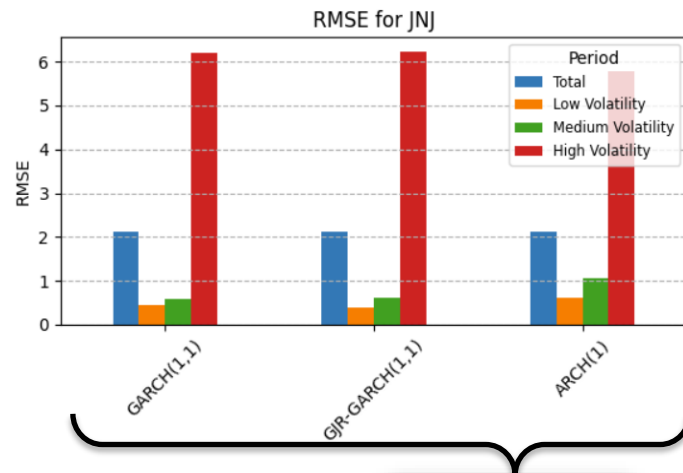
Average RMSE for GARCH(p, q) Combinations Across All Stocks



Analysis – RMSE value



$$\sigma = 3.58$$



$$\sigma = 1.06$$

$$\text{corr}(\sigma, \text{RMSE}) = 0.76$$

Analysis – ARCH performance

4 stocks have this property:

- XOM, CVX, C, BA

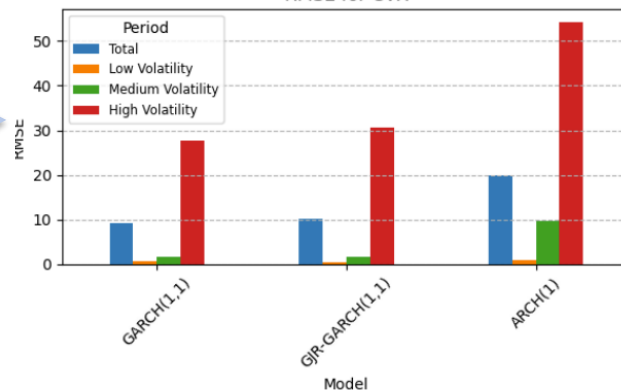
Interpretation:

ACF shape and industry

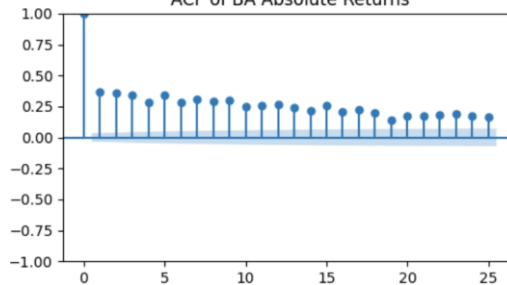
Correlation

Causation?

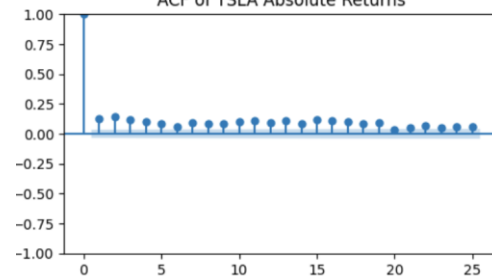
RMSE for CVX



ACF of BA Absolute Returns



ACF of TSLA Absolute Returns



Analysis – GARCH parameters

Insight:

Shocks do not persist over multiple lags, volatility does

