STS2006 (Analytic Geometry and Calculus II)
Quiz 5 Solutions

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1. (5 pts) Rewrite the iterated integral $\int_0^1 \int_y^1 \int_0^y f(x,y,z)\,dz\,dx\,dy$ as an iterated integral in a different order, integrating first with respect to x, then y, then z.

Solution. Let tetrahedron $T = \{(x,y,z) | y \le x \le 1, z \le y \le 1, 0 \le z \le 1\}$. Then

$$\begin{aligned} &\int_0^1 \int_y^1 \int_0^y f(x,y,z)\,dz\,dx\,dy \\ =& \iiint_T f(x,y,z)\,dV \\ =& \int_0^1 \int_z^1 \int_y^1 f(x,y,z)\,dx\,dy\,dz \end{aligned}$$

2. (5 pts) Rewrite the iterated integral $\int_{-2}^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^2 xz\,dz\,dx\,dy$ by changing to cylindrical coordinates. Don't evaluate.

Solution. Let $D = \{(r,\theta,z) | 0 \le z \le 2, 0 \le \theta \le 2\pi, 0 \le r \le z\}$, where $x^2+y^2 = r^2$ and $x = r\cos\theta$. Then

$$\begin{aligned} &\int_{-2}^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^2 xz\,dz\,dx\,dy \\ =& \iiint_D xz\,dV \\ =& \int_0^{2\pi} \int_0^2 \int_0^z z \cdot r^2 \cos\theta\,dr\,dz\,d\theta \end{aligned}$$