

## STS2006 (Analytic Geometry and Calculus II)

### Quiz 2 Solutions

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**1. (5 pts)** Find the approximation of  $\sqrt{(3.02)^2 + (1.97)^2 + (5.99)^2}$  by using the linear approximation method.

*Solution.* Let

$$w = f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$$

then

$$\begin{aligned} & \sqrt{(3.02)^2 + (1.97)^2 + (5.99)^2} \\ & \approx f(3, 2, 6) + \frac{\partial w}{\partial x} dx + \frac{\partial w}{\partial y} dy + \frac{\partial w}{\partial z} dz \\ & = \sqrt{3^2 + 2^2 + 6^2} + \left( \frac{2 \times 3}{2\sqrt{3^2 + 2^2 + 6^2}} \right) dx + \left( \frac{2 \times 2}{2\sqrt{3^2 + 2^2 + 6^2}} \right) dy + \left( \frac{2 \times 6}{2\sqrt{3^2 + 2^2 + 6^2}} \right) dz \\ & = 7 + \frac{3}{7} dx + \frac{2}{7} dy + \frac{6}{7} dz \\ & = 7 + \frac{3}{7} (0.02) + \frac{2}{7} (-0.03) + \frac{6}{7} (-0.01) \\ & = 7 - \frac{6}{700} \end{aligned}$$

**2. (5 pts)** Use the chain rule to find  $\frac{\partial w}{\partial r}$ ,  $\frac{\partial w}{\partial \theta}$  where  $w = xy + yz + zx$ ,  $x = r \cos \theta$ ,  $y = r \sin \theta$ ,  $z = r\theta$ .

*Solution.*

$$\begin{aligned} \frac{\partial w}{\partial r} &= \frac{\partial w}{\partial x} \frac{\partial x}{\partial r} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial r} + \frac{\partial w}{\partial z} \frac{\partial z}{\partial r} \\ &= (y + z) \cos \theta + (x + z) \sin \theta + (x + y) \theta \end{aligned}$$

$$\begin{aligned} \frac{\partial w}{\partial \theta} &= \frac{\partial w}{\partial x} \frac{\partial x}{\partial \theta} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial \theta} + \frac{\partial w}{\partial z} \frac{\partial z}{\partial \theta} \\ &= (y + z) (-r \sin \theta) + (x + z) (r \cos \theta) + (x + y) r \end{aligned}$$