STS2006 (Analytic Geometry and Calculus II)
Quiz 3 Solutions

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1. (5 pts) Find the directional derivative of the function $f(x,y) = x^2y + y^2z$ at the point $(1,2,3)$ in the direction of the vector $\mathbf{v} = (2,-1,2)$.

Solution. Since f is differentiable,

$$\begin{aligned} D_{\mathbf{v}}(x,y,z) &= \nabla f \cdot \frac{\mathbf{v}}{|\mathbf{v}|} \\ &= \left(2xy, x^2 + 2yz, y^2\right) \cdot \frac{(2,-1,2)}{3} \\ &= \frac{1}{3}\left(4xy - x^2 - 2yz + 2y^2\right) \end{aligned}$$

thus $D_{\mathbf{v}}(1,2,3) = \frac{1}{3}\left(4 \cdot 1 \cdot 2 - 1^2 - 2 \cdot 2 \cdot 3 + 2 \cdot 2^2\right) = 1$.

2. (5 pts) Find the local maximum and minimum values, and saddle points of the function $f(x,y) = 2 - x^4 + 2x^2 - y^2$.

Solution. Since f is differentiable,

$$\begin{aligned} D &= \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = f_{xx}f_{yy} - \left(f_{xy}\right)^2 \\ &= \left(-12x^2 + 4\right)(-2) - 0^2 \\ &= 24x^2 - 8 \end{aligned}$$

When $f_x(x) = 0$,

$$\begin{aligned} -4x^3 + 4x &= 0 \\ x^3 &= x \\ \therefore x &= 0, \pm 1 \end{aligned}$$

When $f_y(y) = 0$,

$$-2y = 0$$
$$\therefore y = 0$$

Therefore f has:

x	y	D	f_{xx}	
-1	0	16	-8	local maximum at $(-1,0)$
0	0	-8		saddle point at $(0,0)$
1	0	16	-8	local maximum at $(1,0)$