STS2006 (Analytic Geometry and Calculus II)
Quiz 4 Solutions

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1. (5 pts) Evaluate the integral $\iint_D \sin\left(x^2+y^2\right) dA$, where D is the region in the first quadrant between the circles with the center the origin and radii 1 and 3.

Solution. By definition, $D = \left\{(r,\theta)\middle| 1 \le r \le 3, 0 \le \theta \le \frac{\pi}{2}\right\}$.

$$\begin{aligned}
&\iint_D \sin\left(x^2+y^2\right) dA \\
=&\int_0^{\frac{\pi}{2}} \int_1^3 r\sin r^2 \, dr\, d\theta \\
=&\frac{\pi}{2}\int_1^3 r\sin r^2 \, dr
\end{aligned}$$

Let $r^2 = u$, $2r\,dr = du$. Then

$$\begin{aligned}
&\frac{\pi}{2}\int_1^3 r\sin r^2 \, dr \\
=&\frac{\pi}{4}\int_1^9 du \sin u \\
=&\frac{\pi}{4}\left(-\cos u\right)\Big|_1^9 \\
=&\frac{\pi}{4}\left(\cos 1 - \cos 9\right)
\end{aligned}$$

2. (5 pts) Find the area of the part of the plane $3x+2y+z=5$ that lies in the first octant.

Solution. Let $z = f(x,y) = 5-3x-2y$. Then the area of the plane is the surface integral of f for $D = \left\{(x,y)\middle| 0 \le x \le 2, 0 \le y \le \frac{6-3x}{2}\right\}$.

$$
\begin{aligned}
S &= \iint_D \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 + 1}\, dA \\
&= \int_0^2 \int_0^{\frac{6-3x}{2}} \sqrt{(-3)^2 + (-2)^2 + 1}\, dy\, dx \\
&= \int_0^2 \frac{6-3x}{2} \sqrt{14}\, dx \\
&= \sqrt{14} \left[3x - \frac{3x^2}{4} \right]_0^2 \\
&= 3\sqrt{14}
\end{aligned}
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