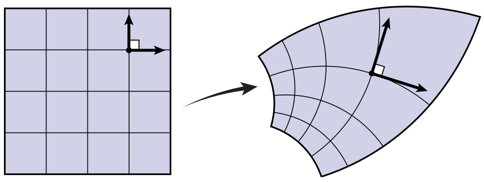
Discrete Differential Geometry 离散差分几何

CARNEGIE MELLON UNIVERSITY 15-458/768 | SPRING 2025 | TUE/THU, TIME TBD | ROOM TBD

Assignment 4: Conformal Parameterization 作业 4：共形参数化

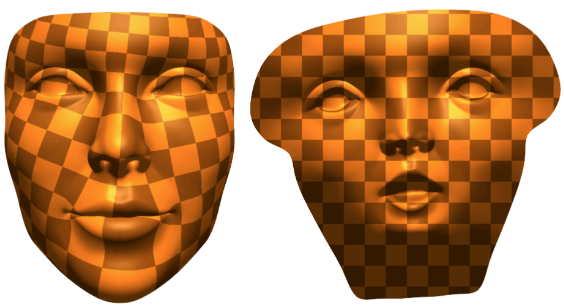
Written 写



The written part of your next assignment, on conformal surface parameterization, can be found here. Conformal parameterization is useful for (among many other things) finding maps between cortical surfaces, or designing shape-shifting matter. You’ ll have the opportunity to implement one of these algorithms in the coding part of the assignment.

您可以在此处找到下一个作业中关于共形面参数化的书面部分。共形参数化对于（除许多其他外）查找皮层表面之间的映射或设计形状变化的物质很有用。您将有机会在作业的编码部分实现其中一种算法。

Coding 编码



For the coding portion of your assignment, you will implement the Spectral Conformal Parameterization (SCP) algorithm, which is detailed in the course notes.

Please implement the following routines in

对于作业的编码部分，您将实施频谱共形参数化 （SCP） 算法，该算法在课程笔记中有详细说明。

请在

- core/geometry.[js/cpp]:
  - complexLaplaceMatrix
- projects/parameterization/spectral-conformal-parameterization.[js/cpp]:
  - buildConformalEnergy
  - flatten
- utils/solvers.[js/cpp]:
  - solveInversePowerMethod
  - residual

Notes

笔记

- The complex version of the cotan-Laplace matrix can be built in exactly the same manner as its real counterpart. The only difference now is that the cotan values of our matrix will be complex numbers with a zero imaginary component. This time, we will work with meshes with boundary, so your Laplace matrix will have to handle boundaries properly (you just have to make sure your `cotan` function returns 0 for halfedges which are in the boundary).

  cotan-Laplace 矩阵的复数版本可以以其与实际矩阵完全相同的方式构建。现在唯一的区别是，我们矩阵的 cotan 值将是虚部为零的复数。这一次，我们将使用具有边界的网格，因此您的拉普拉斯矩阵必须正确处理边界（您只需确保函数 `cotan` 为边界内的半边返回 0）。

- The `buildConformalEnergy` function builds a $|V| \times |V|$ complex matrix corresponding to the conformal energy $E_C(z) = E_D(z) - \mathcal{A}(Z)$ where E_D is the Dirichlet energy (given by our complex cotan-Laplace matrix) and $\mathcal{A}$ is the total signed area of the region $z(M)$ in the complex plane (Please refer to Section 7.4 of the notes for more details). You may find it easiest to iterate over the halfedges of the mesh boundaries to construct the area matrix (Recall that the `Mesh` object has a `boundaries` variable which stores all the boundary loops.

  该 `buildConformalEnergy` 函数构建了一个 $|V| \times |V|$ 对应于共形能 $E_C(z) = E_D(z) - \mathcal{A}(Z)$ 的复矩阵，其中 E_D 是狄利克雷能量（由我们的复数 cotan-Laplace 矩阵给出），$\mathcal{A}$ 并且是复平面中区域 $z(M)$ 的总符号面积（有关详细信息，请参阅注释的第 7.4 节）。您可能会发现迭代网格边界的半边以构建面积矩阵是最容易的（回想一下，该 `Mesh` 对象有一个 `boundaries` 存储所有边界循环的变量。

- The `flatten` function returns a dictionary mapping each vertex to a vector (`linear-algebra/vector.js`) of planar coordinates by finding the eigenvector corresponding to the smallest eigenvalue of the conformal energy matrix. You can compute this eigenvector by using solveInversePowerMethod (described below).

该 flatten 函数返回一个字典，通过查找与共形能量矩阵的最小特征值相对应的特征向量，将每个顶点映射到平面坐标的向量 （ linear-algebra/vector.js ） 。您可以使用 solveInversePowerMethod（如下所述）来计算此特征向量。

- Your solveInversePowerMethod function should implement **Algorithm 1** in Section 7.5 of the course notes with one modification – after computing $Ay_i = y_{i-1}$, center y_i around the origin by subtracting its mean. You don't have to worry about the B matrix or generalized eigenvalue problem. **Important:** Terminate your iterations when your residual is smaller than 10^{-10}.

  您的 solveInversePowerMethod 函数应该实现课程笔记第 7.5 节中的算法 1，并进行一次修改 - 计算 $Ay_i = y_{i-1}$ 后，通过减去其平均值以原点为中心 y_i 。您不必担心 B 矩阵或广义特征值问题。重要说明：当残差小于 时终止迭代 10^{-10} 。

- The parameterization project directory also contains a basic implementation of the Boundary First Flattening (BFF) algorithm. Feel free to play around with it in the viewer and compare the results to your SCP implementation.

  参数化项目目录还包含 Boundary First Flattening （BFF） 算法的基本实现。您可以在查看器中随意使用它，并将结果与您的 SCP 实施进行比较。

Handin instructions can be found in the Assignments section of the main page.

提交说明可以在主页的 Assignments 部分找到。

Carnegie Mellon University | 15-458/858B | Discrete Differential Geometry