STS2006 (Analytic Geometry and Calculus II)
Quiz 1 Solutions

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1. (5 pts) Find the limit

$$\lim_{(x,y)\to(0,0)} \frac{x^3-y^2}{x^2+2y^2}$$

if it exists, or show that the limit does not exist.

Solution. Let

$$f(x,y) = \frac{x^3-y^2}{x^2+2y^2}$$

if $x = 0$,

$$f(0,y) = \frac{0^3-y^2}{0^2+2y^2} = \frac{-y^2}{2y^2} = -\frac{1}{2}$$

$$\therefore f(0,y) \to -\frac{1}{2} \quad \text{when} \quad y \to 0$$

if $y = 0$,

$$f(x,0) = \frac{x^3-0^2}{x^2+2\times 0^2} = \frac{x^3}{x^2} = x$$

$$\therefore f(x,0) \to 0 \quad \text{when} \quad x \to 0$$

thus the limit **does not exist**.

2. (5 pts) Find the first partial derivatives $\dfrac{\partial z}{\partial x}$, $\dfrac{\partial z}{\partial y}$ of the function $z = \cos\left(x^2y+x^2-y^2+1\right)$.

Solution.

$$\begin{aligned}
\frac{\partial z}{\partial x} &= \frac{\partial}{\partial x}\cos\left(x^2y+x^2-y^2+1\right) \\
&= -\sin\left(x^2y+x^2-y^2+1\right)\cdot\frac{\partial}{\partial x}\left(x^2y+x^2-y^2+1\right) \\
&= -\sin\left(x^2y+x^2-y^2+1\right)(2xy+2x) \\
&= -2x(y+1)\sin\left(x^2y+x^2-y^2+1\right)
\end{aligned}$$

$$\begin{aligned}\frac{\partial z}{\partial y} &= \frac{\partial}{\partial y}\cos\left(x^2y+x^2-y^2+1\right)\\ &= -\sin\left(x^2y+x^2-y^2+1\right)\cdot\frac{\partial}{\partial y}\left(x^2y+x^2-y^2+1\right)\\ &= -\left(x^2-2y\right)\sin\left(x^2y+x^2-y^2+1\right)\end{aligned}$$