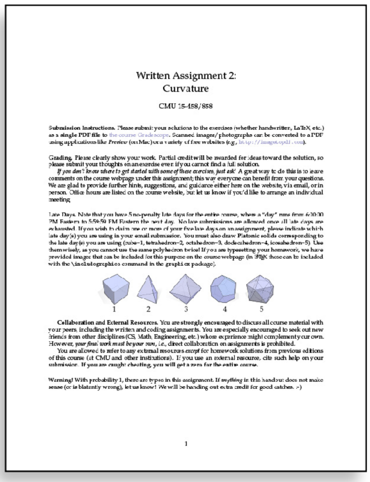
离散微分几何

CARNEGIE MELLON UNIVERSITY 15-458/768 | SPRING 2025 | TUE/THU, TIME TBD | ROOM TBD

作业 2：曲率

书面

作业 2 的笔试部分可以在这里找到。它考察了光滑离散曲面的曲率，并补充了课堂上讲到的一些细节。请注意，这里有相当多的练习，所以最好将它们拆分成作业的一部分。



Written Assignment 2:
Curvature

CMU 15-458/858

Submission Instructions. Please submit your solutions to the exercises (whether handwritten, LaTeX, etc.) as a single PDF file to the course Gradescope. Scanned images/photographs can be converted to a PDF using applications like *Preview* (on Mac) or a variety of free websites (e.g., http://imagetopdf.com).

Grading. Please clearly show your work. Partial credit will be awarded for ideas toward the solution, so please submit your thoughts on an exercise even if you cannot find a full solution.

If you don't know where to get started with some of these exercises, just ask! A great way to do this is to leave comments on the course webpage under this assignment; this way everyone can benefit from your questions. We are glad to provide further hints, suggestions, and guidance either here on the website, via email, or in person. Office hours are listed on the course website, but let us know if you'd like to arrange an individual meeting.

Late Days. Note that you have 5 no-penalty late days for the entire course, where a "day" runs from 6:00:00 PM Eastern to 5:59:59 PM Eastern the next day. No late submissions are allowed once all late days are exhausted. If you wish to claim one or more of your five late days on an assignment, please indicate which late day(s) you are using in your email submission. You must also draw Platonic solids corresponding to the late day(s) you are using (cube–1, tetrahedron–2, octahedron–3, dodecahedron–4, icosahedron–5). Use them wisely, as you cannot use the same polyhedron twice! If you are typesetting your homework, we have provided images that can be included for this purpose on the course webpage (in LaTeX these can be included with the \includegraphics command in the graphicx package).

1 2 3 4 5

Collaboration and External Resources. You are strongly encouraged to discuss all course material with your peers, including the written and coding assignments. You are especially encouraged to seek out new friends from other disciplines (CS, Math, Engineering, etc.) whose experience might complement your own. However, *your final work must be your own*, *i.e.*, direct collaboration on assignments is prohibited.

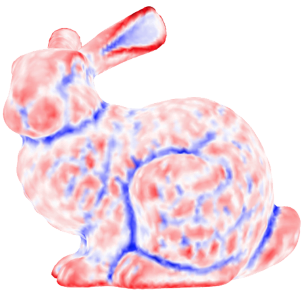
You are allowed to refer to any external resources *except* for homework solutions from previous editions of this course (at CMU and other institutions). If you use an external resource, cite such help on your submission. If you are caught cheating, you will get a zero for the entire course.

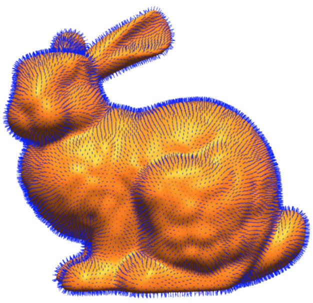
Warning! With probability 1, there are typos in this assignment. If *anything* in this handout does not make sense (or is blatantly wrong), let us know! We will be handing out extra credit for good catches. :-)

1

编码

在本次作业的编码部分，你将实现各种离散曲率和曲面法线的表达式，这些表达式





将在书面作业中推导。（如果你想先进行编码，最终的表达式如下所示。）实现后，你将能够在网格上可视化这些几何量。为简单起见，你可以假设网格没有边界。

入门指南： 请在core/geometry.[js/cpp]

中实现以下例程：

- 角度
- 二面角
- 顶点法线角度加权
- 顶点法线球体内接
- 顶点法线面积加权
- 顶点法线高斯曲率
- 顶点法线平均曲率
- 角度缺陷
- 总角度缺陷

注释

1. 法线之间的二面角N_{ijk}和N_{ijl}两个相邻面ijk和ijl（分别）由下式给出

$$\theta_{ij} := \operatorname{atan2}\left(\frac{e_{ij}}{\|e_{ij}\|} \cdot (N_{ijk} \times N_{jil}), N_{ijk} \cdot N_{jil}\right)$$

在哪里e_{ij}是从顶点开始的向量i到顶点j. 2. 角加权法线、球内接法线、面积加权法线、离散高斯曲率法线和顶点离散平均曲率法线的公式i是

$$N_i^\phi := \sum_{ijk \in F} \phi_i^{jk} N_{ijk}$$

$$N_i^S := \sum_{ijk \in F} \frac{e_{ij} \times e_{ik}}{\|e_{ij}\|^2 \|e_{ik}\|^2}$$

$$N_i^A := \sum_{ijk \in F} A_{ijk} N_{ijk}$$

$$KN_i = \frac{1}{2} \sum_{ij \in E} \frac{\theta_{ij}}{\|e_{ij}\|} e_{ij}$$

$$HN_i = \frac{1}{2} \sum_{ij \in E} \left(\cot\left(\alpha_k^{ij}\right) + \cot\left(\beta_l^{ij}\right)\right) e_{ij}$$

在哪里ϕ_i^{jk}是边之间的内角e_{ij}和e_{ik}， 和A_{ijk}是脸部面积ijk。请注意，求和仅针对包含顶点的元素（边或面）进行i. 在返回所有法向量的最终值之前，先对其进行归一化。3. 顶点处的外心对偶区域i给出的是

$$A_i := \frac{1}{8} \sum_{ijk \in F} \|e_{ik}\|^2 \cot\left(\alpha_j^{ki}\right) + \|e_{ij}\|^2 \cot\left(\beta_k^{ij}\right)$$

4. 顶点处的离散标量高斯曲率（也称为角缺陷）和离散标量平均曲率i由下式给出

$$K_i := 2\pi - \sum_{ijk \in F} \phi_i^{jk}$$

$$H_i := \frac{1}{2} \sum_{ij \in E} \theta_{ij} \|e_{ij}\|$$

注意，这些量是离散的2-形式，即它们表示在顶点邻域内积分的总高斯曲率和平均曲率。

它们可以通过除以顶点的外心对偶面积（即应用离散霍奇星）转换为逐点量（即顶点处的离散0-形式）。5. 你需要推导主曲率的表达式κ_1和κ_2在书面作业的练习4中。你的`principalCurvatures`实现应该返回顶点处（按顺序）的最小和最大主曲率值。

提交说明：

请仅提交源代码文件 `geometry.js` 或 `geometry.cpp` （取决于你使用的是JavaScript还是C++）。这意味着你的代码不应依赖于对任何其他文件的编辑。然后按照常规的提交说明进行操作。

提交说明可在**主页的“作业”部分找到。**

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