STS2006 (Analytic Geometry and Calculus II)
Quiz 6 Solutions

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1. (5 pts) Evaluate the line integral $\int_C y^2 z\, ds$, where C is the line segment from $(3,1,2)$ to $(1,2,5)$.

Solution. C can be expressed as

$$C : t\,(3,1,2) + (1-t)\,(1,2,5) = (1+2t, 2-t, 5-3t) \quad 0 \le t \le 1$$

in vector form. Therefore

$$\begin{aligned}
&\int_C y^2 z\, ds \\
=&\int_0^1 y^2 z \cdot \sqrt{(2)^2 + (-1)^2 + (-3)^2}\, dt \\
=&\sqrt{14} \int_0^1 (2-t)^2 (5-3t)\, dt \\
=&\sqrt{14} \left[-\frac{3}{4} t^4 + \frac{17}{3} t^3 - 16t^2 + 20t \right]_0^1 \\
=&\frac{107}{12} \sqrt{14}
\end{aligned}$$

2. (5 pts) Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x,y) = xy^2 \mathbf{i} - x^2 \mathbf{j}$ and C is given by the vector function $\mathbf{r}(t) = t^3 \mathbf{i} + t^2 \mathbf{j}$, $0 \le t \le 1$.

Solution.

$$
\begin{aligned}
&\int_C \mathbf{F} \cdot d\mathbf{r} \\
&= \int_0^1 \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t)\, dt \\
&= \int_0^1 \mathbf{F}\left(t^3, t^2\right) \cdot \left(3t^2, 2t\right) dt \\
&= \int_0^1 \left(t^7, -t^6\right) \cdot \left(3t^2, 2t\right) dt \\
&= \int_0^1 3t^9 - 2t^7\, dt \\
&= \left[\frac{3}{10}t^{10} - \frac{1}{4}t^8\right]_0^1 = \frac{1}{20}
\end{aligned}
$$